Observation of Quantized Circulation in Superfluid Helium

S. C. WHITMORE* AND W. ZIMMERMANN, JR.
Tate Laboratory of Physics, University of Minnesota, Minneapolis, Minnesota
(Received 28 August 1967)

An experimental study has been made of the circulation of the superfluid component of liquid helium II around a fine wire. The method used was that of Vinen, in which the circulation is measured by means of the influence it has on the transverse vibrations of the wire. The present work extends Vinen's original work in several ways. Measurements were made over a range of temperatures from 1.2 to 1.9°K and a range of wire diameters from 25 to 100 μ. In addition, measurements of the direction of the apparent circulation were made as well as of the magnitude. The experiments have two principal results. The first is that motion of the superfluid around the wire could persist for many hours after any motion of the containing vessel had stopped. However, this motion was not steady; smooth changes in apparent circulation could take place throughout an experimental run, changes which could include reversals in direction. The second principal result is that the apparent circulation tended to show markedly greater stability at the anticipated quantum levels than at other values. Long periods of stability were observed at the levels of 0, 1, 2, and 3 quantum units. An additional significant result is that as the wire diameter was increased the maximum value of metastable circulation observed also increased.

I. INTRODUCTION

THE picture of the flow properties of liquid helium II that is now widely accepted is based on the two-fluid-model equations proposed by Landau[1] and on the proposals of Onsager[2] and Feynman[3] regarding quantization of superfluid circulation and the existence of quantum superfluid vortices. In Landau's model it was proposed that the condition

$$\text{curl}\,\mathbf{v}_s=0\,, \tag{1}$$

where $\mathbf{v}_s$ is the velocity of the superfluid component, holds everywhere in the liquid. Onsager and Feynman proposed the additional restriction on $\mathbf{v}_s$ that in a multiply connected region the circulation κ of the superfluid around any closed path in the liquid takes on only the values given by the quantum condition

$$\kappa\equiv\oint \mathbf{v}_s\cdot d\mathbf{r}=nh/m\,, \tag{2}$$

where n is an integer, h is Planck's constant, and m is the mass of the helium atom.

Onsager and Feynman suggested also that quantum vortices can exist in the superfluid component. They proposed that in general the circulation κ around such a vortex satisfies Eq. (2), although it seems likely that only vortices with $n=\pm1$ will ordinarily be found. The cores of these vortices represent filaments running through the liquid along which curl $\mathbf{v}_s$ is singular, and the existence of such vortices requires some modification of Landau's original equations.

More recently it has been proposed[4] that quantization of superfluid circulation stems from the existence in helium II of a complex single-valued order parameter $\Psi(\mathbf{r},t)$. The superfluid velocity is given in terms of the phase θ of this order parameter by the relation

$$\mathbf{v}_s=(h/m)\,\text{grad}\,\theta\,, \tag{3}$$

from which both Eqs. (1) and (2) follow.

A variety of experiments have given evidence in support of these ideas about the motion of the superfluid and quantization of circulation. Perhaps the most direct support has come from the vibrating wire experiment of Vinen,[5] the vortex-ring experiment of Rayfield and Reif,[6] and the orifice experiment of Richards and Anderson[7] which was repeated recently by Khorana and Chandrasekhar.[8] Because of the importance of Vinen's experiment in testing the ideas concerning quantization of circulation, the present work was undertaken to repeat and, if possible, extend Vinen's work.[9]

In the present experiment, as in Vinen's, a study was made of the motion of the superfluid in a slender cylindrical vessel along whose axis was stretched a fine wire. The circulation of the superfluid around the wire was measured by means of the influence the circulating fluid has on transverse vibrations of the wire. This type of experiment has the virtue of concerning itself directly with circulation around a solid obstacle, unlike the other experiments bearing on quantization of circulation, which have been concerned with the existence of quantum vortices. However, as will be seen, quantum vor-

---

* Present address: Randall Laboratory, University of Michigan, Ann Arbor, Mich.

[1] L. D. Landau, J. Phys. (USSR) **5**, 71 (1941); **8**, 1 (1944).

[2] L. Onsager, Nuovo Cimento **6**, Suppl. 2, 249 (1949).

[3] R. P. Feynman, in *Progress in Low Temperature Physics*, edited by C. J. Gorter (North-Holland Publishing Co., Amsterdam, 1955), Vol. I, Chap. II.

[4] V. L. Ginzburg and L. P. Pitaevskii, Zh. Eskperim. i Teor. Fiz. **34**, 1240 (1958) [English Trans.: Soviet Phys.—JETP **7**, 858 (1958)].

[5] W. F. Vinen, Proc. Roy. Soc. (London) **A260**, 218 (1961).

[6] G. W. Rayfield and F. Reif, Phys. Rev. **136**, A1194 (1964).

[7] P. L. Richards and P. W. Anderson, Phys. Rev. Letters **14**, 540 (1965).

[8] B. M. Khorana and B. S. Chandrasekhar, Phys. Rev. Letters **18**, 230 (1967).

[9] A preliminary account of the present work was given by S. C. Whitmore and W. Zimmermann, Jr., Phys. Rev. Letters **15**, 389 (1965). A detailed account of the work has been given by S. C. Whitmore, Ph.D. thesis, University of Minnesota, 1966 (unpublished).

tices probably played an essential role in determining our results.

II. MOTION OF THE FLUID IN THE ABSENCE OF WIRE VIBRATION

We begin by considering a simple model for the motion of the helium in the cylinder surrounding the wire in the absence of wire vibration. Although this model will prove to be too simple for an understanding of our results, it provides a helpful starting point. Consider the cylinder and wire to be in rotation at an angular velocity Ω about their axis of cylindrical symmetry at some temperature below the λ temperature T_λ. The equilibrium motion of the normal fluid will be solid-body rotation at the angular velocity of the cylinder and wire. The motion of the superfluid will be characterized by some circulation κ around the wire together with some distribution of quantum vortices in the liquid.

The equilibrium motion of the superfluid in a rotating annulus has been considered by Vinen[5] and by Fetter.[10] As the angular velocity is increased from zero the circulation κ increases in a series of equal quantum steps, $0, h/m, 2h/m, \cdots$, with no vortices present in the liquid until a certain critical value of Ω is achieved. At this critical angular velocity vortices aligned parallel to the wire begin to appear in the liquid, and for further increases in Ω, increases in both κ and the number of vortices occur. Eventually, for large enough values of Ω, the vortex density per unit area becomes uniform and equal to $2\Omega\, m/h$, and κ becomes equal to $2\Omega\pi r_1^2$, where r_1 is the radius of the wire. For any given Ω, the equilibrium array of vortices must just rotate in solid-body fashion at the angular velocity of the cylinder and wire, since any motion of the vortices relative to the normal fluid tends to be dissipated.

Now suppose that the cylinder and wire are brought to rest. The normal fluid will tend to coast to a stop because of its viscosity. The vortex array, supposing that the ends of the vortices are not pinned to the walls, will tend to continue rotating. However, the drift of vortices relative to the normal fluid will result in the dissipation of both the energy and angular momentum of the superfluid. As a consequence it is plausible that the vortices will tend to spiral outward toward the cylinder wall where they will tend to disappear.

If Ω is large enough initially, the flow of the superfluid may be fast enough just after the cylinder and wire are stopped for new vortices to be created in the superfluid at the wire and the walls of the cylinder. It seems likely that such new vortices would also contribute to the dissipation of the superfluid flow. One possibility would be for vortices of the same sign of circulation as κ to form at the wire, move outward through the fluid, and disappear at the wall of the cylinder, thus lowering κ. Another possibility would be for vortices of opposite circulation to form at the cylinder wall and to move inward through the fluid. In so doing they might annihilate with other vortices having the same sign of circulation as κ, or they might eventually reach the wire and disappear, also lowering κ.

However, it is plausible that as κ falls to a low enough value, any vorticity which may be present in the liquid eventually tends to disappear leaving the system in a metastable state with superfluid flow around the wire. If, as seems likely, vortex formation and migration is the only mechanism by which κ can change, the energy barrier against this process may be sufficiently high at small enough values of κ to permit the existence of metastable states with quite long lifetimes. The present experiment was undertaken to search for long-lived metastable quantum circulations of this type.

The question arises of how initially to produce a rotational state in which there is circulation around the wire. It seems likely that this result can be achieved either by bringing the cylinder and wire into rapid enough rotation at a temperature below T_λ or by bringing the entire liquid into solid-body rotation above T_λ by means of its viscosity and then cooling it through T_λ in steady rotation.

III. VIBRATION OF THE WIRE

A. Fluid Motion and Forces on the Wire

In this section we consider the influence that the fluid surrounding the wire has on the transverse vibrations of the wire, the influence which enables the superfluid circulation κ to be detected and measured. We assume for sufficiently small amplitudes of wire displacement and velocity that κ remains constant and that $\mathbf{v}_s$ remains curl-free. In this sense κ is assumed to be an adiabatic invariant during the wire motion. In terms of the model discussed in Sec. II these assumptions are equivalent to the assumption that small enough wire motions are insufficient to create vortices in the superfluid.

The forces exerted by the liquid on the vibrating wire may be analyzed in terms of the two-fluid model. We begin by assuming that the motions of the two components occur with constant total density ρ and constant entropy density ρs, where s is the entropy per unit mass. The basis of this assumption is that the velocities of both fluid components in this experiment remained much less than the speeds of first and second sound and that the times required for first and second sound to traverse the cylinder were short compared to the periods of vibration. Under the condition of constant ρ and ρs it can be shown that $\operatorname{div} \mathbf{v}_s = 0$ and $\operatorname{div} \mathbf{v}_n = 0$, where $\mathbf{v}_n$ is the velocity of the normal component.

Under these circumstances the problem of finding the force exerted by the fluid on the wire breaks up into two separate incompressible-flow problems, each one of which has a form familiar in classical fluid mechanics.[11]

[10] A. L. Fetter, Phys. Rev. **152**, 183 (1966); **153**, 285 (1967).

[11] L. D. Landau, J. Phys. (USSR) **8**, 1 (1944); L. D. Landau and E. M. Lifshitz, *Fluid Mechanics* (Pergamon Press, Ltd., London, 1959), Chap. XVI.

