Corrections to ρ Dominance and Pole Dominance of the Divergence of the Axial-Vector Current

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(Received 24 August 1967)

A superconvergence assumption about the matrix elements allows us to discuss corrections to one-meson dominance of a current or its divergence. The deviation of the observed ρ leptonic branching ratio from the ρ-dominance prediction and the correction to the Goldberger-Treiman relation are evaluated, and other predictions of the model are discussed.

THE assumptions, that the ρ meson dominates the isovector hadronic vector current and that the π meson dominates the divergence of the axial current (PDDAC), have led to a considerable understanding of the interactions of these mesons. There are, however, experimental indications that further states need to be considered in the hadronic currents and it is the purpose of this paper to consider these corrections and to show that, with a simple model, the predictions agree with experiment in direction and approximately in magnitude.

We shall first consider in general a form factor $F(t)$ from state A to B which is supposed to be dominated by a meson M. We assume an unsubtracted dispersion relation for $F(t)$,

$$F(t)=\frac{1}{\pi}\int_\sigma^\infty \frac{\mathrm{Im}F(t')dt'}{t'-t}, \tag{1}$$

where σ is the threshold. The contributions to the discontinuity can be separated into the M pole and the correction $tC(t)$,

$$\mathrm{Im}F(t)=\pi f_M{}^{-1}M^2 g_{AMB}\delta(t-M^2)+\pi tC(t)\theta(t-\sigma). \tag{2}$$

The crucial assumption is that the form factor $F(t)$ decreases sufficiently fast asymptotically so that $tF(t)\rightarrow 0$. This is supported by the experimental evidence for the nucleon electromagnetic form factors[1] and we postulate it as a general feature of weak and electromagnetic form factors of all particles. Then, as noted by Balachandran *et al.*,[2] one has a superconvergence relation from (1),

$$f_M{}^{-1}M^2 g_{AMB}+\int_\sigma^\infty tC(t)dt=0. \tag{3}$$

Then evaluating the form factor at $t=0$ one obtains

$$F(0)=f_M{}^{-1}g_{AMB}(1-M^2/m_C{}^2), \tag{4}$$

where

$$m_C{}^2=\int_\sigma^\infty tC(t)dt\Big/\int_\sigma^\infty C(t)dt. \tag{5}$$

Thus the corrections to M dominance of the form factor are given by (4) and (5). Since $C(t)$ is not restricted to be positive, one does not have any general bounds on $m_C{}^2$, furthermore the corrections may be different for different matrix elements of the current since $C(t)$ may depend on A and B. When, however, $C(t)$ itself is dominated by a meson M' one has the simple result $m_C{}^2=M'^2$ which will lead to a universal correction (4) to all matrix elements of the current. In this case, the direction of the correction (4) is also specified since $M'^2>M^2$. Such a simple two-pole model with superconvergence constraint gives a universal t dependence of $F(t)$ for all A and B. If $C(t)$ has contributions from n discrete states and $F(t)$ decreases faster than t^{-2n}, since additional moment superconvergence relations can be used, one will also preserve the prediction of a universal correction to M dominance.

The most familiar application of these considerations is to the ρ-dominance model of the electromagnetic form factors. Taking $A=B=\pi$, one may investigate experimentally the corrections to ρ dominance since the rates $\rho^0\rightarrow\mu^+\mu^-$ and $\rho^0\rightarrow\pi^+\pi^-$ are, respectively, proportional to $f_\rho{}^{-2}$ and $g_{\rho\pi\pi}{}^2$.[3] With a ρ^0 width of 130 ± 10 MeV [4] and a mass of 770 MeV, one derives $g_{\rho\pi\pi}{}^2/4\pi=2.50\pm0.20$, while from the leptonic branching ratios of 6.5 ± 1.4,[5] 5.1 ± 1.2,[6] and 9.7 ± 2.0 [7] (all$\times10^{-5}$) one finds $f_\rho{}^2/4\pi=1.6\pm0.4$, 2.1 ± 0.5, and 1.1 ± 0.2. Thus there are definite indications of a deviation from the universality result $g_{\rho\pi\pi}=f_\rho$, and these are consistent with $m_C/m_\rho\approx1.6$ to 4. When accurate data on $e^+e^-\rightarrow\pi^+\pi^-$ are available, $C(t)$ will be determined directly and a check can be made of any model assumed for it. Since the experimental evidence leads to a correction of ρ dominance such that $g_{\rho\pi\pi}f_\rho{}^{-1}>1$, one has an indication that the simple model for $C(t)$ of a ρ' meson at mass $m_{\rho'}=m_C$ is feasible.

One would expect to see such a contribution in the observed nucleon form factors. The empirical nucleon form-factor dependence of $(1-t/0.71\ \mathrm{BeV}^2)^{-2}$ can be reproduced by a ρ pole at 760 MeV and a ρ' at 930 MeV using the superconvergence constraint. This ρ' cannot be a sharp resonance since no experimental evidence[5–7]

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exists for any ρ' meson below the conjectured ρ' at 1632 MeV.[8] However, an effective contribution in this mass region is suggested by both the form factor and leptonic decay data.

