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Nuclear-Structure Studies of Sr^{88} and Y^{89} by Inelastic Electron Scattering*

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The nuclei Sr^{88} and Y^{89} have been studied by observing the inelastic scattering of 65- and 70-MeV electrons through angles 70°–150°, with an over-all energy resolution of 200 keV. Strongly excited states were found in Sr^{88} at 1.84 MeV (2^+), 2.74 MeV (3^-), 4.0, 6.5, and 7.8 MeV; and in Y^{89} at 1.50 MeV $(\frac{3}{2}^-)$, 1.73 MeV $(\frac{5}{2}^-)$, 2.21 MeV $(\frac{5}{2}^+)$, 2.52 MeV $(\frac{7}{2}^+)$, 2.86 MeV $(\frac{5}{2},\frac{7}{2}^+)$, 3.1, 3.72, 4.0, and 4.16 MeV. The data were analyzed by a distorted-wave calculation to determine multipolarities and reduced nuclear radiative transition probabilities. The results show that the excited states in Y^{89} cannot be described by a weak coupling of the $2p_{1/2}$ proton to core-excited states of Sr^{88}. The sum of the $B(E2)$'s of the low-lying quadrupole states in Y^{89} is found to be about 30% of the $B(E2)$ of the 1.84-MeV state of Sr^{88}, and three strong octupole states are excited in Y^{89}, where only two are expected from the weak-coupling model. No detailed information on the single-particle structure of the levels could be obtained at these momentum transfers.

I. INTRODUCTION

SEVERAL experiments on the nuclei Sr^{88} and Y^{89} have been performed recently using strongly interacting particles. Thus, detailed studies comparing the excited states of the neighboring nuclei by inelastic α-particle scattering by Alster, Shreve, and Peterson,[1] and by inelastic proton scattering by Stautberg, Kraushaar, and Ridley[2] have been carried out. Spin and parity assignments, as well as transition strengths, which can be related to electromagnetic transition rates, have been extracted in these studies from the shapes and magnitudes of the angular distributions.

We have used low-energy inelastic electron scattering to investigate these excitations. The purely electromagnetic interaction of the electron with nuclei leads to an interpretation which is simpler than for strongly interacting particles. Low momentum-transfer electron scattering at high resolution gives the multipolarity of a transition reliably, and for medium-weight nuclei the reduced electric radiative transition probability can be obtained accurately by use of a distorted-wave analysis.[3] A comparison of these results with results obtained by other methods is of interest.

Shafroth, Trehan, and Van Patter[4] have evoked de-Shalit's weak-coupling core-excited-state model[5] in comparing $(n,n'\gamma)$ results from Sr^{88} and Y^{89}. If this model holds for this case, there should be pairs of states in Y^{89}, each pair corresponding to the coupling of the unpaired $2p_{1/2}$ proton to each excited state of the Sr^{88} core at energies given by the center-of-gravity theorem. This model seems to apply very well to the lead isotopes and bismuth where the core consists of the closed proton and neutron shells of Pb^{208}.[6–9] Other possible examples of its applicability have been cited.[1,2]

For the case of Y^{89}, Shafroth *et al.*[4] proposed, as shown in the level diagrams of Fig. 1, that the 1.50-MeV$(\frac{3}{2}^-)$ and 1.73-MeV$(\frac{5}{2}^-)$ states correspond to the 1.84-MeV(2^+) state in Sr^{88}, the 2.21- and 2.52-MeV states in Y^{89} to the 2.74-MeV(3^-) state in Sr^{88}, and the 2.86- and 3.1-MeV states in Y^{89} to the 3.21-MeV(2^+) state in Sr^{88}. The discrepancy of the positions of the states in Y^{89} from those of the center-of-gravity theorem is not serious.

On the other hand, Alster *et al.*[1] showed that a single-particle description of the 1.51- and 1.75-MeV states was more appropriate than the core-excitation model. Furthermore, Shreve *et al.*[10] have used the Zr^{90} (d,He^3) reaction, and Stautberg *et al.*[2] have used the $Sr^{88}(He^3,d)$ reaction, to give further evidence for the single-particle interpretation for these states.

The (α,α') experiment[1] also showed that the sum of the transition strengths of the Y^{89} excitations at 2.22, 2.52, and 2.84 MeV was approximately equal to the Sr^{88} 3^- transition strength. In the core-excitation model, only two levels should give the Sr^{88} strength.

