Long-Range Forces from Neutrino-Pair Exchange*

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The long-range forces arising from neutrino-pair exchange between elementary systems are studied with the use of dispersion-theoretic techniques previously applied to the study of multiphoton exchange. It is shown that the potential decreases as r^{-5} for large separation r, independently of the particles or systems under consideration. The weak lepton-lepton force does not seem to be observable at present in laboratory experiments. The long-range forces *on* neutrinos are also studied. For the case of two neutral molecules, a numerical comparison is made of the three types of long-range forces: gravitational, electromagnetic, and weak.

I. INTRODUCTION

It has been known for some time that the exchange of a pair of neutrinos will produce a long-range force between two particles.[1] However, there has been some disagreement in the literature[1,2] about the dependence of this force on distance. For example, it has been suggested that such forces could be responsible for gravitation. This is not possible since, as we shall see, the force falls off much faster than the inverse square of the distance. In this paper, we will use the techniques previously developed to treat multiphoton exchange[3] to find the force coming from neutrino-pair exchange. It turns out that the power dependence of the force is insensitive to the type of the (nonderivative) neutrino interactions, or to the type of particles exchanging the neutrinos. However, the strength and the sign of the force do depend on these factors.

We treat first, in Sec. II, the long-range potential coming from neutrino-pair exchange between two electrons, assuming a direct four-fermion electron-neutrino interaction. This potential is finite, and behaves as r^{-5} for large r, in agreement with an old result of Ivanenko and Sokolov.[1] We then consider how this result might be modified in an intermediate boson theory of weak interactions. We show that if such a theory is assumed to give finite results for the process $e+\bar{e}\to\nu+\bar{\nu}$ at low energies, then the result for the one neutrino-pair exchange force, expressed in terms of the measurable amplitude for $e+\bar{e}\to\nu+\bar{\nu}$, is the same as in the lowest order four-fermion theory. This result is valid even when the effects of many virtual intermediate bosons are considered.

It is also interesting to consider the long-range force between neutral particles, since in that case it is possible for the neutrino force to asymptotically dominate the multiphoton exchange force. For an ordinary hadron, say, a nucleon N, there is no matrix element of order G_F for $N+\bar{N}\to\nu+\bar{\nu}$, because of the presumed absence of neutral lepton currents. The weak long-range force between two hadrons or a hadron and a lepton is therefore much smaller than that acting between two leptons, at the same distance, although the dependence on distance is the same. This suggests that we consider a mixed hadron-electron system, or molecule, in which case the main contribution to the neutrino-pair exchange potential comes from the electrons. This is analyzed in Sec. III; the neutrino-pair contribution to the long-range potential between two molecules is found to behave asymptotically as r^{-5}, with a coefficient which, for spinless molecules, is simply related to that occurring in the corresponding potential for electrons.

For the sake of justice and completeness we also consider, in Sec. IV, the dominant long-range force *on* neutrinos, which arises from two-photon exchange.

Finally, Sec. V contains a summary and a comparative discussion of the three types of long-range forces acting between neutral molecules: gravitational electromagnetic, and weak.

II. NEUTRINO-PAIR EXCHANGE BETWEEN CHARGED LEPTONS

We consider the force between two electrons arising from ν-pair exchange, first within the framework of the universal four-fermion interaction and then in a W-meson theory of weak interactions. The forces between other pairs of leptons are the same, within the approximations being made, up to a sign change when a lepton is replaced by its antiparticle.

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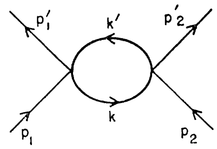

FIG. 1. Lowest-order Feynman diagram for e-e scattering via ν-pair exchange in the four-fermion theory.

A. Four-Fermion Interaction

If we assume the usual form of the current-current weak interaction theory, then there is an electron-neutrino part of the weak interaction Lagrangian of the form[4]

$$L_1=-\tfrac{1}{2}\sqrt{2}G_F\bar{\psi}_e\Gamma_\mu\psi_e\bar{\psi}_\nu\Gamma^\mu\psi_\nu\,, \tag{2.1}$$

where $G_F=10^{-5}M_p^{-2}$ and $\Gamma^\mu=\gamma^\mu(1+\gamma_5)$. The lowest-order matrix element for electron-electron scattering arising from (2.1) is (see Fig. 1)

$$-iM=(-\tfrac{1}{2}\sqrt{2}iG_F)^2H_{\mu\nu}K^{\mu\nu}\,, \tag{2.2}$$

where

$$H_{\mu\nu}=\bar{u}(p_1')\Gamma_\mu u(p_1)\bar{u}(p_2')\Gamma_\nu u(p_2) \tag{2.3}$$

and

$$K^{\mu\nu}=-\int\frac{d^4k}{(2\pi)^4}\operatorname{Tr}\left[\Gamma^\mu\frac{i}{\mathrm{k}}\Gamma^\nu\frac{i}{\mathrm{k}'}\right]. \tag{2.4}$$