The motion of the superfluid component is just that of an ideal incompressible classical fluid of the same density as the superfluid component subject to the same boundary conditions. The motion of the normal component is that of a viscous incompressible classical fluid of the same density and viscosity as the normal component subject to the same boundary conditions. Furthermore, the net force per unit length **f** exerted by the fluid on the wire is just the sum of two terms $\mathbf{f}_s$ and $\mathbf{f}_n$, where $\mathbf{f}_s$ is the force per unit length calculated in the classical ideal fluid problem referred to above and $\mathbf{f}_n$ is the force per unit length calculated in the viscous fluid problem.

The classical problem of the force acting on a vibrating wire stretched down the axis of a circular cylinder filled with fluid is difficult to solve exactly. However, in the present case the amplitude of vibration was of the order of a factor of 10^{-4} smaller than the wavelength of vibration. It thus seems reasonable that in this case a good approximation is to assume that the force per unit length acting at each point along the wire was the same as it would have been for a straight cylinder having the same cross section as the wire moving in the same way as the wire did at that point.

The two-dimensional problem of a circular cylinder of radius r_1 moving arbitrarily in an unbounded ideal fluid, assuming the fluid to be at rest at infinity, has been discussed by Lamb.[12] For a fluid of density ρ_s, the density of the superfluid component, with circulation κ around the cylinder, the force per unit length acting on the cylinder is given by the expression

$$\mathbf{f}_s = -\mu_s d\mathbf{u}/dt + \rho_s \boldsymbol{\kappa}\times\mathbf{u}\,, \tag{4}$$

where $\mu_s = \rho_s \pi r_1^2$, **u** is the instantaneous velocity of the cylinder, and $\boldsymbol{\kappa}$ is a vector of magnitude κ which lies parallel to the axis of the cylinder and whose direction is related to the sense of the fluid circulation by the right-hand rule. The first term on the right represents the contribution by the fluid of an effective mass per unit length μ_s to the cylinder, and the second term represents a Magnus or "lift" force which acts perpendicular to the axis of the cylinder and to **u**.

The two-dimensional problem of a circular cylinder of radius r_1 vibrating in an unbounded viscous fluid, assuming the fluid to be at rest at infinity, has been treated by Stokes.[13] For a fluid of density ρ_n, the density of the normal component, and viscosity η_n, the viscosity of the normal component, the force per unit length in the small-amplitude limit is given by the expression

$$\mathbf{f}_n = -\mu_n k d\mathbf{u}/dt - \mu_n \omega k' \mathbf{u}\,, \tag{5}$$

where $\mu_n = \rho_n \pi r_1^2$, **u** is the velocity of the cylinder having the form $\mathbf{u}_0 \cos\omega t$, ω is the angular frequency of vibration, and k and k' are dimensionless quantities given by the expressions

$$k = 1 - \frac{4}{\beta}\mathrm{Im}\left[\frac{i^{1/2}H_1^{(1)}(i^{1/2}\beta)}{H_0^{(1)}(i^{1/2}\beta)}\right] \tag{6}$$

and

$$k' = \frac{4}{\beta}\mathrm{Re}\left[\frac{i^{1/2}H_1^{(1)}(i^{1/2}\beta)}{H_0^{(1)}(i^{1/2}\beta)}\right], \tag{7}$$

where $\beta = r_1(\omega\rho_n/\eta_n)^{1/2}$, and $H_0^{(1)}$ and $H^{(1)}$ are Hankel functions of the first kind.[14]

The first term on the right of Eq. (5) represents the contribution of an effective mass per unit length $\mu_n k$ to the cylinder, and the second represents a linear damping force. The solution given by Eqs. (5), (6), and (7) is a small-amplitude solution in the sense that it is valid only when the convection terms in the vorticity equation for the fluid can be neglected. These terms are quadratic in the fluid velocity.

If the presence of the outer cylinder is taken into account, correction terms of the order α^{-2} appear in the above formulas for $\mathbf{f}_s$ and $\mathbf{f}_n$, where α is the ratio of the radius r_2 of the outer cylinder to r_1.[15] However, since α^{-2} ranged from 10^{-3} to 10^{-4} in our experiment, these corrections are negligible.

In practice, our wires were somewhat noncircular in cross section. The lift force term in Eq. (4) is fortunately independent of the shape as well as the size of the cylinder.[16] However, the other terms in Eqs. (4) and (5) will in general require correction and become anisotropic when the cross section is noncircular. The effective mass of an elliptical cylinder moving in an unbounded ideal fluid has been discussed by Lamb,[17] and the force on an elliptical cylinder vibrating in an unbounded viscous fluid has been considered by Kanwal when the vibration is along either the major or minor axis of the ellipse.[18]

B. Normal Modes of the Wire

The vibration of the wire is determined by the forces discussed above due to the fluid in combination with those due to tension and stiffness in the wire. Assuming that the wire has a uniform elliptical cross section and choosing Cartesian axes so that the z axis runs along the axis of the wire and the x and y axes lie along the principal axes of the ellipse, we can write the following coupled equations of motion:

$$\mu_x \partial^2 x/\partial t^2 - T\partial^2 x/\partial z^2 + S_x \partial^4 x/\partial z^4 + 2\mu_x\lambda_x \partial x/\partial t + \rho_s\kappa\partial y/\partial t = 0\,, \tag{8}$$

$$\mu_y \partial^2 y/\partial t^2 - T\partial^2 y/\partial z^2 + S_y \partial^4 y/\partial z^4 + 2\mu_y\lambda_y \partial y/\partial t - \rho_s\kappa\partial x/\partial t = 0\,. \tag{9}$$

[12] H. Lamb, *Hydrodynamics* (Dover Publications, Inc., New York, 1945), 6th ed., Article 69.
[13] G. G. Stokes, *Mathematical and Physical Papers* (Cambridge University Press, Cambridge, England, 1901), Vol. 3, p. 38.
[14] E. Jahnke and F. Emde, *Tables of Functions* (Dover Publications, Inc., New York, 1945), 4th ed., pp. 133, 252.
[15] L. A. Segel, Quart. Appl. Math. 18, 335 (1961).
[16] Reference 12, Articles 72b, 370b.
[17] Reference 12, Article 71.
[18] R. P. Kanwal, Z. Angew. Math. Mech. 35, 17 (1955).

Here x and y represent the x and y components of the displacement of the wire from equilibrium, μ_x and μ_y represent the effective-mass-per-unit-length coefficients of the wire, T is the tension in the wire, S_x and S_y are the stiffness coefficients of the wire, and λ_x and λ_y are the damping coefficients.[19] The effective-mass-per-unit-length coefficients include both the mass per unit length μ of the wire itself and the contributions of the fluid. For a wire of circular cross section, μ_x and μ_y are given by the expression

$$\mu_x=\mu_y=\mu+\mu_s+k\mu_n. \tag{10}$$

The damping coefficients are assumed to be due entirely to the viscous damping of the normal fluid and are functions of frequency. For a wire of circular cross section λ_x and λ_y are given by the expression

$$\lambda_x=\lambda_y=\mu_n\omega k'/2(\mu+\mu_s+k\mu_n). \tag{11}$$

We shall assume for the present that μ_x, μ_y, T, S_x, S_y, λ_x, λ_y, and κ are all uniform along the length of the wire.

First, consider the case in which the stiffness of the wire is negligible. In the absence of circulation, the equations of motion are uncoupled and have independent plane-polarized normal-mode solutions of the form

$$x(z,t)=f(z)\ \mathrm{Re}(Ae^{i\eta_x t}), \tag{12}$$

$$y(z,t)=g(z)\ \mathrm{Re}(Be^{i\eta_y t}). \tag{13}$$

The boundary conditions on the wire are that the displacement of the wire must be zero at the ends, which lie at $z=\pm\frac{1}{2}L$, where L is the length of the wire. It is convenient to choose the functions $f(z)$ and $g(z)$ to be real and normalized according to the conditions

$$\int_{-L/2}^{L/2} f^2(z)dz=\int_{-L/2}^{L/2} g^2(z)dz=1. \tag{14}$$

Here, A and B are independent arbitrary complex constants.

For the motion in the x,z plane η_{xm} for the mth normal mode has the form

$$\eta_{xm}=\omega_{xm}+i\lambda_{xm}, \tag{15}$$

where

$$\omega_{xm}=\left[\left(\frac{m\pi}{L}\right)^2\frac{T}{\mu_x}-\lambda_{xm}{}^2\right]^{1/2}. \tag{16}$$

Similar equations hold for the motion in the y,z plane. Note that in general the normal-mode frequencies differ for the two modes having the same value of m. However, $f_m(z)$ and $g_m(z)$ are identical and have the form

$$\begin{aligned} f_m(z)=g_m(z)&=(2/L)^{1/2}\cos(m\pi z/L), \qquad m=1,3,5,\cdots \\ &=(2/L)^{1/2}\sin(m\pi z/L), \qquad m=2,4,6,\cdots. \end{aligned} \tag{17}$$

With uniform circulation κ present the equations of motion are coupled. The normal-mode solutions in this case are of the form

$$x(z,t)=f(z)\ \mathrm{Re}(Ae^{i\eta t}), \tag{18}$$

$$y(z,t)=g(z)\ \mathrm{Re}(Be^{i\eta t}), \tag{19}$$

where the amplitude coefficients and thus the x and y motions are no longer independent. However, the effect of circulation is just to couple the two previously independent modes for each value of m to form two new modes which can conveniently be designated $\pm m$. For the new modes, $f_{\pm m}(z)$ and $g_{\pm m}(z)$ are identical to each other and to $f_m(z)$ and $g_m(z)$ given by Eq. (17).

We can write $\eta_{\pm m}$ for the $\pm m$th mode in the form analogous to Eq. (15),

$$\eta_{\pm m}=\omega_{\pm m}+i\lambda_{\pm m}. \tag{20}$$

However, in this case exact expressions for $\omega_{\pm m}$ and $\lambda_{\pm m}$ do not seem obtainable. Nevertheless, we can make use of the fact that in the present experiment the quantities

$$\Delta\omega_0\equiv\omega_x-\omega_y, \tag{21}$$

$$\Delta\omega_\kappa\equiv\rho_s\kappa/(\mu_x\mu_y)^{1/2}, \tag{22}$$

together with λ_x and λ_y, were all typically of the order of 10^{-3} times smaller than the quantity

$$\omega_0\equiv\tfrac{1}{2}(\omega_x+\omega_y) \tag{23}$$

for the ±1 modes, which are the modes of interest. It is then possible to obtain for these modes in the case $\lambda_x=\lambda_y$ the approximate expressions

$$\omega_\pm\approx\omega_0\pm(\tfrac{1}{2})[(\Delta\omega_0)^2+(\Delta\omega_\kappa)^2]^{1/2}, \tag{24}$$

$$\lambda_\pm\approx\lambda_x=\lambda_y. \tag{25}$$

As a result, the splitting $\Delta\omega$ between the angular frequencies of the $+1$ and -1 modes is given by the important expression

$$\Delta\omega\approx[(\Delta\omega_0)^2+(\Delta\omega_\kappa)^2]^{1/2}. \tag{26}$$

The approximation involves the loss of terms of the order of 1 part in 10^3 and results in negligible error for our purposes. If $\lambda_x\neq\lambda_y$, the damping coefficients $\lambda_\pm$ differ somewhat from each other, and correction terms of the order of $(\lambda_x-\lambda_y)^2/(\Delta\omega_0)^2$ appear in Eq. (26). However, it is believed that in our experiment these terms were small enough to neglect. Thus, where in the absence of circulation we had for $m=1$ an x and a y mode we now have a $+$ and a $-$ mode with a separation in angular frequency which increases with κ.

