Application of a Regge-Pole Model to the Reactions $\pi^- p \rightarrow \pi^0 n$, $\pi^- p \rightarrow \eta n$, and $\pi^+ n \rightarrow \omega p$†‡

M. BARMAWI*
The Enrico Fermi Institute for Nuclear Studies and the Department of Physics, The University of Chicago, Chicago, Illinois

(Received 29 May 1967)

A Regge-pole model with L-S coupling is considered for the reactions $PN \rightarrow P'N$ and $PN \rightarrow VN$. The model is used to analyze the processes $\pi^- p \rightarrow \pi^0 n$, $\pi^- p \rightarrow \eta n$, and $\pi^+ n \rightarrow \omega p$. Good agreement is obtained for the momentum-transfer distribution. The density matrix of the ω meson agrees qualitatively with the available data.

I. INTRODUCTION

In a recent paper,[1] we have developed a Regge-pole model for the reactions $PN \rightarrow P'N$ and $PN \rightarrow VN$ ($P,P' = 0^-$ meson and $V = 1^-$ meson), which is based upon an L-S coupling scheme. This scheme is introduced to take into account the restrictions imposed by the J, P, G selection rules on the various components of the helicity amplitude, which is related to decay density matrix of V.

In this paper we will apply the model to π-N charge exchange scattering, η production and ω production. π-N charge-exchange scattering[2–6] and η production[7–9] have been studied in the context of different models in the literature. We would like to find the restrictions on the validity of our model for these two reactions, which are dominated by a single Regge trajectory. This model is too simple to explain details such as polarization. Therefore, for the last two reactions we restrict ourselves to the momentum-transfer distribution. But for ω production, we study the ω-decay density matrix in its rest frame. While the polarization is determined by the relative phases of the various components of the helicity amplitude, the decay density matrix is characterized by the structure of the crossed-channel amplitude. The decay density matrix is less sensitive to the relative phases than the polarization, therefore, the decay density matrix can be used to test our model.

† Work supported in part by the U. S. Atomic Energy Commission.

‡ Part II of a thesis submitted to the Department of Physics, The University of Chicago, in partial fulfillment of the requirements for the Ph.D. degree.

* Present address: Physics Department, Institute of Technology, Bandung, Indonesia.

[1] M. Barmawi, preceding paper, Phys. Rev. **165**, 1846 (1968). This paper will be referred to as I.

[2] R. K. Logan, Phys. Rev. Letters **14**, 414 (1965); R. J. N. Phillips and W. Rarita, Phys. Rev. **139**, B1336 (1965); B. R. Desai, *ibid.* **142**, 1255 (1966).

[3] F. Arbab and C. B. Chiu, Phys. Rev. **147**, 1045 (1966).

[4] G. Höhler, J. Baacke, H. Schaile, and P. Sonderegger, Phys. Letters **20**, 79 (1966).

[5] R. K. Logan and L. Sertorio, Phys. Rev. Letters **17**, 834 (1966).

[6] R. K. Logan, J. Beaupre, and L. Sertorio, Phys. Rev. Letters **18**, 259 (1967).

[7] R. J. N. Phillips and W. Rarita, Phys. Rev. **140**, B200 (1965); Phys. Rev. Letters **15**, 807 (1965).

[8] R. J. N. Phillips and W. Rarita, Phys. Letters **19**, 598 (1966).

[9] R. K. Logan and L. Sertorio (unpublished).

In Sec. II we present a self-contained summary of the procedure used to calculate the Regge helicity amplitude, the momentum-transfer distribution, and the decay density matrix for $PN \rightarrow P'N$ and $PN \rightarrow VN$. In Sec. III we apply our model to π-N charge exchange scattering, η and ω production. In Sec. IV we summarize and discuss the results.

II. CALCULATION OF THE MOMENTUM-TRANSFER DISTRIBUTION AND THE DECAY DENSITY MATRIX

Let us consider the Regge-pole exchange in the reaction $PN \rightarrow VN$, as shown in Fig. 1. In the L-S coupling scheme, the meson system (P,V) and the $N\bar{N}$ system are coupled to the Reggeized intermediate states with different orbital angular momentum L and total spin S, which are determined by the quantum numbers of the trajectory. For the trajectory with $P = (-1)^J$, where J is the spin of the Reggeized particle, the $N\bar{N}$ system can be coupled with $L = \alpha \pm 1$, and the mesons with $L = \alpha$ to the trajectory. If the parity of the trajectory is $P = (-1)^{J+1}$, then the (P,V) is coupled to the trajectory with $L = \alpha \pm 1$ and the $(N\bar{N})$ with $L = \alpha$. The justification for this model has been discussed in detail in I. For each trajectory we introduce two reduced residues, which determine all the components of the helicity amplitude. The scheme imposes certain restrictions on the residues.