The minus sign in Eq. (2.4) arises because there is a closed loop in Fig. 1. It is convenient to define

$$Q=p_1-p_1'=-(p_2-p_2')$$

and

$$\bar{k}=-k'=Q-k\,,$$

and to rewrite (2.4) in the form

$$K^{\mu\nu}=-\int\frac{d^4k}{(2\pi)^4}\frac{1}{k^2}\frac{1}{\bar{k}^2}L^{\mu\nu}\,, \tag{2.5}$$

where

$$L^{\mu\nu}\equiv\operatorname{Tr}\Gamma^\mu\gamma\cdot k\Gamma^\nu\gamma\cdot\bar{k}$$
$$=8(k^\mu\bar{k}^\nu+k^\nu\bar{k}^\mu-g^{\mu\nu}k\cdot\bar{k})\,.$$

If the quadratically divergent integral (2.5) were supplied with a convergence factor, such as $[\lambda^2/(k^2-\lambda^2)]^2$, the general form of $K^{\mu\nu}$ would be

$$K^{\mu\nu}=Ag^{\mu\nu}+BQ^\mu Q^\nu\,, \tag{2.6}$$

where A and B are (cutoff-dependent) functions of t,

$$t\equiv Q^2\,,$$

analytic in the t plane with a cut starting at $t=0$. The discontinuities $[A]$ and $[B]$ across the cut are finite as $\lambda\to\infty$, and may be obtained by replacing $(k^2)^{-1}$ and $(\bar{k}^2)^{-1}$ by $-2\pi i\delta(k^2)\theta(k^0)$ and $-2\pi i\delta(\bar{k}^2)\theta(\bar{k}^0)$, respectively, in (2.5). Thus with

$$[K^{\mu\nu}]\equiv-(-2\pi i)^2\int\frac{d^4k}{(2\pi)^4}\delta(k^2)\delta(\bar{k}^2)L^{\mu\nu}\,,$$

the θ functions being understood, and using

$$\int d^4k\,\delta(k^2)\delta(\bar{k}^2)[1;k^\mu;k^\mu k^\nu]$$
$$=\tfrac{1}{2}\pi[1;\tfrac{1}{2}Q^\mu;\tfrac{1}{3}(Q^\mu Q^\nu-\tfrac{1}{4}g^{\mu\nu}t)]\,,$$

we find that

$$[K^{\mu\nu}]=-(1/3\pi)(g^{\mu\nu}t-Q^\mu Q^\nu)\,. \tag{2.7}$$

Comparison with Eq. (2.6) yields

$$[A]=-t/3\pi\,,\quad [B]=1/3\pi\,. \tag{2.8}$$

Using (2.2), (2.6), and

$$(p-p')^\mu\bar{u}(p')\Gamma_\mu u(p)=-2m\bar{u}(p')\gamma_5u(p)\,,$$

we get

$$M=\bar{u}(p_2')\bar{u}(p_2')Tu(p_2)u(p_2)\,,$$

where

$$T=-\tfrac{1}{2}iG_F^2(\Gamma^{(1)}\cdot\Gamma^{(2)}A-4m^2\gamma_5{}^{(1)}\gamma_5{}^{(2)}B)\,, \tag{2.9}$$

and the superscript i indicates that the corresponding Dirac matrix acts between the spinors associated with electron i (i=1, 2). Since, in the c.m. system, $\gamma_5{}^{(1)}\gamma_5{}^{(2)}$ taken between Dirac spinors is proportional to

$$[-1/(2m)^2]\boldsymbol{\sigma}^{(1)}\cdot\mathbf{Q}\boldsymbol{\sigma}^{(2)}\cdot\mathbf{Q} \tag{2.10}$$

taken between the corresponding two-component Pauli spinors, only the term proportional to A in (2.9) contributes to the spin-independent part of the potential. Furthermore, for collisions which are nonrelativistic in the c.m. system $[(s-4m^2)/4m^2\ll1]$ the matrix $\Gamma^{(1)}\cdot\Gamma^{(2)}$ is approximately equivalent to unity, spin-dependent terms being dropped. As discussed in Ref. 3, the long-range part of the potential is determined by the absorptive part of the scattering amplitude via

$$V=\frac{1}{4\pi^2r}\int_0^\infty\rho(t)e^{-(\sqrt{t})r}dt\,, \tag{2.11}$$

where $\rho(t)$ is the spectral function, equal to $(2i)^{-1}$ times the discontinuity of the Feynman amplitude. [The normalization factor in (2.11) is appropriate for the scattering of two spin-$\frac{1}{2}$ particles.] Thus, dropping the second term in (2.9) and replacing $\Gamma^{(1)}\cdot\Gamma^{(2)}$ by unity, we get for the spectral function

$$\rho_{\nu\bar{\nu}}(t)=(2i)^{-1}(-\tfrac{1}{2}iG_F^2)[A(t)]$$

or

$$\rho_{\nu\bar{\nu}}(t)=(G_F^2/12\pi)t\,. \tag{2.12}$$

From (2.11) we then get for $V_{\nu\bar{\nu}}$, the long-range, spin-independent part of the electron-electron potential arising from exchange of a neutrino pair,

$$V_{\nu\bar{\nu}}=G_F^2/4\pi^3r^5\,. \tag{2.13}$$

The long range force between two electrons coming from neutrino-pair exchange is, therefore, to lowest order in G_F, repulsive and proportional to r^{-6}. It should be noted that the long-range potential (2.13) is finite

[4] We use Dirac matrices γ^μ with $\gamma^\mu\gamma^\nu+\gamma^\nu\gamma^\mu=2g^{\mu\nu}$, $g^{00}=-g^{11}=-g^{22}=-g^{33}=1$, and $\gamma_5=-i\gamma^0\gamma^1\gamma^2\gamma^3$.

