Existence of Production Amplitudes for a Neutral Scalar Field without Pairing Symmetry*

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A pole-pole contribution alone to the double density function in a neutral scalar field without pairing symmetry is shown to be inconsistent with the unitarity condition, so that the absence of production implies the absence of scattering, even though the pole-pole term gives rise to a Landau curve symmetric in s and t.

I. INTRODUCTION

IT was shown by Aks[1] that production amplitudes are necessary for a neutral scalar field with pairing symmetry; this result was subsequently extended by Cheung and Toll[2] to the scattering of any two stable particles. The method of proof was based on the interconnection between the unitarity and crossing relations. If there are no production processes in one channel, say the s channel, then the Landau curves in the real s-t plane, on which singularities of the double density functions lie, will be asymmetric in s and t; in general, the double density functions, or linear combinations thereof, are symmetric in s and t. These requirements are incompatible with each other unless the double density functions vanish identically. From this it can be shown that the scattering amplitudes themselves are zero. Therefore for nontrivial scattering, production processes must occur.

This line of proof makes the case of a neutral scalar field without pairing symmetry a particularly interesting one, because now there may be a pole singularity in the scattering amplitude, and the pole-pole contribution to the double density function $\rho(s,t)$ gives rise to a symmetric first Landau curve in the real s-t plane, so that the argument based on the incompatibility between the symmetry of $\rho(s,t)$ and the asymmetric character of the Landau curves breaks down as it stands. The usual argument does give the result that all contributions to $\rho(s,t)$ other than the pole-pole term must vanish. We will show in the following that a pole-pole contribution alone to the double density function is incompatible with the unitarity condition. Self-consistency then demands that the pole-pole term, along with the scattering amplitude itself, must vanish if there are no production processes.

II. PROOF THAT THE ABSENCE OF PRODUCTION IMPLIES THE ABSENCE OF SCATTERING

Although our results may follow directly from the axioms of the quantum field theory, we shall assume for simplicity that the scattering amplitude for a neutral scalar particle satisfies the analyticity and crossing property of an unsubtracted Mandestam representation,[3]

$$\phi(s,t)=\frac{g^2}{s-\mu^2}+\frac{g^2}{t-\mu^2}+\frac{g^2}{u-\mu^2}+\frac{1}{\pi^2}\int\!\!\int_{4\mu^2}^{\infty}\frac{ds'dt'\rho(s't')}{(s'-s)(t'-t)}$$
$$+\frac{1}{\pi^2}\int\!\!\int_{4\mu^2}^{\infty}\frac{dt'du'\rho(u't')}{(t'-t)(u'-u)}+\frac{1}{\pi^2}\int\!\!\int_{4\mu^2}^{\infty}\frac{du'ds'\rho(u's')}{(u'-u)(s'-s)},\quad(1)$$

where g is the residue of the pole at μ^2. If we assume that no production processes can occur, then the elastic unitarity in the s channel will be valid for all s with $4\mu^2\leq s<\infty$,[4]

$$\phi_s(s,z(t))=\tfrac{1}{8}\theta(s-4\mu^2)\left(\frac{s-4\mu^2}{s}\right)^{1/2}\int_{-1}^{1}\int_{-1}^{1}dz_1dz_2 \times K'(z,z_1,z_2)\varphi^*(s,z_1)\phi(s,z_2),\quad(2)$$

where s is the c.m. energy squared and z the c.m. scattering angle, with $z=1+2t/(s-4\mu^2)$ and

$$K'(z,z_1,z_2)=\frac{\theta(1+2zz_1z_2-z^2-z_1^2-z_2^2)}{(1+2zz_1z_2-z^2-z_1^2-z_2^2)^{1/2}}$$

also

$$\rho(s,t)=\tfrac{1}{2}\pi^2\left(\frac{s-4\mu^2}{s}\right)^{1/2}\int\!\!\int_{1+8\mu^2/(s-4\mu^2)}^{z>z_1z_2+(z_1^2-1)^{1/2}(z_2^2-1)^{1/2}}dz_1dz_2 \times K(z,z_1,z_2)\phi_t^*(s,z_1)\phi_t(s,z_2),\quad(3)$$

where as usual

$$K(z,z_1,z_2)=\frac{\theta(z^2+z_1^2+z_2^2-1-2zz_1z_2)}{(z^2+z_1^2+z_2^2-1-2zz_1z_2)^{1/2}}$$

and use has been made of the symmetry between the variables t and u. The pole-pole (p-p) contribution to $\rho(s,t)$ can now be obtained by taking only the pole term $\pi g^2\delta(\frac{1}{2}(s-4\mu^2)(1-z)+\mu^2)$ for $\phi_t(s,z)$ in Eq. (3):