The Stautberg *et al.*[2] (p,p') experiment on Sr^{88} and

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Y^{89} is in agreement with the (α,α') results, with the exception that they found that the sum of only two of the $L=3$ excitation strengths in Y^{89} is equal to that of the 3^- excitation in Sr^{88}.

A recent $(n,n'\gamma)$ experiment by Buchanan *et al.*[11] assigned a spin of $\frac{3}{2}^-$ to the 2.87-MeV state in Y^{89} supporting the assignment made by Shafroth. It is interesting to note the consistent difference in spin and parity assignment from charged-particle and neutral-particle scattering. Experiments are now in progress to check the possibility of two very close levels in Y^{89} around 2.8 MeV.

II. EXPERIMENTAL METHOD

The experiment was performed at the Yale University Electron Accelerator Laboratory with the beam from the 75-MeV *L*-band traveling-wave linear accelerator. The beam was analyzed to about 0.3% of the incident energy by an achromatic four-magnet deflection system[12] and focused on the target by a quadrupole triplet magnetic lens. Typical average currents were 2 μA in a 1-mm×2-mm spot on the target. The beam was monitored by a Faraday cup and a digital current integrator. Scattered electrons were received by a 40-cm central-radius-of-curvature, 180°, double-focusing spectrometer.[13] Double coincidences were measured on four sets of counter telescopes consisting of photomultipliers with plastic scintillators. This apparatus will be described more thoroughly in forthcoming articles.[7,14]

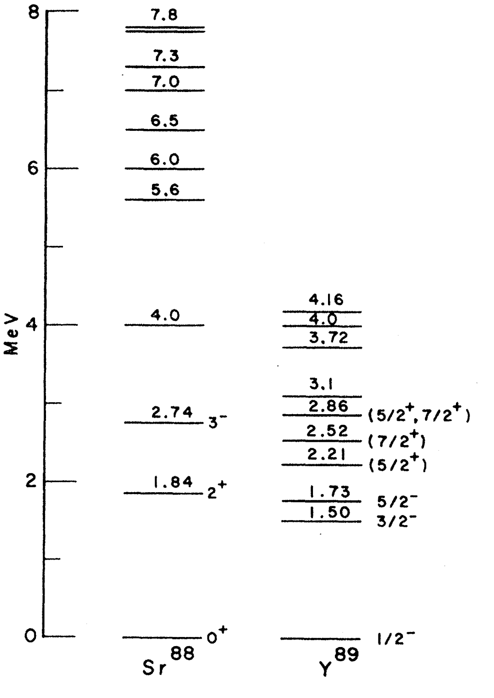

FIG. 1. Level diagrams for Sr^{88} and Y^{89} showing energies, spins, and parities of states observed in this experiment. The energies of Y^{89} levels are taken from Ref. 28.

An improvement in data collection was provided by a PDP-8 computer system[15] which read out scalers and controlled the magnetic field of the spectrometer.[16] A rotating-coil fluxmeter measured the magnetic field of the spectrometer and provided an analog voltage input to an electronic voltmeter that in turn fed a digital signal to the computer. The potentiometer of the spectrometer power supply was driven by a motor stepped by the computer until the proper field was attained. The computer was thus programmed to step the magnetic field after the accumulation of a specified amount of charge in the Faraday cup. Up to 150 data points could be taken automatically by this system, the limiting factor being the amount of data storage space in the computer memory.

An Y^{89} target of 78 mg cm^{-2} and an isotopically enriched (99.84%)Sr^{88} target of 53 mg cm^{-2} were used. In order to prevent the oxidation of the Sr^{88} target, it was loaded into a cell in a He atmosphere, the cell evacuated and transported to the scattering chamber where the cell was removed under vacuum. Targets were rotated in the vacuum in order to average out nonuniformities and to prevent evaporation of the targets.

A typical spectrum of scattered electrons, as recorded by one counter telescope, is shown in Fig. 2 for a Sr^{88} target. All points were corrected for spectrometer dispersion and counter dead-time losses. The widths of 200 keV of the peaks are due to the combined effects of beam-energy spread, detector size, spectrometer aberrations, beam size, and ionization straggling in the target.

