Absorption in High-Energy Photoproduction*

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Absorption in high-energy photoproduction has been derived using a Regge amplitude. It is found that this depends strongly on the square of the four-momentum transfer t, smaller values of $|t|$ giving much less absorption. This behavior differs markedly from that of the Drell peripheral model with a constant absorption factor. The absorption is calculated for s and t values corresponding to energies encountered in a recent SLAC experiment, and is seen to reduce the cross section by factors of 1/220 to 1/25, depending on t. Factors of this order are needed to bring experiment and theory into agreement. The high-energy asymptotic behavior of the amplitude with absorption is found to differ from the usual Regge behavior. The usual absorption using a Born amplitude is also elaborated and generalized and is compared with recent DESY data. The well-known high-energy Regge behavior of the cross section without absorption, $d\sigma/dt=C(t)s^{2\alpha(t)-2}$, is derived for all spins in an Appendix.

I. ABSORPTION FORMALISM

THE peripheral model cannot account for the data in high-energy two-body collisions unless absorption is added.[1-3] Experiments on the high-energy photoproduction of antiprotons at DESY, for example, show that experimental differential cross sections are down from those predicted in a pure peripheral model by a factor of 130.[4] In this paper, the effects of absorption in the high-energy photoproduction of antinucleons (Fig. 1) will be considered. Only this diagram will be included in what follows. Other diagrams, such as Fig. 2, which might be added for gauge invariance, should not contribute appreciably at the very high energies and small angles for the antinucleon under consideration.

The usual absorption formalism[3,5,6] has been generalized to Regge amplitudes by Arnold.[7] He justifies the application of absorptive corrections to Regge amplitudes, using an optical-model point of view. Arnold considers charge exchange reactions in which the incoming particles are the same as the outgoing particles (Fig. 3). We can rewrite Arnold's results as

$$A(s,\sqrt{-t})_{\mathrm{abs}}=\{\{A(s,\sqrt{-t})\}_{\mathrm{F.B.}}\times[1+i\rho(s)\{f_{\mathrm{el}}(s,\sqrt{-t})\}_{\mathrm{F.B.}}]\}_{\mathrm{I.F.B.}},\quad(1)$$

where $\rho(s)$ is a suitable phase-space factor[8,9] and s and t are the usual Mandelstam variables. $f_{\mathrm{el}}(s,\sqrt{-t})$ is the elastic amplitude for the colliding initial or final particles and $A(s,\sqrt{-t})$ is the amplitude for the process under consideration without absorption. The Fourier-Bessel transform and its inverse are defined by

$$m(s,\sqrt{-t})=\int_0^\infty n(s,b)J_0(b\sqrt{-t})bdb\equiv\{n(s,b)\}_{\mathrm{I.F.B.}},\quad(2)$$

$$n(s,b)=\int_0^\infty m(s,\sqrt{-t})J_0(b\sqrt{-t})\,(\sqrt{-t})d(\sqrt{-t})\equiv\{m(s,\sqrt{-t})\}_{\mathrm{F.B.}}.\quad(3)$$

These are equivalent to partial-wave sums for small scattering angles. The amplitude (1) has equal absorption in the initial and final states. $A(s,\sqrt{-t})$ can be either the Born amplitude as in Gottfried and Jackson[6] or a Regge amplitude as in Arnold's work.[7]

For the process in Fig. 1, we must modify (1) since now there is no absorption in the initial state. We find in fact

$$A(s,\sqrt{-t})_{\mathrm{abs}}=\{\{A(s,\sqrt{-t})\}_{\mathrm{F.B.}}\times[1+i\rho(s')\{f_{\mathrm{el}}(s',\sqrt{-t'})\}_{\mathrm{F.B.}}]^{1/2}\}_{\mathrm{I.F.B.}}.\quad(4)$$

Different Mandelstam variables must be used in the elastic part since $f_{\mathrm{el}}(s',\sqrt{-t'})$ refers to the elastic collision of the particles in the final state, which is now different from the initial state. $A(s,\sqrt{-t})$ again can be either a Born amplitude or a Regge amplitude. The spin structure of the final state is unknown for the reaction in Fig. 1, so spin effects have not been included above. In their treatment of spin effects, Gottfried and Jackson neglect the helicity-changing parts of the elastic amplitude and therefore assume that only the amplitude part and not the absorption part has spin dependence. Spin effects are important for the detailed angular distribution.[6] We shall similarly assume that the absorption part of (4) is independent of spin. The amplitude part of (4) can still be spin-dependent.

