Fermion Trajectory Parametrization and Superconvergence

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A parametrization of the nucleon and Δ trajectory is given, using both the position and the width of the resonances in a dispersion relation. The nucleon trajectory requires and accommodates a D-wave πN resonance in the neighborhood of 1670 MeV. Another significant outcome of the scheme is that the Δ trajectory has an intercept, at $W=0$, of -0.69 and hence leads to a superconvergence relation for $\pi^- p$ scattering invariant amplitudes in the nearly backward direction ($u=0$). The resulting sum rules are satisfied only if there exists a strong attractive s-wave interaction (for example, a σ meson) or other equivalent effects (such as an f meson) in the $\pi\pi \to N\bar{N}$ channel.

1. INTRODUCTION

IT has been possible to identify several high-energy resonances in πN scattering as the Regge recurrences of their low-energy counterparts. Three such trajectories seem to be well established and are referred to as N_α ($I=\frac{1}{2}$, even parity, even signature), N_γ ($I=\frac{1}{2}$, odd parity, odd signature), and Δ_δ ($I=\frac{3}{2}$, even parity, odd signature). One of the puzzling features[1] of these trajectories is that they are approximately straight lines in W^2 and hence $\alpha_i(W)$ and $\alpha_i(-W)$ are equal in each case. By MacDowell symmetry,[2] the amplitude of a given parity at W is related to that of the opposite parity at $-W$: thus $\alpha_i(-W)$ describes the Regge pole of opposite parity at $+W$. Then, such a simple formula for $\alpha_i(W)$ would correspond to mass-degenerate resonances of both parities. But, with the possible exception of a $\frac{5}{2}^-$, $I=\frac{1}{2}$ resonance at about 1670 MeV,[3] no opposite-parity partners of the resonances in a fermion trajectory are known. This motivates us to introduce another parametrization of the fermion trajectories which makes use of both the position and the widths of the known resonances in a dispersion relation for $\alpha_i(W)$. Such a parametrization, while reproducing the approximate W^2 behavior in the "low"-energy region, (i) predicts trajectories that continue to rise for quite large energies and (ii) gives a possibility of avoiding the opposite-parity resonances in the low-energy region in cases where they are not found.

In Sec. 2, we introduce a twice-subtracted dispersion relation for the fermion trajectory and obtain a fit for N_α and Δ_δ trajectories as a function of W. The trajectory for N_α appears to be quite similar to the parametrization given by Chiu and Stack,[4] which is based on the backward $\pi^+ p$ elastic scattering. The trajectory for Δ_δ, besides avoiding the realization of any opposite-parity states, suggests a value of $\alpha_\Delta(0)=-0.69$. Then in Sec. 3 we show that this implies at $u=0$ a superconvergent asymptotic behavior for both the invariant amplitudes A and B in $\pi^- p$ elastic scattering (which is pure $I=\frac{3}{2}$ in the u channel). The sum rule that would result from such requirements involves parameters of the resonances in the πN system as well as contributions from the meson resonances that can couple to $\pi\pi$ and $N\bar{N}$ systems. Whereas the superconvergence relation for the amplitude B is satisfied quite simply, the amplitude A requires, in addition, a very strong attractive s-wave interaction, presumably in the form of s-wave resonances or equivalent effects arising from the non-spin-flip part of the higher spin-meson exchanges. The former alternative is reminiscent of several dynamical models for πN and NN scattering, where one finds a need for an attractive s-wave force; the precise magnitude of such a coupling is, however, not in agreement.

