Measurement and Distorted-Wave Born-Approximation Analyses of the 18.7-MeV Cross Sections for the $Al^{27}(\alpha,d_{0,1})Si^{29}$ Reactions*†

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The 18.72-MeV differential cross sections for the $Al^{27}(\alpha,d_{0,1})Si^{29}$ reactions have been measured from 15° to 170° at 5° intervals. Deuteron spectra were measured using an electronic spectrometer system with a particle-identification capability. Both of the measured angular distributions show forward-angle peaking, an absence of any strong oscillatory structure, and weak washed-out undulations at backward angles with a general tendency to rise at angles greater than 130°. The integrated cross sections for transitions to the ground and 1.277-MeV states of Si^{29} are 747±17 μb (16.9° to 171.3°) and 863±20 μb (17.0° to 171.4°), respectively. Distorted-wave Born-approximation analyses of the data have been made in terms of zero-range knockout and stripping models. In the knockout model the initial (final) nuclear state is described as a two-body system consisting of a deuteron (an α particle) bound to a Mg^{25} core, and in the stripping model the final nuclear state is described as a deuteron bound to an Al^{27} core. The gross features of the two angular distributions are equally well reproduced by the knockout and the stripping models. The knockout analysis requires a $2s_{1/2}$ state for the (d,Mg^{25}) system representing the ground state of Al^{27}, and $2d_{1/2}$ and $1g_{7/2}$ states for the (α,Mg^{25}) system representing the ground and 1.277-MeV states of Si^{29}, respectively. The stripping analysis requires $1d_{1/2}$ and $1g_{7/2}$ states for the (d,Al^{27}) system representing the ground and the 1.277-MeV states of Si^{29}, respectively. Both the knockout and the stripping analyses indicate angular momentum transfers of 2 and 4 in the (α,d_0) and (α,d_1) reactions, respectively.

I. INTRODUCTION

THE adequacy of the simple-zero-range distorted-wave Born-approximation (DWBA) knockout and stripping models of Tobocman[1] in the description of 18.7-MeV differential cross sections for the $P^{31}(\alpha,d_{0,1})S^{33}$ and $Na^{23}(\alpha,d_{0,1})Mg^{25}$ reactions has been considered in two previous investigations.[2,3] The P^{31} data were analyzed only in terms of the knockout model, and moderate success was achieved in reproducing the experimental angular distributions. In the analyses of the Na^{23} data, it was found that the general character of the angular distributions could be reproduced for reasonable parameter values in terms of the knockout model only. Although only a qualitative reproduction of the gross features of the experimental angular distribution could be achieved in two of the four cases, the correspondence between the experimental and theoretical results was closer than might be expected for such a simple model of this type of two-nucleon transfer reaction. It is clear that the study of more experimental data is required before the limitations of these models can be understood and/or their ranges of validity, if any, completely defined. These measurements and analyses of the 18.7-MeV Al^{27}-$(\alpha,d_{0,1})Si^{29}$ reaction cross sections were undertaken to partially fulfill this need.

II. EXPERIMENTAL

The 18.7-MeV external α-particle beam from the Purdue University 37-in. cyclotron was used to bombard Al^{27} targets which had been fabricated from commercial 0.1-mil aluminum foil (774±13 μg/cm²). A general description of the cyclotron facility may be found elsewhere.[4] Continuous monitoring of the reaction-particle spectrum at a fixed laboratory angle indicated that the target thickness remained constant throughout the measurements.

The deuteron spectra were measured using an electronic spectrometer system consisting of an $E-\Delta E$ counter telescope, an $E\times\Delta E$ mass identification unit, and a multichannel pulse-height analyzer. The counter telescope was made up of a ΔE counter, a 40-μ transmission-mounted, fully-depleted silicon surface-barrier detector, and an E-ΔE counter, a silicon surface-barrier detector. The spectrometer system had an inherent energy resolution (full width half-maximum) of 80 keV for the 8.780-MeV α-particle group associated with the decay of Po^{212}, but the resolution for the deuteron groups of interest under typical experimental conditions was about 180 keV. A more complete description of the spectrometer system has been presented elsewhere.[2]

The principal experimental difficulty encountered was associated with the fact that the spectrometer system was incapable of operation over a wide energy range with full deuteron detection efficiency and extremely high proton rejection efficiency. This instrumental limitation coupled with the presence of intense proton

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[1] W. Tobocman, *Theory of Direct Nuclear Reactions* (Oxford University Press, London, 1961).
[2] B. B. Srivastava, S. W. Cosper, and O. E. Johnson, Phys. Rev. **153**, 1221 (1967).
[3] B. B. Srivastava and O. E. Johnson, Phys. Rev. **156**, 1219 (1967).
[4] B. T. Lucas, S. W. Cosper, and O. E. Johnson, Phys. Rev. **133**, B963 (1964).

groups which completely dominated the spectra produced experimental circumstances such that only the angular distributions corresponding to the ground- and first-excited-state deuteron groups could be reliably measured.

