well reproduced. Moreover, the method compares favorably with more sophisticated approaches.

ACKNOWLEDGMENTS

We are grateful to Professor A. Bohr and B. R. Mottelson for pointing out to one of us (D.R.B.) the possibilities of the method used in the calculation. Discussions with Professor G. Scharff-Goldhaber, Professor M. Baranger, Professor B. Bayman, and Professor R. Sorenson are greatly appreciated. The assistance of D. Saul in writing the computer programs is gratefully acknowledged. The hospitality of the Brookhaven National Laboratory is much appreciated by D.R.B.

PHYSICAL REVIEW VOLUME 166, NUMBER 4 20 FEBRUARY 1968

Effects of a Soft-Core Potential on the Absorption of Stopped π^- Mesons

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(Received 27 June 1967)

Making use of the correlation function determined on the basis of the Brueckner theory, we calculate total absorption rates, the effects of the short-range N-N correlations on the branching ratio $W(pn\rightarrow nn)/W(pp\rightarrow pn)$, and the distribution of the absorption rate with respect to the opening angle between the emitted nucleons in negative-pion absorption by C^{12} nuclei. Although R^{theor} $(pp;\ 180°/90°)$ is about ten times as small as the experimental value, other theoretical results are in excellent agreement with the experimental results.

1. INTRODUCTION

IT is very interesting to investigate how many effects in pion absorption are given by the short-range N-N correlations due to the repulsive core of the nuclear force.

The π^- mesons are predominantly absorbed by strongly correlated nucleon pairs in the nucleus, i.e., $\pi^-+p+n\rightarrow n+n$ or $\pi^-+p+p\rightarrow p+n$. This can be easily comprehended from the fact that two fast nucleons are mainly emitted back-to-back in the π^--meson absorption by nuclei.[1] Insofar as we are concerned with the two-particle absorption processes mentioned above, we can easily see, by taking into account energy-momentum conservation, that the processes depend on the behavior of the wave function describing the relative motion of the nucleon pair at small internucleon distances ($\lesssim 1$ F),[2] which is the region governed by the repulsive core of the nuclear force. Therefore, we can expect to obtain important information on the short-range N-N correlations by the investigation of the pion-absorption mechanism. On the other hand, the behavior of the repulsive core of the nuclear force in the problems of the nuclear matter can be investigated. through the Brueckner theory. The π^--meson absorption is the most effective tool to test the wave functions obtained on the basis of the Brueckner theory. Recently Akaishi *et al.*[3] numerically calculated the wave function of the relative motion for the singlet S state by solving self-consistently the reaction matrix equation of the Brueckner theory for nuclear matter, assuming an effective central force with a soft core. On the basis of their wave function for the singlet S state, we predict the wave functions for the P and D states from the various experimental data for π^--meson absorption and obtain the total absorption rates and the angular distribution of the absorption rate for C^{12} nuclei. The results obtained in the present paper are compared with the previous work[2] (referred to as I), in which the short-range N-N correlations were introduced phenomenologically. The calculation is performed in the framework of I.

2. THEORY OF THE π^--MESON ABSORPTION

The π-N interaction is taken to be the ordinary pseudovector interaction

$$(f\hbar/\mu c)\bar{\psi}_N\gamma_5\gamma_\mu\tau\psi_N\partial_\mu\phi\,, \tag{1}$$

where the coupling constant $f^2/4\pi\hbar c=0.08$. As was done in I, in the nonrelativistic approximation, we can rewrite the Hamiltonian describing the π^--meson absorption by the strongly correlated nucleon pair in the

[1] S. Ozaki, R. Weinstein, G. Glass, E. Loh, L. Neimala, and A. Wattenberg, Phys. Rev. Letters **4**, 533 (1960).
[2] Il-T. Cheon, Phys. Rev. **158**, 900 (1967).
[3] Y. Akaishi and K. Takada (private communication).

TABLE I. Spin-parity of target nucleus C^{12}.