(cutoff-independent) because the spectral function $\rho_{\nu\bar{\nu}}(t)$ is finite, even though the two-neutrino exchange graph as a whole is infinite. Thus, the formalism being used is especially convenient for the calculation of effective potentials. The wide spectrum of results for neutrino-pair exchange forces given in the literature[1,2] may in part be attributed to the difficulty of separating unambiguously the finite long-range force from the cutoff-dependent short-range forces in other approaches, and in part to the fact that the calculations were based on a variety of Lagrangians, proposed before the four-fermion interaction was understood.

The fact that $V_{\nu\bar{\nu}}$ is repulsive may be understood on qualitative grounds. Within the framework of the $V-A$ theory, the exchange of a neutrino pair is equivalent to a continuous superposition of vector-meson exchanges with a Yukawa-type coupling, as can be seen from Eq. (2.7). It is well known that the exchange of such a vector meson between like particles gives rise to a repulsive force, as for photon exchange. These remarks also serve to explain why, as already stated, changing a particle to its antiparticle changes the sign of the force.

Incidentally, the above line of reasoning shows that, for example, a scalar four-fermion interaction will be equivalent to a superposition of scalar-meson exchanges and so should give rise to an attractive potential. Indeed, with

$$L^{(S)}=-G_S\bar{\psi}_e\psi_e\bar{\psi}_\nu\psi_\nu\,, \qquad (2.14)$$

we get

$$V_{\nu\bar{\nu}}{}^{(S)}=(-3/8\pi^3)G_S{}^2/r^5. \qquad (2.15)$$

Although the effective electron-neutrino interaction Lagrangian may contain either "induced" or "direct" terms other than the $V-A$ term (2.1), in the present picture of the weak interactions the corresponding coupling constants, such as G_S in Eq. (2.14), are expected to be much smaller than G_F.

It should be noted that *any* second-order calculation of the neutrino-pair force which is based on a nonderivative four-fermion interaction and is independent of any cutoff must yield a $V_{\nu\bar{\nu}}\sim r^{-5}$, simply on dimensional grounds. The fermion masses occur only in the external kinematical factors such as $(m/E)^{1/2}$, which are unity in the low-energy limit, so that if we are to have $V_{\nu\bar{\nu}}\propto G_F{}^2r^{-n}$, the proportionality constant must be dimensionless. Since $\dim G_F{}^2=L^4$, $\dim V_{\nu\bar{\nu}}=L^{-1}$ requires $n=5$.

An alternative way of understanding the r^{-5} behavior is to imagine the computation carried out in coordinate space where it can be reduced to the integration over a relative time x^0 of the trace of the square of the neutrino propagator $S_F(x)$, where $x=x_1-x_2$ and x_i is the coordinate of electron i ($i=1, 2$). Since $S_F(x)\propto\gamma\cdot\partial\Delta_F(x)$ $\propto\gamma^\mu x_\mu/(x^2-i\epsilon)^2$, the integrand is proportional to $(x^2-i\epsilon)^{-3}$. On integration over x^0 one therefore gets $|\mathbf{x}|^{-5}$, for $|\mathbf{x}|\neq 0$. Of course, for $\mathbf{x}=0$ the calculation is ambiguous, corresponding to the divergence of the closed loop and the need for subtractions in Eq. (2.5).

To conclude this subsection, we state some results concerning the spin-dependent parts of the long-range interaction. Retention of the terms involving electron spin operators in the reduction of (2.9) yields, apart from spin-other-orbit type interactions which vanish in the nonrelativistic (NR) limit, a spin-spin interaction whose long-range part $V_{\nu\bar{\nu}}{}^{\rm spin}$ is given by

$$V_{\nu\bar{\nu}}{}^{\rm spin}=-\boldsymbol{\sigma}^{(1)}\cdot\boldsymbol{\sigma}^{(2)}V_{\nu\bar{\nu}} +(G_F{}^2/24\pi^3)\boldsymbol{\sigma}^{(1)}\cdot\boldsymbol{\nabla}\boldsymbol{\sigma}^{(2)}\cdot\boldsymbol{\nabla}r^{-3}. \qquad (2.16)$$

The first term in (2.16) arises from the A term in (2.9), on using the equivalence of $\Gamma^{(1)}\cdot\Gamma^{(2)}$ to $1-\boldsymbol{\sigma}^{(1)}\cdot\boldsymbol{\sigma}^{(2)}$ in the NR limit, and the second term in (2.16) arises from the B term in (2.9), on using (2.10). Equation (2.16) may be rewritten in the form

$$V_{\nu\bar{\nu}}{}^{\rm spin}=(\tfrac{5}{2}\boldsymbol{\sigma}^{(1)}\cdot\hat{r}\boldsymbol{\sigma}^{(2)}\cdot\hat{r}-\tfrac{3}{2}\boldsymbol{\sigma}^{(1)}\cdot\boldsymbol{\sigma}^{(2)})V_{\nu\bar{\nu}}; \qquad (2.17)$$

the total electron-electron potential is then

$$V_{\nu\bar{\nu}}{}^{(e,e)}=V_{\nu\bar{\nu}}+V_{\nu\bar{\nu}}{}^{\rm spin}. \qquad (2.18)$$

