Width $\Gamma_{X^0\to 2\gamma}$ as a Test of the Mass Formula for Boson Nonets

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It is generally believed that the physical η and X^0 particles are not pure SU_3 states, but result from octet-singlet mixing. Depending on the assumption of a linear or quadratic mass formula, different values can be obtained for the mixing angle. The latter can be used in turn to estimate the width of $X^0\to 2\gamma$ using the experimental values of $\Gamma_{\eta\to\gamma\gamma}$ and $\Gamma_{\pi^0\to\gamma\gamma}$. Note that the quadratic mass formula predicts values considerably larger than the linear one. An upper bound of 20–30 keV for the $X^0\to 2\gamma$ width, which is not outside the experimental possibilities—e.g., with the Primakoff-effect technique—would be evidence against the use of a quadratic mass formula within the standard SU_3 scheme.

ACCORDING to a recent experiment,[1] the width of the decay $\eta\to 2\gamma$ is $\Gamma_{\eta\to 2\gamma}=1.21\pm0.26$ keV.

We will make some simple considerations on this result. We put[2]

$$X^0=\eta_1\cos\alpha+\eta_8\sin\alpha\,,$$
$$\eta=-\eta_1\sin\alpha+\eta_8\cos\alpha\,. \tag{1}$$

SU_3 then predicts[3]

$$\Gamma_{\eta\to 2\gamma}=\tfrac{1}{3}(m_\eta/m_{\pi^0})^3[\cos\alpha-(A'/M')\sin\alpha]^2\Gamma_{\pi^0\to 2\gamma}\,, \tag{2}$$

$$\Gamma_{X^0\to 2\gamma}=\tfrac{1}{3}(m_{X^0}/m_{\pi^0})^3[\sin\alpha+(A'/M')\cos\alpha]^2\Gamma_{\pi^0\to 2\gamma}\,, \tag{3}$$

where A' is the amplitude $\eta_1\to 2\gamma$ and M' the amplitude $\eta_8\to 2\gamma$.

The mixing angle α can be deduced[2,4,5] from the known masses of the pseudoscalar nonet. Both hypotheses will be left open—that a quadratic or a linear mass formula holds for the boson nonet.

We get

$$\alpha_{\rm sq}=10.4°\,,\quad \alpha_{\rm lin}=23.4°\,.$$

We now use relation (2) to evaluate the A'/M' ratio:

$$A'/M'=22.1\pm2.6\,, \tag{4}$$
$$A'/M'=-11.2\pm1.3\,,\quad \text{using } \alpha_{\rm sq}=10.4°\,, \tag{5}$$

and

$$A'/M'=9.9\pm1.2\,, \tag{6}$$
$$A'/M'=-5.3\pm0.6\,,\quad \text{using } \alpha_{\rm lin}=23.4°\,. \tag{7}$$

We have used $\Gamma_{\pi^0\to 2\gamma}=6\pm0.6$ eV.[6]

The values (4)–(7) can be used as input to (3) to get the width $\Gamma_{X^0\to 2\gamma}$. We obtain

$$\Gamma_{X^0\to 2\gamma}=343\pm90\text{ keV}\qquad A'/M'=22.1\,,\ \alpha_{\rm sq}=10.4° \tag{8}$$
$$=84\pm22\text{ keV}\qquad A'/M'=-11.2\,,\ \alpha_{\rm sq}=10.4° \tag{9}$$
$$=64\pm16\text{ keV}\qquad A'/M'=9.9\,,\ \alpha_{\rm lin}=23.4° \tag{10}$$
$$=14\pm4\text{ keV}\qquad A'/M'=-5.3\,,\ \alpha_{\rm lin}=23.4°\,. \tag{11}$$