$J_{C^{12}}{}^P$	$J_{\text{core}}{}^P$	Spin of (pp) pair	Spin of (pn) pair	$J_{\text{pair}}{}^P$	λ
0^+	0^+	0	0	0^+	0
	2^+	0,1	0,1	2^+	2
	1^-	0,1		1^-	1
	3^+		1	3^+	2
	1^+		1	1^+	0,2

form

$$\mathcal{H}^{NN\pi}=G\sum_{i=1}^{2}\tau^-(i)\boldsymbol{\sigma}(i)\cdot\boldsymbol{\nabla}(i)\phi^-(i)$$
$$=G[\tfrac{1}{2}(T^-\mathbf{S}+\tau^-\boldsymbol{\sigma})\cdot(\boldsymbol{\nabla}_R\Phi^-+\boldsymbol{\nabla}_r\phi^-)+(T^-\boldsymbol{\sigma}+\tau^-\mathbf{S})\cdot(\tfrac{1}{4}\boldsymbol{\nabla}_R\phi^-+\boldsymbol{\nabla}_r\Phi^-)] \quad (2)$$

by separating the c.m. and relative motions of the nucleon pair in the nucleus, where $G=f\hbar^2/\mu(2\mu c^2)^{1/2}$. We use the same notation as in I. The wave function of the initial nucleus is given as follows:

$$|\mathrm{I}\rangle=\sum_{\substack{J_cJ_p\\M_cM_p}}\frac{1}{\sqrt{N_c}}(J_cM_cJ_pM_p|J_0M_0)\sum(\lambda\mu s\nu|J_pM_p)$$
$$\times\langle nlNL;\lambda|n_1l_1n_2l_2;\lambda\rangle(lmLM|\lambda\mu)(T_1\Lambda_1T'\Lambda'|T\Lambda)$$
$$\times\Phi_c(J_cT')\psi_{NLM}(\mathbf{R})\varphi_{nlm}(\mathbf{r})X_{s\nu}\chi_{T_1\Lambda_1}, \quad (3)$$

where N_c is the number of the states of the core nucleus; (|) and ⟨ | ⟩ are Clebsch-Gordan coefficient and transformation brackets; $\Phi_c(J_cT')$ is the wave function of the core nucleus; $\psi_{NLM}(\mathbf{R})$ and $\varphi_{nlm}(\mathbf{r})$ are the harmonic-oscillator wave functions of a nucleon pair for the motion of their c.m. and their relative motion; and X and χ represent the spin- and isotopic-spin-wave functions of the pair. For the radial wave function of the relative motion of a nucleon pair in the nucleus, we take into account the effect of correlations due to a repulsive core in the nucleon-nucleon interaction. For brevity, only the states given in Table I are taken for the core nucleus. The allowed quantum numbers of the pair coupled to the core nucleus are determined through the selection rules included in transformation brackets, $2n_1+2n_2+l_1+l_2=2n+2N+l+L$ and $\mathbf{l}_1+\mathbf{l}_2=\mathbf{l}+\mathbf{L}=\boldsymbol{\lambda}$, and spin-parity conservation. The final state is taken to be

$$|F\rangle=V^{-1}\exp(i\mathbf{K}\cdot\mathbf{R}+i\mathbf{k}\cdot\mathbf{r})\Phi_x(J_cY')X_{s''\nu''}\chi_{T''\Lambda''}. \quad (4)$$

Camac *et al.*[4] measured the ratio of the nuclear absorption of π mesons from the $2P$ state to the probability of radiative transition from the $2P$ state to the $1S$ state W_a/W_r. Their result is $9.5\pm1.4\geq W_a/W_r\geq2.4\pm1.1$. On the basis of this result, we assume that the π mesons are absorbed from the $2P$ state. Furthermore, we assume zero-range absorption, i.e., that a pion is absorbed by a nucleon pair at the c.m.: $\phi(\mathbf{r}_1)=\phi(\mathbf{r}_2)=\phi(\mathbf{R})$. Since the π-meson wave functions for the $2P$ state given in the well-known form

$$\phi(\mathbf{r})=\left(\frac{Z}{2a_0}\right)^{3/2}\frac{Z}{\sqrt{3}a_0}r\exp\left(-\frac{Z}{2a_0}r\right)Y_{1\nu}(\hat{r})$$
$$\simeq(Z/a_0)^{5/2}(1/2\sqrt{6})rY_{1\nu}(\hat{r}) \quad (5)$$

vary slowly in the region of the nucleus, this assumption is reasonable. In (5) the approximation in the second line is for $ZR_0/a_0\ll1$ (with R_0 standing for the nuclear radius), which is fulfilled for light nuclei.