In arriving at Eq. (2.18) we have dropped spin-other-orbit type interactions and terms which fall off faster than r^{-5}. The term $V_{\nu\bar{\nu}}$ then arises *entirely* from the interaction of the vector currents carried by the leptons, whereas $V_{\nu\bar{\nu}}{}^{\rm spin}$ comes *entirely* from the interaction of the axial currents. Since under charge conjugation the vector current is odd, but the axial current is even, it follows that $V_{\nu\bar{\nu}}{}^{(e^-,e^+)}$, the electron-positron potential, is obtainable from (2.18) by changing the sign of the first term;

$$V_{\nu\bar{\nu}}{}^{(e^-,e^+)}=-V_{\nu\bar{\nu}}+V_{\nu\bar{\nu}}{}^{\rm spin}. \qquad (2.19)$$

In the spin singlet state, $V_{\nu\bar{\nu}}{}^{\rm spin}$ is equivalent to $2V_{\nu\bar{\nu}}$, so that the net force is repulsive for both an e^--e^- and an e^--e^+ system. In a spin-triplet state, $V_{\nu\bar{\nu}}{}^{\rm spin}$ is equivalent, on taking a spherical average, to $-\frac{2}{3}V_{\nu\bar{\nu}}$, so that the force is still repulsive for an e^--e^- system but is attractive for an e^--e^+ system.

B. ν-Pair Exchange in the W-Meson Theory

If we consider instead of the four-fermion interaction, the W-meson theory to lowest order, then the matrix element for $e^-+e^+\rightarrow\nu+\bar{\nu}$ is modified by a factor

$$[1-(p_e-p_\nu)^2/M_W{}^2]^{-1}.$$

For the small values of the external momenta that are relevant to the long-range force, this factor is approximately unity, so that the same result is obtained for the longest-range term in $V_{\nu\bar{\nu}}$ as in the four-fermion theory.

Suppose, however, that we consider higher-order effects in this theory, involving more than one virtual W meson. It is well known that individual graphs for such higher-order effects are in general divergent. No systematic way of obtaining finite answers for such

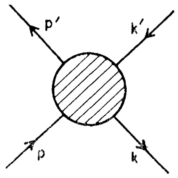

FIG. 2. Symbolic diagram for the reaction $e^-+e^+\rightarrow\nu+\bar{\nu}$, with $-p'$ and $-k'$ the momenta of e^+ and $\bar{\nu}$, respectively.

effects has yet been devised. Several years ago, a method was proposed[5,6] for describing what effects the higher-order terms might produce at low energies if it were indeed possible to obtain finite answers for them. It is of some interest to apply this method to the present problem.

We consider the amplitude F for $e^-+e^+\rightarrow\nu+\bar{\nu}$ with the kinematics of Fig. 2. If we neglect lepton masses in the calculation of higher-order terms, then this amplitude may be written as

$$F=\bar{v}_e(-p')\gamma^\rho(1+\gamma_5)u_e(p)M_{\rho\sigma}\bar{u}_\nu(k)\times\gamma^\sigma(1+\gamma_5)v_\nu(-k'), \quad (2.20)$$

where $M_{\rho\sigma}$ has the form

$$M_{\rho\sigma}=\alpha g_{\rho\sigma}+\beta Q_\rho Q_\sigma+P_{\rho\sigma}.$$

Here α and β are invariants, $Q=k-k'$ as before, and $P_{\sigma\rho}$ is a tensor which vanishes when Q is kept finite and the other linearly independent momenta are set equal to zero. For the calculation of long-range forces, we need $M_{\rho\sigma}$ at small values of the external momenta. According to the "power-counting method" of Ref. 6, in the limit of zero external momenta one has

$$\alpha(0)=(g^2/M_W^2)C+O(g^4),$$
$$\beta(0)=g^2/m_W^2+O(g^4),$$
$$P_{\mu\nu}(0)=O(g^4).$$

Here g is the coupling constant of the W-meson–lepton interaction, and C is an unknown constant, of order one in g^2. If only ladder graphs are considered, then $C=\frac{3}{4}$. We note that the quantity $\alpha(0)$ determines the low-energy electron-pair annihilation into neutrinos. It is therefore in principle experimentally measurable, say, through the astrophysical effects of this interaction.

We may now compute the long-range interactions as in Sec. II A. From (2.20) it can be seen that the net effect is to replace $(G_F/\sqrt{2})\Gamma^\mu$ by $M^{\mu\mu'}\Gamma_{\mu'}$ or, to lowest order in g^2, by

$$\alpha\Gamma^\mu+\beta QQ^\mu. \quad (2.21)$$

Since Eq. (2.7) shows that

$$Q^\mu[K_{\mu\nu}]=Q^\nu[K_{\mu\nu}]=0,$$

the β term in (2.21) does not contribute to the absorptive part, so that $(G_F^2/2)[K_{\mu\nu}]$ is simply replaced by $\alpha^2(0)[K_{\mu\nu}]$. Hence the same result is obtained as in the four-fermion interaction theory taken in lowest order, except that $G_F/\sqrt{2}$ is replaced by $\alpha(0)$. However, since $\alpha(0)$ is according to the above discussion the effective Fermi coupling constant for $e^-+e^+\rightarrow\nu+\bar{\nu}$ at low energies, it follows that even when the higher-order effects are included, the long-range potential between two electrons coming from neutrino-pair exchange can be written

$$V_{\nu\bar{\nu}}=G_{\rm eff}^2/4\pi^3r^5, \quad (2.22)$$

where $G_{\rm eff}$ is the measurable Fermi coupling constant for $e^-+e^+\rightarrow\nu+\bar{\nu}$ at low energy.

