

Also, the systematic overfulfillment of the sum rules has disappeared. The CA strengths are generally just outside the experimental uncertainties assigned to the earlier strengths.

It is of interest to note that the need for an  $l=0$  contribution in the  $^{47}\text{Ti}(d,p)^{48}\text{Ti}$  transition to the first  $6^+$  state (No. 6 in Table III) has disappeared. Since the  $^{47}\text{Ti}$  ground-state spin is  $\frac{5}{2}$ , such an admixture is not allowed, so the argument that a doublet may exist here, is no longer valid.

#### ACKNOWLEDGMENTS

We are indebted to Mrs. Bonnie Andersen, Mrs. Virginia Camp, Mrs. K. Sakamoto, Mrs. M. Nagatani, Mrs. Mieko Kitajima, and Mrs. Carmen Vanegas for carefully scanning the nuclear track plates. The cooperation of the NBI-NORDITA GIER computer center and the help of Dr. J. H. Bjerregaard, Mrs. L. Vistisen, and Mrs. B. Scharff in the  $(d,p)$  reanalysis are gratefully acknowledged.

### Nuclear Spin, Hyperfine-Structure Separation, and Nuclear Magnetic Moment of 18-min $^{88}\text{Rb}^\dagger$

PAUL A. VANDEN BOUT,\* ANTONI DYMANUS,† VERNON J. EHLERS,§ MICHAEL H. PRIOR,  
AND HOWARD A. SHUGART

*Department of Physics and Lawrence Radiation Laboratory, University of California,  
Berkeley, California*

(Received 8 September 1967)

We have used the atomic-beam magnetic-resonance technique to measure the nuclear spin and the hyperfine-structure separation of 18-min  $^{88}\text{Rb}$  in the  $^2S_{1/2}$  electronic ground state. These results, combined with the Fermi-Segrè formula, yield the nuclear magnetic moment. Our results are:  $I=2$ ,  $\Delta\nu = \pm 1186.084(18)$  MHz,  $\mu_I(\text{uncorr}) = \pm 0.506(5)$  nm,  $\mu_I(\text{corr}) = \pm 0.508(5)$  nm. Present nuclear theory favors assignment of the negative sign to the value of the moment.

#### I. INTRODUCTION

DEVELOPMENT of nuclear theories has in the past depended heavily upon the experimental measurement of static nuclear properties, and in particular upon the determination of nuclear spins and multipole moments. Although several nuclear theories are now reasonably well established, such information still serves as a useful guide in the further development of these theories, and allows determination of the ground state of the nuclei involved. As part of our continuing program to measure pertinent nuclear properties, we have measured the nuclear spin, hyperfine-structure separation, and nuclear magnetic moment of  $^{88}\text{Rb}$ .

Two features of  $^{88}\text{Rb}$  make it of special interest. First, it is an odd-odd nucleus, and a measurement of its spin and magnetic moment serves as a good check on various proposed coupling schemes for odd-odd nuclei. Second, it is the heaviest in a series of eight Rb isotopes for which the spins and magnetic moments have been measured. The change in nuclear properties caused by the one-by-one addition of neutrons is particularly amenable to comparison with nuclear theories, and thus it is desirable to measure the nuclear properties of a chain of isotopes.

† Research supported by the U. S. Atomic Energy Commission.  
\* Present address: Columbia University, Physics Department, New York, N. Y.

† On leave from University of Nijmegen, Netherlands.

§ Present address: Calvin College, Grand Rapids, Mich.

#### II. THEORY OF THE EXPERIMENT

The Hamiltonian describing the hyperfine structure (hfs) of  $^{88}\text{Rb}$  is given by

$$\mathcal{H} = h\mathbf{a}\cdot\mathbf{J} - g_I\mu_0\mathbf{J}\cdot\mathbf{H} - g_I\mu_0\mathbf{I}\cdot\mathbf{H}, \quad (1)$$