Thus we can calculate the matrix elements describing the π^--meson absorption from the $2P$ state by the nucleon pair in the same way as in I. For example, for the pion absorption by the relative D-state (pn) pair formed by a $1p$-state proton and a $1p$-state neutron,

$$\langle F|\mathcal{H}^{NN\pi}|J_c{}^P,T_1,S,J_p{}^P,\lambda;\,p_pn_p{}^D\rangle=\langle F|\mathcal{H}^{NN\pi}|3^+,0,1,3^+,2;\,p_pn_p{}^D\rangle$$
$$=GV^{-1}(00T'\Lambda'|T\Lambda)\frac{\pi}{\sqrt{7}}\left(\frac{Z}{a_0}\right)^{5/2}\sum_m(-)^mY_{2m}(\hat{k})\left[\left(\frac{(3+m)(4+m)}{30}\right)^{1/2}\left(\frac{(3-m)(3+m)}{15}\right)^{1/2}+\left(\frac{(3-m)(4-m)}{30}\right)^{1/2}\right]\left(\frac{\pi}{2\alpha^5}\right)^{1/4}\exp\left(-\frac{K^2}{2\alpha}\right)\int j_2(kr)\varphi_{02}(r)r^2dr. \quad (6)$$

The parameter α is the constant of the harmonic-oscillator wave function for the c.m. motion of the pair. It is determined from the electron scattering data: $\alpha=0.374$ F^{-2}.[5]

In order to take account of the effects of the short-range repulsive core in the nucleon-nucleon interaction, we introduced the correlation function of the type $f(r)=1-\exp(-\xi r^2)$, in I. In this framework

$$\varphi_{nl}(r)=A_{nl}f(r)\varphi_{nl}{}^{sh}(r),$$

where $\varphi_{nl}{}^{sh}(r)$ denotes the harmonic-oscillator wave function and A_{nl} is the normalization constant. The correlation parameter ξ depends, of course, on the state of the nucleon pair absorbing the π^- meson. In the present work we will use the wave function obtained by the Brueckner theory for $\varphi_{nl}(r)$. Recently, Akaishi *et al.*[3] numerically calculated the wave function of the relative

[4] M. Camac, A. D. McGuire, J. B. Platt, and H. J. Schulte, Phys. Rev. **99**, 897 (1955).

[5] R. Hofstadter, Ann. Rev. Nucl. Sci. **7**, 231 (1957); Rev. Mod. Phys. **28**, 214 (1956).

TABLE II. The value of the wave function for various k. $u(kr)$ is the wave function calculated by Akaishi *et al.* $a=0.68$, $b=0.6$ F^{-2}, $c=1.35$, $d=1.8$ F^{-2}.

	$k=0.9k_F=1.368$ F^{-1}		$k=0.5k_F=0.76$ F^{-1}		$k=0.1k_F=0.152$ F^{-1}	
r	$f(r)j_0(kr)$	$u(kr)$	$f(r)j_0(kr)$	$u(kr)$	$f(r)j_0(kr)$	$u(kr)$
0	0.33		0.33		0.33	
0.2	0.397	0.3764	0.403	0.4014	0.405	0.4110
0.4	0.575	0.5424	0.592	0.5946	0.601	0.6155
0.6	0.770	0.7327	0.833	0.8336	0.862	0.8757
0.8	0.866	0.8374	1.001	1.0008	1.063	1.0720
1.0	0.826	0.8222	1.046	1.0518	1.150	1.1566
1.2	0.716	0.7249	1.022	1.0201	1.171	1.1620
1.4	0.565	0.5907	0.951	0.9499	1.147	1.1330
1.6	0.418	0.4475	0.866	0.8678	1.114	1.0965
1.8	0.280	0.3091	0.784	0.7850	1.083	1.0638
2.0	0.153	0.1821	0.706	0.7049	1.058	1.0377
2.2	0.0460	0.07003	0.629	0.6277	1.037	1.0179
2.4	−0.0439	−0.02475	0.555	0.5529	1.021	1.0028
2.6	−0.118	−0.10073	0.478	0.4800	1.006	0.9910
2.8	−0.170	−0.1571	0.407	0.4089	0.990	0.9813
3.0	−0.204	−0.1937	0.339	0.3395	0.982	0.9730