The s-channel helicity amplitude of the Regge-pole contribution is given by[1]

$$\langle \lambda_1 \bar{\lambda}_2 | F | \lambda_V \rangle = -\epsilon(2\alpha+1)\alpha' [(1 \pm e^{-i\pi\beta})/2 \sin\pi\beta] (1-z)^{a/2} (1+z)^{b/2} \times N_{\lambda_f \lambda_i}{}^{(\alpha)} P_\beta{}^{(b,a)}(-z) \tilde{R}_{\lambda_f \lambda_i}, \quad (2.1)$$

where $\lambda_i = \lambda_V$, $\lambda_f = \lambda_1 - \bar{\lambda}_2$, and $\tilde{R}_{\lambda_f \lambda_i} \equiv c_{\lambda_f \lambda_i} R_{\lambda_f \lambda_i}$ will be given explicitly below. The λ_1 and $\bar{\lambda}_2$ are, respectively, the helicities of the nucleon and antinucleon in the s channel and λ_V is the helicity of V. In the case of P' production we put $\lambda_V = 0$. For convenience, we list in the Appendix the asymptotic form of $N_{\lambda_f \lambda_i}{}^{(\alpha)} P_\beta{}^{(b,a)}(z)$ and the values of a, b, ϵ, and β corresponding to each set of $(\lambda_V, \lambda_1 \bar{\lambda}_2)$ for $PN \rightarrow P'N$ and $PN \rightarrow VN$. In Eq. (2.1), ϵ is the phase factor which appears in the definition of the d function. Note that the choice of the sign

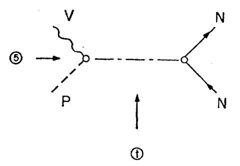

FIG. 1. The s and t channels of Regge-pole exchange for $PN \to VN$. The s-channel c.m. system is the rest frame of the intermediate state. In this frame is defined the orbital angular momentum and the total spin of the initial state (P,V) and the final state $(N,\bar{N})$ which describes the coupling to the trajectory.

in the signature factor in Eq. (2.1) is determined by the parity of β on the mass shell.

The result for $\tilde{R}_{\lambda_f\lambda_i}$ for the reaction $PN \to P'N$ is[1]

$$\begin{aligned}\tilde{R}_{00}&=k[\alpha r_- + (\alpha+1)r_+],\\ \tilde{R}_{10}&=k(p_0/M)[\alpha(\alpha+1)]^{1/2}[r_- - r_+],\end{aligned}\qquad(2.2)$$

where p and k are, respectively, the momenta of $(N\bar{N})$ and (P,V) in the c.m. system of the crossed channel, and where $r_\pm$ is the residue associated with the orbital angular momentum $L=\alpha\pm1$ in the $N\bar{N}$ vertex. The threshold behavior[1] for the lower wave coupling r_- is

$$r_- = b_-(2pk/s_0)^{\alpha-1},\qquad(2.3a)$$

and

$$r_+ = b_+(2pk/s_0)^{\alpha-1}(p/M)^2.\qquad(2.3b)$$

The unit s_0 that appears in the threshold behavior is chosen to be $s_0=2[\Pi_i m_i]^{1/2}$.[2,10]

For the processes $PN \to VN$ the same amplitude (2.1) is valid, with some modification in $\tilde{R}_{\lambda_f\lambda_i}$ due to the difference in the spin configuration.

For the exchange of the trajectory with $T=1$, $J^{PG}=1^{-+}$, 2^{+-} (or $T=0$ and $J^{PG}=1^{--}$, 2^{++}) we have[1]

$$\begin{aligned}\tilde{R}_{10}{}^{\alpha}&=-\tilde{R}_{-10}{}^{\alpha}=[\alpha(\alpha+1)]^{1/2}k(\sqrt{s})(r_-{}^{(4)}-r_+{}^{(4)}),\\ \tilde{R}_{11}{}^{\alpha}&=-\tilde{R}_{1,-1}{}^{\alpha}=(ks/2M)[(\alpha+1)r_-{}^{(4)}+\alpha r_+{}^{(4)}],\qquad(2.4)\\ \tilde{R}_{\lambda_f,0}{}^{\alpha}&=0,\end{aligned}$$

with the following threshold behavior[1]:

$$\begin{aligned}r_-{}^{(4)}&=a_-{}^{(4)}(2pk/s_0)^{\alpha-1},\\ r_+{}^{(4)}&=a_+{}^{(4)}(2pk/s_0)^{\alpha-1}(p/M)^2.\end{aligned}\qquad(2.5)$$

The r_- is the reduced residue corresponding to $L=(\alpha-1)$ wave coupling and r_+ to the $L=(\alpha+1)$ wave coupling at the $N\bar{N}$ vertex.

