Small-Scale Trapped Filaments in Intense Laser Beams

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The properties of small trapped filaments in liquids are discussed in the light of new experimental and theoretical results. The duration of the filaments is shown to be about 10^{-10} sec, after which they "blow up," apparently because of the heating and subsequent expansion of the liquid. The measured diameters range from 4 μ in CS_2 to about 12 μ in nitrobenzene. The power found in a small filament is the order of 10 kW, agreeing with the theoretical trapping threshold for the Kerr effect, and indicating that this is the principal mechanism responsible for initiation of the filaments. For power above the trapping threshold, filaments continue to decrease in size and increase in intensity until higher-order terms in E^2 cause the nonlinear refractive index to saturate. Calculation of the limiting diameters, based on saturation of the Kerr effect, yields values about one order of magnitude smaller than the measured diameters and indicates that some other mechanism determines the size. Finally, the low value of the stimulated Raman gain in most of the length of filaments, which had been observed previously, is explained on the basis of frequency broadening and dispersion.

I. INTRODUCTION

INTENSE filaments of light, only a few microns in diameter, have been shown[1,2] to exist in laser beams transmitted through a variety of liquids. This occurs when the power in the beams exceeds a threshold value which, for most liquids, is in the range of a few tens of kilowatts. For beams having a diameter of about a millimeter on entering a liquid, the filaments form in a distance of a few centimeters and persist for many tens of centimeters. Most of the production of Raman radiation in liquids, and of stimulated Brillouin and Rayleigh scattering, typically takes place in these filaments and, hence, an understanding of their formation and characteristics is important to any explanation of the rich variety of instabilities and stimulated radiation produced when intense beams are propagated through liquids.

Consider now the relation between the trapping phenomena observed previously[3–6] and what we have called "small-scale trapping."[1] Trapping theory based of an index of refraction of the form $n=n_0+n_2E^2$, where E is the electric field, has been substantiated[3] by examination of filaments of diameter 50 to 100 μ. Evidence was also presented[4,6] from the distance in air over which a filamented beam from a first liquid could travel before losing its anomalously high Raman gain properties in a second liquid. This result suggested the formation of trapped filaments of diameter 20 to 50 μ. We believe the small-scale filaments of diameter about 5 μ are not, in fact, substantially different phenomena, although they represent an extreme and interesting case of trapping.

Earlier reports[1,2] have described many of the characteristics of these filaments, but have left some areas of uncertainty, including the mechanism of trapping and the power levels involved. New information concerning the filaments is presented here which, it is believed, leads to a better understanding of the basic phenomena involved. This information includes measurements of the duration, power, and size of small filaments. The importance of both the saturation of the nonlinear refractive index and the build-up of stimulated radiations in determining the size and duration of the filaments is discussed, as well as aspects of these parameters of the filaments which remain to be resolved.

II. FILAMENT DURATION

Perhaps the most crucial single new piece of information about the small-scale trapped filaments is a measurement of their lifetime. Previously, it was determined from observations with a fast photodiode that the filaments did not last more than about 10^{-9} sec.[1,2] Use of a still faster photodiode, kindly made available to us by Dr. de Maria at the United Aircraft Company, allowed measurements on an oscilloscope trace which

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[1] R. Y. Chiao, M. A. Johnson, S. Krinsky, H. A. Smith, C. H. Townes, and E. Garmire, IEEE J. Quantum Electron. **2**, 467 (1966).
[2] R. G. Brewer and J. R. Lifsitz, Phys. Letters **23**, 79 (1966).
[3] E. Garmire, R. Y. Chiao, and C. H. Townes, Phys. Rev. Letters **16**, 347 (1966).
[4] P. Lallemand and N. Bloembergen, Phys. Rev. Letters **15**, 1010 (1965).
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[6] G. Hauchecorne and G. Mayer, Compt. Rend. **261**, 4014 (1965).

showed the filament lifetime in CS_2 to be not more than about 10^{-10} sec. This is an upper limit only, but other considerations discussed below make it unlikely that the pulse associated with transmission through a filament is much less than this. In addition, a measurement of the spectral width of the filamentary laser light sets a lower limit of about 10^{-11} sec.