Thus, within these approximations, there is one effective constant which determines both the long-range force *and* the low-energy annihilation amplitude. This situation is reminiscent of the Thirring theorem in electrodynamics,[7] which states that the low-energy Thompson cross section is given by the same coupling constant which determines the long-range Coulomb force.

This type of conclusion regarding the effects of higher-order corrections to the $e^-e^+\rightarrow\nu\bar{\nu}$ amplitude may also hold in the four-fermion theory, provided that no infrared troubles occur, since the power-counting arguments of Ref. 6 appear also to apply in this case.

III. NEUTRINO-PAIR FORCES ON HADRONS AND MOLECULES

The amplitude for scattering of a hadron h by a real or virtual neutrino, or equivalently, the amplitude for $h+\bar{h}\rightarrow\nu+\bar{\nu}$, has no terms of order G_F in a four-fermion theory or of order g^2 in W-meson theory, as a consequence of the assumed absence of neutral currents. The higher-order graphs are, of course, divergent in either kind of theory. The leading term coming from weak interactions *alone* is formally of order g^4 in a W-meson theory; explicit summation of a certain subset of divergent graphs in this theory gives in fact a finite amplitude of order g^4, for a spin-$\frac{1}{2}$ hadron such as the proton.[6] At low momenta this amplitude is proportional to

$$(g^4/M_W^2)(\bar{u}_h'\gamma_\mu(1+\gamma_5)u_h)(\bar{u}_\nu'\gamma^\mu(1+\gamma_5)u_\nu). \quad (3.1)$$

For an effective $h+\nu\rightarrow h+\nu$ amplitude of the form (3.1), the considerations of Sec. II apply. Using $g^2/M_W^2\approx G_F/\sqrt{2}$, we are led to expect a long-range potential between hadrons of the form

$$V_{\rm weak}{}^{(h,h)}\propto g^4G_F^2/4\pi^3r^5. \quad (3.2)$$

This is a factor of 10^{-6} smaller than the corresponding potential $V_{\nu\bar{\nu}}$ [Eq. (2.13)] between leptons for $M_W/M_p=10$, and still much smaller than $V_{\nu\bar{\nu}}$ for M_W/M_p as large as 100.

However, there is a larger contribution to the long-range hadron-hadron potential coming from ν-pair exchange, arising from the charge form factor[8] of the

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[7] W. Thirring, Phil. Mag. **41**, 1193 (1950).
[8] J. Bernstein and T. D. Lee, Phys. Rev. Letters **11**, 512 (1963).

neutrino. This gives rise to an effective hadron-neutrino interaction proportional to

$$[(4\pi)^{1/2}e]^2(g^2/M_W^2)\bar{\psi}_h\gamma_\mu\psi_h\bar{\psi}_\nu\gamma^\mu(1+\gamma_5)\psi_\nu\,, \qquad (3.3)$$

and hence to a potential

$$V_{\mathrm{weak;\,em}}{}^{(h,h)}\propto[(4\pi)^{1/2}e]^4G_F{}^2/4\pi^3r^5\,, \qquad (3.4)$$

omitting cross terms between (3.1) and (3.3) (of order g^2e^2). Similarly, the long-range hadron-lepton potential arising from ν-pair exchange is

$$V_{\mathrm{weak;\,em}}{}^{(h,l)}\propto[(4\pi)^{1/2}e]^2G_F{}^2/4\pi^3r^5. \qquad (3.5)$$

It seems more interesting to study the ν-exchange force in a case where it may be asymptotically dominant over electromagnetic forces. Such a possibility arises when one considers the ν-pair exchange force between neutral atoms or molecules. It is known[3] that the potential arising from multiphoton exchange between such objects falls off asymptotically at least as fast as $1/r^7$.

The amplitude for the emission of a ν pair by a molecule is the sum of terms corresponding to pair emission by any of the constituent electrons or nucleons. The dominant terms arise from the electrons, since these are of order g^2, whereas the nucleon terms are at best of order g^2e^2. Thus a molecule, although electrically neutral, does *not* act like a neutral system for neutrino-pair emission.

The amplitude for ν-pair emission is therefore the sum of contributions from each of the Z electrons. The axial-vector contribution may be neglected for small momentum transfer, since the electrons are nonrelativistic and we consider only the spin-independent part of the force. The vector contribution is proportional to the electronic part of the charge form factor of the molecule. The molecular neutrino emission amplitude is therefore equal to Z times the amplitude for emission of a ν pair by a free electron, provided that the momentum transfer to the neutrinos is small compared with the inverse molecular radius. The results of Sec. II can thus be applied to obtain for the intermolecular potential arising from neutrino-pair exchange,

$$V_{\nu\bar{\nu}}{}^{\mathrm{mol}}\approx+(Z^2/4\pi^3)G_F{}^2/r^5\,, \qquad (3.6)$$

valid for r much larger than the molecular radius. We note that at suitably large distances this potential will dominate the van der Waals potential, arising from two-photon exchange, since the latter falls off as r^{-7}.