At the target, the α-particle beam energy was 18.72 MeV with an rms spread of 60 keV, and its normal cross section was circular with a diameter of 0.078 in. The defining aperture of the counter telescope was circular, and subtended an azimuthal angle of approximately 2.3° and a solid angle of approximately 0.001 sr with respect to the target center.

III. EXPERIMENTAL RESULTS

The experimental values of the differential cross sections for the $(\alpha,d_{0,1})$ reactions on Al^{27} at 18.72 MeV are indicated in the various figures by the solid circles. The bars on the experimental points represent the probable errors based on counting statistics only. The assignment of a $\pm 15\%$ probable systematic error in the absolute differential cross sections is based on an apprasial of uncertainties in the target thickness, beam current integration, and experimental geometry. In the reduction of the experimental data, finite geometry has been accounted for only to first order.

The $(\alpha,d_{0,1})$ angular distributions are the same in general character. The common features are: an absence of any strong oscillatory structure, an asymmetry with respect to 90°, forward-angle peaking, and weak washed-out undulations at backward angles with a general tendency to rise for angles greater than 130°. The integrated cross sections are 747 ± 17 μb (16.9°–171.3°) and 863 ± 20 μb (17.0°–171.4°) for the ground- and 1.277-MeV-state transitions, respectively.

IV. ANALYSES

A. General

A detailed comparison was made between the experimental angular distributions and those predicted by the zero-range DWBA knockout and stripping models of Tobocman.[1] The theoretical angular distributions were calculated using a computer code developed and described by Gibbs *et al.*[5]

The DWBA amplitude for the reaction $I(\alpha,d)F$ is

$$A_{\alpha d}(\mathbf{K}_\alpha,\mathbf{K}_d)=\langle\Phi_{dF}^{(-)}(\mathbf{K}_d)\varphi_d\varphi_F \times|V_{dF}-\bar{V}_{dF}|\Phi_{\alpha I}^{(+)}(\mathbf{K}_\alpha)\varphi_\alpha\varphi_I\rangle, \quad (1)$$

where $\Phi_{\alpha I}^{(+)}$ and $\Phi_{dF}^{(-)}$ are the optical-model wave functions corresponding to the entrance and exit channels, respectively; each φ is the internal wave function of the entity denoted by the subscript; V_{dF} is the interaction potential between the deuteron and the final nucleus; and $\bar{V}_{dF}$ is the appropriate optical-model potential for the exit channel.

Since detailed descriptions of the various simplifying assumptions and the procedures for the calculation of the theoretical angular distributions in the knockout and the stripping models may be found elsewhere,[2,3] only the essentials will be presented here.

Unless otherwise specified it may be assumed that in attempting to reproduce the experimental angular distributions, sets of parameter values were sought which yielded good agreement over the entire angular range covered by the data.

TABLE I. The sets of OM parameters corresponding to elastic α-particle and deuteron scattering from Al^{27}.

Set No.	Reaction	Energy (MeV)	Angular range used in fit	V_0 (MeV)	W_0 (MeV)	D (F)	r_0 (F)
1	$(\alpha,\alpha)^a$	18.82	10.1°– 82.5°	12.89	7.82	0.469	1.990
2			20.5°– 82.5°	34.86	10.14	0.481	1.849
3			13.1°– 82.5°	63.71	14.61	0.482	1.737
4			13.1°– 82.5°	103.05	20.05	0.480	1.652
1	$(d,d)^b$	9.20	10.7°–170.7°	59.01	17.12	0.499	1.671

[a] These sets of parameters were derived in the present study using experimental data from Ref. 6.
[b] This set of parameters was taken from Ref. 8.