The determination of filament duration immediately helps clarify the mechanism of trapping. The amount of energy transmitted through each filament had already been determined to be in the range of a few to a few tens of ergs,[1,2] so that the power is a few, or a few tens, of kilowatts. This is clearly in the range of the threshold trapping power due to Kerr effects. Additional measurements of the light energy transmitted during the lifetime of a small-scale filament in CS_2 give mean values of about 10 ergs, with deviations of about a factor of 2. This energy is normally distributed about equally between laser light, first Stokes, and higher-order Stokes light, although the relative intensities of these various components may vary considerably along the length of a filament. The threshold calculated for self-trapping due to the Kerr effect is 10 kW,[7,8] and experimental measurements[3,9] for trapping in larger filaments confirm this value. Ten ergs transmitted for 10^{-10} sec would represent just this same power level of of 10 kW, and hence Kerr effects seem an adequate mechanism for producing the trapping.

Destruction of the filament after about 10^{-10} sec is believed to result from either heating and expansion, or from instabilities, as discussed earlier.[1] The energy transmitted in a filament is degraded into heat by successive Raman shifts and, in some materials, probably eventually by direct absorption of the resulting infrared radiation. How rapidly this occurs is difficult to determine exactly. However, estimates of Raman gain make an average distance of 1 cm for degradation of the laser energy seem reasonable. Very likely, the filament is destroyed by heating and expansion in the region where it initiates and where the gain is known to be very high because of the measured rate of build-up of Stokes light. In this part of the filament, its energy may be degraded in a distance as small as 1 or 2 mm. Five ergs of energy converted from the light beam to heat along 1-cm length of the central 2 μ of a filament would raise its temperature about 10°C, and produce an expansion. The expansion would travel at acoustic velocities, and hence move a radial distance of 1 μ in about 6×10^{-10} sec. This is in the right range of times to destroy the filamentary trapping, although somewhat longer than one might expect in view of the filament lifetime of about 10^{-10} sec.

Perhaps the best experimental evidence of the fact that the filament blows up is that at distances the order of a millimeter after the first self-focus, a dark spot in laser light appears, coinciding with a bright spot in first Stokes light.[1] Essentially all of the trapped laser light has been converted to Stokes light. Since each filament lasts only 10^{-10} sec and the exciting laser pulse is at least twenty times as long, one must explain why the dark spot is visible in a photograph which integrates light during the entire pulse. Presumably, the excitation and heat delivered to the region where the filament exists momentarily is sufficient to expand the liquid, thus decreasing the index of refraction and defocusing subsequent light from the same region.

Other mechanisms, involving instabilities such as stimulated Brillouin and Rayleigh scattering, undoubtedly contribute to the finite lifetime of the filaments. These can build up to considerable intensity in about 10^{-10} sec and may reflect or scatter light so much that it cannot propagate sufficiently to maintain the filament.

III. KERR EFFECT AND SATURATION OF THE NONLINEAR INDEX

The Kerr effect, which is now considered to be the principal trapping mechanism, deserves further scrutiny as it can represent more than one physical process. One contribution is of purely electronic origin, involves distortion of the electron charge distribution, and would yield an electric birefringence even for isolated atoms. However, this is not the dominant effect in most molecules. Recent numerical Hartree-Fock calculations of McLean and Yoshimine[10] indicate that the electronic contribution to the molecular Kerr constant is typically about 1% of the total. Nevertheless, for very short, intense pulses, as with a mode-locked laser, only this mechanism might prevail.

The principal contribution for anisotropic molecules is probably their tendency to orient in the direction of the applied electric field. For a nonpolar anisotropic molecules, the theory of the molecular orientational Kerr effect given below, and a measurement of the polarizability anisotropy lead to values for the Kerr constants in reasonable agreement with the experimental value. This would suggest that for this case other contributions to the Kerr effect are normally of lesser importance.