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[1] S. Aks, J. Math. Phys. 6, 516 (1965).

[2] F. K. Cheung and J. S. Toll, Phys. Rev. 160, 1072 (1967).

[3] The results we obtain will be independent of subtractions made in the Mandelstam representation for $\varphi(s,t)$. The Mandelstam representation used here actually follows from the axioms, since we assume that elastic unitarity is valid for all energies in one channel. See Ref. 2.

[4] S. Mandelstam, Phys. Rev. 112, 1344 (1958).

$$\rho_{p-p}(s,t)=\tfrac{1}{2}(\pi^2)\left(\frac{s-4\mu^2}{s}\right)^{1/2}\iint_{1+8\mu^2/(s-4\mu^2)}^{z>z_1z_2+(z_1^2-1)^{1/2}(z_2^2-1)^{1/2}} dz_1dz_2$$

$$\times K(zz_1z_2)\left(\frac{2\pi}{s-4\mu^2}\right)^2 g^4$$

$$\times\delta\left(1+\frac{2\mu^2}{s-4\mu^2}-z_1\right)\delta\left(1+\frac{2\mu^2}{s-4\mu^2}-z_2\right)$$

$$=\frac{4\pi^2g^4\theta(st-4\mu^2t-4\mu^2+12\mu^4)}{\{st[(s-4\mu^2)(t-4\mu^2)-4\mu^4]\}^{1/2}}. \qquad (4)$$

As expected, the pole-pole contribution to $\rho(s,t)$ is manifestly symmetric in s and t and is singular along the curve

$$(s-4\mu^2)(t-4\mu^2)-4\mu^4=0, \qquad (5)$$

which in fact is the Landau curve associated with $\rho_{p-p}(s,t)$. It is also symmetric in s and t and has asymptotes $s=4\mu^2$ and $t=4\mu^2$.

We next show that $\rho_{p-p}(s,t)$ is the only contribution to $\rho(s,t)$ in the whole real s-t plane. If we denote the remaining contributions other than the pole-pole term by $\rho_1(s,t)$, then

$$\rho_1(st)\equiv\rho(s,t)-\rho_{p-p}(s,t). \qquad (6)$$

Since both $\rho(s,t)$ and $\rho_{p-p}(s,t)$ are symmetric in s and t, so must be their difference $\rho_1(s,t)$. Now $\rho_1(s,t)$ becomes zero before the next Landau curve $t_3^+(s)$ is reached. If we assume that no production processes may occur along the t channel, $t_3^+(s)$ will be asymptotic to $s=4\mu^2$ and $t=9\mu^2$, and hence asymmetric with respect to s and t. The usual argument for a neutral scalar field with pairing symmetry or that for pion-pion scattering can now be carried over to $\rho_1(s,t)$. We then see that the asymmetry of $t_3^+(s)$ is incompatible with the symmetry of $\rho_1(s,t)$ unless $\rho_1(s,t)\equiv 0$. We therefore arrive at the result that

$$\rho_{p-p}(s,t)=\rho(s,t) \qquad (7)$$

for all real s and t, as we intended to show.

We will now show that a double density function consisting only of the pole-pole term is inconsistent with the unitarity condition.