Experimentally, one has not much information to compare with. We know that

$$\Gamma_{X^0\to\text{all modes}}<4\text{ MeV},\quad (\text{Ref. 7})$$
$$\Gamma_{X^0\to 2\gamma}<0.15(\Gamma_{X^0\to\text{all modes}})<600\text{ keV},\quad (\text{Ref. 8})$$
$$\Gamma_{X^0\to\rho\gamma}<1\text{ MeV},\quad (\text{Ref. 7}) \tag{12}$$
$$\Gamma_{X^0\to\text{neutrals}}=0.26\pm0.04\Gamma_{X^0\to\text{all modes}},\quad (\text{Ref. 9}) \tag{13}$$
$$\Gamma_{X^0\to\pi^+\pi^-\eta}=0.48\pm0.05\Gamma_{X^0\to\text{all modes}},\quad (\text{Ref. 9}). \tag{14}$$

The upper experimental limit $\Gamma_{X^0\to 2\gamma}<600$ keV is consistent with all the solutions (8)–(11).

A somewhat lower figure can be obtained using isotopic-spin invariance and $\Gamma_{\eta\to\text{neutrals}}\Gamma_{\eta\to\text{all modes}}=0.73$. From (14), one predicts $\Gamma_{X^0\to\pi^0\pi^0\eta(\eta\to\text{neutrals})}=0.18$ $\Gamma_{X^0\to\text{all modes}}$. Comparing with (13), we see that about 8% of the total width is allowed for the $X^0\to 2\gamma$ decay mode. Hence $\Gamma_{X^0\to 2\gamma}\lesssim 400$ keV, which again is consistent with all values (8)–(11).

[1] C. Bemporad, P. L. Braccini, L. Foà, K. Lübelsmeyer, and D. Schmitz (to be published). We are grateful to the authors for having made their result available to us prior to publication.
[2] S. L. Glashow, in *Group Theoretical Concepts and Methods in Elementary Particle Physics*, edited by F. Gürsey (Gorden and Breach Science Publishers, Inc., New York, 1964), p. 303; S. Coleman, S. L. Glashow, and D. J. Kleitman, Phys. Rev. **135**, B779 (1964).
[3] R. H. Dalitz and D. G. Sutherland, Nuovo Cimento **37**, 1777 (1965); **38**, 1945(E) (1965).
[4] M. Gell-Mann, *The Eightfold Way* (W. A. Benjamin, Inc., New York, 1964); Y. Ne'eman, Nucl. Phys. **26**, 222 (1961).
[5] S. Okubo, Progr. Theoret. Phys. (Kyoto) **27**, 949 (1962).
[6] P. Stamer, S. Taylor, E. L. Koller, T. Heutter, J. Grauman, and D. Pandoulas, Phys. Rev. **151**, 1108 (1966), average of Table VI.
[7] A. H. Rosenfeld, A. Barbaro-Galtieri, W. J. Podolsky, L. R. Price, M. Roos, P. Soding, W. J. Willis, and C. G. Wohl, University of California Radiation Laboratory Report No. UCRL-8030, 1967 (unpublished).
[8] G. R. Kalbfleisch, O. I. Dahl, and A. Rittenberg, Phys. Rev. Letters **13**, 349a (1964).
[9] Average of the results of G. W. London *et al.*, Phys. Rev. **143**, 1034 (1966); M. Goldberg *et al.*, Phys. Rev. Letters **13**, 249 (1964); and of Ref. 8.

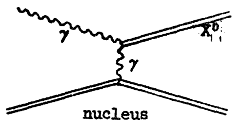

FIG. 1. Diagram for photoproduction of X^0 mesons.

Up to now, we have used only SU_3 and/or experimental data. In order to make a choice between our different solutions (4)–(7), we try to use the pole-dominance model due to Gell-Mann, Sharp, and Wagner (GSW).[10]

In this model, the following relations hold[3,11]:

$$\frac{\Gamma_{X^0\to 2\gamma}}{\Gamma_{X^0\to \rho\gamma}} \approx 0.1\left(\frac{1+(M'/A')\tan\alpha}{1+(M/A)\tan\alpha}\right)^2, \tag{15}$$

$$\frac{\Gamma_{\eta\to\pi^+\pi^-\gamma}}{\Gamma_{\eta\to 2\gamma}} = 0.233\left(\frac{1-(A/M)\tan\alpha}{1-(A'/M')\tan\alpha}\right), \tag{16}$$

$$\Gamma_{\eta\to 2\gamma} = 0.67\times 10^{-3}[\cos\alpha-(A'/M')\sin\alpha]^2\Gamma_{\omega\to\pi^0\gamma}. \tag{17}$$