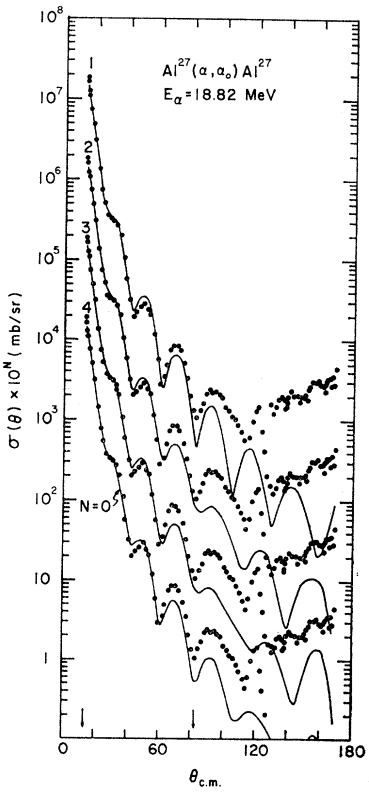

FIG. 1. Comparisons between the experimental (solid circles) and best-fitting OM (solid curves) angular distributions for the $Al^{27}(\alpha,\alpha_0)Al^{27}$ reaction at 18.82 MeV. The number associated with each solid curve identifies the OM set used in its calculation (see Table I). The angular range of the experimental data used in the OM parameter search is indicated by the two small arrows.

[5] W. R. Gibbs, V. A. Madsen, J. A. Miller, W. Tobocman, E. C. Cox, and L. Mowry, National Aeronautics and Space Administration Technical Note No. TN-D2170, 1964 (unpublished).

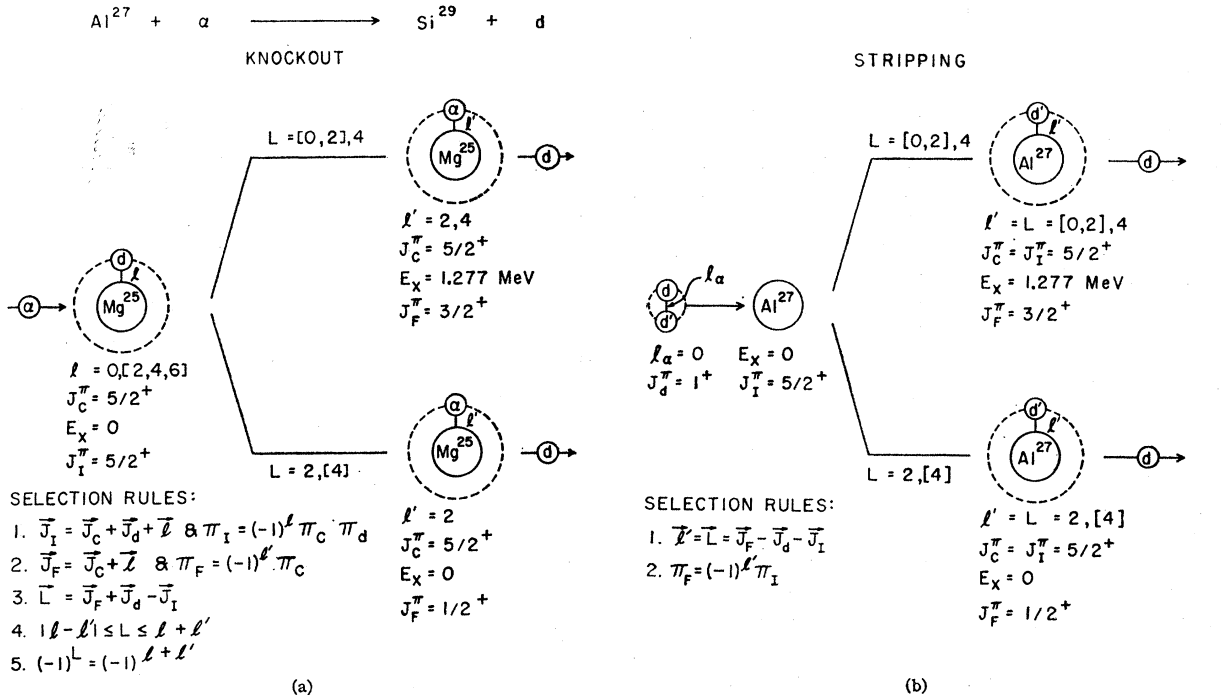

FIG. 2. Graphic summaries of some of the pertinent details of the knockout and the stripping models as applied to the $Al^{27}(\alpha,d_{0,1})Si^{29}$ reactions. Included in each section of the figure are: the excitation energy (E_x), spin, and parity of each nuclear state; the spin and parity of each constituent for each two-body system; the values of the relative orbital angular momenta (l and/or l') for each two-body system, and the angular momentum transfer L, and the rules for their selection. In the stripping model, the zero-range assumption for the (d,d') interaction requires that the two deuterons making up the incident α particle have zero relative orbital angular momentum. The values of l and/or l', and L which are excluded on the basis of these analyses are enclosed in square brackets.