III. ANALYSIS OF DATA

The methods of analysis are similar to those of Ref 7. Inelastic cross sections in units of the Mott cross section were obtained by multiplying the inelastic-to-elastic peak area ratios C by the elastic cross section $\sigma_e(E_0,\theta)$, in units of the Mott cross section:

$$\sigma_I(E_0,\theta)/\sigma_{\text{Mott}}=C[\sigma_e(E_0,\theta)/\sigma_{\text{Mott}}], \tag{1}$$

where

$$\sigma_{\text{Mott}}=(Ze^2/2E_0)^2\cos^2(\tfrac{1}{2}\theta)/\sin^4(\tfrac{1}{2}\theta)$$

is the Mott cross section for the elastic scattering through an angle θ of an electron of energy E_0 from a point spinless nucleus with no recoil. For Sr^{88}, σ_e was

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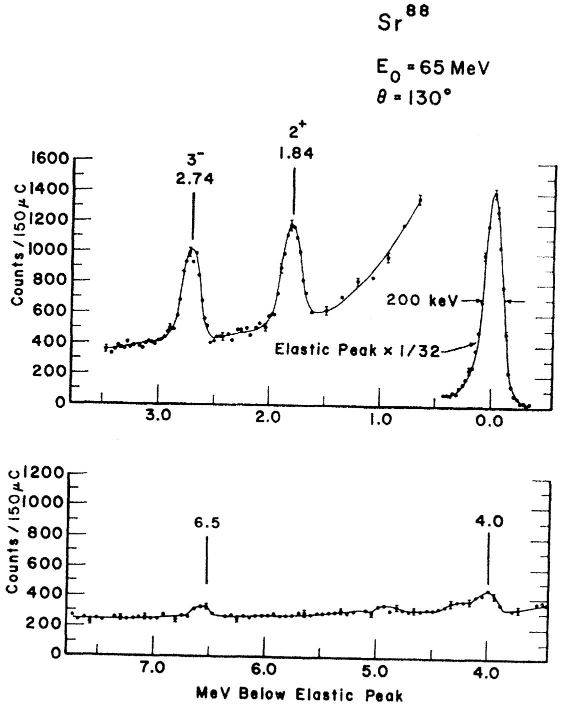

FIG. 2. The energy spectrum of electrons of initial energy 65 MeV scattered through an angle of 130° from a Sr^{88} target as observed by one detector only.

calculated by using the Fischer and Rawitscher[17] distorted-wave computer code with Fermi-distribution parameters determined in the electron-scattering experiment of Helm[18]: the half-density radius $c=4.80\mathrm{F}$, and the 10 to 90% skin thickness $t=2.30\mathrm{F}$, corresponding to a charge distribution of the form

$$\rho=\rho_0\left[1+\exp\left(\frac{r-c}{t/4.4}\right)\right]^{-1}. \tag{2}$$

Since no charge-distribution parameters were known for Y^{89}, it was necessary to determine the Y^{89} elastic scattering cross section. It was convenient to compare the elastic scattering from ${}_{39}\mathrm{Y}^{89}$ between 70° and 150° for $E_0=60$ and 70 MeV to that of Co^{59}, considered as a standard, in order to reduce errors due to equipment instabilities. In this range, the two cross sections are comparable because of size effects, even though the charge of ${}_{39}\mathrm{Y}^{89}$ exceeds that of ${}_{27}\mathrm{Co}^{59}$ by a large amount. The Co^{59} Fermi charge-distribution parameters of Crannell *et al.*[19]; $c=4.09\mathrm{F}$ and $t=2.50\mathrm{F}$, were used. The results are shown in Fig. 3. The parameter c was increased according to $A^{1/3}$ between Sr^{88} and Y^{89}, and a best fit to the data was sought by varying t. The value of t found is $2.50\pm0.15\mathrm{F}$, a slightly larger value than for Sr^{88}. A higher-energy experiment could be more definitive about this matter, as low-energy electron scattering is primarily sensitive to the rms radius, and not to the details of the charge distribution.