For the exchange of trajectories with $T=1$, $J^{PG}=0^{--}$, 1^{++} (or $T=0$, $J^{PG}=0^{-+}$, 1^{+-}), we have[1]

$$\begin{aligned}\tilde{R}_{01}{}^{\alpha}&=\tilde{R}_{0,-1}{}^{\alpha}=[\alpha(\alpha+1)]^{1/2}(p_0/M)p(r_-{}^{(2)}-r_+{}^{(2)}),\\ \tilde{R}_{00}{}^{\alpha}&=[k_0(V)/m_V](p_0/M)\\ &\quad\times p[\alpha r_-{}^{(2)}+(\alpha+1)r_+{}^{(2)}]\sqrt{2},\qquad(2.6)\\ \tilde{R}_{1,\lambda_i}{}^{\alpha}&=0,\end{aligned}$$

where the threshold behavior is given by[1]

$$\begin{aligned}r_-{}^{(2)}&=a_-{}^{(2)}(2pk/s_0)^{\alpha-1},\\ r_+{}^{(2)}&=a_+{}^{(2)}(2pk/s_0)^{\alpha-1}(k^2s/m_0{}^3)\sqrt{2},\end{aligned}\qquad(2.7)$$

[10] M. Barmawi, Phys. Rev. Letters **16**, 595 (1966).

and $k_0(V)$ is the V-meson energy in the s channel center-of-mass (c.m.) frame. In this case, $r_\pm{}^{(2)}$ corresponds to $L=\alpha\pm1$ at the meson vertex and $L=\alpha$ at the $N\bar{N}$ vertex. We note that an additional factor s in the expression for r_+ is required as a consequence of the unequal-mass meson system (P,V) (cf. I, Sec. VI).

Finally, for the exchange of $T=1$, $J^{PG}=1^{+-}$, 2^{-+} (or $T=0$, $J^{PG}=1^{++}$, 2^{--}), we have[1]

$$\begin{aligned}\tilde{R}_{0,\lambda_i}{}^{\alpha}&=0,\\ \tilde{R}_{1,1}{}^{\alpha}&=\tilde{R}_{1,-1}{}^{\alpha}=(p/M)[(\alpha+1)r_-{}^{(1)}+\alpha r_+{}^{(1)}],\qquad(2.8)\\ \tilde{R}_{1,0}{}^{\alpha}&=[k_0(V)/m_V](p/M)[r_-{}^{(1)}-r_+{}^{(1)}][\alpha(\alpha+1)]^{1/2}\sqrt{2},\end{aligned}$$

with the threshold behavior[1]

$$\begin{aligned}r_-{}^{(1)}&=a_-(2pk/s_0)^{\alpha-1},\\ r_+{}^{(1)}&=a_+(2pk/s_0)^{\alpha-1}(k^2s/m_0{}^3),\end{aligned}\qquad(2.9)$$

where $r_\pm$ is the reduced residue associated with $L=\alpha\pm1$ wave coupling in the meson vertex. By using Eqs. (2.1)–(2.8), we can calculate the helicity amplitude. The momenta p and k are given by

$$\begin{aligned}p^2&=(s-4M^2)/4,\\ k^2&=[s-(m_V+m_P)^2][s-(m_V-m_P)^2]/4s,\end{aligned}$$

where M = nucleon mass. Note that z in Eq. (2.1) is obtained by analytic continuation to the physical region of the t channel. We found that z is real and negative, with the modulus given by

$$z=(2t+s-\textstyle\sum_i m_i^2)/4pk,$$

where $t=m_P{}^2+M^2+2M(p_L{}^2+m_P{}^2)^{1/2}$ (p_L = incident momentum of P in the lab system). The momentum-transfer distribution $d\sigma/ds$ and the decay density matrix $\rho_{mm'}$ is given by (2.6) and (2.7) of I, respectively, or by Eqs. (2) and (3) of Ref. 10.

III. APPLICATIONS

For the analysis of the experimental data, we introduce the following parametrization. Each trajectory has four parameters. Two of these specify the trajectory. We shall assume that the trajectories are straight lines, the parameters are the slope α' and the intercept $\alpha(0)$. The two other parameters are for the residues, that is, for $a_\pm$. These parameters are to be determined by the method of least χ^2 fit to the momentum-transfer distribution data. In ω production, the parameters thus determined are then used to calculate the decay density matrix, which can be compared with the experimental data.