For quasi-two-body collisions, the optical theorem suggests that we can write the elastic amplitude in the form

$$\rho(s')f_{\mathrm{el}}(s',\sqrt{-t'})=i[\sigma_{\mathrm{tot}}(s')/4\pi]g(\sqrt{-t'})[1+ih(s',\sqrt{-t'})],\quad(5)$$

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[9] R. C. Arnold, Phys. Rev. **136**, B1388 (1964).

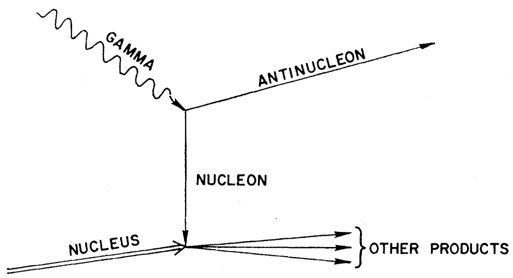

FIG. 1. Antinucleon production diagram used in this paper. In cases where Regge amplitudes are used, the exchanged particle is a Regge pole.

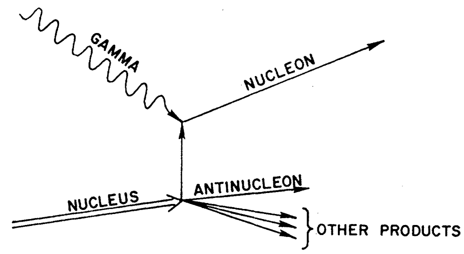

FIG. 2. Antinucleon production diagram. This must be included for gauge invariance but should contribute much less to the cross section than Fig. 1 at the very high energies and momenta for the antinucleon under consideration.

where $g(0)=1$ and $h(s',0)=0$. We shall assume that at high energies and small angles $h(s',\sqrt{-t'})\approx 0$. At energies and angles of interest, this amplitude may have a significant real part which could materially lessen the absorption.[10,11] Expanding both sides of (5) in terms of partial waves and equating the coefficients of the $P_0(\cos\theta)$ term gives

$$\frac{\sigma_T}{4\pi}\int_0^{2k} g(\sqrt{-t'})\,(\sqrt{-t'})d(\sqrt{-t'}) = 1-e^{-2\beta_0}\cos 2\alpha_0. \quad (6)$$

If $g(\sqrt{-t'})$ drops off reasonably at high-momentum transfers, this can be written with the upper integration limit $2k\rightarrow\infty$, to a good approximation. $\delta_0=\alpha_0+i\beta_0$ is the S-wave phase shift. Assuming complete S-wave absorption then leads to

$$\frac{\sigma_T(s')}{4\pi}=\left[\int_0^\infty g(\sqrt{-t'})\,(\sqrt{-t'})d(\sqrt{-t'})\right]^{-1}. \quad (7)$$

Now $g(\sqrt{-t'})=e^{zt'/2}$ provides an accurate parametrization of the elastic scattering for high energies and low-momentum transfers. Then (7) gives $\sigma_T/4\pi=z$, and (4) becomes

$$A(s,\sqrt{-t})_{\mathrm{abs}}=\{\{A(s,\sqrt{-t})\}_{\mathrm{F.B.}} \times[1-e^{-b^2/2z}]^{1/2}\}_{\mathrm{I.F.B.}}. \quad (8)$$

[10] K. J. Foley, R. S. Gilmore, R. S. Jones, S. L. Lindenbaum, W. A. Love, S. Ozaki, E. H. Willen, R. Yamada, and L. S. L. Yuan, in *Proceedings of the Twelfth Annual International Conference on High-Energy Physics, Dubna, 1964* (Atomizdat, Moscow, 1965).

[11] Denoting the absorption factor in (4) as $D^{1/2}$ and including a real part gives

$$D^{1/2}=\left\{\left[1-\frac{\sigma_T}{4\pi}\int_0^\infty (\sqrt{-t'})g(\sqrt{-t'})J_0(b\sqrt{-t'})d\sqrt{-t'}\right]^2 +\left[\frac{\sigma_T}{4\pi}\int_0^\infty (\sqrt{-t'})h(s',\sqrt{-t'})g(\sqrt{-t'})J_0(b\sqrt{-t'})d\sqrt{-t'}\right]^2\right\}^{1/4}.$$

Now if we let $g(\sqrt{-t'})=e^{zt'/2}$, where z is a constant, assume complete S-wave absorption, and let the real part be a constant h times the imaginary part, this becomes

$$D^{1/2}\approx[(1-e^{-b^2/2z})^2+h^2e^{-b^2/z}]^{1/4}\approx\left[\frac{b^4}{4z^2}+h^2\left(1-\frac{b^2}{z}\right)\right]^{1/4},$$

so that h will be important if $h\geq b^2/2z\approx 1$ to 5% depending on z. This is a very crude estimate.

