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Superconvergence Sum Rules for Meson-Meson Scattering

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Superconvergence sum rules have been obtained for pseudoscalar mesons scattered by spin-2^+ mesons. Results for A_2-π and K^{**}-π scattering have been compared with experiments. In general, saturation by low-lying single-particle intermediate states is found to be poor.

I. INTRODUCTION

In a scattering process involving high-spin particles, the kinematical structure of the physical amplitude in terms of invariant amplitudes is such that unitarity, coupled with the assumption of a constant shape of the diffraction peak, demands that some of these invariant amplitudes be superconvergent, i.e., they vanish faster than $1/s$ at large s and fixed momentum transfer t, where s is the energy variable. A fixed-t dispersion relation for superconvergent amplitude, free of kinematical s singularities, allows a superconvergence sum rule (SC rule) which can be used to obtain interesting physical results. In particular, the assumption of saturation of the resulting dispersion integral by a few low-lying single-particle intermediate states gives relations among various coupling constants and masses. de Alfaro *et al.*[1] have obtained bounds for the invariant amplitude of ρ-π scattering from the Regge-pole model, and have also constructed SC rules.[2] In particular, by assuming that these sum rules at $t=0$ are saturated by π, ω, and φ intermediate states, they obtain interesting results,[3] viz., $g_{\rho\varphi\pi}{}^2=0$ and $g_{\rho\omega\pi}{}^2=4g_{\rho\pi\pi}{}^2/m_\rho{}^2$. This technique of extracting physical information from superconvergent invariant amplitudes is very attractive and has recently been the subject of widespread study.

In the present paper, we outline in Sec. II the unitarity arguments necessary to obtain bounds on the invariant amplitudes of a two-body scattering process. In Sec. III, the elastic scattering of pseudoscalar mesons by 2^+ mesons has been considered. The high-energy behavior of the various invariant amplitudes occurring in this process has been studied by using (1) unitarity arguments along with the assumption of a constant shape of diffraction peak, and (2) Regge-pole phenomenology. Possible SC rules from fixed-t dispersion relations for superconvergent amplitudes have explicitly been written down. In particular, unitarity bounds permit three SC rules for K^{**}-π scattering and five SC rules for A_2-π scattering. Regge-pole bounds are stronger, and permit seven SC rules for K^{**}-π scattering and nine sum rules for A_2-π scattering. It is found that pseudoscalar and vector mesons, in general, fail to saturate SC rules. However, the SC rule for the amplitude having strongest convergence seems to be fairly saturated by 0^- and 1^- mesons for both K^{**}-π and A_2-π scattering. The sum rules obtained for K^{**}-π scattering from unitarity bounds when saturated with K and K^* only, give good results. Further, it is argued that consistent solutions to all the sum rules could be obtained if an infinite number of intermediate particle states is used for saturation.

[1] V. de Alfaro, S. Fubini, G. Rossetti, and G. Furlan, Phys. Letters **21**, 576 (1966). This paper will be referred to as AFRF.

[2] AFRF assume that the Regge parameter $\alpha(0)$ is negative for the isotopic-spin-2 channel. This enables them to show that their B^2 amplitude is superconvergent. But the presence of Regge cuts might not allow such superconvergence, because the $I=2$ trajectory will have the leading edge at $\alpha_\rho+\alpha_\rho-1$, which may not be negative. See I. J. Muzinich, Phys. Rev. Letters **18**, 381 (1967); R. J. N. Phillips, Phys. Letters **24B**, 342 (1967).

[3] If AFRF had considered the sum rule $\int\nu\,\mathrm{Im}A^2(\nu)d\nu=0$, they would have obtained the trivial result $g_{\rho\pi\pi}=g_{\rho\varphi\pi}=g_{\rho\omega\pi}=0$.