B. Optical-Model Parameters

The entrance-channel optical-model (OM) parameters used in this study were derived from an analysis of the 18.82-MeV $Al^{27}(\alpha,\alpha_0)Al^{27}$ data of Gailar *et al.*[6] using a Saxon-Woods four-parameter potential of the form

$$U(r)=-(V_0+iW_0)/(e^x+1), \quad (2)$$

where

$$x=(r-r_0A^{1/3})/D, \quad (3)$$

together with a Coulomb potential corresponding to a uniformly charged sphere having a radius given by $1.2A^{1/3}$ F. Details concerning the computer program used and the general procedures followed in the OM analyses were similar to those described previously by Lucas *et al.*[7] This study yielded four theoretical angular distributions whose correspondence to the experimental data were quantitatively nearly equivalent. These sets of OM parameters are given in Table I, and the theoretical angular distributions (solid curves) are compared with the experimental angular distributions (solid circles) in Fig. 1. In the DWBA analyses of the $Al^{27}(\alpha,d_{0,1})Si^{29}$ data, fitting was attempted using OM sets Nos. 1 and 4 only. These sets were selected on the basis of a subjective judgment of the quality of fits to the experimental elastic-scattering data.

The correct exit-channel descriptions would require OM parameters for elastic deuteron scattering from Si^{29} at 11.00 MeV (ground state) and 9.64 MeV (1.277-MeV state). Since this information was not available, the single set of OM parameters (see Table I) derived by Cosper[8] for 9.2-MeV elastic deuteron scattering from Al^{27}, also using the four-parameter potential described above, was adopted.

C. Knockout Analysis

Presented in Fig. 2(a) is a diagrammatic summary of the knockout model, and details concerning the relevant nuclear states and their representation. Also included in the figure are the allowed values of the initial and final bound-state relative orbital angular momenta (l and l'), the angular momentum transfer (L), and their selection rules.

It should be noted that the occurrence of multiple allowed l and l' values is not precluded by purely formal considerations. The nonuniqueness in the orbital angular momentum does however imply a nonuniqueness in the bound-state potential—a circumstance that does

[6] O. H. Gailar, E. Bleuler, and D. J. Tendam, Phys. Rev. **112**, 1989 (1958).

[7] B. T. Lucas, S. W. Cosper, and O. E. Johnson, Phys. Rev. 144, B972 (1966).

[8] S. W. Cosper, Ph.D. thesis, Purdue University, 1965 (unpublished) (Order No. 66-5252, University Microfilms, Inc., Ann Arbor, Mich.).

not lend itself to a simple direct physical interpretation. For this reason it has been assumed that each bound state should be characterized by a single value for the orbital angular momentum quantum number. The problem then becomes one of selection from among those values of l and l' that are admitted by spin and parity considerations. The method of selection will be discussed at another point in this report.

In this model, the bound-state wave functions φ_I and φ_F are calculated assuming a two-body, core and extracore particle, interaction characterized by the Coulomb potential for a uniformly charged sphere and a real nuclear potential with a Saxon-Woods shape. In the calculation of the initial and final bound-state wave functions, the quantities for which some freedom of selection or adjustment may be exercised are: the nuclear radii R_N and diffusivities D_N of the potentials, and the radial (N and N') and orbital angular momentum (l and l') quantum numbers. Full exploitation of this parametrization was not practical; consequently the nuclear radii and diffusivities were assigned the physically reasonable values of 4.250 F [9] and 0.500 F, respectively. Known masses and excitation energies were used to calculate the binding energies of the two-body states. The determination of the well depth and the wave functions for each bound state then only required that the relative angular momentum and radial quantum numbers be specified. It was found that no correspondence between the experimental and theoretical angular distributions could be achieved without using a cutoff radius R_C and a normalization constant.