where  $\mathbf{a}$  is the magnetic-dipole hfs interaction constant,  $\mathbf{I}\hbar$  is the nuclear angular momentum,  $\mathbf{J}\hbar$  is the electronic angular momentum,  $g_I = \mu_I/I$  and  $g_J = \mu_J/J$  are the corresponding  $g$  factors,  $\mathbf{H}$  is the external magnetic field,  $h$  is Planck's constant, and  $\mu_0$  is the Bohr magneton. The energy levels of Eq. (1) for  $J = \frac{1}{2}$  are given by the Breit-Rabi formula,

$$W(F, M_F) = \frac{-h\Delta\nu}{2(2I+1)} - g_I\mu_0 H M_F + (F-I)h\Delta\nu\{1 + 4M_F x / (2I+1) + x^2\}^{1/2}, \quad (2)$$

where  $\Delta\nu = a(I + \frac{1}{2})$ ,  $x = (g_I - g_J)(\mu_0/h)H/\Delta\nu$ , and  $\mathbf{F} = \mathbf{I} + \mathbf{J}$ . These levels are illustrated in Fig. 1 for  $^{88}\text{Rb}$ .

The theory of operation of an atomic-beam apparatus has been described in detail elsewhere<sup>1</sup>; we give only a brief sketch here. Atoms effuse from the slit of an oven at one end of an evacuated chamber and pass through three magnetic fields. The first is strongly inhomogeneous and the atoms suffer deflections due to their mag-

<sup>1</sup> N. F. Ramsey, *Molecular Beams* (Oxford University Press, New York, 1956).

netic moments. The second magnetic field is homogeneous, and in it transitions are induced among the hfs energy levels by an additional radiofrequency (rf) magnetic field. The third magnetic field is identical to the first. If a transition is induced between a level having  $m_J = +\frac{1}{2}$  and one having  $m_J = -\frac{1}{2}$ , these atoms are deflected by the third magnetic field toward the detector. Those atoms which have not undergone a transition are deflected away from the detector.

For  $J = \frac{1}{2}$  and for low magnetic fields ( $x \ll 1$ ), the frequency corresponding to the only  $\Delta F = 0$  transition satisfying the selection rule of the apparatus that  $\Delta m_J = \pm 1$  is given by

$$\nu \approx \frac{-g_J H (\mu_0/h)}{(2I+1)} \quad (3)$$

The spin  $I$  is determined by setting the magnetic field to a few gauss, and making a discrete frequency search for a resonance. By observing this  $\Delta F = 0$  resonance at increasingly higher magnetic fields, information is obtained about  $\Delta\nu$ , as can readily be seen from Eq. (2). Finally, when observation of the  $\Delta F = 0$  resonance at high fields has reduced the uncertainty of  $\Delta\nu$  to about 1 MHz, a search can be made for a  $\Delta F = 1$  transition, determining  $\Delta\nu$  directly.

To a good approximation, the hfs interaction constant of any isotope of a given element is proportional to its nuclear  $g$  factor. This is expressed by the Fermi-Segré relation

$$a(1)/a(2) = g_I(1)/g_I(2) \quad (4)$$

Thus, a measurement of  $a$  for an isotope yields the nuclear magnetic moment, provided  $a$  and  $g_I$  have been measured for another isotope of the same element.

### III. THE EXPERIMENT

#### A. Beam Production and Detection

Radioactive  $^{88}\text{Rb}$  with a half-life of 18 min is easily produced by neutron irradiation of the 28%-abundant naturally occurring  $^{87}\text{Rb}$ . The  $^{88}\text{Rb}$  for these experiments was produced in two different reactors. Most of

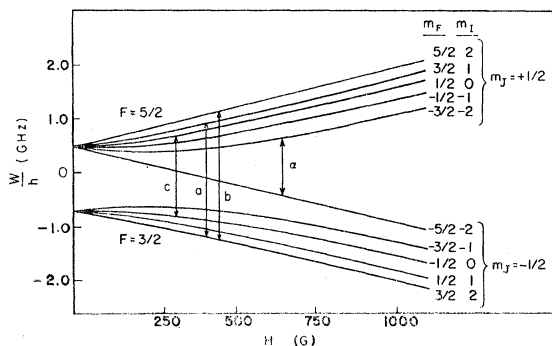


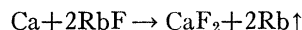
FIG. 1. Breit-Rabi energy-level diagram for  $^{88}\text{Rb}$  ( $I = 2$ ).