The relation between the amplitude coefficients for the $m=1$ modes is given by the approximate expression

$$A_\pm/B_\pm\approx\pm iR^{\pm1}, \tag{27}$$

where

$$R\approx(\Delta\omega+\Delta\omega_0)/\Delta\omega_\kappa. \tag{28}$$

[19] P. M. Morse, *Vibration and Sound* (McGraw-Hill Book Co., New York, 1948), 2nd ed., p. 166.

Thus, in these modes each point on the wire moves in an ellipse for which R is the ratio of the length of one principal axis to that of the other. In the limit of a cylindrically symmetric wire or, more generally, in the limit $\Delta\omega_\kappa \gg \Delta\omega_0$, the modes become circularly polarized.

When the stiffness of the wire is not negligible, the situation is somewhat more complicated. The boundary conditions are that both the displacement and the z derivative of the displacement vanish at the ends of the wire. In the absence of circulation one again gets independent, plane-polarized modes of the form given by Eqs. (12), (13), (14), and (15). However, ω_{xm} and ω_{ym} are now the solutions of transcendental equations, and $f_m(z)$ and $g_m(z)$ are no longer sinusoidal.[19] In general $f_m(z)$ and $g_m(z)$ are not equal to each other as they were in the absence of stiffness.

When uniform circulation is present, the situation is further complicated by the fact that there is coupling not just between pairs of plane-polarized modes of a given m but between each x mode and all of the y modes, and vice versa. However, let us make the plausible approximation that only coupling between modes of the same m is important. This approximation can be justified in terms of the near-orthogonality of $f_m(z)$ and $g_n(z)$ when $m \neq n$, which would hold for a nearly isotropic wire. Then it can be shown that Eqs. (21), (22), and (26) are also valid for this more complicated case if κ in Eq. (22) is replaced by

$$\kappa_1 \equiv \kappa \int_{-L/2}^{L/2} f_1(z) g_1(z) dz . \qquad (29)$$

In our experiment the dimensionless parameters $(2/L)(S_x/T)^{1/2}$ and $(2/L)(S_y/T)^{1/2}$ which characterize the importance of stiffness relative to tension as a restoring force were never larger than 10%. Under these circumstances it is estimated that κ_1 differed from κ by less than one part in 10^3, an effect too small to be detected.

Hence, we conclude that although the anisotropic stiffness present played a role in determining the zero-circulation frequency splitting between the $m=1$ modes, it was not significant in altering the basic equations by which κ is determined from the frequency splitting between these modes.

It is conceivable that some asymmetry in the mounting of the ends of the wire also contributed to the zero-circulation splitting, due effectively to different boundary conditions for the x and y directions. However, we assume that such effects are negligible in regard to determining κ from the frequency splitting.

In practice it proved to be possible and useful to alter $\Delta\omega_0$ by twisting one end of the wire relative to the other. For a wire of elliptical cross-section such twisting leads to a violation of the assumption that μ_x, μ_y, S_x, S_y, λ_x, and λ_y are uniform along the length of the wire and complicates the wire motion. It was assumed, however, that Eqs. (21), (22), and (26) which were used to determine κ from $\Delta\omega$ continue to be a good approximation.

C. Principles of the Measurement Technique

The wire is put into vibration by sending a current pulse through it in the presence of a transverse magnetic field. Assume that the field is oriented at $\frac{1}{4}\pi$ rad to the x and y axes in the x,y plane. Then, neglecting small effects due to stiffness, the two approximately elliptical modes with $m=1$ will be excited equally. The ensuing vibrations are then observed by observing the emf induced in the wire as it moves in the magnetic field. For these two lowest modes the resulting signal is the superposition of two damped oscillations of equal amplitude but of slightly different angular frequency. Hence, for these modes there results a beat-pattern emf of the form

$$\mathcal{E}(t) = \mathcal{E}(0) e^{-\lambda t} \sin(\omega_0 t) \cos(\tfrac{1}{2}\Delta\omega t), \qquad (30)$$

and the measurement of the beat rate gives the desired frequency splitting. Here we have neglected any difference between λ_x and λ_y.

For a uniform magnetic field the displacement amplitude with which the mth pair of modes is excited is approximately proportional to $1/m^3$ for odd m. The modes of even m are not excited. As a consequence the emf resulting from the mth modes is also approximately proportional to $1/m^3$ for odd m, so that the signal from every mode having $m>1$ is reduced by at least a factor of 27 compared to the signal from the $m=1$ modes. Because of this ratio and also because of the increase of damping with ω, the effects of modes with $m>1$ should be small. Despite the nonuniformity of the magnetic field actually used in this experiment, the effects of these modes were never seen, and thus these modes have been neglected.

It proved convenient to measure the beat rate by measuring the time interval τ which elapsed between the pulsing of the wire and the occurrence of the first beat minimum. Then $\Delta\omega$ was obtained from the expression

$$\Delta\omega = \pi/\tau . \qquad (31)$$

The procedure above makes use of a field which is constant in direction throughout the measurement, and yields a measurement only of the magnitude of the circulation, not its direction. However, by pulsing the wire in a field pointing in a direction somewhat different from that in which the emf is observed, a measurement of the direction of circulation can be made.

In order to understand the principle of this measurement, it is helpful to consider the simple situation in which $\Delta\omega_0=0$ and the modes with circulation are circularly polarized. Then the motion following the initial pulse can be thought of as plane-polarized in a plane which is precessing at the angular rate $\frac{1}{2}\Delta\omega_\kappa$ in the direction of the circulation of the fluid. The occurrence of the first beat minimum corresponds to precession through $\frac{1}{2}\pi$ rad. If now the field direction in which the wire is pulsed differs from that in which the beat minimum is observed, then the plane of vibration must precess through an angle different from $\frac{1}{2}\pi$, and the resulting alteration in the time interval between the pulse and the first beat minimum can be used to infer the direction of circulation. In the case of elliptical normal modes the situation is more complicated but qualitatively similar.

IV. APPARATUS

A. Mechanical Apparatus

An over-all sketch of the apparatus is shown in Fig. 1, and a detailed view of the cell containing the wire is shown in Fig. 2. As can be seen in Fig. 1 the cell and wire are located at the bottom of a Dewar and can be immersed in liquid helium. The wire is surrounded by a Pyrex tube of 3-mm i. d. which provides smooth walls for the flow around the wire. The Pyrex tube is in turn enclosed by a brass can. The cell fills with liquid directly from the surrounding bath.

Although both platinum and tungsten wires were tried early in the experiment, the "wires" which proved easiest to handle and which gave the cleanest vibrational behavior were quartz fibers with a conducting coating of evaporated gold of the order of 1000 Å in thickness.

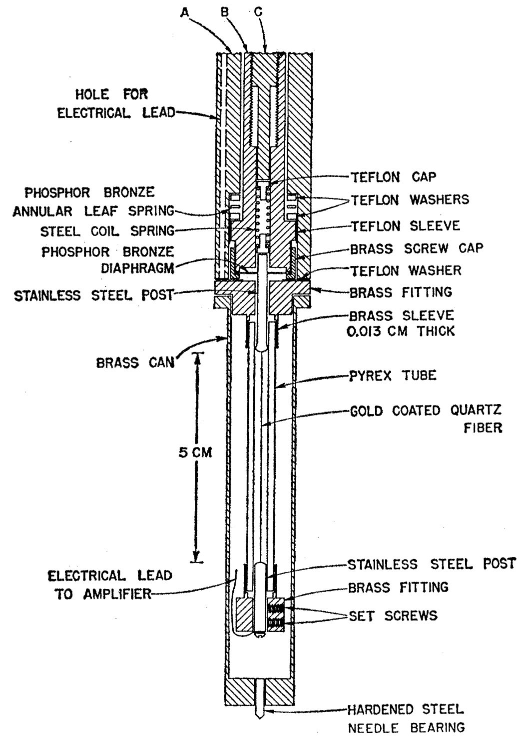

FIG. 2. View of the cell containing the wire.

The results reported here were all obtained using such fibers ranging from 25 to 100 μ in diameter. The fibers used are listed in Table I along with the angular frequency splitting $\Delta\omega_\kappa$ expected for one quantum of circulation at 1.20°K.

The mass per unit length μ of a fiber was found by weighing it on a microbalance and measuring its length with a steel ruler. The diameter of a fiber was determined from the average of the diameters measured at each end of the fiber using a micrometer caliper. It was observed using a microscope that the fibers were in general noncircular in cross section with diameters in different directions differing by as much as 10%.