Since there is only a single allowed l' value for the ground state of Si^{29}, the l value for the ground state of Al^{27} was selected on the basis of the relative quality of of the best theoretical fits to the (α,d_0) angular distribution for each allowed l value. The part of the analysis using the No. 4 set of entrance-channel OM parameter will be described first since it yielded better DWBA fits for the experimental data. The cutoff radius R_C was varied from 0 to 6.0 F and for each allowed (l,l') pair the combinations of (N,N') values explored were (1,1), (1,2), (2,1), and (2,2). For $R_C \geq R_N$, the shapes of the theoretical angular distributions were found to be very insensitive to the values of the radial quantum numbers. As a consequence, for $R_C \geq R_N$, it was only necessary to investigate the theoretical angular distributions for a single (N,N') pair for each allowed (l,l') pair. The theoretical angular distribution that was judged to be in the closest agreement with the experimental data is represented by the solid curve in Fig. 3(a). The corresponding values of the cutoff radius and the bound-state parameters (BSP) are also tabulated in the figure. It should be pointed out that although R_C is less than R_N, in this case, the shape of the theoretical angular distribution was found to be only weakly dependent on the values of the radial quantum numbers. The reason for the selection of (N,N') to be (2,2) will be given below.

On the basis of the best theoretical fit to the (α,d_0) angular distribution, $l=0$ was selected for the ground state of Al^{27}. A procedure similar to that described above was followed using the (α,d_1) data in an attempt to select the l' value for the 1.277-MeV state of Si^{29}. The best theoretical fit to the (α,d_1) data is shown in Fig. 4(a) by the solid line. In this case the shape of the theoretical angular distribution depended strongly on the values of the radial quantum numbers. The reproduction of the angular distribution requires the radial quantum numbers 2 and 1 for the ground state of Al^{27} and the 1.277-MeV state of Si^{29}, respectively. Thus the states of the two-body systems representing the ground state of Al^{27} and the 1.277-MeV state of Si^{29} were determined to be $2s_{5/2}$ and $1g_{3/2}$, respectively. It is physically reasonable to require very nearly the same bound-state potential-well depths for the final nuclear state in both the (α,d_0) and (α,d_1) reactions. To achieve this end it was necessary to assign $N'=2$ to the ground state of Si^{29} and to identify the state of the two-body system representing the ground state of Si^{29} as $2d_{1/2}$.

The theoretical fits achieved in this study indicate that the (α,d_0) and (α,d_1) reactions correspond to angular momentum transfers of 2 and 4, respectively.

Identical procedures were followed using the No. 1 set of OM parameters (see Table I), a physically less realistic set, for the entrance channel. The quality of the best theoretical fits achieved was found to be much poorer compared to that of the fits obtained using set No. 4. Moreover, these analyses indicated $l=2$ for the ground state of Al^{27}, but could not unambiguously identify the l' value for the 1.277-MeV state of Si^{29}. In addition, it was found that the (α,d_0) reaction involves an $L=2$ and 4 admixture with neither dominant, and that the (α,d_1) reaction involves either an $L=2$ and 4 admixture with $L=4$ overwhelmingly dominant, or an $L=0$, 2, and 4 admixture with the $L=2$ component negligible. Because the shallow well characterized by OM set No. 1 is not physically credible, and because the resulting fits are of definitely inferior quality, there will be no further discussion of these analyses.

D. Stripping Analysis

A digrammatic summary of the stripping model, and details concerning the relevant nuclear states and their representation are presented in Fig. 2(b). Also included in the figure are the allowed values of the final bound-state relative orbital angular momentum l', the angular momentum transfer L, and their selection rules.

The bound-state wave function φ_F is calculated for the two-body system consisting of the captured deuteron d' and Al^{27} in its ground state. The method of

[9] Radius parameters for A^{27} and Si^{29} were calculated taking $R_N=1.4A^{1/3}$ F. Because the difference between these two values is much smaller than the grid size used in calculating the radial integrals, their mean value of 4.250 F was adopted for both nuclei in the knockout analysis.