The inelastic peaks of Fig. 2 are situated on a continuum spectrum of scattered electrons, usually referred to as the radiation tail, resulting mainly from electrons that have lost energy by radiation processes. For the case of the 1.84- and the 2.74-MeV peaks of Sr^{88}, the inelastic peak areas were determined relative to the elastic peak by using an empirical least-squares fitting procedure for the radiation tail suggested first by Helm.[7,18] For the case of Y^{89}, where the peaks were not widely separated, as shown in Fig. 4, it was necessary to calculate the radiation-tail contribution of each peak separately. Assuming no recoil and the Schiff peaking approximation, the cross section for the radiation tail for a target of thickness t radiation lengths is given by[20]

$$\frac{d^2\sigma}{d\Omega dE}=\frac{\alpha}{\pi}\left\{\ln\left[\frac{2E_0}{mc^2}\sin(\tfrac{1}{2}\theta)\right]-\frac{1}{2}\right\}\left\{\left[\frac{1}{E_0-E}\right]\left[1+\frac{E^2}{E_0{}^2}\right]\right.$$
$$\left.\times\left[\frac{d\sigma}{d\Omega}(E_0)+\frac{d\sigma}{d\Omega}(E)\right]\right\}+\frac{1}{E_0-E}t\left[\frac{d\sigma}{d\Omega}(E_0)+\frac{d\sigma}{d\Omega}(E)\right], \tag{3}$$

where it was found necessary to include distorted-wave

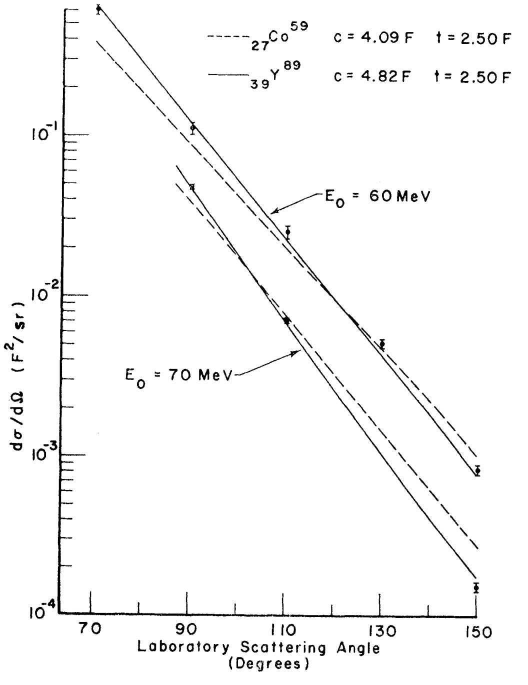

FIG. 3. Elastic scattering cross sections for ${}_{27}\mathrm{Co}^{59}$ and ${}_{39}\mathrm{Y}^{89}$ for 70- and 60-MeV electrons versus laboratory angle of scattering for the Fermi-distribution parameters c and t given. The experimental points shown for Y^{89} were measured relative to Co^{59} cross sections calculated as described in Sec. III.

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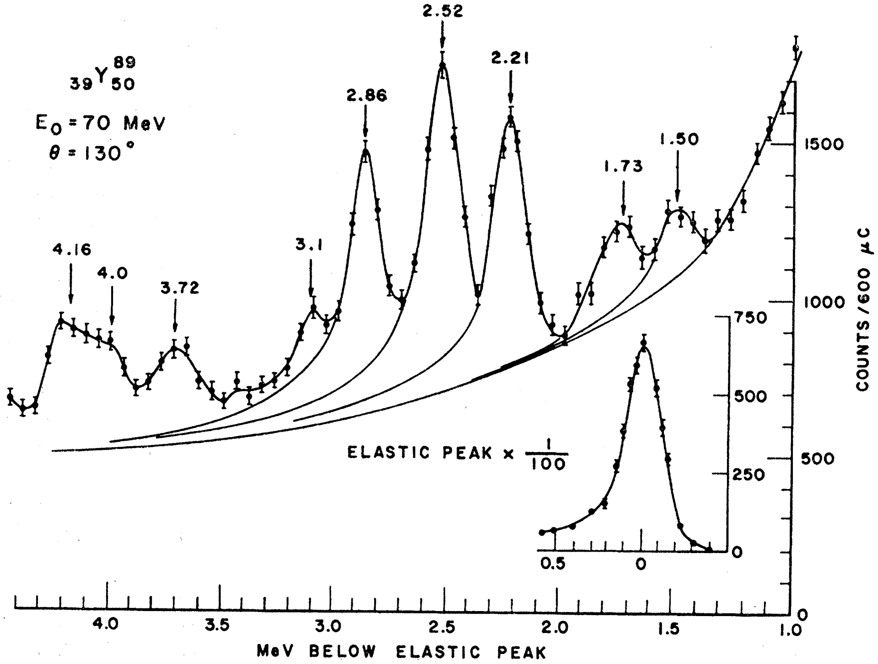