2. FERMION TRAJECTORIES

In this section we summarize the details of the parametrization fermion trajectory $\alpha_i(W)$ obtained earlier in order to explain the πN charge-exchange polarization.[5] $\alpha_i(W)$, where i stands for the family label, is expected to satisfy a dispersion relation in W, where W is the total center-of-mass (c.m.) energy, with cuts along $W_0(=m+u)$ to ∞ and $-W_0$ to $-\infty$. If the first cut is associated with $l=J-\frac{1}{2}$ partial waves, then the second is related to $l=J+\frac{1}{2}$. The information about the imaginary part of $\alpha_i(W)$ is available at the resonances through the relation

$$\mathrm{Im}\alpha_i(W)=\tfrac{1}{2}\Gamma\frac{d}{dW}\mathrm{Re}\alpha_i(W)\,|_{W=W_R}\,, \quad (1)$$

where Γ is the resonance width. We parametrize $\mathrm{Im}\alpha_i(W)$, consistent with the threshold behavior, through

$$\mathrm{Im}\alpha_i(W)=(W-W_0)^\lambda P_1(W)/P_2(W)\,, \quad (2)$$

$$\lambda=\alpha_0\equiv\alpha_i(W_0)\,,\ \text{for } \Delta_\delta$$

and

$$\lambda=\alpha_0+1\,,\ \text{for } N_\alpha \text{ and } N_\gamma\,, \quad (3)$$

where $P_1(W)$ and $P_2(W)$ are polynomials in W. Since

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[1] V. Barger and D. Cline, Phys. Rev. Letters 16, 913 (1966).

[2] S. W. MacDowell, Phys. Rev. 116, 774 (1959); see also B. R. Desai, Phys. Rev. Letters 17, 498 (1966).

[3] A. H. Rosenfeld *et al.*, Rev. Mod. Phys. 39, 1 (1967).

[4] C. B. Chiu and J. D. Stack, Phys. Rev. 153, 1575 (1967).

[5] B. R. Desai, D. T. Gregorich, and R. Ramachandran, Phys. Rev. Letters 18, 565 (1967).

in each trajectory the resonances are found to occur only in either the $l=J-\frac{1}{2}$ or the $l=J+\frac{1}{2}$ set of partial waves, the information regarding the imaginary part of the trajectory is available for only one of the two cuts. We shall ignore the other cut and write for $\alpha_i(W)$

$$\alpha_i(W)=\alpha_0+b_i(W-W_0)+\frac{(W-W^0)^2}{\pi}\int_{W_0}^{\infty}\frac{dW'\,\mathrm{Im}\alpha_i(W')}{(W'-W_0)^2(W'-W)}, \quad (4)$$

where α_0 and b_i are subtraction constants. We find that at least two subtractions are necessary, since a single subtraction with the known $\mathrm{Im}\alpha_i$ is unable to sustain the rising nature of $\mathrm{Re}\alpha_i$ even at the low energies. The parameters are fixed using the known resonances (listed in Table I) in Eqs. (1), (2), and (4). It was possible to obtain a good fit with the polynomials $P_1(W)$ and $P_2(W)$ of order no higher than quadratic. The $\mathrm{Re}\alpha_i$ for N_α and Δ_δ are plotted in Fig. 1.

We believe that the subtraction constants partly contain some of the effects of the cut we have ignored. Furthermore, since we have made two subtractions, our results are insensitive to the precise behavior of $\mathrm{Im}\alpha_i$ at infinity.[6] The trajectory so obtained predicts further resonances at higher energies and these were used to explain the nonvanishing polarization in the πN charge exchange at 6 and 11.2 GeV/c.[5] We wish to concentrate, in this paper, on the prediction of α_i near $W=0$, which is, in fact, related to the asymptotic behavior for πN scattering in the almost backward directions.

Indeed, since $W=0$ is equidistant from the two cuts, the contributions from both cuts are equally important there. The absence of one set of resonances could perhaps be argued as an indication that the contribution of the corresponding cut is of less importance. Though the procedure in (4) is not completely justifiable here, we shall be content to examine whether the consequent results are plausible.

We find that all the trajectories turn around in the W plane and could correspond to possible resonances of opposite parity. In particular, the nucleon trajectory is such as to give a $\frac{5}{2}^-$ resonance in the neighborhood of 1600 MeV, which is consistent with a D-wave resonance at 1670 MeV, now rather well established.[3] The trajectory almost exactly reproduces that of Chiu and Stack,[4] including the intercept

$$\alpha(W=0)=-0.34\,, \quad \text{for } N_\alpha\,. \quad (5)$$

The analysis of Chiu and Stack was based on π^+p backward scattering, and thus the above value of α_{N_α} should be consistent with the asymptotic behavior in the crossed channel due to the exchange of the nucleon trajectory.