Still another part of the Kerr effect[11] arises from the clustering of molecules in preferred configurations under the action of an applied field. A quantitative assessment of this contribution is difficult, but it is apparently not a dominant effect for molecules with large anisotropic polarizabilities.

Previous theoretical studies have given the threshold of power required for trapping, and hence for maintaining a steady-state trapped filament. It was found

[7] R. Y. Chiao, E. Garmire, and C. H. Townes, Phys. Rev. Letters **13**, 479 (1964).
[8] P. L. Kelley, Phys. Rev. Letters **15**, 1005 (1965).
[9] C. C. Wang, Phys. Rev. Letters **16**, 344 (1966).
[10] A. D. McLean and M. Yoshimine, J. Chem. Phys. **46**, 3682 (1967).
[11] R. W. Hellwarth, Phys. Rev. **152**, 156 (1966).

that if $n=n_0+n_2E^2$, the threshold power is independent of the filament diameter. However, if the power is above threshold, there is no steady-state solution in this approximation, but rather the light is focused until it becomes so concentrated that higher-order terms in E^2 are important and the nonlinear index saturates.

We now formulate an expression for the index change based on the molecular orientational Kerr effect in order to examine its saturation behavior. Our starting point is the Lorenz-Lorentz relation which relates the refractive index to the average molecular polarizability $\langle\alpha\rangle$:

$$(n^2-1)/(n^2+2)=\tfrac{4}{3}\pi N\langle\alpha\rangle. \quad (1)$$

Assume a symmetric top molecule such as CS_2, for simplicity, with principal polarizabilities α_3 and α_1, where α_3 makes an angle θ with an electric field E and $\alpha_3>\alpha_1$. The general case where all three principal polarizabilities are different will be similar in its general characteristics. Projection of the induced moments of α_3 and α_1 onto E then yields the average polarizability in the E direction

$$\langle\alpha\rangle=\alpha_3\langle\cos^2\theta\rangle+\alpha_1\langle\sin^2\theta\rangle=(\alpha_3-\alpha_1)\langle\cos^2\theta\rangle+\alpha_1. \quad (2)$$

The averaging is over all angles θ in the statistical distribution. Hence

$$\langle\cos^2\theta\rangle=\int_0^\pi \cos^2\theta\exp(a\cos^2\theta)\sin\theta d\theta \Big/ \int_0^\pi \exp(a\cos^2\theta)\sin\theta d\theta, \quad (3)$$

where $a=(\alpha_3-\alpha_1)\langle E^2\rangle/2kT$ and isothermal behavior is assumed. With the change of variable $\cos\theta=t/a^{1/2}$ and integration by parts, Eq. (3) becomes

$$\langle\cos^2\theta\rangle=\left[2(a)^{1/2}e^{-a}\int_0^{a^{1/2}}e^{t^2}dt\right]^{-1}-(1/2a): \quad (4)$$

The expression

$$e^{-a}\int_0^{a^{1/2}}e^{t^2}dt$$

is known as the Dawson integral and is tabulated.[12]

The change in index for arbitrary field strength can now be obtained from Eqs. (1), (2), and (4) as

$$\left(\frac{n^2-1}{n^2+2}\right)-\left(\frac{n_0^2-1}{n_0^2+2}\right)=\frac{4}{3}\pi N(\alpha_3-\alpha_1)[\langle\cos^2\theta\rangle-\tfrac{1}{3}], \quad (5)$$

where in zero field the index is n_0 and $\langle\cos^2\theta\rangle=\frac{1}{3}$.

In Fig. 1, the variation of $\langle\cos^2\theta\rangle$ with the field parameter a is shown. The saturation effect is quite evident and becomes pronounced for $a>2$. For liquid CS_2, $a=2$ implies electric fields of about 4×10^7 V/cm, a power of 35 kW for a filament diameter of 1 μ, and an index change of $\Delta n=0.2$. In the limit of complete saturation, $\Delta n=0.5$.