II. UNITARY REQUIREMENTS

The S matrix for the two-body scattering process $A+B\to C+D$ is defined by

$$S=1-i(2\pi)^4(p_0k_0p_0'k_0')^{-1/2}\delta(p+k-p'-k')T\,, \quad (1)$$

where T is the reaction matrix, p and k are the four-momenta of the initial particles A and B, and k' and p' are those of the final particles C and D. The usual Mandelstam variables s, u, and t are defined by

$$s=(p+k)^2,\quad u=(p-k')^2,\quad t=(p-p')^2. \quad (2)$$

The differential cross section in the c.m. system is given by

$$\frac{d\sigma}{d\Omega}=\frac{K}{s}\sum_{\text{spins}}|T|^2,$$

where, for fixed t, K is a constant in the limit $s\to\infty$. So, for large s and fixed t, we have

$$\int\sum_{\text{spins}}|T|^2dt<\text{const}\times\sigma^{\text{tot}}s^2. \quad (3)$$

Inequality (3) is just the requirement of unitarity and can be used for obtaining bounds on the invariant amplitudes for the process $A+B\to C+D$. AFRF have used this inequality along with the assumption of a constant shape of the diffraction peak for obtaining bounds on the invariant amplitudes of ρ-π scattering. We shall use similar arguments for obtaining unitarity bounds for the invariant amplitudes of 0^--2^+ elastic scattering.

III. ELASTIC SCATTERING OF PSEUDOSCALAR AND TENSOR MESONS

A. Invariant Amplitudes

From the invariance arguments, one can easily see that there will be nine independent helicity amplitudes for 0^--2^+–meson elastic scattering. A simple analysis enables us to write the following decomposition of the T matrix:

$$T=\sum_{i=1}^{9} A_i(s,u,t) I_i^{\mu\nu\lambda\rho} e_{\mu\nu} e_{\lambda\rho}'^{\dagger}, \qquad (4)$$

where $e_{\mu\nu}$ and $e_{\lambda\rho}'$ are the polarization tensors of initial and final spin-2 mesons.[4] The nine invariant amplitudes $A_i(s,u,t)$ are the scalar functions of the usual Mandelstam variables s, u, and t defined by Eq. (2). Now k and k' stand for the four-momenta of the initial and final pseudoscalar mesons, and p and p' for the four-momenta of the corresponding 2^+ mesons. $I_i^{\mu\nu\lambda\rho}$ are the Lorentz covariants constructed from the available four-momenta (k, k', p, and p') and the covariant tensor $g^{\mu\nu}$, and are as defined below:

$$\begin{aligned}
I_1^{\mu\nu\lambda\rho}&=P^\mu P^\nu P^\lambda P^\rho,\\
I_2^{\mu\nu\lambda\rho}&=R^\mu R^\nu R^\lambda R^\rho,\\
I_3^{\mu\nu\lambda\rho}&=P^\mu g^{\nu\lambda} R^\rho+R^\mu g^{\nu\lambda} P^\rho,\\
I_4^{\mu\nu\lambda\rho}&=P^\mu g^{\nu\lambda} P^\rho,\\
I_5^{\mu\nu\lambda\rho}&=R^\mu g^{\nu\lambda} R^\rho,\\
I_6^{\mu\nu\lambda\rho}&=P^\mu R^\nu P^\lambda R^\rho,\\
I_7^{\mu\nu\lambda\rho}&=P^\mu R^\nu R^\lambda R^\rho+R^\mu R^\nu R^\lambda P^\rho,\\
I_8^{\mu\nu\lambda\rho}&=P^\mu P^\nu P^\lambda R^\rho+R^\mu P^\nu P^\lambda P^\rho,\\
I_9^{\mu\nu\lambda\rho}&=g^{\nu\lambda} g^{\mu\rho},
\end{aligned} \qquad (5)$$

where

$$P=\tfrac{1}{2}(p+p') \quad\text{and}\quad R=\tfrac{1}{2}(k+k').$$

Note that such a choice of Lorentz covariants does not introduce any kinematical s or t singularity in an obvious manner because of the following identity[5]:

$$\begin{aligned}
\tfrac{1}{4}Atg^{\mu\nu}g^{\lambda\rho}
&=\tfrac{1}{4}Atg^{\nu\lambda}g^{\mu\rho}+2t(4m^2-t)R^\mu g^{\nu\lambda}R^\rho\\
&\quad+2[t(4m_\pi^2-t)+A]P^\mu g^{\nu\lambda}P^\rho+16(2m^2-t)P^\mu R^\nu P^\lambda R^\rho\\
&\quad-2\nu t[P^\mu g^{\nu\lambda}R^\rho+P^\rho g^{\nu\lambda}R^\mu]\\
&\quad-16\nu(P^\mu P^\nu P^\lambda R^\rho+R^\mu P^\nu P^\lambda P^\rho)\\
&\quad+16m^2(P^\mu P^\nu R^\lambda R^\rho+R^\mu R^\nu P^\lambda P^\rho)\\
&\qquad+16(4m_\pi^2-t)P^\mu P^\nu P^\lambda P^\rho=0,
\end{aligned} \qquad (6)$$

where

$$A=\nu^2-(t-4m_\pi^2)(t-4m^2)$$

and

$$\nu=s-u.$$

It is easy to see that the amplitudes A_3, A_7, and A_8 with symmetric combinations of isotopic spin in the t channel, and the others with antisymmetric combinations, will be odd under crossing from the s to the u channel.

B. Unitarity Bounds

Using the requirement of unitarity, Eq. (3), along with the assumption of the constant shape of the diffraction peak (under this assumption, the total cross section at high energies will be constant), for large s and at $t=0$, we obtain the following bounds for $A_i(s,0)$:

$$\begin{aligned}
|A_2(s,0)|&<\text{const}\times s^{-3},\\
|A_7(s,0)|&<\text{const}\times s^{-2},\\
|A_{5,6}(s,0)|&<\text{const}\times s^{-1},\\
|A_{3,8}(s,0)|&<\text{const},\\
|A_{1,4,9}(s,0)|&<\text{const}\times s.
\end{aligned} \qquad (7)$$

We have used heuristic arguments for obtaining the bounds (7). However, we hope that as in the spinless case, systematic application of unitarity will give similar results apart from logarithmic factors on the right-hand side. In view of this, we note that $A_2(s,0)$, $sA_2(s,0)$, and $A_7(s,0)$ are superconvergent.

C. Regge Behavior

For obtaining SC rules from fixed-t dispersion relations, we must consider the amplitudes which are free of kinematic s singularities. The problem of kinematic-singularitity-free amplitudes has been studied in a general manner,[6] although the procedure involved is somewhat cumbersome. Hara and Wang[7] have shown that the amplitude

$$A_{\lambda_c\lambda_b;\,\lambda_a\lambda_d}=F_{\lambda_c\lambda_b;\,\lambda_a\lambda_d}/(\cos\tfrac{1}{2}\theta)^{|\lambda+\mu|}(\sin\tfrac{1}{2}\theta)^{|\lambda-\mu|} \qquad (8)$$

is free of kinematical singularities in s, where $F_{\lambda_c\lambda_b;\,\lambda_a\lambda_d}$

[4] For the spin-2 polarization tensor $e_{\mu\nu}$, we have $e_{\mu\nu}=e_{\nu\mu}$, $e_\mu{}^\mu=0$, $e_{\mu\nu}p^\nu=0$; p^ν is the four-momentum of the corresponding spin-2 particle.

[5] After the completion of our investigations, we received a copy of work by N. J. Papastamatiou and S. Pakvasa [Phys. Rev. **161**, 1554 (1967)] in which they have independently noted a similar identity and hence the choice of Lorentz covariants. However, in their identity, a few numerical factors are incorrect [for this, compare our Eq. (6) with their expression for $tA\delta_{\mu\rho}\delta_{\nu\sigma}$].

[6] D. Hall and A. S. Wightman, Kgl. Danske Videnskab. Selskab, Mat.-Fys. Medd. **31**, No. 5 (1957). For a more recent discussion of the problem, see also G. C. Fox, Phys. Rev. **157**, 1493 (1967), and the references quoted therein.