motion for the singlet S state by solving self-consistently the reaction matrix equation of the Brueckner theory for nuclear matter, assuming an effective central force with soft core in the form[6]

$$V(r)=\sum_{i=1}^{3} V_i \exp[-(r/a_i)^2],$$

where the term with $i=1$ corresponds to the one-pion-exchange potential (OPEP), that with $i=2$ to the attraction at intermediate distances, and that with $i=3$ to the strongly repulsive forces around the origin. The effective central force used by Akaishi *et al.* is shown in Fig. 1.

Introducing the correlation function of the following type

$$f(r)=1+a\exp(-br^2)-c\exp(-dr^2), \tag{7}$$

we can obtain, approximately, the wave function given by Akaishi *et al.*:

$$u(kr)\simeq f(r)j_0(kr),$$

where $j_0(kr)$ is the spherical Bessel function. Of course, the parameters a, b, c, and d depend on the relative momentum k. The validity of our approximation is displayed in Table II. Among these parameters, only one set is used for various momenta in the present paper. If we consider the k dependence of these parameters, we can obtain much better results. However, for an approximation, we may neglect the k dependence of the correlation function.[7] Though the wave functions obtained by Akaishi *et al.* are for nuclear matter, there seems to be no difference between the effects of the repulsive core in the two-body force on the wave function for nuclear matter and that for the finite system. It has been shown[8] that in the region of $r<0.3$ F the following relation exists in good approximation between the harmonic-oscillator wave function and the spherical Bessel function:

$$\varphi_{nl}^{sh}(r)=\left[\frac{z\Gamma(n+l+\frac{3}{2})}{\pi n!l(n+\frac{1}{2}l+\frac{3}{4})}\right]^{1/2} 2^{l+1}j_l(kr), \tag{8}$$

$$k=[4\gamma(2n+l+\tfrac{3}{2})]^{1/2}\ (\gamma\equiv M\omega/4\hbar).$$

If we take $\gamma=0.0935$ F^{-2}, which is determined from the electron-scattering data, then $k=0.75$ F^{-1} for $n=l=0$ and $k=1.14$ F^{-1} for $n=1$, $l=0$. Thus, it is a good approximation to use the correlation function

$$f(r)=1+0.68\exp(-0.6r^2)-1.35\exp(-1.8r^2) \tag{9}$$

for the singlet S state in our problem. It is possible to predict approximately the wave functions for the P and D state from the experimental data of the π^--meson ab-

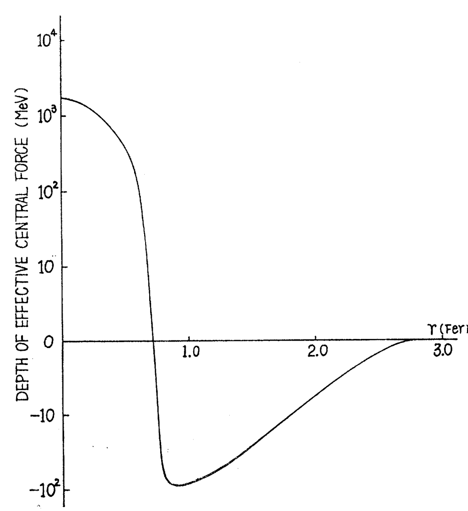

FIG. 1. The effective central force with soft core used by Akaishi *et al.*

[6] M. Harada, R. Tamagaki, and H. Tanaka, Progr. Theoret. Phys. (Kyoto) **36**, 1003 (1966).
[7] Y. Akaishi and K. Takada, Progr. Theoret. Phys. (Kyoto) **37**, 847 (1967).
[8] A. Kallio, Phys. Letters **18**, 51 (1965).

sorption by using the function (7) on the basis of the knowledge, mentioned above, of the S state.