A. $\pi^- p \to \pi^0 n$

The π-N charge exchange is dominated by the ρ Regge pole. It can explain the momentum-transfer distribution. This simple ρ-trajectory exchange model has been studied extensively in the literature.[2–6] The aim of the present analysis is to examine the restrictions

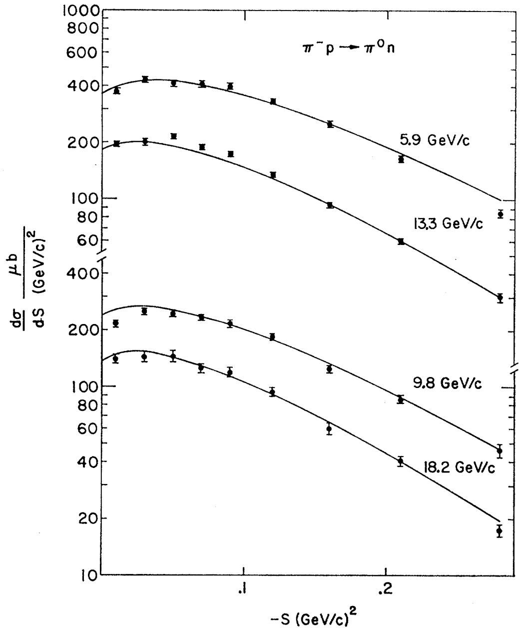

FIG. 2. Comparison of the momentum-transfer distribution $d\sigma/ds$ for $\pi^- p \to \pi^0 n$ with the data at 5.9, 9.8, 13.3, and 18.2 GeV/c.

on the validity of our model, and to obtain the parameters of the ρ Regge pole to be used in the analysis of ω production.

The data analyzed are those of Stirling *et al.*[11,12] We restrict ourself to the momentum-transfer distribution at 5.9, 9.8, 13.3, and 18.2 GeV/c to minimize the effect of the resonances in the direct channel. We find the trajectory to be

$$\alpha_\rho(s) = 1.1s + 0.58\,,$$

which is consistent with the result of Höhler *et al.* However, our model can explain the momentum-transfer distribution only up to $s = -0.3(\text{GeV}/c)^2$. The peak near the forward direction at $s \approx -0.04(\text{GeV}/c)^2$ is reasonably reproduced. The fit is shown in Fig. 2. The χ^2 is 99.5 for 36 data points, which is comparable to that obtained by Logan and Sertorio. The residues are $b_+ = 69.38$, and $b_- = 29.23$.

Our model fails to explain the appearance of a secondary minimum at $s \approx -0.6(\text{GeV}/c)^2$. The calculated $d\sigma/ds$ is found to be monotonically decreasing beyond $s = -0.3(\text{GeV}/c)^2$. In the present model the helicity-flip amplitude is not large enough to dominate $d\sigma/ds$ at larger momentum transfers. The explanation of the secondary peak requires the helicity-flip amplitude to be larger than the helicity-conserving amplitude. This might mean that our simple ansatz of the slowly varying residue $b_\pm$ does not hold in a larger-momentum-transfer interval.

B. $\pi^- p \to \eta n$

The reaction $\pi^- p \to \eta n$ is another simple reaction dominated by a single Regge pole.[7–9] The selection rules require that the quantum numbers of the trajectory are $P = (-1)^J$, $T = 1$, and $G = -1$. Only the A_2 meson has these quantum numbers. This reaction has been studied by Phillips and Rarita,[7,8] and Logan and Sertorio[9] in their models.

Since the A_2 meson has an even-signature trajectory, and the trajectory $\alpha_{A_2} = 0$ at a negative value of s, it will develop a ghost in the $(L = \alpha + 1)$-wave coupling part. To eliminate this ghost, we add a ghost-killing factor α to the reduced residue associated with $L = \alpha + 1$. The residue for A_2 is then

$$\begin{aligned} \tilde{R}_{00} &= k[\alpha r_- + \alpha(\alpha+1) r_+]\,, \\ \tilde{R}_{10} &= k(p_0/\simeq)[\alpha(\alpha+1)]^{1/2}[r_- - \alpha r_+]\,, \end{aligned} \tag{3.1}$$

instead of Eq. (2.2).

Using Eqs. (3.1), (2.1), and (2.9), we analyze the momentum-transfer-distribution data. The data on $d\sigma/ds$ are those of Guisan *et al.*[13] at incident momenta of 5.9, 9.8, 13.3, and 18.2 (GeV/c). The data are less accurate than those for the π-N charge exchange scattering. By fitting the momentum-transfer distribution, we obtain for the trajectory

$$\alpha_{A_2}(s) = 0.44s + 0.38\,,$$

and the residues are

$$a_+^{(4)} = 262.8, \quad a_-^{(4)} = 216.8\,,$$

with $\chi^2 = 39$ from 39 data points. The result is shown in Fig. 3. The momentum-transfer distribution in this graph is from the reaction

$$\pi^- + p \to n + \eta \searrow 2\gamma\,.$$

For the branching ratio $\Gamma(\eta \to 2\gamma)/\Gamma(\eta)$ we take 0.386. Our model explains the momentum-transfer distribution of this reaction better than in the case of π-N charge exchange scattering. It explains the momentum-transfer distribution over a wider range $s > -0.85$ $(\text{GeV}/c)^2$. The A_2 trajectory parameters are consistent with those of the other authors.[7]