[7] Y. Hara, Phys. Rev. **136**, B507 (1964); L.-L C. Wang, *ibid.* **142**, 1187 (1966). See also T. L. Trueman, Phys. Rev. Letters **17**, 1198 (1966).

is the t-channel helicity amplitude, θ is the scattering angle in the t channel, $\lambda=\lambda_a-\lambda_d$, and $\mu=\lambda_c-\lambda_b$.

For large s and fixed t,[8]

$$|A_{\lambda_c\lambda_b;\,\lambda_a\lambda_d}|\sim\text{const}\times F_{\lambda_c\lambda_b;\,\lambda_a\lambda_d}/s^{n(\lambda,\mu)}\,, \tag{9}$$

where $n(\lambda,\mu)$ is the maximum between λ and μ. The factor $s^{n(\lambda,\mu)}$ in Eq. (9) is responsible for the better convergence of some amplitudes, since all F's behave in the same way for large s and fixed t. In the Appendix, we have given the relations between various $A_{\lambda_c\lambda_b;\,\lambda_a\lambda_d}$ (or B_i) and the invariant amplitudes $A_i^{(s,u,t)}$. Following standard Reggeization procedure,[9] it is straightforward to obtain the following asymptotic behavior for the kinematical s-singularity-free amplitudes B_i (or $A_{\lambda_c\lambda_b;\,\lambda_a\lambda_d}$):

$$B_i\sim\textstyle\sum_j \gamma_{ij}(t)s^{\alpha_{ij}(t)-\Delta_i}\,, \tag{10}$$

where Δ_i is the helicity change ($\Delta_1=4$, $\Delta_2=3$, $\Delta_3=\Delta_6=2$, $\Delta_4=\Delta_7=1$, $\Delta_5=\Delta_8=\Delta_9=0$), and $\gamma_{ij}(t)$ are the Regge residues which are functions of t alone. $\alpha_{ij}(t)$ are the usual Regge parameters ($0\leq\alpha_{ij}(t)\leq1$) depending upon the internal quantum numbers of the trajectory. To this end, we remark that for A_2-π scattering, the $I=2$ trajectory does not necessarily demand $\alpha_{ij}<0$.[10] In fact, if Regge cuts are present, then it may be that $\alpha_{ij}\geq0$.[11]

D. Superconvergence Sum Rules (SC Rules)

It is easy to see that for a superconvergent amplitude $B(\nu,t)$ which does not have any kinematical s singularity and is odd under crossing from s to u channel,[12] a sum rule of the type

$$\int_0^\infty \text{Im}B(\nu,t)d\nu=0\,, \tag{11}$$

called a superconvergence sum rule, would follow from the fixed-t dispersion relation. In the preceding sections, we have studied the high-energy bounds of the amplitudes from unitarity arguments and Regge-pole phenomenology. Certainly, unitarity bounds are more reliable, and we expect three sum rules for K^{**}-π scattering, namely,

$$\int_0^\infty \text{Im}A_2^1(\nu,t)d\nu=0\,, \tag{12a}$$

$$\int_0^\infty \nu\,\text{Im}A_2^0(\nu,t)d\nu=0\,, \tag{12b}$$

and

$$\int_0^\infty \text{Im}A_7^0(\nu,t)d\nu=0\,, \tag{12c}$$

where the superscript on the invariant amplitudes $A_i(\nu,t)$ refers to the isotopic spin in the t channel. For A_2-π scattering, in addition to these three SC rules, we have two more sum rules, namely,

$$\int_0^\infty \nu\,\text{Im}A_2^2(\nu,t)d\nu=0 \tag{12d}$$

and

$$\int_0^\infty \text{Im}A_7^2(\nu,t)d\nu=0\,. \tag{12e}$$

These sum rules have been written down for $t=0$ only.