The behavior immediately before saturation is best understood by expanding the two exponentials in Eq. (3) and integrating term by term. This gives

$$\langle\cos^2\theta\rangle=\tfrac{1}{3}+4a/45+8a^2/945-16a^3/14\,175. \quad (6)$$

The corresponding change in index is

$$\Delta n=\frac{(n_0^2+2)^2}{6n_0}(\tfrac{4}{3}\pi N)(\alpha_3-\alpha_1)\times\left[\frac{4a}{45}+\frac{8a^2}{945}-\frac{16a^3}{14\,175}\cdots\right], \quad (7)$$

where again $a=(\alpha_3-\alpha_1)\langle E^2\rangle/2kT$ and the local field $E=\frac{1}{3}(n^2+2)E'$ for a Lorentz cavity and an external field E'. E^2 is understood to be averaged over an optical period. Alternatively, Eq. (7) may be written

$$n=n_0+n_2E^2+n_4E^4+n_6E^6+\cdots.$$

These expressions are useful in considering below the effect of index saturation on beam trapping and filament diameter.

IV. THEORETICAL FILAMENT DIAMETER

Kelley and Gustafson,[13] and also Wagner and Haus,[14] have examined the dynamic behavior of a circularly symmetric beam of light whose total intensity is above the threshold for trapping. The beam is of course initially focused toward its axis, then it broadens out before refocusing again, as indicated by Talanov.[15] However, they find that higher-order nonlinearities, which become important for small diameters of the

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[14] H. A. Haus and W. G. Wagner (to be published).

[15] V. I. Talanov, Zh. Eksperim. i Teor. Fiz., Pis'ma v Redaktsiyu. **2,** 218 (1965) [English transl.: Soviet Phys.—JETP Letters **2,** 138 (1965)]; S. A. Akhmanov, A. P. Sukhorukov, R. V. Khokhlov Zh. Eksperim. i Teor. Fiz. **50,** 1537 (1966) [Soviet Phys.—JETP **23,** 1025 (1966)].

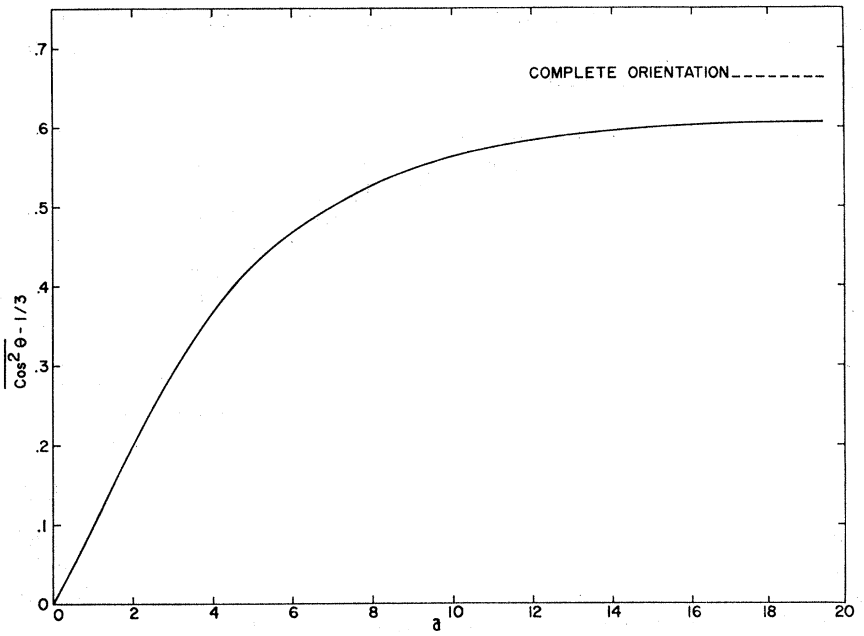

FIG. 1. The molecular orientation function $[\langle \cos^2\theta \rangle - \frac{1}{3}]$ versus electric field parameter $a = (\alpha_3 - \alpha_1) E^2/2kT$.

beam, damp out multiple refocusing, so that the beam eventually stabilizes with a diameter determined by the higher-order nonlinearities. The stable condition, corresponding usually to a very small diameter beam, is similar to that discussed by Piekara[16] for trapping when high-order nonlinearities are present.