The fibers were mounted in the stainless steel posts at the ends of the Pyrex tube using first an application of silver paint or silver-impregnated epoxy cement[20] to establish electrical contact, followed by an application

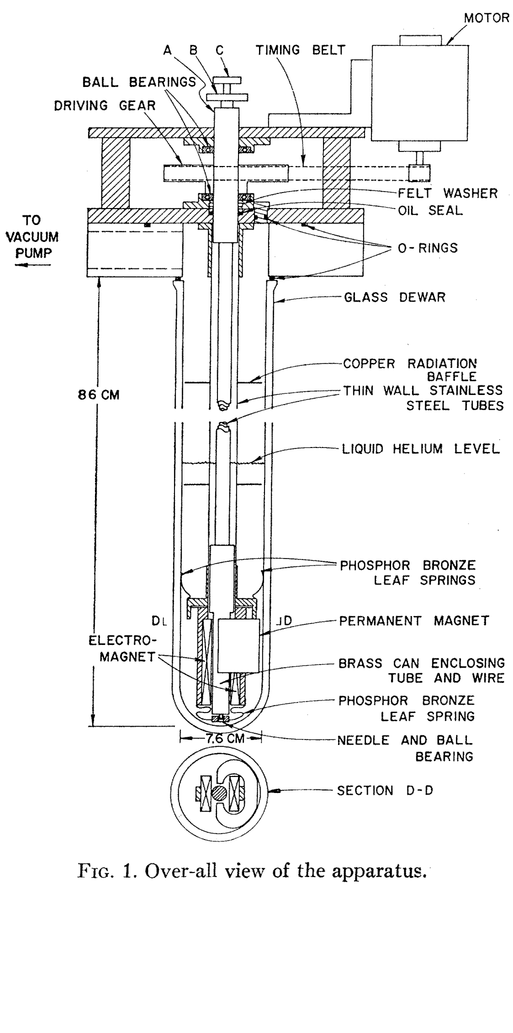

FIG. 1. Over-all view of the apparatus.

TABLE I. Representative experimental parameters. The entries in the fourth column indicate the range of values of $\Delta\omega_0$ over which measurements of stable nonzero circulation were made during the various runs with a given wire.

Wire	Wire diameter (μ)	$\Delta\omega_\kappa(n=1,\ 1.20°\mathrm{K})$ (rad sec^{-1})	$\Delta\omega_0$ (rad sec^{-1})	λ(500Hz, 1.20°K) (sec^{-1})
C	25	6.1	9.8 –10.3	9.1
D	39.4	3.52	4.95–9.03	5.3
E	75.0	1.24	2.41–4.05	2.9
G	100	0.650	1.81–1.87	2.1
H	95.7	0.787	1.93	2.0

[20] E-Solder 3021, Epoxy Products, Inc., Irvington, N. J.

of clear epoxy cement[21] for additional strength and to provide a smooth and symmetric fillet between post and wire.

Concentric thin-wall stainless steel tubes running most of the length of the apparatus connect the parts labeled A, B, and C, respectively, in Figs. 1 and 2. Tube B allows the upper end of the wire to be twisted relative to the lower end, permitting $\Delta\omega_0$ to be adjusted, and C allows adjustment of the tension in the wire and thus of the wire frequency during a run. The entire assembly including the cell and connecting tubes can be put into rotation at angular speeds up to 80 rad sec^{-1} by means of the drive motor at the top.

The cell is located in the gap of a stationary permanent magnet supplying an average field of 1350 G at the wire. The cell is also located in the gap of a stationary electromagnet which was used to apply a field to the wire of about 250 G perpendicular to that of the permanent magnet. The electromagnet, wound from superconducting wire and requiring 1 A to provide the above field, could be pulsed on and off and was used during measurements of the direction of circulation to rotate the total field at the wire.

The temperature of the bath was determined from vapor pressure measurements by means of an Octoil-S manometer using the 1958 temperature scale.[22] The bath temperature was stabilized using a carbon resistance thermometer and an electronic regulator similar in principle to the one described by Blake and Chase.[23]

B. Electrical Apparatus

The wire, having a resistance at low temperature of between 10 and 100 Ω, was located in one arm of a simple bridge providing both resistance and capacitance balance. The bridge was arranged so that a driving pulse could be supplied to the wire with very little direct effect on the circuit used to amplify the small vibrational emf subsequently induced in the wire. This arrangement prevented undesirable saturation of the amplifier with the driving pulse. It was observed that the sensitivity of the system was limited not by electrical noise but rather by mechanical noise picked up by the wire. Care was taken so that the circuit seen by the wire provided negligible electrical damping compared to the mechanical damping of the normal fluid.

The driving pulses were supplied by a pulse generator which delivered single pulses periodically, typically every 5 sec. This period allowed the vibrations excited by one pulse to decay to a negligible level before the occurrence of the next pulse. The pulse length was usually chosen to be one-half the period of vibration. For the typical wire frequency of 500 Hz the usual driving pulse was therefore 1 msec in length.

The signal from the wire and bridge was passed through a tunable selective amplifier[24] and displayed against time on the face of an oscilloscope. The horizontal sweep of the scope was triggered by the driving pulse. The time interval between the driving pulse and the occurrence of the first beat minimum was read directly from the scope face. The time base provided by the oscilloscope could be checked using an audio oscillator and an interval timer.

During the run it was essential to monitor the frequency of vibration of the wire from time to time and to readjust the tension in the wire to correct for any drifts. For this purpose use was made of an interval timer to measure a ten-period average of the period of vibration to an accuracy of 1 μsec shortly following the driving pulse. The amplifier was used in a flat-response mode for these measurements.

The use of the selective amplifier was imperative for the reduction of noise but introduced some complications into the measurements. Because a beat pattern is the result of the superposition of two slightly different frequencies, and because the selective amplifier introduces a frequency-dependent phase shift, the output beat pattern of the amplifier was delayed in time relative to the input pattern by an amount we shall call $\Delta\tau$. In principle the beat period might still be found by measuring the interval $2\pi/\Delta\omega$ between successive beat minima, which is unaffected by the amplifier. However, in practice, due to the damping of the wire it proved expedient to measure the time τ from the initial pulse to the first beat minimum. The amplifier increased this interval by an amount $\Delta\tau$ from its true value $\pi/\Delta\omega$, an increase which in the present experiment was as large as 10%.

Another effect of the selective amplifier was to introduce a transient signal in response to the sudden onset of the wire's vibration. Care had to be taken that this signal had decayed to a negligible value by the time the first beat minimum occurred in order that the beat minimum would not be further shifted.

In order to understand these effects and to compute an estimate for $\Delta\tau$ it was assumed that the amplifier behaved approximately like a damped harmonic oscillator. It was found as a result that $\Delta\tau$ is maximum when the amplifier is tuned to ω_0 and is given to first order by the expression

$$\Delta\tau \approx 1/(\gamma-\lambda)\,, \tag{32}$$

where λ is the damping constant of the signal and γ is the damping constant of the amplifier. The constant γ is equal to the angular frequency to which the amplifier is tuned divided by twice the Q of the amplifier. Equation (32) is accurate to 1% or better if as in the present experiment $(\gamma-\lambda)/\Delta\omega > 3$. In this case $\Delta\tau$ is no larger than about 10% of τ. Hence, in order to determine $\Delta\tau$, γ and λ must be determined and the tuning of the selective amplifier must accurately match ω_0.

[21] Epoxy 220, Hughes Associates, Excelsior, Minn.

[22] F. G. Brickwedde, H. van Dijk, M. Durieux, J. R. Clement, and J. K. Logan, Natl. Bur. Std. (U. S.) Monograph **10** (1960).

[23] C. Blake and C. E. Chase, Rev. Sci. Inst. **34**, 984 (1963).

[24] Type 1232-A, General Radio Co., West Concord, Mass.

It was customary to measure γ directly by measuring on the scope face the free decay rate of ringing in the amplifier after pulsed excitation. However, γ was also on occasion measured by measuring the angular frequency separation $2\sqrt{3}\gamma$ between the half-maximum amplitude points of the response curve of the amplifier, a method which gave values in good agreement with the ringing method. Measurements of λ were made by measuring on the scope face the decay rate of the signal from the wire in the absence of circulation with the wire oriented so that only one mode was excited.

The most satisfactory way to achieve good tuning was to measure the angular frequency of ringing in the amplifier using the interval timer and to set the tuning so that this angular frequency agreed with ω_0. It was verified with the wire vibrating in vacuum, when the beat pattern could accurately be observed, that this method gave proper tuning. That is, $\Delta\tau$ was maximum and the over-all amplitude of the pattern was maximum.

A check on the validity of the above procedure for determining $\Delta\tau$ was made with the wire vibrating in a vacuum. In this case, because of the reduced wire damping, it was possible to measure $\Delta\tau$ directly by comparing the time it took the first beat minimum to occur with the time interval between successive minima. The calculated time delays were generally about $(10\pm5)\%$ smaller than the measured values. It was assumed that this same discrepancy applied when liquid helium was present.

When measurements of the direction of circulation were to be made, a circuit was used which turned on the electromagnet before the driving pulse was applied, held it on until that pulse had passed, and then turned it off before the beat minimum was reached. The magnet was held on for a total time interval of several tens of milliseconds.

V. EXPERIMENTAL PROCEDURE

At the beginning of a typical experimental run the inner dewar was filled with liquid helium, and the bath was then cooled from 4.2 to 1.2°K by pumping. In many cases the cell and wire were maintained in steady rotation during this cooldown, the rate of cooling above T_λ being slow enough so that the liquid within the Pyrex tube had ample time to come into solid-body rotation at the angular velocity of the cell. The longest time constant associated with the fluid coming into rotation above T_λ was estimated to be about 45 sec.

Once the bath temperature was stabilized at 1.2°K it was necessary to make a number of preliminary adjustments and measurements. The first adjustment was of the angular frequency of the wire ω_o, which was made by means of the tension control. Because the damping constant of the wire λ depends on ω approximately as $\omega^{1/2}$ it was advantageous to use as low an ω_0 as possible.

On the other hand, ω_0 could not be reduced indefinitely. At not too low an angular frequency the fibers were all found in the absence of circulation to vibrate as if the lowest modes were mutually perpendicular plane-polarized modes. That is, there existed four orientations of the wire in the magnet separated by $\frac{1}{2}\pi$ rad in which only one of the modes was seen, and midway between these positions fully modulated beat patterns were observed. This behavior, which is in accord with the analysis of Sec. III, was seen even when the wire was twisted, a case not covered in that section. However, if ω_0 was made too small, the wire no longer behaved in this simple fashion. For example, the beat pattern would be only partially modulated but the modulation would not depend on the orientation of the wire in the magnetic field, or the damping of the wire would show a large dependence on orientation. Satisfactory behavior of the wire was generally found at a frequency of about 500 Hz, which is typical of the frequencies used.