From Regge-pole high-energy behavior and fixed-t dispersion relations for the superconvergent amplitudes $\nu^\alpha B_i^T(\nu,t)$, where α is a positive integer (including zero) and T the isotopic spin in the t channel, we have seven sum rules for K^{**}-π scattering, namely,

$$\begin{aligned}
&\int_0^\infty \text{Im}B_1^1(\nu,t)d\nu=0\,,\\
&\int_0^\infty \nu\,\text{Im}B_1^1(\nu,t)d\nu=0\,, \quad \int_0^\infty \text{Im}B_2^0(\nu,t)d\nu=0\,,\\
&\int_0^\infty \nu^2\,\text{Im}B_1^1(\nu,t)d\nu=0\,, \quad \int_0^\infty \nu\,\text{Im}B_2^1(\nu,t)d\nu=0\,,\\
&\int_0^\infty \text{Im}B_3^1(\nu,t)d\nu=0\,, \quad \int_0^\infty \text{Im}B_6^1(\nu,t)d\nu=0\,.
\end{aligned} \tag{13a}$$

For A_2-π scattering, besides the above, we have two more, namely,

$$\int_0^\infty \nu\,\text{Im}B_1^2(\nu,t)d\nu=0\,, \quad \int_0^\infty \text{Im}B_2^2(\nu,t)d\nu=0\,. \tag{13b}$$

Further, for A_2-π scattering, if the $I=2$ trajectory has $\alpha_{ij}(t)<0$ for some range of t, then six more sum rules would follow, namely,

$$\begin{aligned}
&\int_0^\infty \nu^3\,\text{Im}B_1^2(\nu,t)d\nu=0\,, \quad \int_0^\infty \nu^2\,\text{Im}B_2^2(\nu,t)d\nu=0\,,\\
&\int_0^\infty \nu\,\text{Im}B_3^2(\nu,t)d\nu=0\,, \quad \int_0^\infty \nu\,\text{Im}B_6^2(\nu,t)d\nu=0\,,\\
&\int_0^\infty \text{Im}B_4^2(\nu,t)d\nu=0\,, \quad \int_0^\infty \text{Im}B_7^2(\nu,t)d\nu=0\,.
\end{aligned} \tag{14}$$

E. Saturation

It would be hard to expect to get saturation of all SC rules by a few one-particle intermediate states. Further, it is very difficult to see how one can obtain a solution for various values of t with only single-particle states. In spite of these well-known difficulties, we discuss here

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[8] For large s and at fixed t, $\cos\theta\to s$, so the denominator of Eq. (8) goes as $s^{\frac12|\lambda+\mu|+\frac12|\lambda-\mu|}$.

[9] M. Gell-Mann, M. Goldberger, F. E. Low, E. Marx, and F. Zachariasen, Phys. Rev. **133**, B145 (1964).

[10] In many recent papers (see, e.g., Ref. 1), an assumption $\alpha_i<0$ for $I=2$ trajectory has been made. If Regge cuts are present, appropriate care should be taken.

[11] I am thankful to Professor V. N. Gribov for a discussion on this point.

[12] If the amplitude is even under crossing symmetry [i.e., $B(s,u,t)=B(u,s,t)$], then sum rule (11) would be trivially satisfied.

the various SC rules independently. Sum rules obtained from unitarity bounds, Eqs. (12a)–(12e), are more reliable, and if we assume that they are saturated by 0^- and 1^- mesons, we obtain the following results for K^{**}-π scattering:

$$g_{K^{**}K\pi}^2 - m_{K^{**}}^2 g_{K^{**}K^*\pi}^2 = 0\,, \quad (15a)$$

$$\nu_K g_{K^{**}K\pi}^2 - \nu_{K^*} m_{K^{**}}^2 g_{K^{**}K^*\pi}^2 = 0\,, \quad (15b)$$

$$g_{K^{**}K\pi}^2 - \tfrac{1}{2}(m_{K^{**}}^2 - m_{K^*}^2 + m_\pi^2) g_{K^{**}K^*\pi}^2 = 0\,, \quad (15c)$$