Either a Born or a Regge amplitude can be used for $A(s,\sqrt{-t})$. If we use the former, the integrations must be carried out on a computer in general. A Regge amplitude is easier to handle, especially if we make a simplifying assumption. The differential cross section can be written, in general, at high energies

$$d\sigma/dt=C(t)s^{2\alpha_N(t)-2}, \quad (9)$$

where $\alpha_N(t)$ is the Regge trajectory of the exchanged particle. This is independent of the spin structure of the reaction (see Appendix). $C(t)$ contains Regge residues and some kinematics and should be a slowly varying function of t. We shall assume that $C(t)$ is a constant. Then the Regge amplitude is

$$A(s,\sqrt{-t})_R=B(s/s_0)^{\alpha_N(t)-1}, \quad (10)$$

where s_0 is the energy scale factor. In general, $\alpha(t)-1=dt+f(\sqrt{t})+e$, where d, e, and f are constants and $d>0$. The f term will contribute to the phase of $A(s,\sqrt{-t})_R$ since $t<0$. Absorption will change this phase when we insert $A(s,\sqrt{-t})_R$ into (8), and the f term will contribute to the cross section. In what follows, I shall neglect this f term which should be small, especially for the N_α Regge trajectory. The following can be easily generalized to include the effects of the f term.

Using this amplitude in (8) gives

$$\{A(s,\sqrt{-t})_R\}_{\mathrm{F.B.}}=Be^{-b^2/4r}(s/s_0)^e/2r \quad (11)$$

and

$$A(s,\sqrt{-t})_{\mathrm{abs}}=B\left(\frac{s}{s_0}\right)^{\alpha_N(t)-1} \times\left(\sum_{n=0}^{\infty}(-1)^nR_n\frac{(2n-3)(2n-5)\cdots(1)}{2^n n!}\right), \quad (12)$$

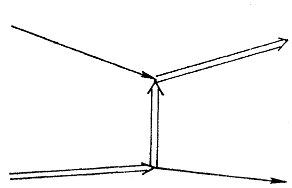

FIG. 3. Charge-exchange-reaction diagram.

TABLE I. SLAC experimental data.

Primary electron energy (GeV)	Production angle (deg)	Secondary momentum (GeV/c)	Antiproton yield per 10^7 incident electrons/sr (GeV/c)
17.85 ± 0.15	2	10	$2.9\ \pm0.7$
17.85 ± 0.15	2	12	$1.2\ \pm0.4$
17.85 ± 0.15	3	12	0.26 ± 0.07
$16.0\ \pm0.08$	3	10	$1.1\ \pm0.2$
$16.0\ \pm0.08$	3	12	0.17 ± 0.10

where

$$R_n=\frac{1}{1+nM}\exp\left[-tr\left(\frac{nM}{1+nM}\right)\right],$$

$$r=d\ln(s/s_0)\,,\quad \text{and}\quad M=2r/z\,.$$

The summation part in (12) represents the absorption factor, which is s- and t-dependent. This amplitude has the usual asymptotic Regge behavior only if $t\gtrsim0$. ($t\to0$ as $s\to\infty$ if the peak scattering angle is $\theta\lesssim M_3/E_3$,[12] where M_3 and E_3 are the mass and energy of the antinucleon in Fig. 1.) The fact that the asymptotic behavior of the amplitude with absorption present is different from $s^{\alpha_N(t)-1}$ for $t<0$ means that we are no longer dealing with a simple Regge pole but perhaps also with cuts in the J plane.[7] This different asymptotic behavior represents a good test of the Reggeized absorption formalism.