The next adjustment concerned the intrinsic angular frequency splitting $\Delta\omega_0$. If the position of the first minimum of the beat pattern was to shift under the influence of circulation by an amount which was large enough to measure accurately, it was necessary for $\Delta\omega_0$ to be comparable in size to the expected splitting $\Delta\omega_\kappa$ due to circulation, or smaller. On the other hand, $\Delta\omega_0$ could not be so small that the first beat minimum associated with zero circulation was lost in noise as the vibrations died away. Usually after cooling a first examination showed a value of $\Delta\omega_0$ which was too large, and the wire had to be twisted to reduce $\Delta\omega_0$ to a proper size. It was also found that $\Delta\omega_0$ usually tended to drop as the tension was increased, and sometimes tension changes were used to achieve the desired $\Delta\omega_0$. In fact, the dependence of $\Delta\omega_0$ on the wire tension was strong enough so that it was necessary during the run to maintain the wire frequency constant to about one part in 10^3 in order to avoid drifts in $\Delta\omega_0$ due to this source. The range of values of $\Delta\omega_0$ used for each wire and the damping constant λ for that wire at 500 Hz and 1.20°K are listed in Table I.

It should be mentioned that after stopping rotation or after an adjustment of the wire's twist it was usually true that the planes of the wire's intrinsic modes were not aligned at $\frac{1}{4}\pi$ rad to the magnetic field. In this case the modes of the wire were not equally excited and the resulting beat pattern was not fully modulated. In such a case the cell was always rotated slowly by hand until the pattern became fully modulated, since measurements could then be made with greatest accuracy.

In order to establish a certain intrinsic angular frequency splitting $\Delta\omega_0$ it was, of course, necessary to know when the circulation around the wire went to zero. This was by no means a trivial problem, since it was found that large amounts of circulation appeared spontaneously around the wire even when the apparatus was cooled to 1.2°K without rotation. It might be supposed that $\Delta\omega_0$ could be measured above T_λ, where the fluid was entirely normal and at rest, or at 1.2°K in helium vapor. However, at T_λ and above the damping of the wire was much too large to permit measurement of a suitable $\Delta\omega_0$, and it was found that $\Delta\omega_0$ was in general very different when the wire was immersed in liquid helium at 1.2°K than when the wire was surrounded by helium vapor at the same temperature. This last effect might well have been due to a noncircular cross section, since the contribution of the liquid to the effective mass per unit length of the wire could then have been anisotropic.

Vinen found it to be a general rule in his experiment that the observed total angular frequency splitting $\Delta\omega$ always decreased to a well-defined minimum value "just before the helium drains out of the space surrounding the wire."[5] He took this value of $\Delta\omega$ to be equal to $\Delta\omega_0$. In the present experiment $\Delta\omega$ was observed to take on an unusually steady value during almost every run when the surface of the helium bath had fallen to a level about one centimeter above the top of the wire. However, except in a few cases this steady value was not the minimum value for that run. These unusual levels were regarded as spurious effects of some kind. It was impossible to regard them as defining $\Delta\omega_0$.

Our principal method of determining the angular frequency splitting with zero circulation was just to excite the wire repeatedly and observe $\Delta\omega$ over a period of time. It was found that near the beginning of a run, during the first few hours after the liquid-helium transfer, $\Delta\omega$ would decrease to a well-defined minimum value several times an hour. It was sometimes possible to drive $\Delta\omega$ to a minimum by heating the wire briefly with a direct current. This value of $\Delta\omega$ was taken, provisionally, to be equal to $\Delta\omega_0$. If later in the run $\Delta\omega$ fell to a lower minimum, then $\Delta\omega_0$ had to be revised to be equal to this new value. In practice such revisions as had to be made were very minor, amounting at most to 2% of the assumed value. What was striking was the reliability with which the assumed minimum would reappear throughout a run lasting as long as 22 h. The small changes that did occur were usually brought on by warming and cooling the apparatus or by a fast rotation.

Early in the experiment, as a means of determining that the behavior of the wire had not changed during a run, and by inference that $\Delta\omega_0$ had remained stable, it was customary to transfer liquid helium twice during a run and each time let the liquid drain completely away from the wire. Measurements of the frequency splitting in helium vapor were made each time, and compared. The measurements usually agreed to within 2%. After several runs of this kind the apparatus was felt to be reliable enough that this practice could be abandoned.

In preparation for the measurements of circulation the width of the current pulse was set to one-half the period of the wire, and the amplitude of the pulse was chosen to be as small as possible and yet yield a signal from the wire which was sufficiently large compared to noise so that the position of the first beat minimum could accurately be determined. There were several reasons why it was thought desirable to keep the wire vibration amplitude as small as possible. The principal reason was to ensure that the wire vibration should not disturb the circulation around the wire. We have no clear idea of what conditions this criterion imposes. Conditions which might conceivably be relevant are that the wire displacement amplitude should be small compared to the wire radius and that the wire velocity amplitude should be small compared to the superfluid velocity at the surface of the wire due to circulation. In practice, the first condition was well satisfied while the second was not. The displacement amplitude of the wire usually lay between 2 and 8 μ. However, the smallest current pulse that could be used with fiber D, for example, resulted in a velocity amplitude at 500 Hz of about 0.16 cm sec^{-1}, twice the fluid velocity at the surface of the wire for one quantum of circulation.

However, an observation was made which tends to cast doubt on the importance of this velocity criterion. It was found that even if the current pulse was increased ten times from its lowest practical value, the behavior of the apparent circulation did not change. That is, stabilities occurred at the same quantum levels as before and, qualitatively, circulation changed with time in the same way. With this observation in mind it was customary to use a fairly large current pulse, such that the maximum wire velocity was about twenty times the fluid velocity at the wire with one quantum of circulation.

Another reason for keeping the amplitude of the pulse small was that the power dissipated in the wire due to Joule heating should be small enough not to disturb the circulation. It is not known *a priori* what a suitable power limit might be. In most cases the power was of the order of 10 or 100 μW. However, power dissipated in runs with wire G was of the order of 1000 μW, and the circulation observed with wire G was much less stable than with other wires of about the same diameter. Of course, it would have been possible to reduce the power dissipation by evaporating a thicker layer of gold onto the fiber, reducing its electrical resistance, or by using a superconducting coating.

Finally, it was considered desirable to satisfy the small-amplitude condition under which Eq. (5) was derived, although a mild violation would probably not have had a significant effect on our results. The violation of this condition would presumably lead to an alteration of the parameters k and k', but $\Delta\omega_\kappa$ for a given κ depends only weakly on k and is quite insensitive to k'. Segel indicates that in order to satisfy this condition it is sufficient for the displacement amplitude of the wire to be small compared to r_1.[15] For the runs with wires E and H the ratio of the displacement amplitude to r_1 was never greater than 0.1, so for these runs this condition was very well satisfied.

During the circulation measurements themselves the wire was pulsed typically every 5 sec, and the position of the first beat minimum was read by eye and recorded by hand as a point on a roll of chart paper. When desired the direction of the circulation was measured by pulsing the wire several times with the electromagnet alternately on and off. Then the polarity of the magnet was reversed and the same procedure repeated. Measurements could be continued essentially without interruption for as long as the observer's stamina would permit or until the helium bath level fell to the top of the wire.

The time base provided by the oscilloscope was measured before and after the run. In order to determine the time delay $\Delta\tau$ introduced by the amplifier, it was necessary to measure λ and γ. The damping constant λ of the wire had to be measured only once for a given ω_0 and temperature. However, the damping constant γ of the amplifier was checked from time to time during the run because it could drift by as much as 10%.

Making use of $\Delta\tau$, $\Delta\omega$ was computed from the time of occurrence of the first beat minimum. Once $\Delta\omega_0$ was known, $\Delta\omega_\kappa$ was determined from $\Delta\omega$ using Eq. (26). Finally, what we shall call the apparent circulation $\bar{\kappa}$ was determined from $\Delta\omega_\kappa$ using Eq. (22) in connection with the effective mass per unit length of the wire and the superfluid density ρ_s at the particular temperature involved.[25]

Although most measurements were made at 1.2°K, some measurements were made at higher temperatures up to 1.9°K. The measurements at higher temperatures were less accurate because as the temperature increases $\Delta\omega_\kappa$ decreases in proportion to ρ_s for a given κ, the damping of the wire gets larger, and as the damping increases $\Delta\tau$ increases.

VI. RESULTS

The apparent circulation in units of h/m as a function of time for three of the most interesting runs, E-6, E-7, and H-2, is shown in Figs. 3, 4, and 5. The letter in the designation of the run indicates the particular wire being used, as identified in Table I.

Because of the compressed time scale required to fit each curve into a single figure it seemed wise to plot averages of circulation made over intervals of about 60 sec instead of the actual apparent circulation point by point. An idea of how much information is lost in this averaging process can be gained by examining Fig. 6. This figure presents data covering about 12 min of run E-7. The top panel is an actual point-by-point record of the time interval $\tau+\Delta\tau$ between the initial excitation of the wire and the appearance of the first minimum of the beat pattern on the screen of the oscilloscope. The second panel is a point-by-point transcription of these time measurements into values of apparent circulation. The third panel shows the average apparent circulation for this segment of the run. When the time scale for this panel is compressed to fit the time scale for Fig. 4, which is the plot of the whole of run E-7, this segment occupies just the very small region between hours 1.3 and 1.5 near the beginning of the plot. It is evident that Figs. 3, 4, and 5 present only the broad features of a very large quantity of data.

Nevertheless, the broad features of the data have some very interesting properties. One of them, which is the first principal result of this experiment, is that motion of the superfluid around the wire could persist for long periods of time even though the assembly

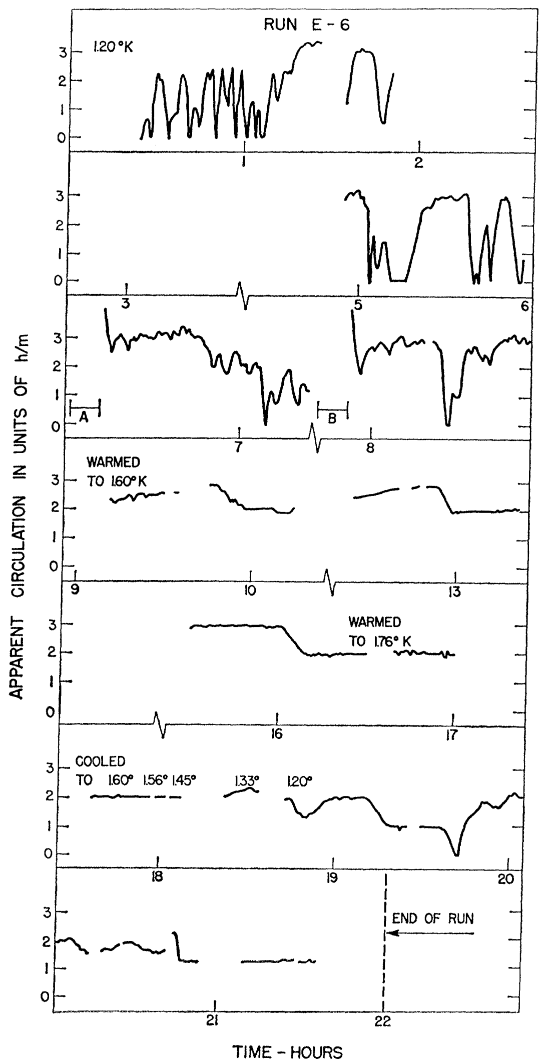

FIG. 3. Apparent circulation as a function of time for run E-6. The cell and wire were initially cooled to 1.20°K in steady rotation at 6.3 rad sec^{-1}. The lettered horizontal bars denote periods during which the cell and wire were rotated at about 6 rad sec^{-1} with temperature constant at 1.20°K.