Any beam with more than the threshold power thus collapses to a small size, dependent on the power level, with beam intensities so high that terms in the expansion $n = n_0 + n_2E^2 + n_4E^4 + n_6E^6 \cdots$ higher than n_2E^2 become important. In fact, since both n_2 and n_4 are positive for molecular orientation effects, as shown in Eq. (7), it is the term n_6E^6 which must be sufficiently large to prevent further collapse of the beam. The result, for power levels not very much larger than threshold, is a small filament about one-third of a wavelength in diameter. Irwin, Chiao, and Gustafson[17] have examined carefully the limiting size of filaments carrying power a little above the threshold value. They show that molecular orientation alone, under either isothermal or adiabatic conditions, gives a limiting diameter of less than 1 μ.

As will be discussed below, the observed filaments are not quite as small as predicted by the Kerr saturation. However, they are so small that the field has a sufficiently high intensity for many instabilities to occur. This presumably makes the filament heat up, expand, and then no longer trap light. The resulting beam then collapses into a new small filament in a slightly different location, and this process continues during the life of the pulse in accordance with experimental observation that individual filaments occur sequentially when the power is just above threshold. For a very powerful beam, a number of filaments may be formed simultaneously.

V. MEASUREMENTS OF FILAMENT DIAMETER

Direct microscope measurements of the half-intensity diameter of small-scale filaments give about 4 μ in CS_2 and 12 μ in nitrobenzene. These values are obtained after small corrections for the resolution of the microscope, which was about 2 μ.

The rate of diffraction of the small-scale filaments has also been measured by noting the increase in the image size when the microscope is defocused by known amounts. Such measurements show that if one assumes a diffraction angle of $1.22\lambda/D$ after the light emerges from the end of the cell, the calculated diameter D is three or four times larger than that measured directly. The discrepancy is associated with the radial distribution of intensity. A diffraction angle $\theta = 1.22\lambda/D$ assumes a uniformly illuminated circular aperture of diameter D. The actual intensity distribution is more like a Gaussian curve, which is known to give the smallest diffraction for a given mean-square diameter. As is familiar from the uncertainty principle, this limit is $\theta = \lambda/2\pi D$ if rms values are taken for θ and D.

Chiao and Dodson[18] have calculated the diffraction angle using the theoretical intensity distribution in a trapped filament whose diameter at half-maximum intensity is D. They find that the diffraction angle (half-width at half-intensity) is a factor 2.88 smaller than that for a circular aperture of the same diameter.

[16] A. Piekara, IEEE J. Quantum Electron. **2**, 249 (1966).
[17] R. Irwin, R. Y. Chiao, and K. Gustafson (to be published).
[18] R. Y. Chiao and J. Dodson (to be published).

When the refractive index is appreciably saturated, the distribution of energy is such as to make this factor still larger, and the calculated value for a typical filament is 5.9. Allowing for such factors, the measured diameter and that deduced from diffraction data agree reasonably well and the above discrepancy is removed.

It seems clear that the small-scale trapping observed here is similar to the trapped filaments deduced from indirect experiments by Mayer and Hauchecorne[6] and by Bloembergen and Lallemand.[4] These authors concluded that the filaments had diameters of several tens of microns in nitrobenzene because of their apparent rate of diffraction, and such diameters are rather different from those given above. However, if one uses the above more accurate diffraction formula, the resulting diameters are consistent with those measured in nitrobenzene directly with a microscope.

In most respects, the observed properties of trapping in liquids fit the expected behavior. However, the filament size appears to be more than ten times larger than the sub-micron diameter predicted from the more obvious higher-order terms in the refractive index. Moreover, the Kerr effect is of comparable magnitude in CS_2 and nitrobenzene, and yet the size of the small-scale filaments is considerably larger in the latter liquid. Furthermore, the lifetime, as described above, is a little shorter than might be expected for the size of filaments measured, although the calculation cannot be made precise enough to show that there is a real inconsistency.