the irradiations were done in the General Electric Test Reactor at Vallecitos, Calif. The samples were flown by helicopter the approximately 25 miles to Berkeley; this took about one half-life. The latest experiments, however, used samples irradiated in the new Triga Mk III reactor, recently placed in operation on the University of California Berkeley campus. Although the flux of the Triga is estimated to be  $\approx \frac{1}{3}$  that of the GE reactor, one half-life is saved between removal from irradiation and beginning the experiment; little loss of signal intensity was observed upon changing to the campus reactor.

$^{87}\text{Rb}$  was irradiated in 100-mg samples of RbF salt. The RbF was dried thoroughly and encapsulated in quartz vials for delivery to the reactor. The fluorine contributed no significant activity to the sample since only (11-sec)  $^{20}\text{F}$  was produced and this had decayed by the time the experiment was begun. The principal background activities produced (besides the  $^{88}\text{Rb}$ ) were  $^{86}\text{Rb}$  with an 18.6-day half-life, and  $^{134\text{m}}\text{Cs}$  with a 2.9-h half-life.  $^{86}\text{Rb}$  was produced from neutron capture by 72%-abundant  $^{85}\text{Rb}$ , and  $^{134\text{m}}\text{Cs}$  was produced from the 100%-abundant  $^{133}\text{Cs}$  present as an impurity in the RbF sample. From the observed decay of the samples, we estimate that a typical sample contained 140 mCi  $^{88}\text{Rb}$ , 0.5 mCi  $^{86}\text{Rb}$ , and 2.5 mCi  $^{134\text{m}}\text{Cs}$ . This corresponds to a 0.2% Cs contamination in our supply of RbF salt.

Upon delivery of the sample to the laboratory it was immediately opened and loaded into a tantalum oven together with some freshly made filings of calcium metal. An excess of Ca was used, usually 1.5 to 2 times the RbF in volume. The RbF and Ca were mixed in the oven, which was then sealed and loaded into the atomic beam apparatus.

Once in the beam machine the oven was heated initially by electron bombardment. The electron bombardment served to heat the oven quickly so that the chemical reaction



could be started. Once it had begun, only the thermal radiation from a nearby tungsten filament was required to sustain it.

The radioactive beam was detected as follows: A sulfur surface or "button" was placed at the detection position of the apparatus and exposed to the beam for 2.5 min. This surface was then placed in a Geiger counter shielded against extraneous counts by an anti-coincidence counter, and its counting rate was measured. The beam intensity was monitored by following the same procedure with a button placed to one side of the first, and exposed simultaneously to a portion of the unflopped beam. The signal then consisted of the ratio of the counting rate of the center button to the counting rate of the side button. In calculating this ratio, only the portion of the counting rate due to  $^{88}\text{Rb}$  should be used. This was obtained by taking the difference between the counting rate immediately after exposure to the atomic

TABLE I. Experimental data.

Run number	<sup>86</sup> Rb calibrating isotope data		Transition <sup>a</sup>	<sup>88</sup> Rb data	
	$\nu$ (MHz)	Calculated $H$ (G)		$\nu$ (MHz)	$\nu_{\text{obs}} - \nu_{\text{calc}}$ (kHz)
415	2.150(20)	4.590(43)	$\alpha$	2.600(75)	$\mu_I > 0$ 5
526	3.500(25)	7.455(53)	$\alpha$	4.225(75)	$\mu_I < 0$ 4
540	5.184(15)	11.013(32)	$\alpha$	6.300(100)	-12
547	7.106(15)	15.049(31)	$\alpha$	8.700(100)	-1
879	7.020(10)	14.869(21)	$\alpha$	9.550(40)	22
884	19.262(15)	40.006(30)	$\alpha$	24.200(50)	18
887	37.140(15)	75.031(29)	$\alpha$	48.650(75)	-21
888	61.595(15)	120.016(27)	$\alpha$	85.288(50)	-12
908	61.565(15)	119.962(27)	$\alpha$	85.288(50)	-27
896	109.630(10)	199.978(16)	$\alpha$	167.575(75)	-23
914	0.705(25)	1.509(53)	$a$	1188.700(300)	24
914	0.705(25)	1.509(53)	$b$	1189.500(300)	0
916	0.925(15)	1.979(32)	$b$	1190.600(300)	74
922	0.880(16)	1.883(34)	$c$	1186.090(20)	31
932	0.900(25)	1.925(53)	$c$	1186.100(15)	75
938	0.987(16)	2.111(34)	$c$	1186.095(15)	-5
939	2.342(25)	4.999(53)	$c$	1186.165(30)	5
					-3
					3
					$\chi^2 = 1.2$
					$\chi^2 = 2.7$