From Eq. (1), the Mandelstam representation for $\phi(s,t,u)$ now becomes

$$\phi(s,t,u)=\frac{g^2}{s-\mu^2}+\frac{g^2}{t-\mu^2}+\frac{g^2}{u-\mu^2}+\frac{1}{\pi^2}\iint_{4\mu^2}^{\infty}\frac{ds'dt'\rho_{p-p}(s't')}{(s'-s)(t'-t)}$$

$$+\frac{1}{\pi^2}\iint_{4\mu^2}^{\infty}\frac{dt'du'\rho_{p-p}(t'u')}{(t'-t)(u'-u)}$$

$$+\frac{1}{\pi^2}\iint_{4\mu^2}^{\infty}\frac{du'ds'\rho_{p-p}(u's')}{(s'-s)(u'-u)}, \qquad (8)$$

from which the s-channel absorptive part may be obtained:

$$\phi_s(s,t)=\frac{1}{\pi}\int_{4\mu^2}^{\infty}dt'\frac{\rho_{p-p}(t',s)}{t'-t}+\frac{1}{\pi^2}\int_{4\mu^2}^{\infty}du'\frac{\rho_{p-p}(u',s)}{u'-u}, \qquad (9)$$

where we note that the pole at $s=\mu^2$ does not contribute to the absorptive part $\phi_s(s,t)$ because it is below the $4\mu^2$ threshold.

When the value of ρ_{p-p} in Eq. (4) is substituted into Eq. (9), we get

$$\phi_s(s,t)=g^4\left[4\pi\int_{4\mu^2}^{\infty}\frac{d\xi}{\{\xi s[(\xi-4\mu^2)(s-4\mu^2)-4\mu^4]\}^{1/2}}\right.$$

$$\left.\times\frac{2\xi+s-4\mu^2}{(\xi-t)(\xi+s+t-4\mu^2)}\right]. \qquad (10)$$

The point to be made here is that the integral is independent of g. We denote it by $h(s,t)$, so that

$$\phi(s,t)=g^4h(s,t), \quad s\geq 4\mu^2. \qquad (11)$$

On the other hand, $\phi_s(s,t)$ is also given by the elastic unitarity integral (2), which together with Eq. (8) gives

$$\phi_s(s,t)=\tfrac{1}{8}(s-4\mu^2)\left(\frac{s-4\mu^2}{s}\right)^{1/2}\int_{-1}^{1}\int_{-1}^{1}dz_1dz_2K'(z,z_1,z_2)$$

$$\times[g^2a(sz_1)+g^4b(sz_1)][g^2a(sz_2)+g^4b(sz_2)], \qquad (12)$$

where

$$a(s,z)=\frac{1}{\frac{1}{2}(4\mu^2-s)(1-z)-\mu^2}+\frac{1}{\frac{1}{2}(4\mu^2-s)(1+z)-\mu^2}+\frac{1}{s-\mu^2},$$

$$b(s,z)=\frac{1}{\pi^2}\int_{4\mu^2}^{\infty}\frac{ds'}{s'-s}\int_{4\mu^2}^{\infty}\frac{dt'}{t'-\frac{1}{2}(4\mu^2-s)(1-z)}\rho_{p-p}'(s't')$$

$$+\frac{1}{\pi^2}\int_{4\mu^2}^{\infty}\frac{ds'}{s'-s}\int_{4\mu^2}^{\infty}\frac{du'}{u'-\frac{1}{2}(4\mu^2-s)(1+z)}\rho_{p-p}'(u's')$$

$$+\frac{1}{\pi^2}\int_{4\mu^2}^{\infty}\frac{du'}{u'-\frac{1}{2}(4\mu^2-s)(1+z)}$$

$$\times\int_{4\mu^2}^{\infty}\frac{dt'}{t'-\frac{1}{2}(4\mu^2-s)(1-z)}\rho_{p-p}'(u't'),$$

and

$$\rho_{p-p}'(\xi\eta)=\frac{4\pi^2}{\{\xi\eta[(\xi^2-4\mu^2)(\eta-4\mu^2)-4\mu^4]\}^{1/2}}.$$