IV. LONG-RANGE FORCES ON NEUTRINOS

Since the neutrino has been the agent responsible for the forces on other particles that we have considered here, it seems only just to inquire whether there may be long-range forces acting on the neutrinos themselves. This could be of interest in connection with the propagation of neutrinos across astronomical distances, as well as in connection with their interaction in the laboratory.

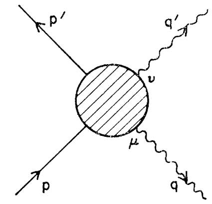

FIG. 3. Symbolic diagram for two-photon emission by a neutrino.

Although the neutrino is a spin-$\frac{1}{2}$ particle, the fact that it occurs with a definite helicity forbids it from carrying any magnetic moment. The single-photon vertex of the neutrino therefore reduces to a charge distribution, which leads to exponentially decreasing forces.[9] Therefore, as in the case of a spinless neutral particle, we are driven to consider forces arising from two-photon exchange between neutrinos and charged particles.

We thus need to know the amplitude M for emission of two photons by a neutrino. The general form of M, consistent with current conservation, CP invariance, and the identity of the two photons, may be obtained by the same methods used in Ref. 3 to find the amplitude for two-photon emission by a spin-zero particle. Using also the γ_5 invariance of the neutrino interactions and the Dirac equation for the neutrinos, $pu=p'u'=0$, one finds that on the neutrino and photon mass shells, but with $\epsilon\cdot q$ and $\epsilon'\cdot q'$ not necessarily equal to zero, one may write, neglecting terms which vanish more rapidly than the third power of momentum in the low-energy limit,

$$M=(\bar{u}'\Gamma^\alpha u)\epsilon^\mu\epsilon'^\nu(F_1T_{\alpha\mu\nu}{}^{(1)}+F_2T_{\alpha\mu\nu}{}^{(2)})\,. \qquad (4.1)$$

Here

$$T_{\alpha\mu\nu}{}^{(1)}=\Delta_\alpha(P_\mu q_\nu-P_\nu q_\mu')+P\cdot\Delta(g_{\alpha\mu}q_\nu-g_{\alpha\nu}q_\mu') \\ -\Delta^2(g_{\alpha\mu}P_\nu+g_{\omega\nu}P_\mu)-P\cdot\Delta\Delta_\alpha g_{\mu\nu}$$

and

$$T_{\alpha\mu\nu}{}^{(2)}=\Delta_\alpha(P_\mu q_\nu'-P_\nu q_\mu)+P\cdot\Delta(g_{\alpha\nu}q_\mu-g_{\alpha\mu}q_\nu')\,,$$

with

$$\Delta=q-q'\,,\quad P=p+p'\,,$$

and the kinematics is given by Fig. 3. The F_1 are scalar functions of the invariants $P\cdot\Delta$ and Δ^2.

The term proportional to F_1 in Eq. (4.1) corresponds to an effective Hamiltonian density $H^{(1)}$ for the neutrino-photon interaction of the form

$$H^{(1)}=\lambda_1J_{\alpha\beta}g_{\rho\sigma}F^{\alpha\rho}F^{\beta\sigma}\,, \qquad (4.2)$$

where $F^{\mu\nu}=\partial^\nu A^\mu-\partial^\mu A^\nu$ is the electromagnetic field tensor and $J_{\alpha\beta}$ is a neutrino tensor current defined by

$$J_{\alpha\beta}=i[\bar{\psi}_\alpha\Gamma_\alpha\partial_\beta\psi-(\partial_\beta\bar{\psi})\Gamma_\alpha\psi]\,,$$

[9] A discussion of single-photon exchange forces between neutral particles is given in J. Soffer and J. Sucher, Phys. Rev. 161, 1664 (1967).

where ψ is the neutrino field operator. The term proportional to F_2 in Eq. (4.1) corresponds to an effective Hamiltonian density

$$H^{(2)}=\lambda_2 J_{\alpha\beta}F^{\alpha\beta}\partial_\mu A^\mu. \qquad (4.3)$$

We wish to consider the effect on the neutrino of an external electromagnetic field. To this end we replace the photon field A_μ by an external (c number) field in Eqs. (4.2) and (4.3). For a static field $H^{(2)}$ vanishes,[10] so that in this case we need only consider $H^{(1)}$.