FIG. 4. The energy spectrum of electrons of initial energy 70 MeV scattered through an angle of 130° from an Y^{89} target as observed by one detector only. The solid lines are radiation tails calculated according to Eq. (3).

elastic cross sections[17] for $d\sigma/d\Omega$ as functions of the incident electron energy E_0 and the scattered electron energy E, instead of Born-approximation cross sections. Agreement between the experimental and calculated radiation tail was found for angles of 130° and forward, in the region of excitations up to about 4 MeV. For higher excitations experimental background effects resulting from elastically scattered electrons striking the spectrometer vacuum chamber become important. The solid lines of Fig. 4 show the radiation tails calculated by Eq. (3).

Because of the large charges of Sr^{88} and Y^{89}, and because of the relative low electron energy used in this experiment, it was necessary to use distorted-wave calculations for the inelastic scattering as well as the elastic scattering. We have used the numerical distorted-partial-wave analysis of Griffy, Biedenharn, Reynolds, Onley, and Wright, referred to as GBROW,[3,21] with a transition charge density for an incompressible and irrotational fluid[22] with the same values of the half-density radius c and of the surface thickness t as the ground-state Fermi-distribution parameters. This calculation applies to electric multipole excitations and includes both longitudinal and transverse[23] contributions. Electron energy loss and electron rest mass are taken into account.

It is expected that the calculations are largely model-independent for the low-energy electrons of this experiment because the cross sections are largely insensitive to the details of the interior of the wave functions. A spherical Bessel function $j_L(qr)$, of order L and argument qr, where q is the momentum transferred to the nucleus and r is the nuclear radial coordinate, multiplies other parts of the matrix element.[24] For low q, this Bessel function largely governs the q dependence of the cross section. Thus, the cross section as a function of q is insensitive to the model. The validity of this discussion is supported by detailed checks, using the GBROW code,[7] for excitations in Pb^{208} where qr, for the same q, is larger than in this experiment.

The reduced nuclear radiative transition probability $B(EL)$ is obtained by normalizing the calculated cross sections to the experimental points using a least-squares fitting procedure as shown in Figs. 5 and 6. The electromagnetic transition strength G is given in Weisskopf single-particle units (W.u.):

$$G = B(EL)/B(EL)_{\mathrm{sp}}, \qquad (4)$$

where

$$B(EL)_{\mathrm{sp}}/e^2 = [(2L+1)/4\pi][3R_0{}^L/(3+L)]^2,$$

and $R_0 = 1.20A^{1/3}$F.

IV. RESULTS AND DISCUSSION

Sr^{88}

The most prominent peaks observed in the Sr^{88} spectrum correspond to the excitation of the 2^+ level at 1.84 MeV and the 3^- level at 2.74 MeV, as shown in Fig. 2. Inelastic scattering cross sections for these excitations were measured for momentum transfers q, ranging from 0.35 to 0.65F^{-1} and are shown in Fig. 5,

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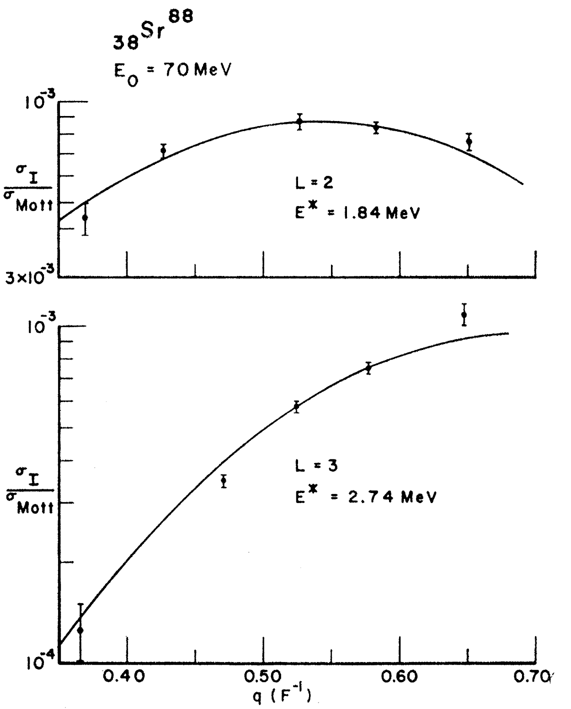