Equating Eq. (11) to Eq. (12), we obtain

$$g^4\Big[-h(s,t)+\int_{-1}^{1}\int_{-1}^{1}dz_1dz_2K'(zz_1z_2)a(tz_1)a(tz_2)\Big]$$

$$+g^6\int_{-1}^{1}\int_{-1}^{1}dz_1dz_2K'(zz_1z_2)[a(tz_1)b(tz_2)+b(tz_1)a(tz_2)]$$

$$+g^8\int_{-1}^{1}\int_{-1}^{1}dz_1dz_2K'(zz_1z_2)b(tz_1)b(tz_2)\equiv 0\,. \quad (13)$$

If we rewrite this as

$$g^4[g^4m(s,t)+2g^2n(s,t)+l(s,t)]=0\,, \quad (14)$$

then either

$$g^2=0 \quad (15)$$

or

$$g^2=\frac{n(s,t)}{m(s,t)}\pm\left[\left(\frac{n(s,t)}{m(s,t)}\right)^2-\frac{l(s,t)}{m(s,t)}\right]^{1/2}. \quad (16)$$

For Eq. (16) to be true, the right-hand side must be independent of s and t, which requires at least that for all s and t

$$n(s,t)=k_1m(s,t)=k_2l(s,t)\,. \quad (17)$$

That this is not the case can be seen from the explicit expressions for $a(s,z)$, $b(s,z)$, and $h(s,z)$. We therefore must have $g=0$. As a consequence, the pole-pole contribution $\rho_{p-p}(s,t)$, along with the scattering amplitude and its absorptive part, is identically zero, and we regain the result that the absence of production implies the absence of scattering.

III. AN ALTERNATIVE PROOF THAT THE ABSENCE OF PRODUCTION IMPLIES THE ABSENCE OF SCATTERING

The fact that a pole-pole contribution alone to the double density function is inconsistent with elastic unitarity is also clear from a diagrammatic approach, because low-order singularities will generate higher-order singularities through the unitarity relation. Conversely, if all singularities higher than the pole-pole term are zero, the pole-pole term itself must also be zero. Thus a pole term in the scattering amplitude will lead to the box and higher-order ladder diagrams whose contributions to the double density function can be computed by Cutkosky's method.[5] We will show that

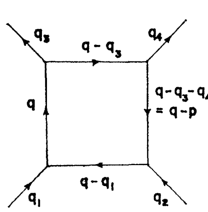


FIG. 1. Kinematics of the box diagram.

[5] R. E. Cutkosky, J. Math. Phys. **1**, 429 (1960).

the sixth-order contribution is not zero unless the pole-pole contribution vanishes also, and that under such an assumption the double density function, along with the scattering amplitude itself, is also zero.

The contribution to the full density function from the box diagram, Fig. 1, is given by

$$\rho_0(s,t)=16\pi^2g^4\int d^4q\,\delta(q^2-\mu^2)\delta((q-q_1)^2-\mu^2)$$

$$\times[(q-q_3)^2-\mu^2]\delta((q-p)^2-\mu^2)\,, \quad (18)$$

where g is the "renormalized" coupling constant of the 3ϕ vertex, and all the lines are considered to lie on the mass shell,

$$q^2=q_1^2=q_2^2=q_4^2=\mu^2\,.$$

In the c.m. system of the s channel, $P=p_3+p_4=(\sqrt{s},0,0,0)$, so that $\delta(2q\cdot P-s)=\delta(2q_0\sqrt{s}-s)$. The integration over $|\mathbf{q}|^2$ and q_0 can be carried out immediately to give

$$\rho_0(s,t)=2\pi^2g^4\left(\frac{s-4\mu^2}{s}\right)^{1/2}\int d\Omega_q$$

$$\times\delta(\tfrac{1}{2}s-\mu^2-2|\mathbf{q}|^2x_1)\delta(\tfrac{1}{2}s-\mu^2-2|\mathbf{q}|^2x_2)\,, \quad (19)$$

where $\cos\theta=x_1=\hat{q}\cdot\hat{q}_1$ and $x_2=\hat{q}\cdot\hat{q}_3$. The angular integral $d\Omega_q$ can be evaluated using a coordinate system defined by

$$\hat{q}_1=(0,0,1)\,,$$
$$\hat{q}_3=(0,\,(1-x^2)^{1/2},\,x)\,,$$
$$\hat{q}=(\sin\theta\cos\phi,\,\sin\theta\sin\phi,\,\cos\theta)\,,$$