The result (12) holds for any γ reaction of the form in Fig. 1 in which only one Regge trajectory is important. For reactions of the form of Fig. 3, in which the initial and final absorptions are equal (not necessarily charge exchange), the analogous result is

$$(A_i)_{\text{abs}}=B\left(\frac{s}{s_0}\right)^{\alpha_N(t)-1}\left\{1-\left[\frac{e^{-(rtM)/(1+M)}}{(1+M)}\right]\right\}.\quad(13)$$

For very crude order-of-magnitude estimates, (8) can be written as

$$A(s,\sqrt{-t})_{\text{abs}}\approx(1-e^{-b^2/2z})^{1/2}A(s,\sqrt{-t})\,,\quad(14)$$

where $b\approx1/M_{\text{ex}}$ and M_{ex} is the mass of the exchanged particle. When used with a Born amplitude, this amplitude blows up at high energies.[13]

II. COMPARISON WITH EXPERIMENT

Specializing now to the antinucleon photoproduction problem, we need the N_α Regge trajectory. Chiu and Stack[14] give $d=1.052$ and $e=-1.340$. The N_γ trajectory should not be important.

A recent experiment at SLAC[15] on the photoproduction of antiprotons with the electron beam impinging on a 0.3-radiation-length beryllium target gave the results in Table I. A detailed match of (12) to these SLAC data is beside the point since the predicted differential cross section would have to be folded with the bremsstrahlung spectrum to give the yield, and this cannot be done because we do not know the t dependence of the factor B well enough. Equation (12) predicts the asymptotic s dependence as a function of t. To obtain a general idea of the behavior of (12), the absorption factor can be calculated for the values of s and t obtained above if the primary beam consisted of photons rather than electrons. For data points 1, 2, and 5 this gives the results in Table II. The final column is the absorption factor in (12) calculated for these s and t values. This factor enters the cross section squared, so that the absorption is fairly strong. In calculating s and t, a quasi-two-body final state was assumed. z was calculated for $z=\sigma_T/4\pi$, where σ_T is the total cross section for the final antiproton reacting with the rest of the final products. σ_T for protons on N^{14} is ≈250 mb.[16] Thus, crudely, $\sigma_T\approx250(10/14)^{2/3}$ mb for antiprotons on (Be^9+p). Then $z\approx40$ BeV^{-2}. The absorption factor in (12) does not depend very strongly on z or s but does depend strongly on t. It is interesting that the crude estimate (14) gives an absorption factor of

$$b/\sqrt{(2z)}\approx(1/0.938)/\sqrt{80}=1/8.4$$

for the amplitude. This compares quite well with the absorption factors above and is considerably less work. It is, of course, s- and t-independent. We can see, from the behavior of (12), that absorption can reduce the cross sections at these high energies by factors of the order of 1/220 or 1/25 depending on t. Smaller values of $|t|$ give much less absorption. This should be of interest to experimentalists designing photoproduction experiments. This behavior differs markedly from that of the Drell peripheral model with a constant absorption factor.

TABLE II. Absorption factors calculated for selected s and t values

s	t	Antiproton yield per 10^7 incident electrons/sr (GeV/c)	Absorption factor
371.7	-0.91	$2.9\ \pm0.7$	1/14.9
371.7	-0.695	$1.2\ \pm0.4$	1/5.3
340.6	-0.82	0.17 ± 0.10	1/7.7

It is also interesting to look briefly at the DESY data mentioned at the beginning of this paper. The DESY data at 6 BeV on the photoproduction of antiprotons are at too low an energy to use a Regge amplitude. The calculation of the integrals in (8) using the Born amplitude, calculated by Kohaupt using the Drell peripheral model but without the high-energy-limit approximation,[17] would be difficult. We can make the crude ap-

[12] S. D. Drell, Phys. Rev. Letters **5**, 278 (1960).
[13] S. D. Drell, in *Proceedings of the International Symposium on Electron and Photon Interactions at High Energies*, edited by G. Höhler *et al.* (Deutsche Physikalische Gesellschaft, Hamburg, 1965), p. 71.
[14] C. Chiu and J. Stack, Phys. Rev. **153**, 1575 (1967).
[15] S. M. Flatté, R. A. Gearhart, T. Hauser, J. J. Murray, R. Morgado, M. Peters, P. R. Klein, L. H. Johnston, and S. E. Wojcicki, Phys. Rev. Letters **18**, 366 (1967).
[16] Result quoted in Ref. 4.
[17] Formula given in Ref. 4.

proximation (14), however, and obtain an absorption factor of

$$\frac{b^2}{2z}=\frac{(1/0.938)^2}{(2)50}\cong\frac{1}{90}$$

on the cross section. Applying this to Kohaupt's result at 6 GeV gives a cross section of

$$d^2\sigma/d\Omega dp=450\ \mu\text{b/sr (GeV}/c)\times 1/90=5\ \mu\text{b/sr (GeV}/c),$$

in good agreement with experimental result of 3.5 μb/sr (GeV/c).[18] This does not test (8) or (12), but it does show that applying absorption even crudely can bring prediction and experiment much closer together.