Both of these deviations might indicate that the filaments are in fact somewhat smaller than they appear to be. They are small enough that the resolution of microscopes used leaves some uncertainty about their exact size. However, it does not seem likely that the filaments in CS_2 can have an appreciably smaller diameter than 3 or 4 μ. Furthermore, small-scale trapped filaments in nitrobenzene are at least as large as 12 μ, and it is unlikely that their measured diameters can be in error by any large fractional amount.

Some other nonlinearities are present which have not been allowed for. Nonlinear losses may play a major role in determining filament size, just as they influence the duration of the filaments. The amount of heating in the filament which results from all losses is difficult to calculate, and clearly influences their size. However, an increase in temperature without expansion of the material usually increases the index of refraction for most substances, and hence, heat alone should decrease the filament size. For the filaments to persist in space, the energy lost per unit length from the light traveling along the filament must equal the energy per unit length which can be fed into the filament from the surrounding laser light. Elementary calculations indicate that such effects may indeed be dominant in preventing the filament from collapsing to diameters less than a few microns, since then energy would be too rapidly converted by a variety of instabilities.

VI. EXPERIMENTAL EVIDENCE FOR KERR EFFECT SATURATION

From the above discussion and the observation that most small-scale filaments, in a given liquid, are all about the same size, there is little doubt that the index increase is limited in some manner. Perhaps the most quantitative evidence of index saturation comes from the observation of spatial beats in the standing-wave patterns of the more intense small-scale filaments.[19] For CS_2, these measurements give index changes of about 0.2, and according to Sec. III, this is in the region where (partial) saturation of the orientational Kerr effect is important. Moreover, this value is comparable to the maximum index change of about 0.5 which can be produced when all molecules are oriented.

VII. SATURATION OF RAMAN GAIN

Since any beam slightly above the trapping threshold and in a sufficiently long cell can be expected to collapse to a filament where the field strengths are exceedingly high, many nonlinearities and instabilities set in almost simultaneously. Just which effects occur most prominently depends on the detailed properties of the material, but the expected ones clearly include Raman effects of all orders, stimulated Brillouin scattering, and stimulated Rayleigh scattering. Sometimes these occur separately, and sometimes simultaneously, producing a complex variety of radiative and energy transfer effects.

One of the puzzling questions about small-scale trapping has been why the Raman gain is not even greater than it is, since the field strength would indicate enormously high gains. For example, a field strength of 10^6 V/cm, somewhat lower than that calculated above, should give a gain constant for Stokes light of 10/cm in CS_2. This high gain would very quickly convert all laser radiation to Stokes light; after the Stokes intensity becomes comparable with that of laser light, it would utilize all the laser power in a distance of about 1 mm. This is in contrast to the experimental observation that after the first centimeter of filament, Stokes radiation increases by no more than two orders of magnitude in a distance of a number of centimeters.

The Stokes gain is in fact high at the initiation of the filament, and appears to agree roughly with calculated values for the first few millimeters, after which the Stokes gain appears to decrease. It was the high calculated gain, the high initially observed Stokes gain, and the smaller subsequent gain which led to the previous suggestion[1] that the gain was "saturated." Thus, in a small filament a large fraction of the molecules might be expected to make transitions to excited vibrational levels. However, it is now clear that the total energy in the filament is not sufficient to so excite molecules in the entire filament. Even if the entire power in the

[19] R. G. Brewer and C. H. Townes, Phys. Rev. Letters **18**, 196 (1967).

beam flows into the filament, as may be a reasonable approximation in some cases, the filament's lifetime is not long enough for this type of saturation to occur throughout its length.

Saturation of gain by molecular excitation does seem to occur in the region immediately following initiation of the filament. One observes that in a distance of the order of 1 mm essentially all of the trapped laser light is converted to Stokes light. In this region, a picture of the beam in laser light shows a dark spot where the filament occurs. Ten ergs absorbed in 1 mm of filament, approximately what is observed, is just sufficient to put an equal number of molecules in the exicted and ground vibrational states and thus substantially reduce the Raman gain. One expects in this case that molecular relaxation will occur during 10^{-10} sec only to vibrational levels which have transitions of about the same energy. Since it appears that the initial, high Stokes gain region of the filament does saturate by molecular excitation, this saturation may make some contribution to the beam trapping in this region as suggested earlier. However, this cannot play a substantial role in the remaining length of the filaments where the field intensities seem to be comparably large, but the Stokes gain very much lower.