and changing variables from (x_1,ϕ) to (x_1,x_2), where $x_2=\hat{q}\cdot\hat{q}_3=(1-x^2)^{1/2}\sin\theta\sin\phi+x\cos\theta$. Then

$$dx_1d\phi=\frac{\partial(x_1\phi)}{\partial(x_1x_2)}dx_1dx_2$$

$$=dx_1dx_2\frac{\theta(1-x^2-x_1^2-x_2^2+2xx_1x_2)}{(1-x^2-x_1^2-x_2^2+2xx_1x_2)^{1/2}}\,,$$

and we obtain

$$\rho_0(s,t)=2\pi^2g^4\left(\frac{s-4\mu^2}{s}\right)^{1/2}\left(\frac{1}{2|\mathbf{q}|^2}\right)^2$$

$$\times\frac{\theta(1-x^2-x_1^2-x_2^2+2xx_1x_2)}{(1-x^2-x_1^2-x_2^2+2xx_1x_2)^{1/2}}\bigg|_{x_1,x_2=1+\mu^2/2|\mathbf{q}|^2}$$

$$=4\pi^2g^4\left(\frac{1}{st}\right)^{1/2}\frac{\theta((t-4\mu^2)(s-4\mu^2)-4\mu^4)}{((t-4\mu^2)(s-4\mu^2)-4\mu^4)^{1/2}}\,, \quad (20)$$

where we have used the kinematic relations

$$s=4(|\mathbf{q}|^2+\mu^2)\,,$$
$$t=-2|\mathbf{q}|^2(1-x)\,.$$

As expected, $\rho_0(s,t)$ is manifestly symmetric in s and t, and it is seen to be equal to ρ_{p-p} in Eq. (4). The Landau curve associated with ρ_0 is given by

$$(t-4\mu^2)(s-4\mu^2)-4\mu^4=0$$

or

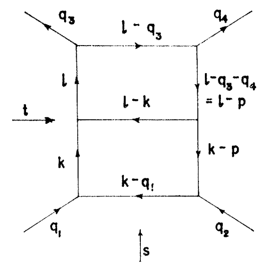
$$t=4\mu^2+4\mu^4/(s-4\mu^2)\,, \tag{21}$$

which has asymptotes $t=4\mu^2$ and $s=4\mu^2$, and is also symmetric in s and t.

If we now subtract $\rho_0(s,t)$ from $\rho(s,t)$ and call the resulting double density function $\rho_1(s,t)=\rho(s,t)-\rho_0(s,t)$, then $\rho_1(s,t)$ is again symmetric in s and t, and is zero until the next Landau curve is reached. From our premise that there are no production processes in the s channel, it follows that contributions to $\rho_1(s,t)$ from Fig. 2 will be the only lowest-order term of $\rho_1(s,t)$. We shall call it $\rho_{10}(s,t)$. By a method similar to that used for $\rho_1(s,t)$, the function $\rho_{10}(s,t)$ can be calculated by Cutkosky's method:

$$\rho_{10}(s,t)=4\pi^2g^6\int d^4k\delta(k^2-\mu^2)\delta((k-q_1)^2-\mu^2)$$
$$\times\delta((k-P)^2-\mu^2)\int d^4l\delta(l^2-\mu^2)\delta((l-q_3)^2-\mu^2)$$
$$\times\delta((l-P)^2-\mu^2)\delta((l-k)^2-\mu^2)\,, \tag{22}$$

FIG. 2. Kinematics of the sixth-order ladder diagram.