The absorption rates are given by Fermi's golden rule

$$W(NN;J_c{}^P)=\frac{2\pi}{\hbar}\int|\langle F|\mathcal{H}^{NN\pi}\times|J_c{}^P,T_1,S,J_p{}^P,\lambda;NN\rangle|^2\rho_f, \quad (10)$$

where

$$\rho_f dE_f=V^2(2\pi)^{-6}d\mathbf{k}d\mathbf{K}$$

and

$$E_f=\frac{\hbar^2k^2}{M}+\frac{\hbar^2K^2}{4M}+\frac{\hbar^2K^2}{2M_c}.$$

In order to get the distributions for the opening angle Θ between the emitted nucleons, it is convenient to use the coordinate system given in Fig. 2 of I, integrated with respect to $\mathbf{k}$ and $\mathbf{K}$. Under the assumption that the energy is shared equally by the two emitted nucleons in the pion absorption, we have $|\mathbf{k}_1|=|\mathbf{k}_2|$; in other words, $\beta=\frac{1}{2}\pi$ in Fig. 2 of I. When $\beta=\frac{1}{2}\pi$,

$$\cos\Theta=[(\tfrac{1}{4}+M/2m)K^2-D]/[(\tfrac{1}{4}-M/2m)K^2+D],$$

where

$$1/m=(1/2M)+(1/M_c),\quad D=(M/\hbar^2)(\mu c^2-B-E_x).$$

B is the separation energy, E_x is the excitation energy of the residual nucleus, and M_c is the mass of the core nucleus. The distribution of the absorption rates with respect to the projection of the opening angle Θ between the emitted nucleons are then given as follows:

$$\frac{dW(p_sp_s{}^{1S};0^+)}{d\cos\Theta}=\frac{\Gamma}{128}C(p_sp_s{}^{1S})A_{00}{}^2\times\exp\left(-\frac{K^2}{2\alpha}\right)BL^2(-\tfrac{3}{2}), \quad (11a)$$

$$\frac{dW(p_sn_s{}^{3S};1^+)}{d\cos\Theta}=\frac{3\Gamma}{128}C(p_sn_s{}^{3S})A_{00}{}^2\times\exp\left(-\frac{K^2}{2\alpha}\right)BL^2(-\tfrac{3}{2}), \quad (11b)$$

$$\frac{dW(p_pp_p{}^{1S};0^+)}{d\cos\Theta}=\frac{\Gamma}{16}C(p_pp_p{}^{1S})\exp\left(-\frac{K^2}{2\alpha}\right)B\times\left[A_{10}\left\{\frac{1}{4}L(-\tfrac{3}{2})-\tfrac{1}{2}\gamma L(-\tfrac{5}{2})+\frac{\gamma k^2}{12}L(-\tfrac{7}{2})\right\}+\tfrac{1}{4}(\surd\tfrac{3}{2})A_{00}\times(1-K^2/3\alpha)L(-\tfrac{3}{2})\right]^2, \quad (11c)$$

$$\frac{dW(p_pp_p{}^{1P};0^+)}{d\cos\Theta}\simeq 0(\langle 0101;0|0101;0\rangle\simeq 0) \quad (11d)$$

$$\frac{dW(p_pn_p{}^{3S};1^+(\lambda=0))}{d\cos\Theta}=\frac{\Gamma}{16}C(p_pn_p{}^{3S})\exp\left(-\frac{K^2}{2\alpha}\right)B\times\left[A_{10}\left\{\frac{1}{4}L(-\tfrac{3}{2})-\frac{\gamma}{2}L(-\tfrac{5}{2})+\frac{\gamma k^2}{12}L(-\tfrac{7}{2})\right\}+\tfrac{1}{4}(\surd\tfrac{3}{2})A_{00}\times\left(1-\frac{K^2}{3\alpha}\right)L(-\tfrac{3}{2})\right]^2, \quad (11e)$$

$$\frac{dW(p_pp_p{}^{1S};2^+)}{d\cos\Theta}=\frac{\Gamma}{45\cdot 2^8}C(p_pp_p{}^{1S})A_{00}{}^2\frac{K^4}{\alpha^2}\times\exp\left(-\frac{K^2}{2\alpha}\right)BL^2(-\tfrac{3}{2}), \quad (11f)$$