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APPENDIX

We have used the well-known result that the differential cross section can be written as $d\sigma/dt=C(t)s^{2\alpha_N(t)-2}$ at very high energies, where $\alpha_N(t)$ is the Regge trajectory of the exchanged particle. It is interesting that this form of the differential cross section holds independent of the spin structure of the reaction products. For completeness, we will show this explicitly. We write

$$\frac{d\sigma}{dt}=\frac{P_s'}{P_s}\frac{t}{s^2}(\text{const})\sum_{\lambda_{\bar{2}},\lambda_4}\sum_{\lambda_1,\lambda_{\bar{3}}}\frac{\pi^2[2\alpha_N(t)+1]^2}{(2s_1+1)(2s_3+1)}$$
$$\times\frac{|\beta_{\lambda_1,\lambda_{\bar{3}}}(1\bar{3}\to N)|^2|\beta_{\lambda_{\bar{2}},\lambda_4}(\bar{2}4\to N)|^2}{16|P_t|^2\sin^2\pi\alpha_N(t)}$$
$$\times|d_{\lambda,\mu}{}^{\alpha_N(t)}(\theta_t)\pm(-1)^{-\lambda}d_{\lambda,-\mu}{}^{\alpha_N(t)}(\pi-\theta_t)|^2,$$

where we are using Fiset's notation.[19] Asymptotically, $\mathbf{P}_l(x)\approx P_l(x)$[20] so these d's can be considered to be just the usual d's of Jacob and Wick.[21] Then

$$d_{\lambda\mu}{}^j(\theta)=d_{-\mu-\lambda}{}^j(\theta)=(-1)^{\lambda-\mu}d_{\mu\lambda}{}^j(\theta),$$
$$d_{\lambda\mu}{}^j(\theta)=(-1)^{j+\lambda}d_{\lambda-\mu}{}^j(\pi-\theta),$$

and the d factor above becomes

$$|d_{\lambda,\mu}{}^{\alpha_N}(\theta_t)[1+(-1)^{-2\lambda-\alpha_N}]|^2,$$

where $\lambda=\lambda_1-\lambda_{\bar{3}}$ and $\mu=\lambda_{\bar{2}}-\lambda_4$. Now, asymptotically, $(P_s'/P_s)(1/|P_t|^2)$ is only a function of t, so that all of the s dependence is in the factor

$$(1/s^2)|d_{\lambda,\mu}{}^{\alpha_N}(\theta_t)[1\pm(-1)^{-2\lambda-\alpha}]|^2.$$

Now all $d_{\lambda\mu}{}^{\alpha_N}(\cos\theta_t)$, for $\cos\theta_t\gg 1$, where μ and λ are either both $\pm\frac{1}{2}$ odd integers or both $\pm$ integers (this includes all possible reactions), go asymptotically in s as $(\cos\theta_t)^{\alpha_N}\sim s^{\alpha_N}$ independent of μ and λ. Thus, asymptotically, $d\sigma/dt=C(t)s^{2\alpha_N-2}$, independent of the spin structure of the reaction products.

[18] Correcting for virtual nucleon effects at the vertex, as in the work of F. Selleri [Nuovo Cimento 39, 1122 (1965)], reduces our result a little further. Using Selleri's notation, $|K_-|\approx|K_+|$ for the DESY data so $\Gamma^\alpha=|K_+|\times\{\text{vertex on the mass shell}\}$, and we get a further reduction of the cross section by a factor of $|K_+|^2\approx(1/1.5)^2$.

[19] E. O. Fiset, Nuovo Cimento 35, 472 (1964).

[20] M. Gell-Mann, in *Proceedings of the International Conference on High-Energy Physics, Geneva, 1962*, edited by J. Prentki (CERN Scientific Information Service, Geneva, 1962), p. 533.

[21] M. Jacob and E. C. Wick, Ann. Phys. (N. Y.) 7, 404 (1959).