Although the agreement for incident momenta between 5.9 and 18.2 GeV/c with the data is good, below 4 GeV/c there is a systematic deviation of the calculated $d\sigma/ds$ from the data. Logan and Sertorio[9] have shown these are due to the effects of the resonances in the direct channel.

[11] A. V. Stirling, P. Sonderegger, J. Kirz, P. Falk-Vairant, O. Guisan, C. Bruneton, P. Borgeaud, M. Yvert, J. P. Guillaud, C. Caverzaisio, and B. Amblard, Phys. Rev. Letters **14**, 763 (1965).

[12] P. Sonderegger, J. Kirz, O. Guisan, P. Falk-Vairant, G. Bruneton, P. Borgeaud, A. V. Stirling, C. Caverzasio, J. P. Guillaud, M. Yvert, and B. Amblard, Phys. Letters **20**, 75 (1966).

[13] O. Guisan, J. Kirz, P. Sonderegger, A. V. Stirling, P. Borgeaud, G. Bruneton, P. Falk-Vairant, B. Amblard, C. Caverzasio, J. P. Guillaud, and M. Yvert, Phys. Letters **18**, 200 (1965).

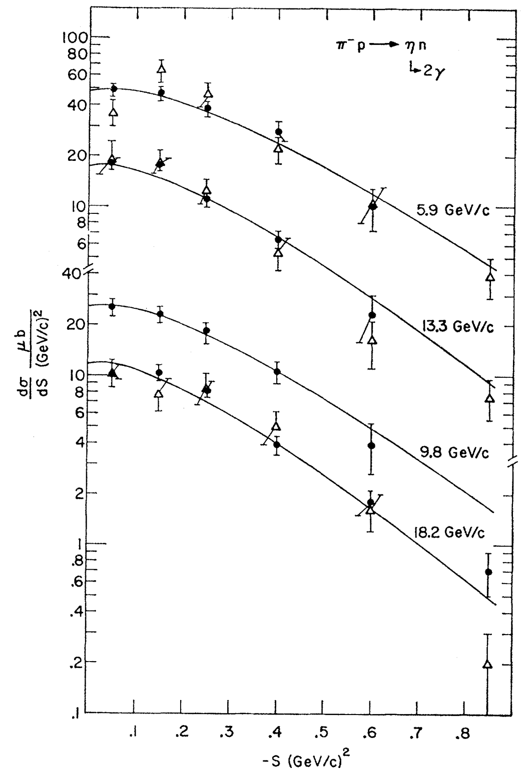

FIG. 3. Comparison of the momentum-transfer distribution $d\sigma/ds$ for $\pi^- p \to \eta n$ with the data at 5.8, 9.8, 13.3, and 18.2 GeV/c.

C. $\pi^+ n \to \omega p$

We have analyzed this reaction before with a more naive model.[10] In this paper we reanalyze it with the present model, where the residues are parametrized according to the L-S coupling scheme. The ω production is dominated by a trajectory with $T=1$ and $G=+1$. The only well-established resonance with these quantum numbers is the ρ meson. The ρ-trajectory exchange leads to $\rho_{00}=0$, as can be seen from Eq. (2.3), and Eq. (2.9) of I. It is found experimentally that $\rho_{00}\neq 0$. For this reason, we proposed in Ref. 10 that in ω production another trajectory with unnatural parity is exchanged. A candidate for this trajectory is that of the B meson, an ω-π resonance. The enhancement of the ω-π system produced in π-p collisions at higher energies has been observed in many experiments.[14] However, the interpretation of this enhancement as a resonant state has been questioned. Deck proposed a model of a kinematical enhancement[15] for the ρ-π system. It is extended by Maor to the ω-π enhancement.[16] This model applies only to enhancement in π-p scattering at higher energies. Recently, Baltay *et al.*[17] has observed an ω-π enhancement in the $p\bar{p}$ annihilation at rest, interpreted as a B meson. The observation of the B meson in $p\bar{p}$ annihilation at rest has the advantage of fewer complications due to other effects. The assumption of the above-mentioned kinematical enhancement does not apply to this situation, and there are fewer channels which contribute to the background, so that it can be taken into account in a simple fashion.[16,18] Therefore, the assumption of a B meson is not unreasonable. The possible J^{PG} assignments for the B meson are 1^{++} and 2^{-+}. We shall take the simplest assignment 1^{++}. The expression for the residue is given by (2.5), where $k_0(V) = (m_B^2+m_\omega^2-m_\pi^2)/(2m_B)$.