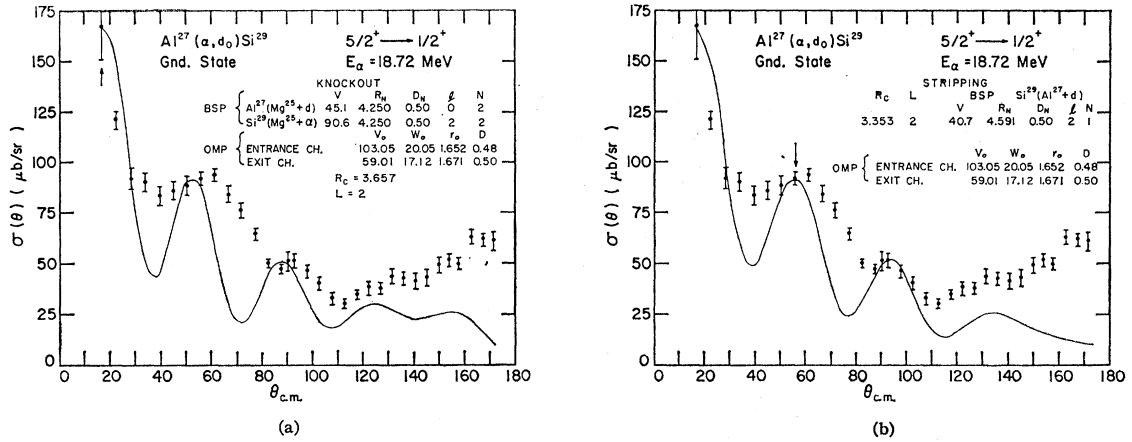

FIG. 3. The experimental differential cross sections for the $Al^{27}(\alpha,d_0)Si^{29}$ reaction (solid circles) and the best-fitting DWBA knockout-model (part a) and stripping-model (part b) angular distributions (solid curves). The information given in each section of the figure includes: the incident α-particle energy E_α, the spin and parity of each nuclear state, the BSP, the entrance- and exit-channel OM parameters, the cutoff radius R_C, and the value of the angular momentum transfer L. The points at which the theoretical angular distributions are normalized to the experimental data are indicated by small arrows.

calculating this bound-state wave function and the general considerations regarding multiple values for l' are the same as were discussed in the knockout analysis (see Sec. IV C). The best fits to the experimental data in the stripping model were also obtained using set No. 4 of the entrance-channel OM parameters. Since there are only two allowed l' values for the ground state of Si^{29}, the possibility of fitting the (α,d_0) angular distribution was investigated first. The radius parameter R_N and the diffusivity D_N for the Si^{29} bound-state potential were assigned the fixed values 4.301 F [9] and 0.500 F, respectively. The adjusted quantities were: the radial and orbital angular momentum quantum numbers N' and l', respectively; the cutoff radius R_C; and a normalization constant. The cutoff radius was varied from 0 to 6.0 F, and the N' values 1 and 2 were investigated for each l' value. For reasons similar to those described in the knockout analysis (see Sec. IV C), when $R_C \geq R_N$, it was only necessary to investigate the theoretical angular distributions for a single N' value for each allowed l' value. Since the stripping formalism involves fewer parameters than the knockout formalism, some searches were made in which R_N was also varied within reasonable limits. It was found that the shape of the theoretical angular distribution was insensitive to a variation of R_N in the range 4.00 to 4.60 F. As described below, the choice of $R_N = 4.591$ F was dictated by the analysis of the (α,d_1) angular distribution. On the other hand, the shape was strongly dependent on the values of the cutoff radius and the radial and orbital angular momentum quantum numbers. The solid curve in Fig. 3(b) represents the best fitting theoretical angular distribution that was found for the (α,d_0) reaction. The adjusted values of the parameters are tabulated in the figure.