[6] The two subtractions also imply trajectories that go to infinity as W goes to infinity. That the trajectories may rise indefinitely, as a consequence of inelastic effects, has been pointed out by S. Mandelstam (unpublished). Here, however, the trajectory goes to either $+\infty$ or $-\infty$, depending on the degree of convergence of $\mathrm{Im}\alpha_i$, without seriously affecting the energy region of interest.

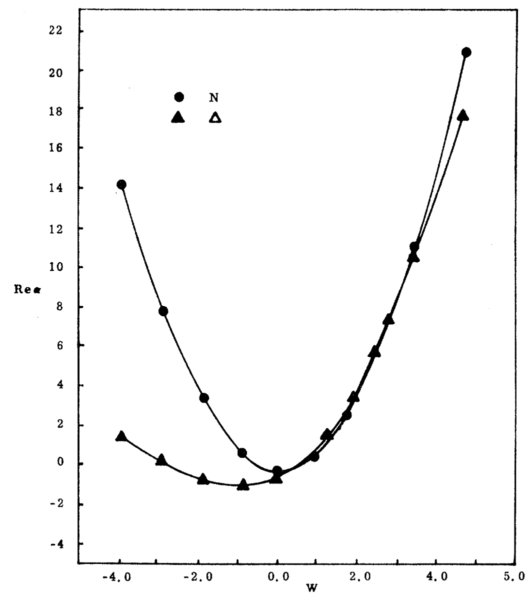

FIG. 1. Real part of α for N_α and Δ trajectories versus c.m. energy in GeV.

The Δ_δ trajectory, it is clear from Fig. 1, does not lead to an opposite-parity resonance until about 3500 MeV and hence is compatible with the present experimental situation. Further, the feature to be particularly noted is that it corresponds to

$$\alpha(W=0)=-0.69\,, \quad \text{for } \Delta_\delta\,. \quad (6)$$

The π^-p backward-scattering results are, however, not good enough to establish such an intercept unambiguously. But Eq. (6), as we shall see in the next section, implies a superconvergent asymptotic behavior and hence leads to a sum rule, the verification of which is an alternative check of the equation.

3. SUPERCONVERGENCE AND SUM RULES

The exchange of fermion Regge poles in the u channel gives rise to an asymptotic behavior[7]:

$$A^{(I)}\sim s^{[\alpha_i(\sqrt{u})-1/2]}\,, \quad s\to\infty \quad (7a)$$

$$B^{(I)}\sim s^{[\alpha_i(\sqrt{u})-1/2]}\,, \quad s\to\infty \quad (7b)$$

where A and B are the invariant amplitudes and I specifies the isospin in the u channel and that of the trajectories α_i exchanged. When $I=\frac{3}{2}$, the contribution is given entirely by the Δ_δ trajectory and by Eq. (6); this implies that A and B have a behavior faster than

[7] See, for example, S. C. Frautschi, M. Gell-Mann, and F. Zachariasen, Phys. Rev. **126**, 2204 (1962).

TABLE I. Resonance parameters and their contributions to the superconvergence relations (8a) and (8b).