or

$$\rho_{10}(s,t)=4\pi g^6\int dk_0d\Omega_k\tfrac{1}{2}(|\mathbf{k}|d|\mathbf{k}|^2)$$
$$\times\delta(k^2-\mu^2)\delta(2kq_1-\mu^2)\delta(2k\cdot P-s)I(x_2x_3k)\,, \tag{23}$$

where

$$I(x_2x_3k)=\frac{\theta(1-x^2-x_2^2-x_3^2+2xx_2x_3)}{(1-x^2-x_2^2-x_3^2+2xx_2x_3)^{1/2}}\bigg|_{x_1=x_3=(\frac{1}{2}s-2\mu^2)/2l^2}\,,$$

and we are in the c.m. system of the s channel defined by $P=q_1+q_2=q_3+q_4=(s^{1/2},0,0,0)$. The integration on d^4k may be carried out similarly to what has just been done for the box diagram, and we obtain

$$\rho_{10}(s,t)=4\pi g_9$$
$$\times\left(\frac{1}{64}\frac{s-4\mu^2}{s}\right)\int_{-1}^{1}dx\frac{\theta(1-y^2-x_1^2-x^2+2yx\,x_1)}{(1-y^2-x_1^2-x^2+2yx\,x_1{}^{1/2})}\,\frac{\theta(1-x^2-x_2^2-x_3^2+2xx_2x_3)}{(1-x^2-x_2^2-x_3^2+2xx_2x_3)^{1/2}}\bigg|_{x_1=x_2=x_3=1+(\mu^2/2|\mathbf{q}|^2),\,y=1+t/2|\mathbf{q}|^2}. \tag{24}$$

Singularities of $\rho_{10}(s,t)$ will occur if the singularities of the two factors in the integrand pinch together, which happens when

$$1-y^2-x_1^2-x^2+2yx_1x=0\,,$$
$$1-2x_1^2-x^2+2xx_1^2=0\,, \tag{25}$$

so that

$$x=(x_1+y)/2x_1\,, \tag{26}$$

and Eq. (25) becomes

$$\left(\frac{x_1+y}{2x_1}\right)^2-2\left(\frac{x_1+y}{2x_1}\right)x_1^2+2x_1^2-1=0 \tag{27}$$

or

$$(x_1-y)(4x_1^3-3x_1-y)=0.$$

$x_1-y=0$ gives $t=\mu^2$, which is not interesting becaues $\rho_{10}(s,t)$ is known to be zero at $t=\mu^2$, as is evident from the step functions in the integrand of Eq. (24).

For $4x_1^3-3x_1-y=0$, we have

$$4\left(1+\frac{2\mu^2}{4q^2}\right)^3-3\left(1+\frac{2\mu^2}{4q^2}\right)-\left(1+\frac{2t}{4q^2}\right)=0$$

or

$$t_3{}^+(s):\ \ t=9\mu^2+\frac{12\mu^4}{s-4\mu^2}+\frac{16\mu^6}{(s-4\mu^2)^2}. \tag{28}$$

This is the explicit form of the first Landau curve pertaining to $\rho_1(s,t)$, and $\rho_1(s,t)$ is zero before this Landau curve is reached. As expected, it is asymmetric in s and t, and has asymptotes

$$t=9\mu^2\,,$$
$$s=4\mu^2.$$

From the asymmetry of the Landau curve just obtained, and the symmetric character of $\rho_1(s,t)$, it follows from the usual analysis that $\rho_1(s,t)\equiv0$. In particular, $\rho_{10}(s,t)=0$, since it is the only contribution to $\rho_1(s,t)$ before the next Landau curve for $\rho_1(s,t)$ is reached.

On the other hand, we have obtained for $\rho_{10}(s,t)$ Eq. (24), which in general can be transformed into the standard form of an elliptic integral,

$$\rho_{10}(s,t)=4\pi g^6\tfrac{1}{64}\frac{s-4\mu^2}{s}\frac{1}{(1-x_1^2)^{1/2}}\frac{1}{(1-y^2)^{1/2}}\frac{\beta(\alpha+\gamma)}{\alpha}$$
$$\times\frac{(\alpha^2-1)^{1/2}}{[\beta^2(\alpha+\gamma^2)-1]^{1/2}}\int_0^{2\pi}\frac{d\xi}{(1-\xi^2)^{1/2}[1-\beta^2(1-\gamma/\alpha)^2\xi^2]^{1/2}}\,, \tag{29}$$

where

$$\alpha=\frac{1-x_1^2-y^2-2x_1^3y\pm[(x_1^2+y)^2-4x_1^2(x_1-y)]^{1/2}}{x_1(x_1-y)},$$