$$\frac{dW(p_pp_p{}^{1D};2^+)}{d\cos\Theta}=\frac{\Gamma}{2^{12}}C(p_pp_p{}^{1D})A_{02}{}^2\times\exp\left(-\frac{K^2}{2\alpha}\right)k^4BL^2(-\tfrac{7}{2}), \quad (11g)$$

$$\frac{dW(p_pn_p{}^{3S};3^+)}{d\cos\Theta}=\frac{0.837\Gamma}{15\cdot 2^8}C(p_pn_p{}^{3S})A_{00}{}^2\frac{K^4}{\alpha^2}\times\exp\left(-\frac{K^2}{2\alpha}\right)BL(-\tfrac{3}{2}), \quad (11h)$$

$$\frac{dW(p_pn_p{}^{3D};3^+)}{d\cos\Theta}=\frac{2.69\Gamma}{2^{12}}C(p_pn_p{}^{3D})A_{02}{}^2\times\exp\left(-\frac{K^2}{2\alpha}\right)k^4BL^2(-\tfrac{7}{2}), \quad (11i)$$

$$\frac{dW(p_pn_p{}^{3S};1^+(\lambda=2))}{d\cos\Theta}=\frac{0.4722\Gamma}{15\cdot 2^8}C(p_pn_p{}^{3S})A_{00}{}^2\frac{K^4}{\alpha^2}\times\exp\left(-\frac{K^2}{2\alpha}\right)BL^2(-\tfrac{3}{2}), \quad (11j)$$

$$\frac{dW(p_pn_p{}^{3D};1^+)}{d\cos\Theta}=\frac{1.73\Gamma}{2^{12}}C(p_pn_p{}^{3D})A_{02}{}^2\times\exp\left(-\frac{K^2}{2\alpha}\right)k^4BL^2(-\tfrac{7}{2}), \quad (11k)$$

$$\frac{dW(p_sp_p{}^{3S};1^-)}{d\cos\Theta}=\frac{\Gamma}{2^8}C(p_sp_p{}^{3S})A_{00}{}^2\frac{K^2}{\beta}\times\exp\left(-\frac{K^2}{2\alpha}\right)BL^2(-\tfrac{3}{2}),\quad(11l)$$

$$\frac{dW(p_sp_p{}^{3P};1^-)}{d\cos\Theta}=\frac{3\Gamma}{2^{10}}C(p_sp_p{}^{3P})A_{01}{}^2\times\exp\left(-\frac{K^2}{2\alpha}\right)k^2BL^2(-\tfrac{5}{2}),\quad(11m)$$

where

$$B=(k^2/D)[(\tfrac{1}{4}-\mu/m)K^2+D]^2$$

and

$$L(x)=\gamma^x\exp(-k^2/4\gamma)+d(b+\gamma)^x\exp[-k^2/4(b+\gamma)]-C(d+\gamma)^x\exp[-k^2/4(d+\gamma)].$$

The definition of Γ is

$$\Gamma=(G^2M/\pi\hbar^3)(Z/a_0)^5(\pi/2\alpha^3)^{1/2},$$

and C denotes the weighting factors

$$\begin{aligned}
&C(p_pn_p{}^{3S})=C(p_pn_p{}^{3D})=\tfrac{3}{4}NZ=12,\\
&C(p_pn_p{}^{1S})=C(p_pn_p{}^{1D})=\tfrac{1}{4}NZ=4,\\
&C(p_sn_s{}^{3S})=\tfrac{3}{4}N'Z'=3,\\
&C(p_sn_s{}^{1S})=\tfrac{1}{4}N'Z'=1,\\
&C(p_pp_p{}^{3P})=C(p_pp_p{}^{3S})=\tfrac{3}{4}\cdot[\tfrac{1}{2}Z(Z-1)]=\tfrac{9}{2},\quad(12)\\
&C(p_pp_p{}^{1P})=C(p_pp_p{}^{1S})=\tfrac{1}{4}\cdot[\tfrac{1}{2}Z(Z-1)]=\tfrac{3}{2},\\
&C(p_pp_p{}^{1D})=\tfrac{1}{4}Z(Z-1)=3,\\
&C(p_pp_s{}^{3S})=C(p_pp_s{}^{3P})=\tfrac{3}{4}ZZ'=6,\\
&C(p_pp_s{}^{1P})=\tfrac{1}{4}ZZ'=2,\\
&C(p_sp_s{}^{1S})=\tfrac{1}{4}[\tfrac{1}{2}Z'(Z'-1)]=\tfrac{1}{4},
\end{aligned}$$