Resonances	I	J^P	Γ in GeV	x	Contributions to the integral $\left[\frac{1}{\pi}\int ds\, \mathrm{Im}A^{3/2}(4\pi)^{-1}\right]$ in GeV	Contributions to the integral $\left[\frac{1}{\pi}\int ds\, \mathrm{Im}B^{3/2}(4\pi)^{-1}\right]$
$N_\alpha(938)$	$\frac{1}{2}$	$\frac{1}{2}^+$	...	...	0	$29.2(=2g^2/4\pi)$
$N_\alpha(1688)$	$\frac{1}{2}$	$\frac{5}{2}^+$	0.10	0.80	−0.52	4.52
$N_\gamma(1525)$	$\frac{1}{2}$	$\frac{3}{2}^-$	0.12	0.76	0.44	−1.34
$N_\gamma(2190)$	$\frac{1}{2}$	$\frac{7}{2}^-$	0.20	0.15	0.74	−0.94
$N_\gamma(2650)$	$\frac{1}{2}$	$11/2^-$	0.36	0.07	0.98	−1.12
$N_\gamma(3030)$	$\frac{1}{2}$	$15/2^-$	0.40	0.01	0.18	−0.26
$\Delta_\delta(1238)$	$\frac{3}{2}$	$\frac{3}{2}^+$	0.12	1.00	4.68	−6.20
$\Delta_\delta(1924)$	$\frac{3}{2}$	$\frac{7}{2}^+$	0.17	0.35	0.44	−0.72
$\Delta_\delta(2420)$	$\frac{3}{2}$	$11/2^+$	0.31	0.11	0.38	−0.64
$\Delta_\delta(2840)$	$\frac{3}{2}$	$15/2^+$	0.40	0.03	0.24	−0.34
Total s-channel contributions					7.6	22.2
$\rho(760)$	1	1^-	0.13	...	5.6	−22.4
$\sigma(?)$[a]	0	0^+	?	...	$m_\sigma(g_{\sigma\pi\pi}g_{\sigma NN}/4\pi)$	0

[a] See the text.

s^{-1}, requiring therefore at $u=0$[8]:

$$\frac{1}{\pi}\int_{-\infty}^{\infty} ds\, \mathrm{Im}A^{(3/2)}(s,t,u=0)=0\,, \quad (8a)$$

$$\frac{1}{\pi}\int_{-\infty}^{\infty} ds\, \mathrm{Im}B^{(3/2)}(s,t,u=0)=0\,. \quad (8b)$$

The rest of this section will be on the verification of the validity of (8a) and (8b).

The integrals in (8a) and (8b) get contributions from both s-channel and t-channel (arising from the left-hand cut) poles. In the direct channel these correspond to the same families of resonances that made the Regge trajectories. Their contributions can be evaluated in the narrow-width approximation as follows[9]:

$$\frac{1}{\pi}\int ds\, \mathrm{Im}A^{(3/2)}=\tfrac{2}{3}\Big(\sum_{N_i} A_{N_i}+\sum_{\gamma_i} A_{\gamma_i}\Big)+\tfrac{1}{3}\sum_{\Delta_i} A_{\Delta_i}\,, \quad (9a)$$

$$\frac{1}{\pi}\int ds\, \mathrm{Im}B^{(3/2)}=\tfrac{2}{3}\Big(\sum_{N_i} B_{N_i}+\sum_{\gamma_i} B_{\gamma_i}\Big)+\tfrac{1}{3}\sum_{\Delta_i} B_{\Delta_i}\,. \quad (9b)$$

The summations N_i, γ_i, and Δ_δ are over the set of N_α, N_γ, and Δ_δ resonances. A typical contribution of a resonance at $W=W_i$ is evaluated as

$$A_i=\pm(4\pi X_i\Gamma W_i/k_i^3)[(W_i\pm m)(E_i\mp m)P_{J+\frac{1}{2}}'(z) +(W_i\mp m)(E_i\pm m)P_{J-\frac{1}{2}}'(z)]\,, \quad (10a)$$

$$B_i=(4\pi X_i\Gamma W_i/k_i^3) \times[(E_i\mp m)P_{J+\frac{1}{2}}'(z)-(E_i\pm m)P_{J-\frac{1}{2}}'(z)]\,, \quad (10b)$$

where the upper signs are appropriate for the resonances that belong to the Δ_δ trajectory, while the lower signs are for those in the N_α and N_γ families. W_i, k_i, and E_i are, respectively, the total energy (mass of the resonance), the momentum, and the energy of the nucleon in the c.m. system evaluated at the resonance. X_i and Γ are the elasticity and the width of the resonance. Finally, when $u=0$, at the resonances,

$$z=-1+(m^2-\mu^2)^2/2k_i^2W_i^2\,.$$

The contribution from each resonance so calculated is listed in Table I.