FIG. 5. Distorted-wave inelastic cross section in units of the Mott cross section versus momentum transferred to the Sr^{88} nucleus for the electric quadrupole excitation at 1.84 MeV and for the electric octupole excitation at 2.74 MeV. An incompressible and irrotational hydrodynamical model was assumed.

together with the curves calculated by the method described in the previous section. The extracted excitation $B(EL)$ values are given in Table I, and are in fairly good agreement with the quoted mean lifetimes of Helm,[25] and also with the (p,p') experiment of Stautberg *et al.*[2] and earlier work quoted therein. The $B(EL)$ values extracted from the (α,α') experiment[1] are lower than the values obtained in this experiment; however, a distorted-wave Born-analysis (DWBA) calculation for the (α,α') case, including the Coulomb-excitation contribution, raises the $B(E3)$ of the 2.74-MeV state from 6.2 to 20 W.u., in better agreement with the value 25.0 W.u. obtained in this work.

A peak was observed at 4.0 MeV which fits a GBROW curve corresponding to a 2^+ state and the strength is approximately 1.6 W.u. Stautberg *et al.*[2] also observed this state and were able to make a 2^+ or 4^+ assignment. Helm[18] also saw a partially resolved peak at approximately this energy by high-energy (187-MeV) inelastic electron scattering.

Several peaks at higher excitation energies were observed, as shown in Fig. 7. The cross sections for the 6.5-MeV level, taken over the limited range of momentum transfer, do not permit a definite assignment of spin and multipolarity. An assignment of 2^+ or 3^- to this level is possible, with a slight preference for 2^+. The 2^+ assignment would result in a value for $B(E2)$ of 1.1 W.u., whereas a 3^- assignment gives a value for $B(E3)$ of 4.0 W.u. A 3^- transition at this energy and strength is consistent with (α,α') experiments[26,27] in Zr^{90} where higher excited 3^- states have been observed with strengths of approximately 20% of the first ex-

TABLE I. Experimentally determined values of the reduced nuclear radiative transition probabilities $B(EL)$ for the excitation of a nucleus to a level at energy E^* above its ground state by a transition of electric character and multipolarity L, in units of $e^2\,F^{2L}$, where 1 F$=10^{-13}$ cm. The last column gives $B(EL)$ in single-particle Weisskopf units according to Eq. (4).

E^*	L	$B(E0\rightarrow L)$ ($e^2\,F^{2L}$)	G (W.u.)
		Sr^{88}	
1.84	2	990±50	8.5
2.74	3	80 600±3000	25.0
4.0	2	190±40	1.6
6.5	(2)	130±30	1.1
	(3)	13 000±3000	4.0
		Y^{89}	
1.50	2	120±50	1.0
1.73	2	140±40	1.2
2.21	3	33 700±3000	10.2
2.52	3	37 800±3000	11.4
2.86	3	32 300±3000	9.8

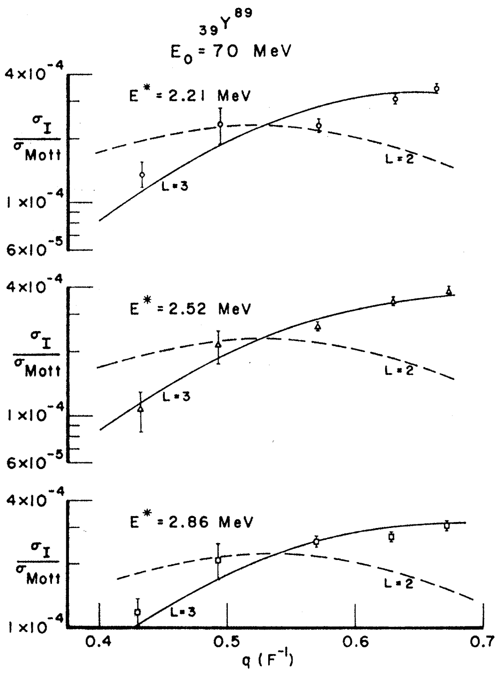

FIG. 6. Distorted-wave inelastic cross section in units of the Mott cross section versus momentum transferred to the Y^{89} nucleus for electric octupole (solid lines) excitations at 2.21, 2.52, and 2.86 MeV. An incompressible and irrotational hydrodynamical model was assumed. For comparison, curves are shown for electric quadrupole excitations.

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[25] The $B(EL)$ values in WU quoted by Helm in Ref. 17 are not consistent with his values for the mean lifetime.