where

$$\nu_\alpha = (m_\alpha^2 - m_{K^{**}}^2 - m_\pi^2)\,.$$

It is straightforward to write all of the five sum rules, Eqs. (12a)–(12e), for A_2-π scattering. For the sake of clarity, below we write down for Eqs. (12a) and (12e), respectively,

$$g_{A_2\eta\pi}^2 + g_{A_2X^0\pi}^2 - m_{A_2}^2 g_{A_2\rho\pi}^2 = 0\,, \quad (16a)$$

$$g_{A_2\eta\pi}^2 + g_{A_2X^0\pi}^2 + \tfrac{1}{2}(m_{A_2}^2 - m_\rho^2 + m_\pi^2) g_{A_2\rho\pi}^2 = 0\,. \quad (16b)$$

Here, the coupling constants $g_{2^+PP'}$ and g_{2^+VP} have been defined through the following interaction Hamiltonians[13]:

$$H_{2^+PP'} = g_{2^+PP'} \Psi_{\mu\nu} \partial^\nu \phi_P \partial^\mu \phi_{P'} + \text{H.c.}$$

and

$$H_{2^+VP} = g_{2^+VP} \Psi_{\mu\alpha} \partial_\nu V_\lambda \partial_\sigma \partial^\alpha \phi_P \epsilon^{\mu\nu\lambda\sigma} + \text{H.c.}\,,$$

where $\phi_{P,P'}$, V_λ, and $\Psi_{\mu\nu}$ are the wave functions for $J^P = 0^-$, 1^-, and 2^+ mesons, respectively.

It is interesting that the sum rules (15a)–(15c) predict the ratio of decay widths $\Gamma(K^{**} \to K^* + \pi)/\Gamma(K^{**} \to K + \pi)$ as 0.19, 0.28, and 0.64, respectively. The experimental value for this ratio, as reported at the Berkeley Conference,[14] is 1.10 ± 0.86. For A_2-π scattering, the sum rule (16a) gives[15]

$$\Gamma(A_2 \to \pi + \eta)/\Gamma(A_2 \to \pi + \rho) \simeq 0.22\,,$$

which is to be compared with the experimental value[16] 0.12 ± 0.08. However, the sum rules (12d) and (12e) are badly violated. Inclusion of the 1^+ meson does not improve the situation. The other two sum rules for A_2-π scattering when saturated with 0^- and 1^- single-particle states only give reasonably good results.

The saturation of the sum rules obtained from Regge-pole phenomenological bounds, in general, is poor. Further, a finite number of intermediate single-particle states cannot saturate the SC rules for the full range of t. However, an infinitely large number of single-particle intermediate states could give nontrivial solutions. We hope that further understanding of the saturation problem of SC rules will give us a good way of understanding strong interactions.

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APPENDIX

We give here the explicit expressions of the amplitudes $A_{\lambda_c\lambda_b;\,\lambda_a\lambda_d}$ free of kinematical s singularities, in terms of the invariant amplitudes $A_i(s,u,t)$. Explicit wave functions for spin-2 particles for possible helicity states have been given by Gotsman and Frishman.[17] We will work in the c.m. system of the t channel. Let p and q be the magnitudes of c.m. momentum of 2^+ and 0^- mesons, respectively. It is easy to see that $4pq\cos\theta = u - s$, $p = (\frac{1}{4}t - m^2)^{1/2}$, $q = (\frac{1}{4}t - m_\pi^2)^{1/2}$, and $\omega = (\frac{1}{4}t)^{1/2}$, where θ is the scattering angle, and ω is the meson energy. It is straightforward to obtain the following relations:

$$B_1(s,t) = A_{00;\,2-2} = 4q^4 A_2\,,$$

$$B_2(s,t) = mA_{00;\,2-1} = q^3\omega(4q\cos\theta A_2 + 2pA_7)\,,$$

$$B_3(s,t) = (\sqrt{\tfrac{3}{2}})A_{00;\,20} = q^2[aq^2 A_2 + A_5 - (\nu\omega^2/2m^2)A_7]\,,$$