<sup>a</sup> The quantum numbers for the observed transitions are as follows:

Transition	$F_1$	$M_1$	$F_2$	$M_2$
$\alpha$	$5/2 \leftrightarrow 3/2$	$3/2 \leftrightarrow 1/2$	$5/2 \leftrightarrow 3/2$	$3/2 \leftrightarrow 1/2$
$a$	$5/2 \leftrightarrow 3/2$	$3/2 \leftrightarrow 1/2$	$5/2 \leftrightarrow 3/2$	$3/2 \leftrightarrow 1/2$
$b$	$5/2 \leftrightarrow 3/2$	$3/2 \leftrightarrow 1/2$	$5/2 \leftrightarrow 3/2$	$3/2 \leftrightarrow 1/2$
$c$	$5/2 \leftrightarrow 3/2$	$3/2 \leftrightarrow 1/2$	$5/2 \leftrightarrow 3/2$	$3/2 \leftrightarrow 1/2$

beam and the counting rate about 1 h later, when the <sup>88</sup>Rb had decayed away.

As an additional monitor of beam intensity, a hot-wire surface-ionization detector and an electrometer were used to measure the <sup>86</sup>Rb carrier beam. This procedure also allowed us to calibrate the magnetic field by observing the  $\Delta F = 0$ ,  $\Delta m_F = \pm 1$  transition in <sup>86</sup>Rb.

### B. Experimental Procedure

After the spin was determined, the transition ( $F = \frac{5}{2}$ ,  $M_F = -\frac{3}{2}$ )  $\leftrightarrow$  ( $F = \frac{5}{2}$ ,  $M_F = -\frac{5}{2}$ ) was observed at magnetic fields up to 200 G. The rf source for this transition was a Hewlett-Packard model 608C oscillator followed by an Instruments-for-Industry model 500 amplifier or a Boonton model 230-A amplifier. At low magnetic fields a Tektronix model 190B oscillator was used directly. The rf power was typically 750 mW for this transition.

A search for a  $\Delta F = 1$  resonance yielded one which we identified with the transition ( $\frac{5}{2}, \frac{5}{2}$ )  $\leftrightarrow$  ( $\frac{3}{2}, \frac{3}{2}$ ); the transition was identified by changing the magnetic field slightly and measuring the field dependence of the resonant frequency. Subsequently, the field-independent transition ( $\frac{5}{2}, \pm\frac{1}{2}$ )  $\leftrightarrow$  ( $\frac{3}{2}, \mp\frac{1}{2}$ ) was observed. This line had a full width at half-maximum of about 50 kHz and provided an accurate direct measure of  $\Delta\nu$ . The rf source for these  $\Delta F = 1$  transitions was a Schlumberger model DO 1001 oscillator, with either a model 0800D or a model 01500 plug-in oscillator, followed by a Microwave Labs uhf triode power amplifier. The optimum rf power was 600 mW for this transition.

To ensure the correct identification of the  $\Delta F = 1$  resonances, the transition ( $\frac{5}{2}, \pm\frac{1}{2}$ )  $\leftrightarrow$  ( $\frac{3}{2}, \mp\frac{1}{2}$ ) was ob-

served to have essentially the same peak radiofrequency at 5 G as at 2 G. This is the only  $\Delta F = 1$  transition with a frequency independent of the magnetic field at low field strengths.