A similar procedure was followed in an effort to fit the

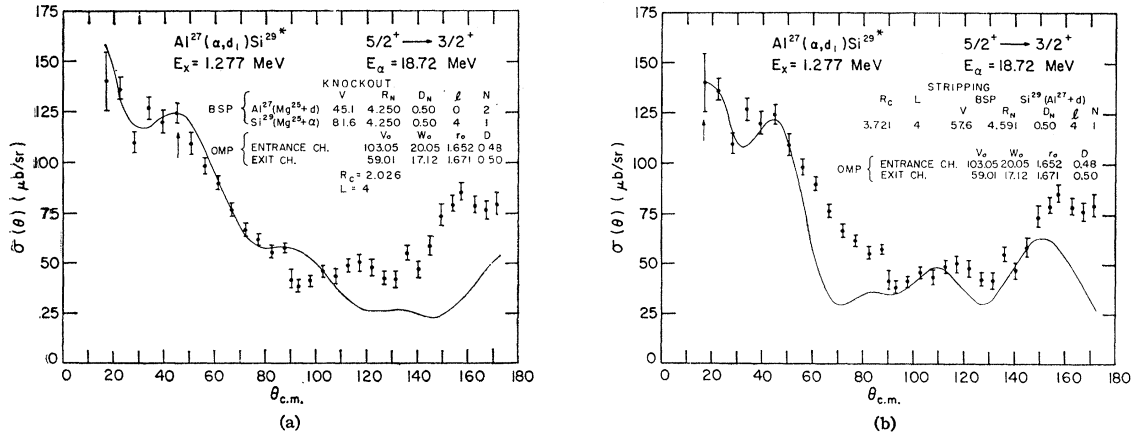

FIG. 4. The experimental differential cross sections for the $Al^{27}(\alpha,d_1)Si^{29}$ reaction (solid circles) and the best-fitting DWBA knockout-model (a) and stripping-model (b) angular distributions (solid curves). E_x is the final-state excitation energy. The remainder of the format of this figure is the same as that of Fig. 3.

(α,d_1) data. The solid curve in Fig. 4(b) is the best-fitting theoretical angular distribution that could be found. It was necessary to adjust the value of R_N to 4.591 F to obtain this fit. The radial and orbital angular momentum numbers for the final bound state were required to have the values of 1 and 4, respectively. The values of other parameters are tabulated in Fig. 4(b). A value of 4.591 F for R_N was also adopted for fitting the (α,d_0) data, because it is physically reasonable to require the same value of R_N for both the ground and 1.277-MeV states of Si^{29}.

On the basis of this analysis, the states of the two-body system (Al^{27},d) representing the ground and 1.277-MeV states of Si^{29} are identified as $1d_{1/2}$ and $1g_{3/2}$, respectively. In agreement with the results of the knockout analysis, the stripping analysis also indicates angular momentum transfers of 2 and 4 in the (α,d_0) and (α,d_1) reactions, respectively.

When identical fitting procedures were followed using the No. 1 set of the entrance-channel OM parameters, fits inferior in quality to those just described resulted. However, these analyses did also indicate that the (α,d_0) and the (α,d_1) reactions correspond to angular momentum transfers of 2 and 4, respectively.

V. DISCUSSION

The only other reported angular-distribution measurements for the (α,d) reaction on Al^{27} were made at an incident energy of 28.4 MeV.[10] Unfortunately, they cover a very limited angular range (23°–55°) and consist of four data points each; therefore, a meaningful intercomparison of shapes cannot be made. There is, however, one similarity between the 18.7- and 28.4-MeV results; for the angular ranges covered, the reduced cross section, $\sigma_{\exp}(2J_I+1)/(2J_F+1)$, for producing the ground state of Si^{29} is about 1.6 times as large as that for producing the 1.277-MeV state.

Just as was found in the P^{31} and Na^{23} investigations,[2,3] the present attempts to fit the experimental angular distributions for the $Al^{27}(\alpha,d_{0,1})Si^{29}$ reactions using these simple DWBA formalisms of Tobocman have been moderately successful. Although both the knockout- and the stripping-model fits to the (α,d_0) angular distribution are of the same general quality, the stripping-model fit does appear to be somewhat better. While, in the case of the (α,d_1) reaction, the knockout model yields an excellent fit to the experimental data for angles forward of about 90°, the correspondence between the experimental and theoretical angular distributions at backward angles is rather poor. On the other hand, although the stripping model provides the poorer description of the (α,d_1) data at forward angles, it does yield the better representation of the data at angles greater than about 90°. Consequently, it appears that the knockout and the stripping models are about equally successful in reproducing the present data.

The results of the present and previous[2,3] studies indicate that the simple knockout and/or stripping models of Tobocman yield angular distributions which reproduce somewhat more than the gross structure of the experimental angular distributions at forward angles and, in some cases, yield similar agreement at backward angles. Although there are a number of possible reasons why the predictions of these models do not agree in detail with the experimental results, the validity of the supposition that a direct-reaction mechanism is at least dominant, is the most suspect. While the shapes of the various angular distributions almost certainly indicate a direct-reaction contribution, there still remains the question of its strength relative to a compound-nucleus contribution. Consequently, before further conclusions regarding the applicability of these models for the description of these reactions can be drawn, it seems essential that this point be carefully investigated.

[10] S. Kakigi, J. Phys. Soc. Japan **20**, 1967 (1965).

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