VIII. FREQUENCY SPREADING, DISPERSION, AND LOSS OF RAMAN GAIN

The remaining question about the general nature of the filament is why the Stokes gain is in fact so low through the last part and most of the filament. The explanation appears to be in the frequency spreading of radiation in the filament, due to stimulated Brillouin and Rayleigh scattering or the formation of optical shocks associated with these scattering processes.[20] This broadening effect makes the intensity per frequency bandwidth sufficiently small that the gain is reduced. Both a frequency spread of the radiation and a dispersion is necessary in order to reduce the gain.

It is easy to show that the Stokes gain associated with a given molecular vibration is dependent essentially on the total power of exciting light and not on its frequency distribution as long as there is no dispersion.[21] This statement assumes the frequency of light does not vary so much that there is a large fractional change in the optical frequency, since the gain is proportional to the product of frequency and power.[22] If there is dispersion, the driving waves of various frequencies may get out of phase after some distance, no longer driving the molecular oscillations synchronously and hence the gain is decreased.

Consider two driving optical waves of frequencies ν_0 and ν_0', and hence wave vectors $k_0=2\pi n_0\nu_0/c$ and $k_0'=2\pi n_0'\nu_0'/c$. Hence n_0 and n_0' are the indices of refraction. These waves generate Stokes frequencies ν_{-1} and ν_{-1}', respectively, which differ from their parent waves by the molecular vibrational frequency, and have wave vectors $k_{-1}=2\pi n_{-1}\nu_{-1}/c$ and $k_{-1}'=2\pi n_{-1}'\nu_{-1}'/c$. Their driving forces on the molecular vibration get out of phase in a distance L if

$$(k_0-k_{-1})L=(k_0'-k_{-1}')L\pm\pi.$$

If $k_0'<k_0$, then normally the plus sign applies in this equation. If the index of refraction is expanded in the form $n=n_0+n_1(\nu-\nu_0)+n_2(\nu-\nu_0)^2$, the above equation leads to the relation

$$L\cong c/[4\nu_m(\nu_0-\nu_0')(n_1+n_2\nu_0)].$$

Here ν_m is the molecular frequency, $\nu_0-\nu_{-1}$ or $\nu_0'-\nu_{-1}'$. For CS_2, $n_1=1.5\times10^{-16}$ sec and $n_2=2.3\times10^{-31}$ sec^2.[23] Hence L equals 5 mm if $\nu_0-\nu_0'$ is about 30×10^{11} hZ or 100 cm^{-1}. That is, if the frequency spread of the driving radiation is as great as 100 cm^{-1}, the maximum gain cannot extend over a distance more than about 5 mm. For filaments in CS_2, a few hundred cm^{-1} is the frequency broadening commonly observed in the Stokes light while the laser light is broadened $\sim$100 cm^{-1} or less. Probably this is laser light which is continuously captured from the surrounding main laser beam along the length of the filament. This gives it less path length in the filament for modulation or frequency smearing to take place. The total intensity of laser light must also in that case be somewhat less than that of Stokes light in order for their rates of conversion to lower frequency to be comparable.

IX. SUMMARY

The general features and many details of "small-scale" trapping are described and seem now to be understood. However, the variety of phenomena which can occur is great and the range of properties of liquids is large enough that a rich mixture of effects is possible. Furthermore, such properties as the lifetime and size of the filaments are still not quantitatively predictable.

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[21] This was first pointed out to one of us (C.H.T.) by N. M. Kroll some time ago (private communication).

[22] R. Y. Chiao, E. Garmire, and C. H. Townes, *Internationa School of Physics, "Enrico Fermi," Course 31*, edited by C. H. Townes and P. A. Miles (Academic Press Inc., New York, 1964), p. 326.

[23] V. G. Cooper and A. D. May, Appl. Phys. Letters **7**, 74 (1965).