### C. Data

Table I summarizes the observations made at all magnetic fields. Two observations were made at 120 G (runs 888 and 908) with the magnetic field in opposite

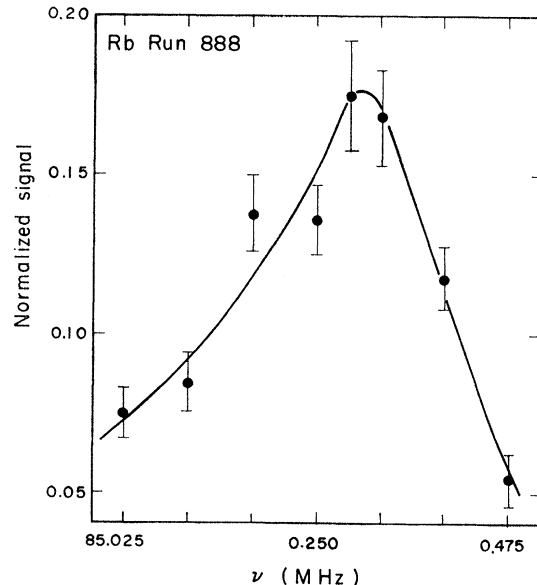


FIG. 2. Resonance corresponding to the transition ( $\frac{5}{2}, -\frac{3}{2}$ )  $\leftrightarrow$  ( $\frac{3}{2}, -\frac{3}{2}$ ) in <sup>88</sup>Rb at 120.016 G with the external magnetic field in the normal direction.

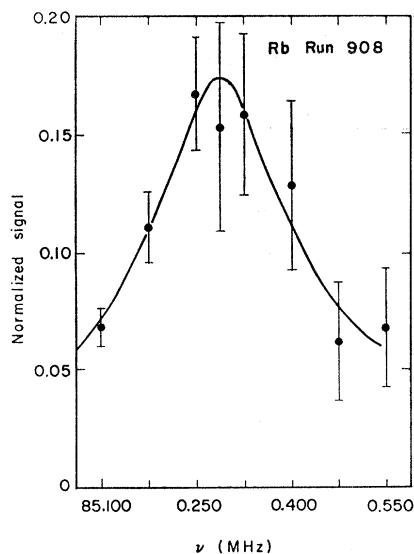


FIG. 3. Resonance corresponding to the transition  $(\frac{5}{2}, -\frac{3}{2}) \leftrightarrow (\frac{3}{2}, -\frac{5}{2})$  in  $^{85}\text{Rb}$  at 119.962 G with the external magnetic field in the reverse direction.

directions to check for a possible Millman effect.<sup>1</sup> Such an effect would bias the data, depending on the direction of the field. The observed shift is not significant with respect to the uncertainty assigned the resonant frequencies. These two resonances are illustrated in Figs. 2 and 3. The  $\Delta F=1$  transition from run 922 is illustrated in Fig. 4. The full width of this resonance at half-maximum is 40 kHz.

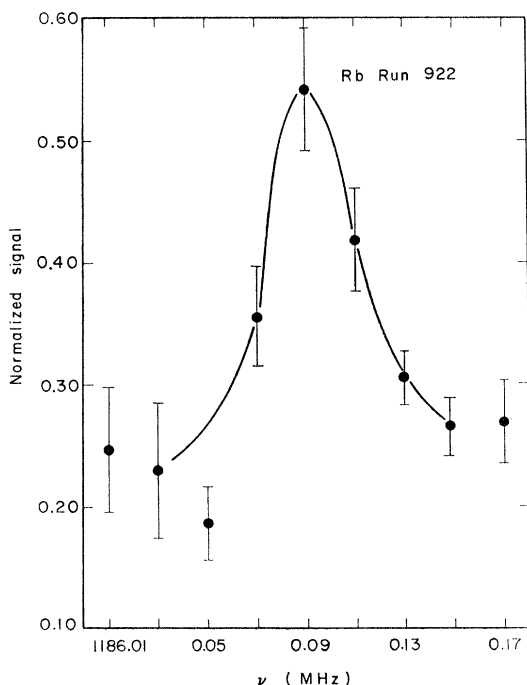


FIG. 4. Resonance corresponding to the transition  $(\frac{5}{2}, \pm\frac{1}{2}) \leftrightarrow (\frac{3}{2}, \mp\frac{1}{2})$  in  $^{85}\text{Rb}$  at 1.883 G.