[25] R. G. Hussey, B. J. Good, and J. M. Reynolds, Phys. Fluids **10**, 89 (1967).

carrying the cell and wire was stationary. A good example of this effect is shown in Fig. 4. Previous to the beginning of this plot the apparatus was filled with liquid helium and the assembly carrying the cell and wire set into steady rotation at an angular speed of 3.0 rad sec^{-1} at a bath temperature above T_λ. While in rotation the apparatus was slowly cooled through T_λ to 1.19°K, where the rotation was brought to a stop. No further rotation was carried out during the run, which lasted until the helium bath level fell to the top of the wire. Circulation measurements were made only during those periods for which the curve is shown. The technique for measuring the direction of the apparent circulation was not in use by the time this run was made, so only the magnitude of the apparent circulation is plotted in this figure.

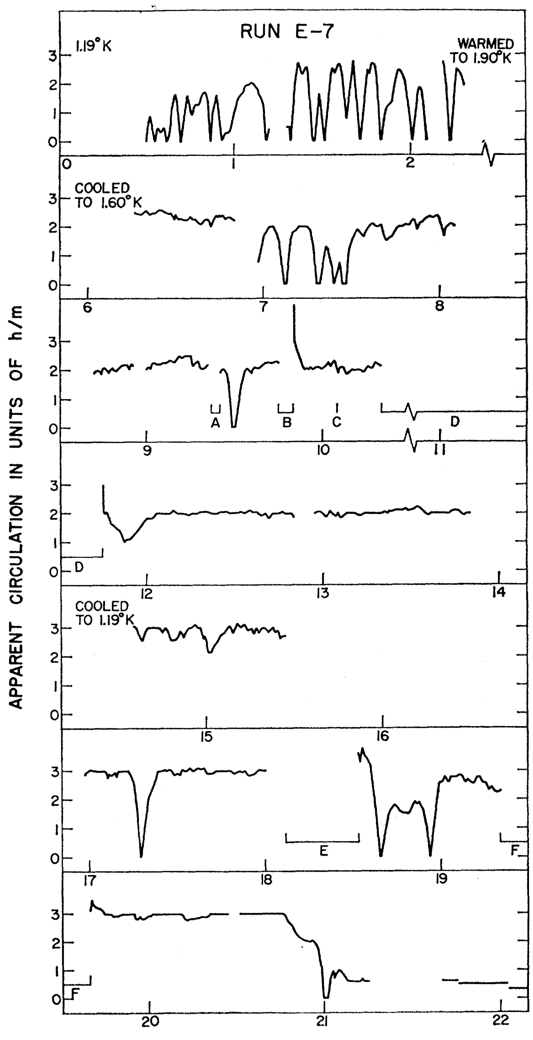

FIG. 4. Apparent circulation as a function of time for run E-7. The cell and wire were initially cooled to 1.19°K in steady rotation at 3.0 rad sec^{-1}. The lettered horizontal bars denote periods during which the wire was heated at the level of 3 mW by a direct current.

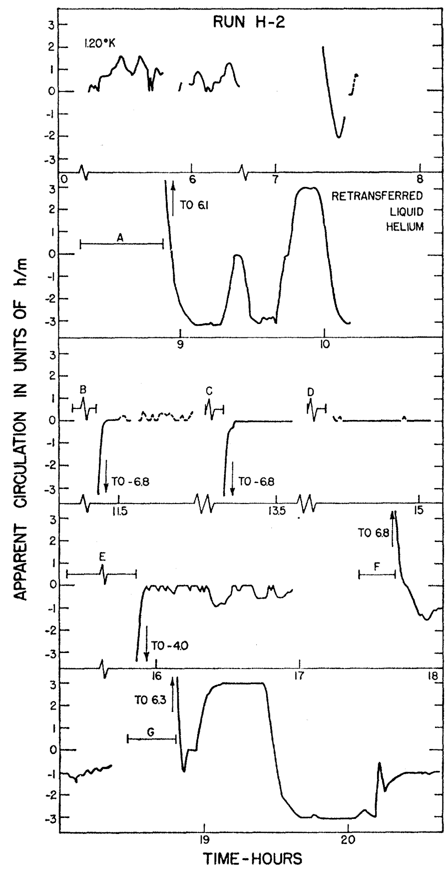

FIG. 5. Apparent circulation as a function of time for run H-2. The cell and wire were at rest during the initial cooldown to 1.20°K. The lettered horizontal bars denote periods during which the cell and wire were rotated at about 3 rad sec^{-1}. Except during period D, the cell was warmed above T_λ and then cooled in rotation to 1.20°K. Rotation was in the positive sense during periods A, D, F, G, and in the negative sense during periods B, C, E.

It is apparent from Fig. 4 that the persistent motion of superfluid around the wire was in general not steady. Smooth changes in apparent circulation took place spontaneously throughout the run. It may seem from the

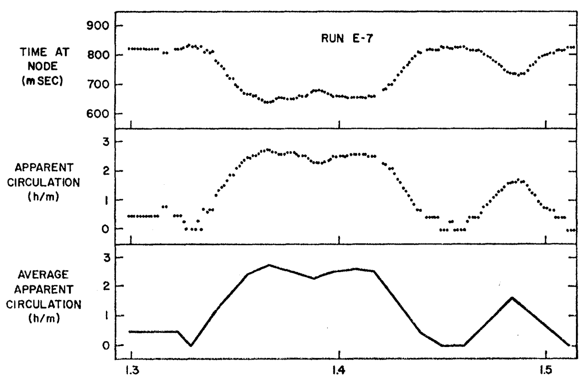

FIG. 6. Comparison of actual data with averaged data for part of run E-7.

top panel of the figure that the spontaneous changes which occurred there were rapid, noisy fluctuations. However, this is an illusion created by the compressed time scale of the drawing. Reference to the segment of the original data from this run plotted in Fig. 6. shows that in fact the changes in circulation were smooth and gradual.

Perhaps a still more striking example of the way in which the apparent circulation could drift spontaneously in time is shown in Fig. 5. By the time this run was made the electromagnet was in use, so the direction of the apparent circulation was measured as well as its magnitude. The sign convention used is that circulation directed counterclockwise around the wire looking down along the wire is positive. Several times during this run the apparatus and helium bath were warmed to a temperature above T_λ, the assembly carrying the cell and wire was set into steady rotation at about 3 rad sec^{-1}, and then the apparatus and bath were cooled back through T_λ to 1.20°K, where the rotation was brought to a stop. As in every other run performed in this experiment, all measurements of the apparent circulation were made with the apparatus at rest.

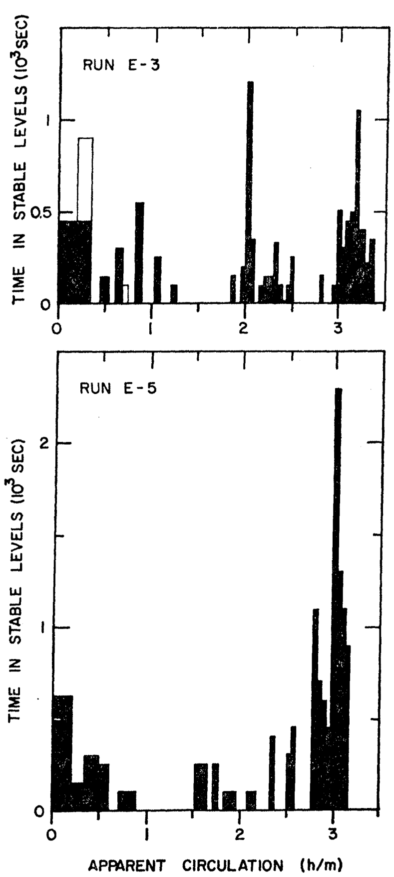

FIG. 7. Time in stable levels versus apparent circulation for runs E-3 and E-5. The unshaded columns represent stable circulations observed during the last hour of the run.

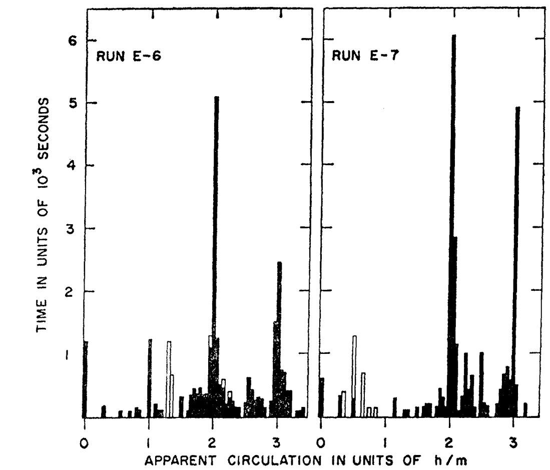

FIG. 8. Time in stable levels versus apparent circulation for runs E-6 and E-7. The unshaded columns represent stable circulations observed during the last hour of each run.

Three separate times during run H-2 the apparent circulation drifted spontaneously from a level of $3h/m$ in one direction, through zero, to a level of $3h/m$ in the other direction. In each case the drift was smooth and monotonic, and took about 10 min. It is remarkable that although the apparent circulation was more or less stable at both ends of each transition, at $\pm 3h/m$, there is no evidence of stability at the intervening nonzero quantum levels. In another case the apparent circulation drifted from a level of $3h/m$ to zero, then back to $3h/m$ in the same direction. Again the circulation was stable at the end points of the transition and at zero, but not in between.