At present, we have data on the momentum-transfer distribution $d\sigma/ds$ and the decay density matrix[19] in ω production at 3.25 GeV/c. The momentum-transfer-distribution data are unnormalized, and we shall keep the same normalization as in the previous analysis,[10] corresponding to a total cross section of about 0.27 mb. The ρ Regge trajectory has been fixed in the analysis of π-N charge exchange scattering. The π-N charge exchange scattering differs from the ω production in the meson vertex. If we use the residue factorization, and assume that the residue of the same Regge pole is proportional to the coupling constants as determined from the decay width, then we can estimate the ρ Regge residue in ω production from that of the π-N charge-exchange scattering. The residues obtained in the Born term are $r_{CE}\propto 2f_{\pi\pi}$ and $r_\omega \propto f_{\omega\rho\pi}/\sqrt{2}m_\omega$, where r_{CE} and r_ω are the residues of π-N charge exchange and of the ω production, respectively. Then $r_{CE}/r_\omega = 2m_\omega\sqrt{2} \times (f_{\rho\pi\pi}/f_{\omega\rho\pi})$. This gives the following residue for ω production: $a_{\rho^-}=71.80$, $a_{\rho^+}=30.24$. Now we are left with the parameters of the B Regge pole. These are obtained by fitting the momentum-transfer distribution. We obtain $\alpha_B=0.505+0.40$, and the residues are $a_{B^-}=187.4$, and $a_{B^+}=6.39$, with $m_0=m_B=1.2$ GeV in Eq. (2.7). The χ^2 is 1.9 for 9 data points. The momentum-transfer distribution is shown in Fig. 3. Using these parameters we calculate the ω-decay density matrix, and the result is shown in Fig. 4. We obtain a qualitative agreement with the data. The deviations in ρ_{00} and $\rho_{1,-1}$ at larger momentum-transfer distribution might be due to the underestimation of the ρ Regge-pole contribution for $s<-0.3(\text{GeV}/c)^2$ as observed in the π-N charge-exchange scattering.

IV. DISCUSSIONS AND CONCLUSIONS

We have applied our Regge-pole model to the reactions $\pi^- p \to \pi^0 n$, $\pi^- p \to \eta n$, and $\pi^+ n \to \omega p$. The

[14] M. Aboline, R. L. Lander, W. A. W. Mehlhop, N.-h. Xuong, and P. M. Yager, Phys. Rev. Letters **11**, 381 (1963); G. Goldhaber, S. Goldhaber, J. A. Kadyk, and B. C. Shen, *ibid.* **15**, 118 (1965); S. U. Chung, M. Neveu-René, O. I. Dahl, J. Kirz, D. H. Miller, and Z. G. T. Guiragossián, *ibid.* **16**, 481 (1966).

[15] R. T. Deck, Phys. Rev. Letters **13**, 169 (1964).

[16] U. Maor and T. A. O'Halloran, Jr., Phys. Letters **15**, 281 (1965); U. Maor, Ann. Phys. (N.Y.) **41**, 456 (1967); see also M. Parkinson, Phys. Rev. Letters **18**, 270 (1967).

[17] C. Baltay, J. C. Severiens, N. Yeh, and D. Zanello, Phys. Rev. Letters **18**, 93 (1967).

[18] Y. Hara, Phys. Rev. **136**, B507 (1964); L. Wang, *ibid.* **142**, 1187 (1966).

[19] H. O. Cohn, W. M. Brigg, and G. T. Condo, Phys. Letters **15**, 334 (1965).

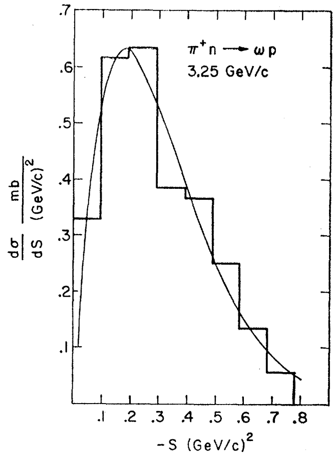

FIG. 4. Comparison of the momentum-transfer distribution $d\sigma/ds$ for $\pi^+ n \to \omega p$ with the data at 3.25 GeV/c [W. M. Brigg, H. O. Cohn, G. T. Condo, N. Gelfand, and G. Lüfjens (private communication)].

scheme for the calculations has been summarized in Sec. II. The parameters obtained from fitting the momentum-transfer distributions are summarized in Table I, together with the number of experimental points used and the corresponding χ^2. We found a good fit for the momentum-transfer distributions of the η and the ω production up to $s \simeq -0.85$ (GeV/c)^2 and for $\pi^- p \to \pi^0 n$ up to $s \approx -0.3$(GeV/c)^2.