TABLE II.  $^{85}\text{Rb}$  final results.

$I=2$
$\Delta\nu = \pm 1186.084(18)$ MHz
$\mu_I$ (uncorr) = $\pm 0.506(5)$ nm
$\mu_I$ (corr) = $\pm 0.508(5)$ nm

That the observed signals were due to  $^{85}\text{Rb}$  and not some other activity was established by a decay analysis of all buttons exposed during each run. In addition, the following analysis was applied to the peak button of run 879: The decay curves of the center and side buttons were plotted. The side-button decay curve was normalized to equal the center-button decay curve at  $t \gg 0$ . Then the normalized side-button decay curve was subtracted from the center-button decay curve. The result is the decay plot of the enhancement in short-half-life activity on the center button. A half-life was fitted to this curve, illustrated in Fig. 5, with the result  $T_{1/2} = 17.5 \pm 2.4$  min.

#### IV. RESULTS AND DISCUSSION

##### A. Results

The Breit-Rabi formula was fitted to the data of Table I by least-squares analysis, varying the hfs interaction constant  $a$ . Two fits were made, one assuming  $\mu_I > 0$ , and another assuming  $\mu_I < 0$ . For the first assumption  $\chi^2 = 1.2$  and for the latter assumption  $\chi^2 = 2.7$ , with 17 observations. The results are given in Table II; in Table III we have listed the constants assumed in deriving the results. The error quoted for  $\mu_I$  has been fixed at 1% to allow for a possible hfs anomaly. These results are in agreement with preliminary findings.<sup>2,3</sup>

The small  $\chi^2$  values (shown in Table I) for the two fits indicate that our original choices of uncertainties for the data were very conservative. However, these choices reflect considerations of possible systematic errors. The final results presented in Table II have uncertainties representing two standard deviations of the least-squares analysis.

TABLE III. Constants assumed in calculating results.

$a$ ( $^{85}\text{Rb}$ ) = 1011.910813(2) <sup>a</sup>
$g_I$ ( $^{85}\text{Rb}$ ) <sub>uncorr</sub> = 0.0002937 <sup>a</sup>
$g_I$ (Rb) = -2.002332 <sup>b</sup>
$\mu_0/h$ = 1.399613
$m_p/m_e$ = 1836.12
$(1-\sigma)^{-1}$ = 1.00334 <sup>c</sup>

<sup>a</sup> See Ref. 6.

<sup>b</sup> From  $g_I$  ( $^{85}\text{Rb}$ )/ $g(e) = 1.0000063(10)$  by L. C. Balling and F. M. Pipkin, Phys. Rev. 139, A19 (1965); and  $g(e) = -2.002319244(54)$  by D. T. Wilkinson and H. R. Crane, Phys. Rev. 130, 852 (1963).

<sup>c</sup> W. C. Dickinson, Phys. Rev. 80, 563 (1950).

<sup>2</sup> V. J. Ehlers and H. A. Shugart, Bull. Am. Phys. Soc. 5, 503 (1960).

<sup>3</sup> V. J. Ehlers, M. H. Prior, H. A. Shugart, and P. A. Vanden Bout, Bull. Am. Phys. Soc. 12, 132 (1967).

TABLE IV. Calculated magnetic moments for possible  $^{88}\text{Rb}$  configurations.