$$B_4(s,t) = (m/\omega q)A_{00;\,21} = q\cos\theta(-q^2\sin\theta A_2 + A_5) + pA_3 - \tfrac{1}{2}pq^2\sin^2\theta A_7\,,$$

$$B_5(s,t) = 4A_{00;\,22} = q^2\sin^2\theta(q^2\sin^2\theta A_2 - 2A_5) + 4A_9\,,$$

$$B_6(s,t) = (m^2/\omega^2 q^2)A_{00;\,1-1} = 4q^2\cos^2\theta A_2 + p^2 A_6 - [(p^2+\omega^2)/\omega^2]A_5 - \nu A_7\,,$$

$$B_7(s,t) = (\sqrt{6})mA_{00;\,10} = 2q^4 a\omega\cos\theta A_2 - \omega q[2(p^2+\omega^2)/m^2 - 1](pA_3 + q\cos\theta A_5) + \omega p q^3(4\omega^2\cos^2\theta/m^2 + a)A_7 + 2(\omega^3 p^2 q/m^2)(q\cos\theta\, A_6 + pA_8)\,,$$

$$B_8(s,t) = 4m^2 A_{00;\,11} = -\omega^2\{q^2\sin^2\theta[4q^2\cos^2\theta A_2 + p^2 A_6 + (m^2/\omega^2)A_5] - 2q^2 A_5 - p^2 A_4 + [4(p^2+\omega^2)/\omega^2]A_9 + \nu(A_3 - q^2\sin^2\theta A_7)\}\,,$$

$$B_9(s,t) = 6A_{00;\,00} = (2\omega^2/m^2)\{(2p^2 A_1 - \nu A_8)\omega^2 p^2/m^2 - [(p^2+\omega^2)/m^2](2p^2 A_4 - \nu A_3 + 2q^2\cos^2\theta A_5) + \tfrac{1}{2}\nu[(\omega^2\nu/4m^2)A_6 - aq^2 A_7]\} + q^4 a^2 A_2 - q^2\sin^2\theta A_5 + 2[2[(\omega^2+p^2)/m^2]^2 - 1]A_9\,,$$

where

$$\nu = s - u$$

and

$$a = -\sin^2\theta + (2\omega^2/m^2)\cos^2\theta\,.$$

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[13] We have suppressed the isotopic-spin structure of the interactions, which can be easily written for the specific cases. The coupling constants $g_{2^+PP'}$ and g_{2^+VP} can be related to the partial decay widths, namely,

$$\Gamma(2^+ \to P + P') = (q^5/60\pi)(g_{2^+PP'}^2/m_{2^+}^2)$$

and

$$\Gamma(2^+ \to V + P) = (q^5/40\pi) g_{2^+VP}^2\,.$$

[14] See G. Goldhaber, in *Proceedings of the Thirteenth Annual International Conference on High-Energy Physics, Berkeley, 1966* (University of California Press, Berkeley, Calif., 1967), p. 103.

[15] To estimate the coupling constant $g_{A_2X^0\pi}$, we use the nonet model [see, e.g., R. J. Rivers, Phys. Rev. **150**, 1256 (1966)], i.e., taking octet and singlet together as a nonet $P_9 = P_8 + (1/\sqrt{3})X_u 1$, where P_8 is the 0^- octet in which η^0 has been replaced by $\eta^0\cos\theta - X^0\sin\theta$, and $X_u = \eta^0\sin\theta + X^0\cos\theta$. Here θ is the mixing angle, which is known to be small ($\simeq 10^0$).

[16] S. U. Chung, O. I. Dahl, L. M. Hardy, R. I. Hess, J. Kirz, and D. H. Miller, Phys. Rev. Letters **18**, 100 (1967).

[17] Y. Frishman and E. Gotsman, Phys. Rev. **140**, B1151 (1965).