To compute the effect of $H^{(1)}$ we need to know λ_1. For small momentum transfer to the neutrino and at not too high neutrino energies, we may identify λ_1 with $F_1(0)$, the value of F_1 for zero value of $P\cdot\Delta$ and Δ^2. Although F_1 (and F_2) vanishes to order G_F in the four-fermion theory, in the W-meson theory the value of $F_1(0)$ is both nonzero and finite (cutoff-independent) in lowest-order perturbation theory, as has been recently shown by Tesoro.[11] To order e^2g^2 the result of Tesoro is

$$F_1(0)=(e^2g^2/3\pi)\,(1/M_W{}^4)[\ln(M_W{}^2/m_e{}^2)-1]. \qquad (4.4)$$

The effective potential U for a neutrino of momentum $\mathbf{p}$ and energy $p^0=|\mathbf{p}|$ arising from $H^{(1)}$ is then found to be

$$U=-F_1(0)p^0[(\mathbf{E}\times\hat{p})^2+(\mathbf{H}\times\hat{p})^2+2\mathbf{E}\times\mathbf{H}\cdot\hat{p}], \qquad (4.5)$$

where we have used Eq. (4.1) and the identities

$$\bar{u}(p)\Gamma^\alpha u(p)=2p^\alpha/p^0,$$
$$(p_\alpha F^{\alpha\rho})=(\mathbf{p}\cdot\mathbf{E},\, p^0\mathbf{E}-\mathbf{p}\times\mathbf{H}).$$

Using (4.4) and (4.5), we may estimate the angular deflection $\Delta\theta$ of a neutrino beam passing through, for example, an electrostatic field $\mathbf{E}(\mathbf{r})=(Q/r^2)\hat{r}$. The result is

$$\Delta\theta=\Delta p_\perp/p\sim 10^{-10}(Q/e)^2(\lambdabar_W/b)^4(M_W/M_p)^2, \qquad (4.6)$$

where $\lambdabar_W$ is the W-meson Compton wavelength and b is the impact parameter of the beam with respect to the charge Q. It can be seen from (4.6) that one would have to be able to pack an enormous charge Q into a small volume to get an appreciable deflection. For example, if $M_W=10M_p$, $b=1$ cm, and $Q=1$ C, (4.6) gives $\Delta\theta\sim 10^{-26}$. However, although it seems unlikely, the possibility of astrophysical effects may not be out of the question.

We note finally that in a Coulomb field such as the one just considered, the two-photon potential on a neutrino falls off as r^{-4}, as for other neutral particles, but that, unlike the other cases, the force is neither central nor velocity-independent.

V. SUMMARY AND DISCUSSION

A. Summary

In Sec. I A, the long-range potential $V_{\nu\bar{\nu}}$ arising from ν-pair exchange between two electrons was first computed, within the framework of a four-fermion $V-A$ interaction. [Eq. (2.13)]. It was pointed out that both the power law and the sign of the force could be understood on the basis of simple arguments. The spin-dependent part of the long-range potential was also obtained [Eq. (2.17)], and the e^--e^- and e^--e^+ forces were compared. In Sec. II B, the problem was reconsidered in the framework of a W-meson theory. Although allowance was made for an arbitrary number of virtual W mesons essentially the same result [Eq. (2.22)] was obtained, reminiscent of Thirring's theorem in quantum electrodynamics.

The dominant ν-pair forces between two hadrons and between a hadron and a lepton were considered in Sec. III, and were found to depend on the charge form factor of the neutrino [Eqs. (3.4) and (3.5)]. The weak long-range force between molecules was also obtained [Eq. (3.6)]. This force comes mainly from the interactions between the electrons, so that $V_{\nu\bar{\nu}}{}^{\mathrm{mol}}$ increases as Z^2. For spinless molecules the weak force will dominate the van der Waals force at sufficiently large distances.

The long-range force *on* neutrinos was studied in Sec. IV. This force arises predominantly from two-photon exchange between a charged particle and a neutrino and so depends on the amplitude for $\gamma+\gamma\to\nu+\bar{\nu}$. Although this amplitude vanishes in lowest order in the four-fermion theory, this is not the case in the W-meson theory. Using the recent work of Tesoro,[11] we were able to calculate the effective potential seen by a neutrino in an external electromagnetic field [Eq. (4.5)] and estimate the magnitude of the effect in a typical case—extremely small, as was to be expected.

B. Comparison of the Long-Range Forces Between Molecules

It is interesting to compare the strength of the ν-pair exchange force between (identical) molecules with the two other long-range forces known to act between such systems: the van der Waals force and the gravitational force. We have, from Eq. (3.6),

$$V_{\nu\bar{\nu}}(r)=(1/4\pi^3)(ZG_F)^2/r^5 \qquad (5.1)$$

for the ν-pair exchange potential. For the van der Waals potential between two identical molecules we write

$$V_{2\gamma}(r)=-(23/4\pi)(\alpha_M{}^{\mathrm{eff}})^2/r^7, \qquad (5.2)$$

[10] The fact that the photons were put on the mass shell in deriving (4.1) is reflected in the fact that (4.3) is invariant only under the restricted gauge transformations, $A_\mu\to A_\mu+\partial_\mu\chi$, with $\Box\chi=0$.

[11] A. Tesoro, Columbia University Ph.D. thesis, 1967 (unpublished).

where α_M^{eff} is an *effective* polarizability.[12] The gravitational potential is, of course,

$$V_{1g} = -GM^2/r, \tag{5.3}$$

where M is the molecular mass and G is the gravitational constant.