$$\beta=\left(\frac{1-x_1^2}{1-y^2}\right)^{1/2},$$

$$\gamma=\frac{x_1^2-x_1y}{1-x_1^2},$$

$$\xi=\frac{\alpha(x-x_1^2)-1+x_1^2}{\alpha(1-x_1^2)-x+x_1^2}.$$

Evidently, this expression for $\rho_{10}(s,t)$ is not a zero function. In order that $\rho_{10}(s,t)\equiv 0$ for all x and y, it is required that the coupling constant g for the 3ϕ vertex be zero. As a consequence, there can be no pole term and no box-diagram singularity either. This means that $\rho_0(s,t)\equiv 0$. Therefore, the entire double density function is also zero, $\rho(s,t)=\rho_0(s,t)+\rho_1(s,t)=0$. From this point on, the usual procedure will lead us to the conclusion that there can be no scattering at all, provided there are no production processes in the s channel.

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Gribov-Pomeranchuk Phenomenon in N/D Approach

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A detailed study of the left-hand function of the Froissart-Gribov representation enables the integral equation for the N function to be reduced to a form studied by Tamarkin. This enables us to show explicitly that the N function develops an essential singularity and causes an infinite number of Regge poles to accumulate at $l=-1$. When moving cuts are introduced, the integral equation, when again reduced to Tamarkin form, gives conditions on the discontinuity across the cuts for eliminating the essential singularity. The technique in the present form, however, is applicable only to the right-most singularity in the complex angular momentum plane. Extensions of these techniques for π-N scattering (in the s channel) are given as an example of the inclusion of spin effects.

1. INTRODUCTION

SEVERAL interesting relations have recently been derived on the basis of superconvergence of strong-interaction amplitudes.[1] These have also been recognized as necessary, at least for the amplitudes involving large helicity flip, in order to satisfy the Froissart bound[2] (a consequence of direct-channel unitarity) for the total amplitude. The asymptotic behavior of these amplitudes has been known for some time to be related to the analyticity properties in the angular momentum plane of the crossed channel. The superconvergence is a consequence of a holomorphy domain in the angular momentum plane larger than that suggested by the Froissart-Gribov (F-G) representation. It was first observed by Gribov and Pomeranchuk[3] that if one attempts to continue the partial-wave amplitude defined by the F-G representation, the presence of fixed poles of the Q_J function conflicts with elastic unitarity and causes an accumulation of an infinite number of Regge poles of positive signature at $l=-1$. A pole at $l=-1$ normally does not contribute to the asymptotic behavior, owing to the wrong signature factor, but, if it becomes an essential singularity, will prevent analytic continuation beyond this point and necessarily force an asymptotic behavior $1/s^{1-\epsilon}$ for any amplitude. Thus, in order to ensure superconvergence, it is necessary to take the mechanism that prevents essential singularities more seriously, as it implies restrictions imposed by unitarity in the direct as well as the crossed channels.

Mandelstam[4] has shown that certain sets of diagrams can produce moving branch points in the complex angular momentum plane, and that by the inclusion of

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[1] V. de Alfaro, S. Fubini, G. Rossetti, and G. Furlan, Phys. Letters **21**, 576 (1966); B. Sakita and K. C. Wali, Phys. Rev. Letters **18**, 29 (1967); L. K. Pande, Nuovo Cimento Letters **48**, 839 (1967); R. Ramachandran, Phys. Rev. **166**, 1528 (1968).

[2] M. Froissart, Phys. Rev. **123**, 1053 (1961); Y. Hara, *ibid.* **136**, B507 (1964).

[3] V. N. Gribov and I. Ya. Pomeranchuk, Phys. Letters **2**, 239 (1962).

[4] S. Mandelstam, Nuovo Cimento **30**, 1113 (1963); **30**, 1127 (1963); **30**, 1148 (1963); V. N. Gribov, I. Ya. Pomeranchuk, and K. A. Ter-Martirosyan, Phys. Rev. **139**, B184 (1965); J. C. Polkinghorne, J. Math. Phys. **6**, 1960 (1965).