Here A is the amplitude for $\eta_1 \to \rho\gamma$ and M that for $\eta_8 \to \rho\gamma$. As a consequence of the model $2A/M = A'/M'$.[3]

Using (12) and (15), we get $\Gamma_{X^0\to 2\gamma} \lesssim 100$ keV, quite independently of the values inserted for α and A'/M'. On this basis, the solution (8) should be rejected.

From (16), with the different solutions (4)–(7), we find

$$\begin{aligned} \Gamma_{\eta\to\pi^+\pi^-\gamma}/\Gamma_{\eta\to 2\gamma} &= 0.026, \quad \text{using (4)} \\ &= 0.10, \quad \text{using (5)} \\ &= 0.028, \quad \text{using (6)} \\ &= 0.10, \quad \text{using (7)}. \end{aligned}$$

A comparison with the experimental value[7]

$$\Gamma_{\eta\to\pi^+\pi^-\gamma}/\Gamma_{\eta\to 2\gamma} = 0.15\pm 0.03$$

shows that the solutions (4) and (6) do not fit the model.

[10] M. Gell-Mann, D. Sharp, and W. Wagner, Phys. Rev. Letters 8, 261 (1962).

[11] M. Veltman and J. Yellin, Phys. Rev. 154, 1469 (1967).

However, on the basis of the GSW model, one can also obtain the prediction

$$\Gamma_{\omega\to\pi\gamma}/\Gamma_{\pi^0\to 2\gamma} = 3.3\times 10^4,$$

independently of $\Gamma_{\eta\to 2\gamma}$, A'/M', and α [i.e., from the formulas (2) and (17) reported above].

Considering that the experimental value is

$$\Gamma_{\omega\to\pi\gamma}/\Gamma_{\pi^0\to 2\gamma} = (19.2\pm 3.5)\times 10^4,$$

one must conclude tha the GSW model does not work satisfactorily in this case. This can cast some doubts on the reliability of our previous results based on GSW. Some other evidence that solution (4) should be rejected can be obtained if one believes in the current estimates[3,12] $\Gamma_{X^0(\text{total})} < 1$ MeV. In fact, using our previous result $\Gamma_{X^0\to 2\gamma}/\Gamma_{X^0\to \text{all modes}} < 8$–$10\%$, one would obtain $\Gamma_{X^0\to 2\gamma} < 100$ keV.

To summarize: Solution (4) is probably to be rejected; the width $\Gamma_{X^0\to 2\gamma}$ is in the range 10–100 keV; there is some reason to exclude solution (6); there is not enough information to discriminate between a quadratic and a linear mass formula for the boson nonets.

In view of this it seems to us that a measurement of the width $\Gamma_{X^0\to \text{total}}$, or better $\Gamma_{X^0\to 2\gamma}$, is highly desirable, since an upper bound like $\Gamma_{X^0\to 2\gamma} \lesssim 20$ keV would provide definite evidence against the quadratic mass formula.

With the same experimental method used to determine the width $\Gamma_{\eta\to 2\gamma}$, i.e., the "Primakoff effect,"[1,13] an upper limit of 20 keV for $\Gamma_{X^0\to 2\gamma}$ should not be out of experimental possibility. For $\Gamma_{X^0\to 2\gamma} = 20$ keV, the production cross section according to the diagram in Fig. 1 would be ~ 3 times larger than for η production.

The other experimentally well-known nonet is the 1^-. For this nonet, the use of a linear rather than quadratic mass formula would change the mixing angle by $\sim 2°$, an effect which is up to now outside the possibility of experimental checking.

[12] L. M. Brown and H. Faier, Phys. Rev. Letters 13, 73 (1964).

[13] G. Bellettini, C. Bemporad, P. L. Braccini, L. Foà, and M. Toller, Phys. Letters 3, 170 (1963).