The second principal result of this experiment is that the circulation around the wire tended to show markedly greater stability at the anticipated quantum levels than at other values. It can be seen in Figs. 3 and 4 that during runs E-6 and E-7 there were long periods of stability at the levels of 2 and 3 quantum units, and during run E-6 stable circulation also occurred at the level of one quantum unit. During run H-2 there were long periods when there was no circulation around the wire, but there were other periods when the circulation was stable at the levels of both $\pm 3h/m$.

This stability is shown in another way by histograms compiled for runs E-3, E-5, E-6, E-7, and H-2 which are shown in Figs. 7, 8, and 9. These histograms represent for a given run the total time the apparent circulation remained stable at each value of the circulation. The criterion for stability was that during a period of at least 100 sec the position of the first minimum of the beat pattern on the screen of the oscilloscope should not drift by more than ± 0.5 mm, which was the smallest displacement that could be estimated by eye. The widths of the columns in the histograms show for each run and each value of circulation how much change in apparent circulation corresponds to a 0.5 mm change in position of the beat minimum. Roughly speaking, for most runs at 1.2°K the criterion for stability required that the circulation not drift by more than $\pm 5\%$ of one quantum unit. However, exception must be made for small circulations. For circulations of one quantum unit or less the allowed drift was more like 10% of one unit.

Not every peak in the histograms appears exactly at a quantum level. In run E-3 a distinct peak occurs at $3.2h/m$, and in run E-5 a peak occurs at $2.8h/m$, although in the latter case a higher peak occurs at $3.0h/m$.

In most of the histograms there are some columns which are drawn only in outline instead of solid black. These columns represent circulations measured during the last hour of the run, when the surface of the helium bath was within one centimeter of the top of the wire. At some point during the last hour of nearly every run the apparent circulation was observed to shift rather suddenly to a very quiet, stable plateau, in general not a quantum level, and then slowly drift to smaller values. This kind of stability appears near the end of the circulation versus time plots of runs E-6, E-7, and H-2. It seems not to be related to the stability at the quantum levels. As evidence, it was possible during these periods near the end of a run to shift the measured circulation to a different plateau, still not a quantum level, by heating

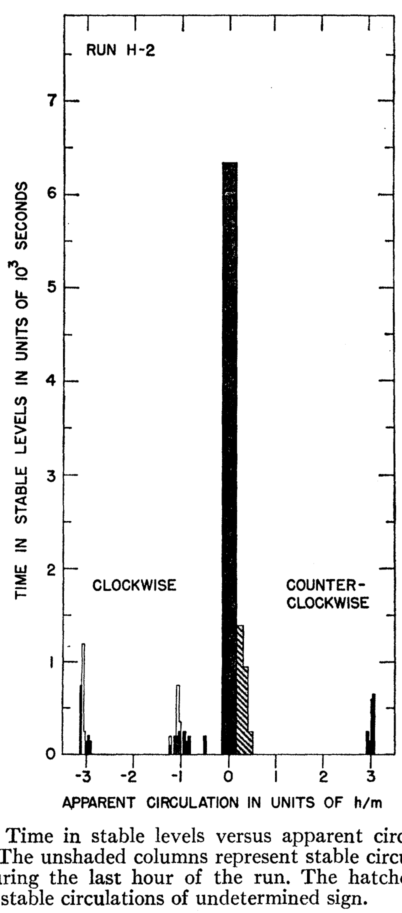

FIG. 9. Time in stable levels versus apparent circulation for run H-2. The unshaded columns represent stable circulations observed during the last hour of the run. The hatched columns represent stable circulations of undetermined sign.

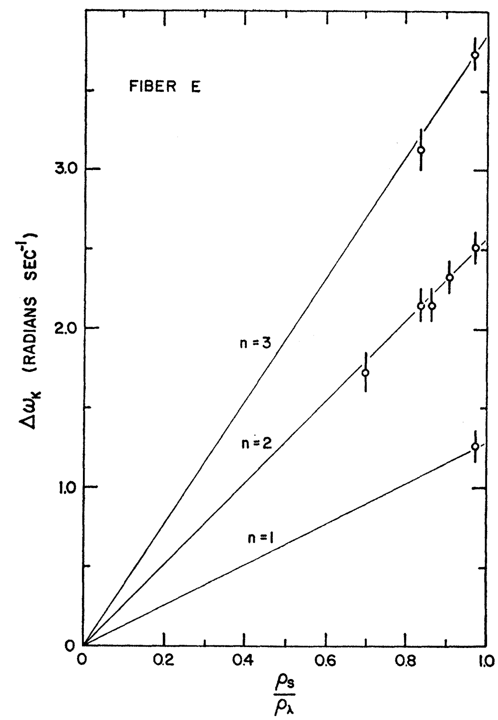

FIG. 10. Values of $\Delta\omega_\kappa$ for stable levels versus ρ_s/ρ_λ for runs E-6 and E-7.

the wire briefly with a direct current. It was not possible to manipulate circulation this way at the quantum levels.

In order to avoid bias, all periods of stable circulation which occurred during the last hour of a run have been plotted on the histograms as open columns, whether or not the circulation behaved in the peculiar manner described above. Because of this practice some genuine quantum levels have almost certainly been included in the open columns. For example, the negative circulation near the end of run H-2 which is stable at the level of $-3h/m$ clearly deserves a solid column in Fig. 9 but gets an open column because it occurred during the last hour of the run.

There are several supplementary observations which indicate that our apparatus was responding in the expected way to the presence of superfluid circulation, and which support the conclusion that our measurements reveal the effects of quantized circulation. First, measurements of apparent circulation were made at a number of temperatures. During runs E-6 and E-7 stable circulation appeared at the anticipated quantum values over a range of temperatures from 1.19 to 1.76°K. This is to say that $\Delta\omega_\kappa$ for the respective stable levels depended linearly on ρ_s as expected, as ρ_s/ρ_λ varied from 0.97 to 0.70. Here, ρ_λ is the liquid density at T_λ. The evidence for this agreement appears in Fig. 10, where $\Delta\omega_\kappa$ is plotted as a function of ρ_s/ρ_λ for the three different quantum levels observed during runs E-6 and E-7. It can also be seen in Fig. 3 that during run E-6 the circulation remained stable for the better part of two hours as the helium bath was warmed from 1.60 to 1.76°K, then cooled to 1.45°K.

Second, during run E-1, at a time when the apparent circulation was fairly stable at $3h/m$, an attempt was made to measure the expected ellipticity of the normal modes of the wire. The assembly carrying the cell and wire was turned $\frac{1}{4}\pi$ rad from its usual position, and photographs were taken of the decaying beat pattern. In such an orientation, where the axes of the normal modes lie parallel and perpendicular to the direction of the magnetic field, the modulation of the beat pattern is a minimum. The envelope of the beat pattern, after the transient response from the amplifier has died away, has the form

$$\{1+R^4+2R^2\cos[\Delta\omega(t-\Delta\tau)]\}^{1/2}e^{-\lambda t}. \qquad (33)$$

For run E-1, with a circulation of $3h/m$ present at 1.2°K, R was expected to equal 1.83. The envelope function was computed for this value of R and is plotted against time in Fig. 11 as a solid curve. Amplitudes measured from a photograph of the beat pattern are also plotted in Fig. 11, having been normalized to the scale of the calculated curve. The agreement is quite good. If no circulation had been present, the curve would have followed the dashed line in Fig. 11.

Third, the shift in node position during the direction measurements gave supporting evidence for the presence of circulation. The shift seen was of the expected magnitude in relation to the departure of $\Delta\omega$ from $\Delta\omega_0$, and no shift was seen when $\Delta\omega$ was equal to $\Delta\omega_0$.

Finally, as additional evidence that the apparatus really measured quantized circulation in the fluid, it should be pointed out that stable apparent circulation was observed at the quantum levels using wires of different diameter and mass per unit length, and using different values of $\Delta\omega_0$ with the same wire.

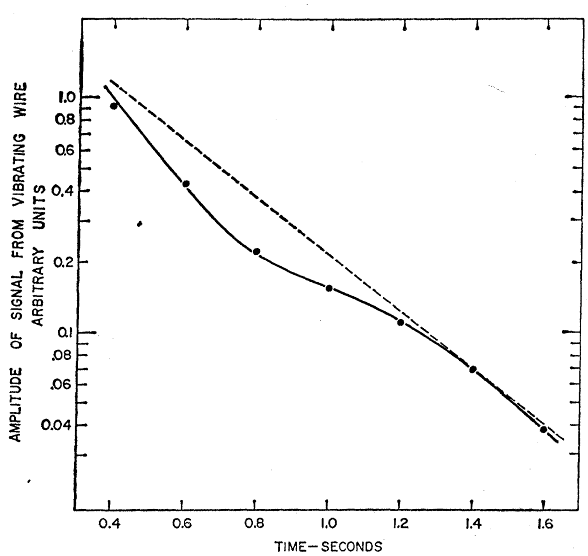

FIG. 11. Signal amplitude as a function of time for unequally excited normal modes during run E-1. The solid curve was calculated assuming a circulation of $3h/m$. The dashed line was calculated assuming zero circulation.

A third significant result of this experiment is that the maximum value at which the apparent circulation around the wire was observed to remain stable depended markedly on the diameter of the wire. As the diameter of the wire was increased, the maximum value of stable circulation observed also increased. With wire C, of diameter 25 μ, stable circulations larger than one quantum unit were rarely observed. With wire D, of diameter 39 μ, stable circulations larger than two units were rarely observed. With wire E, of diameter 75 μ, and wire H, of diameter 96 μ, stable circulations larger than about 3 units were rarely observed. Stable circulations at the level of 4 units were never observed. The fact that Vinen, using a wire 25 μ in diameter, saw stable circulation only at the one-quantum level is consistent with our results.[5]

The effect of steady rotation of the cell and wire on the apparent circulation is exemplified in Fig. 5. Previous to the beginning of this plot the apparatus was cooled through T_λ to 1.20°K with the assembly carrying the cell and wire stationary. At about the eighth hour the apparatus was warmed to a temperature above T_λ, where the cell and wire assembly was set into rotation in the positive sense at 3.2 rad sec^{-1}. Then the apparatus was cooled back through T_λ to 1.20°K, where the rotation was brought to a stop. At about the tenth hour the apparatus was warmed to 4.2°K and more liquid helium was transferred into the Dewar. Then, the cell and wire assembly was set into rotation in the negative sense at 2.8 rad sec^{-1} and cooled through T_λ to 1.20°K, where again the rotation was brought to a stop. Four more times during the run the apparatus was warmed to a temperature above T_λ, then cooled back to 1.20°K in rotation at about 3.2 rad sec^{-1}. During the time intervals marked C and E the rotation was in the negative sense. During the intervals marked F and G the rotation was in the positive sense. One time, during the interval marked D, the apparatus was rotated in the positive sense at 2.7 rad sec^{-1} while the temperature was held steady at 1.20°K.