Our simple ρ Regge-pole model for π-N charge exchange scattering can explain the momentum-transfer distribution. In this model the π-N charge exchange polarization is zero. This does not agree with the recent measurements, which give $\approx 15\%$ polarization.[20] Several alternatives to explain this small polarization have been given by Logan and Sertorio,[5,6] and recently a refinement has been given by Desai *et al.*[21]

In the analysis of ω production we have used data at 3.25 GeV/c where the cross section $d\sigma/ds$ and the ω decay density matrix are available. The absolute cross section is not known, and it is estimated by a comparison to the better-known process of ρ production. It has been shown in π-N charge exchange scattering and in η production that, at incident momenta <4 GeV/c the resonances in the direct channel change the distribution $d\sigma/ds$ only slightly. In ω production, at 3.25 GeV/c, the direct-channel resonances may change the $d\sigma/ds$ and the ω decay density matrix a little. Therefore, the agreement in ω production should be considered as a qualitative agreement. Further measurements at incident momenta above 4 GeV/c would be very useful in providing a test of our model.

TABLE I. Regge-pole parameters.

Reaction	$\alpha(s)$	a_-	a_+	No. of experimental points	χ^2
(1) $\pi^- p \to \pi^0 n$	$\alpha_\rho = 1.1s + 0.58$	69.38	29.23	36	99.5
(2) $\pi^- p \to \eta n$	$\alpha_{A_2} = 0.44s + 0.38$	262.8	216.81	39	39.2
(3) $\pi^+ n \to \omega p$	$\alpha_B = 0.50s + 0.40$	187.4	6.39	9	1.9

The parameters in Table I are obtained from a χ^2 fit of the data. The trajectories for ρ and A_2 are consistent with the results of the model-independent determination for ρ by Höhler *et al.*,[4] and for A_2 by Phillips and Rarita.[8] The ρ and the B trajectories do not pass exactly through the value of the spin at the corresponding mass of the particle, but we find $\alpha(m_\rho^2) \approx 1.2$, $\alpha_B(m_B^2) \approx 1.1$. The deviation in the ρ and B trajectories is less than 20%, indicating that the assumption that the trajectories are straight lines is reasonably satisfied. However, for the A_2 trajectory the situation is quite different. We find $\alpha_A(m_A^2) \approx 1.2$ instead of $=2$. This problem has already been mentioned by Phillips and Rarita, who pointed out that the A_2 must have a considerable curvature and that the A_2 and ρ trajectories are rather dissimilar. The intercept $\alpha_A(0)$ is consistent with the value obtained by Barger and Olsson,[22] from the total cross-section data. To test the sensitivity of the shape of the momentum-transfer distribution to the slope α_A', we have calculated $d\sigma/ds$ under the condition $\alpha_A(m_A^2) = 2$. We find that the momentum-transfer distribution is much steeper and disagrees significantly with the data. It is also possible that these problems with the A_2 trajectory indicate that there are other contributions to the η-production amplitude, which are not negligible. However, with the data presently available, it is difficult to test this assertion. It would be desirable to test the slope of A_2 in other reactions. Un-

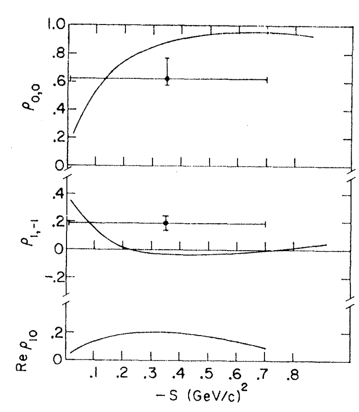

FIG. 5. Comparison of the decay density matrix $\rho_{m m'}$ with the data at 3.25 GeV/c (Ref. 20).

[20] P. Bonamy, P. Borgeaud, C. Bruneton, P. Falk-Vairant, O. Guisan, P. Sonderegger, C. Caverzasio, J. P. Guillaud, J. Schneider, M. Yvert, I. Mannelli, F. Sergiampietri, and L. Vincelli, Phys. Letters 23, 501 (1966).

[21] B. R. Desai, D. T. Gregorich, and R. Ramachandran, Phys. Rev. Letters 18, 565 (1967).

[22] V. Barger and M. Olsson, Phys. Rev. Letters 18, 294 (1967).

TABLE II. Table for a, β, and ϵ for $PN \to P'N$.