Odd neutron	Odd proton	$\mu_I$ (nm)
$d_{5/2}$	$p_{1/2}$	-1.1
$d_{5/2}$	$p_{3/2}$	-0.7
$d_{5/2}$	$f_{5/2}$	+0.1
$g_{7/2}$	$p_{3/2}$	-2.8
$g_{7/2}$	$f_{5/2}$	-0.8

### B. Discussion of Results

The  $^{88}\text{Rb}$  nucleus contains 37 protons and 51 neutrons. The single odd neutron beyond the major closed shell at  $N=50$  most probably occupies the  $2d_{5/2}$  subshell. The nine odd protons are not so unambiguously placed, however. It is possible that they may be distributed in a number of ways among the  $2p_{3/2}$ ,  $1f_{5/2}$ ,  $2p_{1/2}$ , and  $1g_{9/2}$  subshells. A simple proton configuration which yields the observed spin  $I=\frac{3}{2}$  for  $^{87}\text{Rb}$  places three in the  $2p_{3/2}$  and the remaining six in the  $1f_{5/2}$  subshells. Assuming that the same proton configuration exists in  $^{88}\text{Rb}$ , the observed spin of  $I=2$  must arise from the coupling of an odd  $2d_{5/2}$  neutron with an odd  $2p_{3/2}$  proton hole. This is in agreement with the weak particle-hole coupling rule given by Brennan and Bernstein.<sup>4</sup>

It is possible to calculate  $\mu_I$  ( $^{88}\text{Rb}$ ) by using the observed magnetic moments of  $^{91}\text{Zr}$  ( $Z=40$ ,  $N=51$ ,  $I=\frac{5}{2}$ ) and  $^{87}\text{Rb}$  ( $Z=39$ ,  $N=50$ ,  $I=\frac{3}{2}$ ). The values used were  $\mu_I$  ( $^{91}\text{Zr}$ ) = -1.3 nm,<sup>5</sup> and  $\mu_I$  ( $^{87}\text{Rb}$ ) = +2.8 nm.<sup>6</sup> The calculated value for  $\mu_I$  ( $^{88}\text{Rb}$ ) is then -0.7 nm, which is in fair agreement with the observed value of  $\pm 0.5$  nm, provided one chooses the negative sign.

The results of the experiment do not distinguish between the two possible signs for  $\mu_I$  ( $^{88}\text{Rb}$ ); the factor of

<sup>4</sup> M. H. Brennan and A. M. Bernstein, Phys. Rev. **120**, 927 (1960).

<sup>5</sup> E. Brun, J. Oeser, and H. H. Staub, Phys. Rev. **105**, 1929 (1957).

<sup>6</sup> S. Penselin, T. Moran, V. W. Cohen, and G. Winkler, Phys. Rev. **127**, 524 (1962).

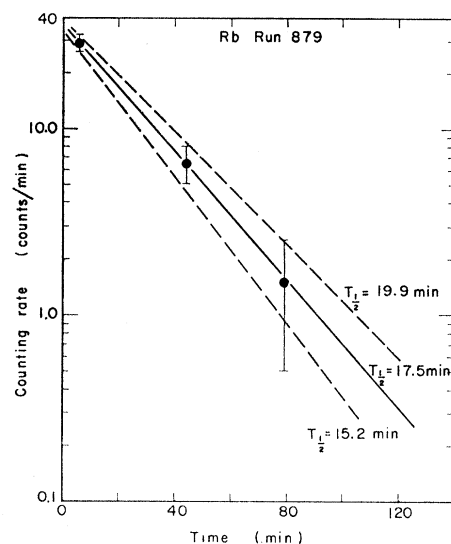


FIG. 5. Decay of enhanced short-half-life activity on the peak button of the resonance observed in run 879. The solid line is the fitted half-life and the dashed lines indicate the error in the fit.

2 between the  $\chi^2$  of the computer fits for positive and negative  $\mu_I$ , which favors assignment of the + sign, is not considered at all conclusive. Linewidth (40 kHz) limitations and the short half-life have thus far prevented the determination of the magnetic-moment sign.

It should be pointed out that it is extremely difficult to calculate a positive magnetic moment of the proper magnitude by using the observed magnetic moments for any of the other proton configurations, including those which do not predict the spin properly. The same holds when one assumes that the odd neutron occupies the  $1g_{7/2}$  subshell. As can be seen from Table IV, all assumed configurations predict either an extremely small positive moment,  $\approx +0.1$  nm, or negative magnetic moments ranging from -0.7 to -2.8 nm. It would seem, then, that theory strongly requires that  $\mu_I$  be negative.