Figure 5 shows that immediately after the rotation of the cell and wire assembly was brought to a stop the apparent circulation was found each time at an abnormally high value, up to $6.8h/m$, and with the same sense as the rotation. (Following rotation D measurements were delayed too long to see large circulation.) However, the circulation was not stable; in each case it decayed steadily to zero in about 4 min. Thereafter, it either remained stable at zero or drifted to some other value in a direction seemingly unrelated to the direction in which the cell and wire had just been rotated.

The observation of large decaying values of circulation immediately after stopping rotation is consistent with the fact that the rotation speeds of several rad sec^{-1}

used here were much greater than those needed to maintain 3 quantum units of circulation around the wire in equilibrium. In fact, the speeds were comparable to those proposed by Griffiths as being necessary to produce several quantum units of circulation around the wire.[26]

It should also be pointed out that it was quite possible for circulation to exist around the wire even though no steady rotation of the apparatus had taken place. For example, Fig. 5 shows that in run H-2 after the apparatus was cooled from 4.2 to 1.20°K the first time, with the cell and wire assembly stationary, circulations as large as $2h/m$ were observed in both directions. Moreover, stable circulations at the level of $3h/m$ were observed in runs with fiber E when no steady rotation of the apparatus had taken place.

On some occasions, for example several times during run E-7, the wire was heated with a direct current in attempts to see whether or not heating would influence the circulation around the wire in some understandable way. It was found that currents dissipating 3 mW of power, if turned on for a minute or more, did seem to affect the apparent circulation, but that the effect was not lasting. For example, during the time interval marked B in Fig. 4 wire E was heated at the level of 3 mW, and when the direct current was turned off the circulation was found to have a value of roughly four quantum units, the largest value observed during that run. However, the circulation quickly decayed back to the level of two quantum units, where it had been before the current was turned on. In general, it was not possible to predict just what effect heating the wire would have on the circulation.

It was generally true of long runs, lasting about 10 h or more, that the apparent circulation drifted more rapidly near the beginning of the run than toward the end. Figures 3 and 4 show good examples of this effect. In each case the circulation tended to be more stable after the eighth or ninth hour of the run than it had been before. During some short runs, no relative stability appeared until the last hour of the run, when the anomalous kind of stability described earlier would set in.

The uncertainties in our determinations of the apparent circulation $\bar{\kappa}$ from $\Delta\omega$ using Eqs. (21), (22), and (26) are thought to arise principally from the following sources: the determination of $(\mu_x\mu_y)^{1/2}$, the measurement of the time of occurrence of the first beat minimum $\tau+\Delta\tau$, the determination of $\Delta\tau$, and the uncertainties in the values of ρ_s used. There is also a contribution due to the use of Eq. (26) under the assumption that $\lambda_x=\lambda_y$.

In practice $(\mu_x\mu_y)^{1/2}$ was computed as if the wire were circular and as if k were unity, in which case $(\mu_x\mu_y)^{1/2}$ is simply equal to $\mu+\rho\pi r_1^2$. The contribution due to the fluid amounted to from 3 to 6% of the total. Measurements of the mass of the wire itself were considered accurate to 2 μg and the length to 0.2%, yielding an uncertainty in $\bar{\kappa}$ of from 1% for wire D down to 0.2% for wire H. The percentage error in $\bar{\kappa}$ introduced here by ignoring an ellipticity of cross section in which the lengths of the major and minor axes differed by as much as 10% was estimated to range from 0.3 to 0.6%. The error in $\bar{\kappa}$ due to assuming $k=1$ was estimated to range from 0.1 to 0.3%.

The determination of $\tau+\Delta\tau$ was limited by the resolution of the graticule of the oscilloscope to an accuracy of from 0.2 to 0.5%. This uncertainty leads to an uncertainty in $\bar{\kappa}$ due to the independent uncertainties in both $\Delta\omega$ and in $\Delta\omega_0$. For wires E and H the uncertainty in $\bar{\kappa}$ contributed by each of these terms is about 1% at a circulation of $2h/m$ and was as large as 3% at a circulation of h/m.

The time delay in the amplifier $\Delta\tau$ was computed from λ and γ, each of which was measured to about 3%. Since $\lambda \ll \gamma$, the error in λ made a negligible contribution compared to that in γ. Since $\Delta\tau$ was subject to a correction of (10 ± 5)%, the total error in $\Delta\tau$ was about 6%. An uncertainty in $\Delta\tau$ affects $\bar{\kappa}$ both through $\Delta\omega$ and $\Delta\omega_0$, but not independently. For wires E and H, the resulting uncertainty in $\bar{\kappa}$ ranged from 0.2 to 0.7%.

The error in ρ_s was assigned by comparing the values computed by Hussey *et al.*[25] with values obtained from data of Dash and Taylor.[27] The two sets of values for ρ_s differ by about 1% of the total density ρ over the temperature range covered in this experiment and contribute a 1% uncertainty in $\bar{\kappa}$.

Finally, although unfortunately a careful check was never made, it seems quite unlikely that λ_x and λ_y ever differed by more than 10%. The resulting uncertainty in $\bar{\kappa}$ is thought to be less than 0.5%.

Taking into account all of the above sources of uncertainty the over-all uncertainty in $\bar{\kappa}$ is thought to amount to a few percent.

VII. DISCUSSION AND CONCLUDING REMARKS

The results of the previous section show that it has been possible to repeat the Vinen experiment successfully and to extend it in several ways. The sensitivity of our electrical system permitted us to make virtually continuous records of apparent circulation for periods of several hours and to make measurements over a temperature range from 1.2 to 1.9°K. Measurements were made with wires ranging from 25 to 100 μ in diameter, and the direction of circulation around the wire was measured as well as its magnitude.

We believe that our observation of metastable circulations occurring preferentially at the levels of $1h/m$, $2h/m$, and $3h/m$ lends additional support to the hypothesis of quantization of superfluid circulation in helium II. It might be commented here that we do not

[26] D. J. Griffiths, Proc. Roy. Soc. (London) **A277**, 214 (1964).

[27] J. G. Dash and R. D. Taylor, Phys. Rev. **105**, 7 (1957).

believe that our data give any consistent evidence for half-integral quantization.[28] However, the behavior of the apparent circulation with time is very different from that suggested in Sec. II, and there is much concerning the detailed hydrodynamics that remains to be understood.

First, consider the observation of apparent circulations intermediate to the quantum levels, for which our analysis has not provided. Vinen suggested that such an observation might be accounted for by supposing that one end of a free vortex was attached to the wire at some point along its length, and Griffiths has considered the stability of such configurations.[5,26] If the vortex is singly quantized, the circulation around the wire will change by one quantum unit at the point of attachment. An approximate method of handling this situation is to assume that Eqs. (8) and (9) continue to hold with κ now a function of z. If the approximation that κ couples only x and y modes of the same m is still valid, then it can be shown that $\Delta\omega_\kappa$ for the $m=1$ modes is still given by Eq. (22) but now with

$$\kappa_1=\int_{-L/2}^{L/2}\kappa(z)f_1(z)g_1(z)dz \qquad (34)$$

in place of κ. Thus, if the point of attachment of a single quantized vortex were to move from one end of the wire to the other, κ_1 would change smoothly by one quantum unit.

However, it was not unusual to see smooth changes of circulation of more than one quantum unit without any sign of the intermediate quantum levels. A striking example was provided in run H-2, where $\bar{\kappa}$ made spontaneous transitions between the stable levels of $\pm 3h/m$ with almost no trace of stability at the intermediate quantum levels. In terms of Vinen's picture, such changes would require either the cooperation of more than one attached vortex or the involvement of a multiply quantized attached vortex. However, we do not have a model for the former possibility, and the latter possibility seems unlikely in view of the probability that only singly quantized vortices normally exist in the liquid.

As an alternative to Vinen's picture, it seems possible that intermediate values of circulation may arise from the mere proximity to the wire of vortices in the liquid. The results of a calculation made by Bickley when applied to the present problem indicate that if a rectilinear vortex lying parallel to the wire moves with the superfluid in the vicinity of the wire, it will contribute a fraction of its own circulation to the apparent circulation around the wire.[29] This fraction will vary from zero for a vortex at a distance large compared to the wire's radius to unity for a vortex at the surface of the wire.

We do not understand why the level of stable circulation preferred tends to rise with increased wire size. It may be that the upper limit of stable circulation observed with a given wire is a critical velocity effect associated with the circulating flow. If $n_{\max}$ is the quantum number of the highest circulation level observed with a wire of radius r_1, then the critical velocity v_c at the surface of the wire is given approximately by the expression

$$v_c\approx \hbar n_{\max}/mr_1. \qquad (35)$$

The value for v_c which fits our results for all wire sizes is roughly 0.1 cm sec^{-1}. However, in view of the fact that the wire velocity in vibration usually exceeded this value by about an order of magnitude and that we did not observe amplitude-dependent effects, this explanation may be tenable only if the critical velocity for oscillatory flow is considerably greater than that for steady flow. It is not clear why with the larger wires circulation at the one quantum level should have occurred so infrequently.

Whatever the detailed mechanism for the interaction between the wire and vortices in the liquid, the spontaneous changes in apparent circulation that take place during a run suggest that a considerable number of vortices are formed in the liquid upon cooling through T_λ with or without rotation and that these vortices persist for long periods of time. The spontaneous changes in direction of circulation that we have seen suggest that even after cooldown in rotation vortices of both signs of circulation are present. Perhaps for these reasons we have been unable to discern any clear relation between the rotation used and the resulting circulation except for the agreement between the direction of rotation and the direction of circulation immediately after rotation is stopped.

It would have been very interesting to make measurements of the circulation in rotation as Vinen did, but we were unable to do so because of noise in the apparatus.

ACKNOWLEDGMENTS

The support given this work in its early stages by the U.S. Air Force Office of Scientific Research and in its later stages by the U. S. Atomic Energy Commission is gratefully acknowledged. We also want to thank Stephen Kral for assistance with some of the measurements.

[28] C. Di Castro, Phys. Letters **24A**, 191 (1967).

[29] W. G. Bickley, Proc. Roy. Soc. (London) **A119**, 146 (1928).