λ_1	λ_2	a	b	β	ϵ
$+\frac{1}{2}$	$+\frac{1}{2}$	0	0	α	$-$
$+\frac{1}{2}$	$-\frac{1}{2}$	1	1	$\alpha-1$	$+$

TABLE III. Table for a, b, β, and ϵ for $PN \to VN$.

λ_V	λ_1	λ_2	a	b	β	ϵ
$+1$	$+\frac{1}{2}$	$+\frac{1}{2}$	1	1	$\alpha-1$	$-$
	$+\frac{1}{2}$	$-\frac{1}{2}$	0	2	$\alpha-1$	$+$
0	$+\frac{1}{2}$	$+\frac{1}{2}$	0	0	α	$+$
	$+\frac{1}{2}$	$-\frac{1}{2}$	1	1	$\alpha-1$	$+$
-1	$+\frac{1}{2}$	$+\frac{1}{2}$	1	1	$\alpha-1$	$+$
	$+\frac{1}{2}$	$-\frac{1}{2}$	2	0	$\alpha-1$	$+$

fortunately, except for $\pi^+p \to \eta N^*$, the A_2 trajectory is masked by the π exchange in $\pi N \to \rho N$; $\pi N \to \rho N^*$ and $KN \to K^*N^*$, or it is accompanied by ρ exchange such as in $KN \to KN^*$. It would, therefore, be very helpful to have more detailed data for the reaction $\pi^+p \to \eta N^*$.

ACKNOWLEDGMENTS

It is a pleasure to express my gratitude to Professor Reinhard Oehme, who suggested the present subject, for stimulating discussions, and for continued guidance and encouragement. I also would like to thank the Institute of Technology, Bandung, and the Center for Developmental Change, University of Kentucky, for their arrangements.

APPENDIX

In Tables II and III we list values of a, b, β, and ϵ for $PN \to P'N$ and $PN \to VN$, respectively.

Asymptotic forms for $N_{\lambda_i,\lambda_f}{}^{\alpha}P_{\beta}{}^{(a,b)}(z)$ are the following:

$$N_{0,0}{}^{\alpha}P_{\alpha}{}^{(0,0)}(z) \sim n_\alpha(2z)^\alpha,$$
$$N_{1,0}{}^{\alpha}P_{\alpha-1}{}^{(1,1)}(z) \sim 2[\alpha/(\alpha+1)]^{1/2}n_\alpha(2z)^{\alpha-1},$$
$$N_{1,1}{}^{\alpha}P_{\alpha-1}{}^{(0,2)}(z) \sim 2[\alpha/(\alpha+1)]n_\alpha(2z)^{\alpha-1},$$
$$N_{1,-1}{}^{\alpha}P_{\alpha-1}{}^{(2,0)}(z) \sim 2[\alpha/(\alpha+1)]n_\alpha(2z)^{\alpha-1},$$

where

$$n_\alpha = (1/\sqrt{\pi})[\Gamma(\alpha+\tfrac{1}{2})/\Gamma(\alpha+1)].$$

Errata

Radiative Corrections. I. High-Energy Bremsstrahlung and Pair Production, KJELL MORK AND HAAKON OLSEN [Phys. Rev. **140**, B1661 (1965)]. The numerical values for F_1 given in Table I are incorrect. The correct values are as follows:

ω_1/ϵ_1	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
$F_1 \times 10^2$	0.095	0.19	0.30	0.43	0.60	0.80	1.08	1.50	2.30

We are indebted to Dr. H. D. Schulz for pointing out these errors to us.

$K_{\mu3}$ and K_{e3} Form Factors at Finite Momentum Transfer, M. FITELSON AND E. KAZES [Phys. Rev. **159**, 1236 (1967)]. Equation (23a) should read

$$(a_1{}^2C_1+a_2{}^2C_2)/\sqrt{2} = \langle\pi^0|J''{}_3{}^1(0)|K^+\rangle_{p=\infty}.$$

Equation (26) should read

$$-\frac{\cos Ma_1}{\cos Ma_2} = \frac{B_2 - b_2C_2M\tan Ma_2}{B_1 - b_1C_1M\tan Ma_1}.$$

Castillejo-Dalitz-Dyson Poles and Asymptotic Fields, STANLEY JERNOW AND EMIL KAZES [Phys. Rev. **160**, 1428 (1967)]. A typographical error appeared in Eq. (2.2) which gave the form of the interaction Hamiltonian. The equation should read

$$G^\dagger \equiv \int d\mathbf{p}\, f(\omega_p)\theta^\dagger(p), \quad \omega_p = (\mu^2+p^2)^{1/2}. \quad (2.2)$$

Also, the left-hand side of Eq. (2.21) should be the time derivative of the field and should read $-\dot{\beta}_j{}^\dagger(t)$.