The contributions to the integrals from the left-hand cut comes from the meson trajectories. By Bose statistics, the $\pi\pi$ system can couple only to $I=1$ odd-signature (e.g., ρ) and $I=0$ even-signature (σ and f) trajectories. Since the information concerning the higher members is too scanty, unlike the case of fermion trajectories, we are forced to consider the low-lying members on a special footing. The ρ-pole contribution to the A and B amplitudes is given by

$$A^{(3/2)}(s,t)=-(g_{\rho\pi\pi}g_m/2m) \times(2s+t-2m^2-2\mu^2)/(m_\rho^2-t)\,, \quad (11a)$$

$$B^{(3/2)}(s,t)=2(g_{\rho\pi\pi}(g_e+g_m)/(m_\rho^2-t))\,, \quad (11b)$$

where g_e and g_m are the ρ-meson coupling to $N\bar{N}$ of the electric (γ_μ) and magnetic ($\sigma_{\mu\nu}$) type.[10] On the as-

[8] Superconvergence relations at $t=0$ have been obtained and saturated with low-lying resonances in various processes: V. de Alfaro, S. Fubini, C. Rossetti, and G. Furlan, Phys. Letters **21**, 576 (1966); B. Sakita and K. C. Wali, Phys. Rev. Letters **18**, 29 (1967); P. Babu, F. Gilman, and M. Suzuki, Phys. Letters **24B**, 65 (1967). There is, however, considerable doubt as to the meaning of the saturation of these relations by a few low-lying resonances [V. de Alfaro, S. Fubini, F. Furlan, and C. Rossetti, Ann. Phys. (N. Y.) 44, 165 (1967)]. The fortuitous circumstance, when such a saturation scheme yields a good agreement, may merely be a reflection of some higher symmetry relation valid in the low-energy region. This, for example, appears to happen in static strong-coupling symmetry scheme [see L. K. Pande, Phys. Letters **24B**, 243 (1967)].

[9] The resonance parameters are taken from Ref. 1.

[10] Our definition for ρ-meson coupling constants g_e and g_m corresponds to that of a and b of M. Parkinson, Phys. Rev. **143**, 1359 (1966).

sumption that the nucleon isovector form factors are dominated by the ρ mesons, the ratio g_m/g_e is given by

$$g_m/g_e \approx 1.85/0.5 = 3.70. \quad (12)$$

Further, consistent with the hypothesis of universality of isovector coupling, it has been found that

$$g_e \approx g_{\rho\pi\pi}, \quad g_{\rho\pi\pi}^2/4\pi = 2.4. \quad (13)$$

With Eqs. (12) and (13) in Eq. (11), the ρ-meson contributions to the integrals (8a) and (8b) are given by

$$\left[\frac{1}{\pi}\int ds\, \mathrm{Im}A^{(3/2)}\right]_\rho = 4\pi\times 3.70\frac{(2m^2+2\mu^2-m_\rho^2)}{2m}\frac{g_{\rho\pi\pi}^2}{4\pi}$$

$$\approx 4\pi 5.65 \text{ GeV}, \quad (14a)$$

$$\left[\frac{1}{\pi}\int ds\, \mathrm{Im}B^{(3/2)}\right] = -4\pi\times 9.40\frac{g_{\rho\pi\pi}^2}{4\pi}$$

$$\approx -4\pi\times 22.4. \quad (14b)$$

On adding up the contributions from the s channel and (14b), we notice that the sum rule for the amplitude B is satisfied quite well. The amplitude A, however, cannot yet satisfy the sum rule, since it receives contributions with the same sign from both the s channel and the ρ pole. We would like to regard this failure as an indication of a strong attractive s-wave force in the t channel for πN scattering. Representing this s-wave effect by a σ resonance, we may write, projecting out its $I=\frac{3}{2}$ part in the u channel,

$$A_\sigma = -m_\sigma g_{\sigma N\bar{N}} g_{\sigma\pi\pi}/(m_\sigma^2 - t). \quad (15)$$