We first compare V_{1g} and $V_{\nu\bar{\nu}}$. Using $M \sim 2ZM_p$ for a typical molecule, we find that $-V_{1g} \gg V_{2\nu}$ for $r \gg 10^{-7}$ cm, more or less independent of the molecule under consideration. Thus the ratio of $V_{\nu\bar{\nu}}$ to V_{1g} is

$$V_{\nu\bar{\nu}}/V_{1g} \approx -[(5\times10^{-8}\text{ cm})/r]^4, \tag{5.4}$$

for r large compared to the molecular radius. We consider next the comparison between V_{1g} and $V_{2\gamma}$. From (5.2) and (5.3), we get

$$V_{2\gamma}/V_{1g} \approx [(2\times10^{-2}\text{ cm})/r]^6\eta, \tag{5.5}$$

where

$$\eta \equiv (\alpha_M^{\text{eff}} M_{\text{H}}/\alpha_{\text{H}} M)^2 \tag{5.6}$$

and α_{H} and M_{H} are the *electric* polarizability $(\frac{9}{2}a_0^3)$ and mass of a hydrogen atom, respectively. Finally, from (5.4) and (5.5) we get

$$V_{2\gamma}/V_{\nu\bar{\nu}} \approx -[(5\times10^{9}\text{ cm})/r]^2\eta. \tag{5.7}$$

The situation can be summarized as follows. From (5.4), we see that the gravitational interaction catches the neutrino-pair exchange interaction at $r \sim 10^{-7}$ cm. If we take $\eta = 1$ in (5.5), the gravitational interaction catches the van der Waals interaction at $r \sim 10^{-2}$ cm, for those molecules for which (5.2) is a good approximation at this latter distance. On the other hand, with $\eta = 1$ in (5.7), the ν-pair interaction does not catch the van der Waals interaction until r is about 7 earth radii. This distance is decreased if $\eta < 1$, but substantial reduction does not seem likely. For example, for the case of two xenon atoms with nuclear spin zero, a crude estimate gives $\eta \sim \frac{1}{4}$. Although estimates of molecular polarizability indicate that η is a decreasing function of molecular weight, only for fantastically small η could the ν-pair force dominate the van der Waals force in a region where these forces are not swamped by the gravitational force.

C. Concluding Remarks

We have seen that the weak long-range forces between elementary systems have rather novel properties. Especially for leptons, these properties are very directly related to the nature of the weak Lagrangian, so that experimental confirmation would be of great interest. Unfortunately, to say that detection of the lepton-lepton weak force in laboratory experiments is hopeless at present would seem to be an understatement.[13,14] Nevertheless, it is important to realize that a system containing electrons will not be neutral with regard to ν-pair forces, even it it is electrically neutral. Therefore, in spite of the weakness of these forces, they may play some role in astrophysical phenomena, where large particle numbers and densities are involved. This possibility is under investigation.

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[12] We write (5.2) by analogy with the result of H. Casimir and C. Polder, Phys. Rev. **73**, 360 (1948). These authors find $V_{2\gamma} = -(23/4\pi)\alpha_e^2/r^7$, for identical atoms with electric polarizability α_e. Contrary to the opinion sometimes stated, this form for the coefficient of r^{-7} is *not* universally valid; this point will be discussed in detail in J. Chem. Phys. (to be published). Incidentally, if charge is measured in Heaviside units, and if correspondingly one puts $\alpha_e = (4\pi)^{-1}\alpha_e'$, then the powers of π appearing in the expressions for $V_{2\gamma}$ and $V_{\nu\bar{\nu}}$ become the same, as one would expect.

[13] If we cut off the potential $V_{\bar{\nu}}^{(e,e)}$ [Eq. (2.19)] at small distances, we may estimate the magnitude of a level shift ΔE in positronium arising from the weak force. Thus, with $V_L(r) = G_F^2/4\pi^3 r^5 = 10^{-3}(\lambda_p/r)^5$ eV for $r \geq L$ and $V_L(r) = 10^{-3}(\lambda_p/L)^5$ for $r < L$, we have $\Delta E \sim \langle\phi|V_L|\phi\rangle \sim 10^{-3}(\lambda_p^5/2a_0^3L^2)$ eV $= 3\times10^{-20}(\lambda_p/L)^2$ eV, for the ground state. The most optimistic value of L is perhaps that associated with the value of r at which higher-order weak interactions may be expected to become important, i.e., $G_F^2/L^2 \sim 1$ or $(\lambda_p/L)^2 \sim 10^5$; this gives $\Delta E \sim 10^{-15}$ eV. For a $(\mu^+\mu^-)$ atom there is enhancement by a factor $(m_\mu/m_e)^3 \sim 10^7$, giving $\Delta E \sim 10^{-8}$ eV, which is 10^{-7} of the fine-structure splitting and might be on the verge of detectability; unfortunately such atoms are hard to come by.

[14] The lepton-baryon weak potential has an additional power of $\alpha = 1/137$ relative to the lepton-lepton weak potential, as seen from Eq. (3.5). Furthermore, because the former involves the emission of a photon by a baryon, it is damped at distances shorter than the nucleon Compton wavelength by the nucleon electromagnetic form factor, unlike the lepton-lepton force, which is not damped in this way. As a result, the lepton-baryon potential gives a contribution to the hfs of a μ^--p atom of the order of a few Hertz, which seems beyond the limit of feasible measurement for this quantity. We thank Dr. L. Wolfenstein for pointing out to us the role of the nucleon form factor in this connection.