The ρ' model for $C(t)$ leads to a universal correction to ρ dominance, so that ratios between strong ρ couplings will be the same as for exact ρ dominance, however the ρ-nucleon couplings are not known sufficiently well[3] to test this prediction critically. The strong couplings of the ρ' are not predicted on our model since only the product with $f_{\rho'}$ is specified by the superconvergence condition.

The assumption that the matrix element of the divergence of the axial current satisfies an unsubtracted dispersion relation has been exploited by Nambu[9] in order to derive the Goldberger-Treiman relation. In keeping with the spirit of superconvergence, we shall investigate the consequences of this matrix element decreasing faster than t^{-1} so that there is an additional constraint. Then the π pole must be accompanied by some contribution of mass $>3m_\pi$. No evidence for a π' resonance exists but the only property we require is an effective mass of the contribution.

The dispersion relation for the divergence of the axial current, taken between baryons A and B, is

$$(m_A+m_B)G^A(t)-tG^P(t) = \frac{\sqrt{2}c_\pi g_{A\pi B}m_\pi^2}{m_\pi^2-t}+\int_{9m_\pi^2}^{\infty}\frac{t'C(t')dt'}{t'-t}, \quad (6)$$

where G^A and G^P are the axial and induced pesudoscalar form factors. Then the correction to the Goldberger-Treiman relation will be given by $(m_A+m_B)\times G^A(0)=\sqrt{2}c_\pi g_{A\pi B}(1-m_\pi^2/m_{\pi'}^2)$. The lifetime 26.08 $\pm$0.15 nsec[10] of $\pi\rightarrow\mu\nu$ together with $G^V=(1.403\pm0.008)\times10^{-49}$ erg cm^3 [11] gives $|c_\pi|=92.7$ MeV with error of 1% due to uncertainties in radiative corrections. With $A=B=$nucleon; $G^A=1.18\pm0.02$ [12] and $g_{\pi^0NN}^2/4\pi=14.7\pm0.3$,[13] which lead to $c_\pi=81.4\pm2.4$ MeV if $m_\pi/m_{\pi'}=0$. This 14% discrepancy is in the direction predicted by our considerations and may be reconciled if $m_{\pi'}/m_\pi=2.9\pm0.5$, which is near to the start of the 3π cut. The agreement of direction and order of magnitude of the correction supports the conjecture of a universal correction to PDDAC and this is the attitude implicit in most current algebra calculations which use the value of c_π obtained from the Goldberger-Treiman relation. In particular, the Adler-Weisberger relation would be expected to be exact with Weisberger's evaluation[14] (which leads to $G^A=1.16$), if the continuim is dominated by resonances, so that one can apply PDDAC to the couplings $\pi N\rightarrow N^*$ which will all be corrected by the same universal factor. One direct application is to predict $g_{\pi\Sigma\Lambda}^2=11.8$ from the direct experimental value[15] $G_{\Sigma\Lambda}^A=-0.93\pm0.09$. This value is consistent with predictions from superconvergence relations of 10 ± 3.[16] We also predict the correction to G_{NN}^P, as measured in μ capture, and this turns out to be a reduction of $4\frac{1}{2}\%$ on the uncorrected value, although such precision has not been obtained experimentally.

Evidence of a possible π' contribution in strong interactions has been found by Bugg[17] using nucleon-nucleon dispersion relations. With a mass of $3m_\pi$ he finds $g_{\pi'NN}^2/4\pi\approx4$ which, together with our superconvergence constraint, determines $c_{\pi'}/c_\pi\approx-0.2$. This π' weak coupling would only contribute a 5% correction to the sum rule of Weinberg.[18]

Kaon pole dominance of the divergence of the strangeness changing axial current may be approached in the same manner. Since the direct experimental measurement of $G_{\Lambda p}^A=(-1.14_{-0.33}^{+0.23})G_{\Lambda p}^V$ [19] is very approximate, we use Cabibbo theory for the hyperon decays which leads to $G_{\Lambda p}^A=(-0.218\pm0.007)G_{NN}^V$.[20] Then, since $c_K=0.273c_\pi$ from $K\rightarrow\mu\nu$ one finds $g_{NK\Lambda}^2/4\pi=(12.6\pm1.0)(1-m_K^2/m_{K'}^2)^{-2}$, while recent KN forward dispersion relation analyses give estimates for this coupling of 6.0 ± 2.0,[21] 7.4 ± 1.2,[22] and 16.0 ± 2.5.[23] The systematic differences between different groups are due mainly to different parametrizations of the low-energy and unphysical regions. The ΣN matrix element similarly leads to $g_{NK\Sigma}^2/4\pi=(1.6\pm0.4)(1-m_K^2/m_{K'}^2)^{-2}$ while Zovko[24] predicts 2.1 ± 0.9. From these data one cannot definitely assert that the δ-function K' correction model, which requires $m_{K'}/m_K>1.56$, is insufficient but this seems to be indicated unless Kim's result is substantiated or $G_{\Lambda p}^A$ is much smaller.

It is a pleasure to thank Dr. P. K. Kabir, Dr. S. Y. Lo, and Dr. G. Shaw for helpful discussion.

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