Since the s-wave part in the t channel cannot contribute to B, the corresponding sum rule is not affected. For A, this gives a further contribution to the integral (8a):

$$\left[\frac{1}{\pi}\int ds\, \mathrm{Im}A^{(3/2)}\right]_\rho = 4\pi m_\sigma\left(\frac{g_{\sigma N\bar{N}}g_{\sigma\pi\pi}}{4\pi}\right). \quad (16)$$

If (8a) is satisfied with the inclusion of σ, then we must require

$$g_{\sigma N\bar{N}}g_{\sigma\pi\pi}/4\pi \approx -13.1/m_\sigma, \quad m_\sigma \text{ in GeV}. \quad (17)$$

While this lends further support to the possibility of a strong attractive s-wave force, if it were to correspond to the σ meson conjectured in various other contexts,[11] we have to ascribe to it a coupling strength much larger than expected.

As an alternative or in addition to the σ mesons, we can consider the possibility of the higher-spin mesons[12] contributing to the sum rules. These mesons normally contribute to both spin-flip and non-spin-flip partial-wave amplitudes in the t channel. These will contribute only to A if their coupling is predominantly of non-spin-flip type. Such an effect, arising from the f meson, can be regarded as supplementing the σ-meson contributions. We shall not, however, go into further details, since these introduce such an element of arbitrariness that no precise consequence of the sum rule is then derivable.

4. CONCLUSION

To summarize, we have given a parametrization of Regge trajectories, making use of both the position and the widths of resonances. Such a parametrization suggests superconvergence of both $\pi^- p$ invariant amplitudes in the nearly backward direction. The known πN resonances together with the ρ meson satisfy the relation for the B amplitude. The superconvergence for A demands, in addition, the existence of a σ meson and/or equivalent non-spin-flip contributions of higher-spin mesons such as f. If we were to regard the superconvergence as essentially the consistency condition between the forces and the resonances such forces produce, the requirement of the σ meson or an attractive s-wave force would be an old story.[13] In the dynamical calculations of πN scattering and NN scattering, the need for such a term has always existed, and perhaps this conclusion is only to be expected.

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[11] L. M. Brown and P. Singer, Phys. Rev. Letters **8**, 460 (1962); A. N. Mitra and S. Ray, Phys. Rev. **135**, B146 (1964); C. Lovelace, R. M. Heinz, and A. Donnachie, Phys. Letters **22**, 332 (1966).

[12] While the work was being done, we came across a report on the $u=0$ superconvergence relations [D. Griffiths and W. Palmer, Phys. Rev. **161**, 1606 (1967)]. Here the authors appear to take the point of view expressed in Ref. 8 and demand the saturation of "superconvergence" relations for *all* πN amplitudes by N, N^*, ρ, and f. For example, at $t=0$, they saturate B^+ with N and N^*, and find it to be in good agreement with the superconvergence assumption, even though the Regge asymptotic behavior would suggest otherwise. At $u=0$, they demand superconvergence of both $I=\frac{1}{2}$ and $\frac{3}{2}$, in contrast with our relations for only $I=\frac{3}{2}$. They point out the need for a t-channel contribution in addition to the ρ meson in order to avoid inconsistencies among the sum rules. If this is taken to be the f meson, they also obtain results which indicate a large contribution to the amplitude A and a negligible amount to B. In view of the relatively small contributions from the higher πN resonances the substantial agreement between their results and ours is not surprising. We thank Professor B. Sakita and Professor K. C. Wali for bringing this paper to our attention.

[13] A. Scotti and D. Y. Wong, Phys. Rev. **138**, B145 (1965); J. S. Ball and D. Y. Wong, *ibid.* **133**, B179 (1964); A. Donnachie and J. Hamilton, *ibid.* **133**, B1053 (1964), and further references therein.