[26] H. W. Broek and J. L. Yntema, Phys. Rev. **138**, B334 (1965).

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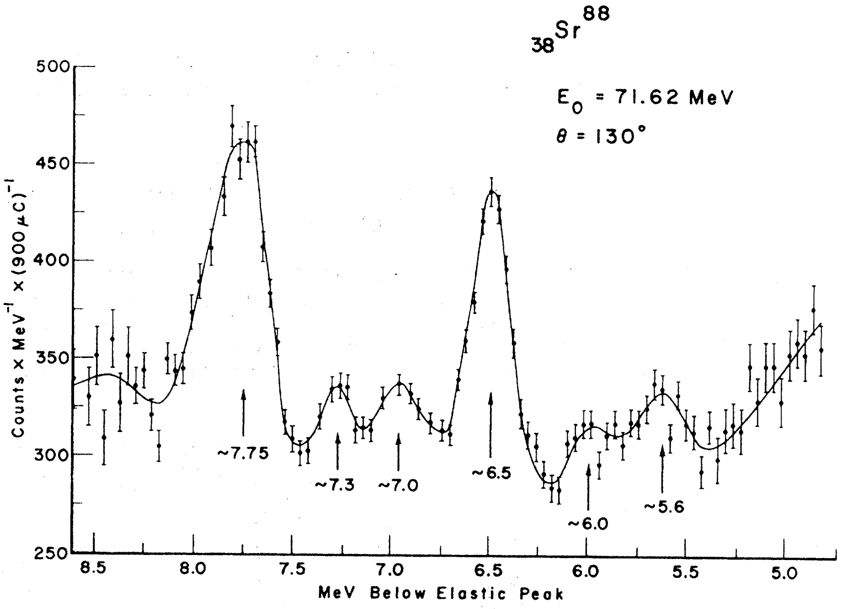

FIG. 7. A portion of the differential energy spectrum of electrons of initial energy 71.62 MeV scattered through an angle of 130 deg from a Sr^{88} target. Experimental points are combined data of all detectors.

cited 3^- state. The 2^+ assignment could be explained by assuming a single-particle transition (see Fig. 14 of Ref. 1). The weak 2^+ single-particle transition at 3.2 MeV was not seen in this experiment because of the high radiation tail and the poorer statistics in that region compared to the 6-MeV region.

No spin assignments can be made for the broad peak at 7.8-MeV excitation, since data were taken only for one momentum transfer.

Y^{89}

A spectrum for Y^{89} is shown in Fig. 4. The three most prominent peaks appear at excitation energies of 2.21, 2.52, and 2.86 MeV. Two smaller peaks appear at 1.50 and 1.73 MeV. The cross sections for these peaks were measured for momentum transfers ranging from 0.42 to 0.68 F^{-1}. The data for the stronger peaks are presented together with the calculated curves in Fig. 6. Large errors were associated with the cross sections for the 1.50- and 1.73-MeV states, since the radiation tail of the elastic peak is very large at these excitation energies and could not be subtracted with sufficient accuracy. The extracted values for the excitation $B(EL)$ of all the measured levels are given in Table I. States at higher excitation were found at 3.1, 3.7, 4.0, and 4.16 MeV, but no complete angular distributions could be obtained for these weakly excited states. All these states were seen before.[1,2,28] In the light of the weak-coupling core-excitation model it can be seen from Table I that the sum of the 1.50- and 1.73-MeV transition strengths is about one-fourth to one-third of the strength of the 1.84-MeV state in Sr^{88}. This is in agreement with the inelastic α particle[1] and the inelastic proton[2] scattering experiments. This ratio was calculated by Alster *et al.*[1] using a simple shell-model configuration. We find the sum of two of the $L=3$ transition strengths falls below the transition strength of the 3^- state in Sr^{88}, whereas the sum of all three $L=3$ states is higher than the 3^- transition strength in Sr^{88}. Alster *et al.*[1] found that all three $L=3$ states summed together equaled the $L=3$ strength in Sr^{88}, whereas Stautberg *et al.*[2] needed only two states. This experiment, therefore, has not resolved the question of which of these octupole states, if any, should be considered to be the states of the weakly-coupled core-excited-state model. It is difficult to understand, in the context of this model, the appearance of three strongly excited octupole states since the model predicts only two such states.[1]

A high resolution experiment with electrons of higher energy, to attain large momentum transfers, is necessary to examine the excitation modes of these states in detail.

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