These weights are determined by multiplying the spin probability and the number of nucleon pairs in the nucleus. In Eq .(11), the normalization constants are

$$A_{00}{}^{-2}=\tfrac{1}{4}\pi^{1/2}N(-\tfrac{3}{2}),\quad A_{10}{}^{-2}=\tfrac{1}{4}\pi^{1/2}N(-\tfrac{3}{2})-\pi^{1/2}\gamma N(-\tfrac{5}{2})+(5\pi^{1/2}/3)\gamma^2N(-\tfrac{7}{2}),$$

$$A_{02}{}^{-2}=(15/16)\pi^{1/2}N(-\tfrac{7}{2})\quad\text{and}\quad A_{01}{}^{-2}=\tfrac{3}{8}(\pi)^{1/2}N(-\tfrac{5}{2}),$$

where

$$N(x)=(2\gamma)^x+2^xa^2(b+\gamma)^x+2^xc^2(d+\gamma)^x+2a(b+2\gamma)^x-2c(d+2\gamma)^x-2ac(b+d+2\gamma)^x.$$

3. NUMERICAL RESULTS AND DISCUSSION

In this section we obtain the branching ratios in the case that the two nucleons are emitted back to back, $R_\pi=W(pn\to nn;\,\Theta=180°)/W(pp\to pn;\,\Theta=180°)$, $R(pn;\,180°/90°)=W(pn\to nn;\,\Theta=180°)/W(pn\to nn;\,\Theta=90°)$, $R(pp;\,180°/90°)=W(pp\to pn;\,\Theta=180°)/W(pp\to pn;\,\Theta=90°)$ and the distribution of the absorption rate with respect to the projection of the opening angle between the emitted nucleons.

TABLE III. The parameter sets of the correlation function.

set	a	b(F^{-2})	c	d(F^{-2})
SI	0.68	0.6	1.35	1.8
PI	0	...	1	0.25
PII	0	...	1	0.18
DI	2.1	1.5	3	2.7

The ratio of the absorption rates at $\Theta=180°$ to that at $\Theta=90°$ is measured by Ozaki *et al.*[1] and by Demidov.[9] The results obtained by Ozaki *et al.* are

$$R^{\text{expt}}(pn;\,180°/90°)=6,$$
$$R^{\text{expt}}(pp;\,180°/90°)=12.5,$$

and the result obtained by Demidov *et al.* is

$$R^{\text{expt}}(pn;\,180°/90°)=4.0.$$

The theoretical values of the branching ratios can be obtained by calculating the ratios of summation of the corresponding terms in Eq. (11). Assuming that the triplet-state correlations are equal to the singlet-state correlations, we try to determine the correlation parameters, using various experimental results. The excitation energy of the residual nuclei is taken to be the average value, 52 MeV.[10] Initially, with the values of the parameters $a=2.1$, $b=1.5$ F^{-2}, $c=3$, and $d=2.7$ F^{-2} for the relative D-state pairs and with the function (9) for the relative S states, we obtain

$$R^{\text{theor}}(pn;\,180°/90°)=5.$$

This is the average value of the results measured by two groups. Next, the correlation parameters for the relative P state are determined by comparing R_π with the experimental results. Using scintillation counters, Ozaki *et al.* evaluated this ratio.[1] Their result was

$$R_\pi=5.0\pm1.5.$$

It is, however, pointed out by Fedotov[10] that this work was subject to criticism from a methodological point of view. In view of the low threshold for nucleons (~9 MeV), coincidences could be recorded which were due to neutrons from decay of the residual nucleus. Taking into account the scattering of the primary nucleons by the nucleons of the residual nucleus with transfer of an energy greater than 30 MeV to a secondary nucleon and also the absorption of primary nucleons by the nucleus, Fedotov determined the ratio R_π.
His result is

$$R_\pi=4\pm1.3.$$

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[9] V. S. Demidov, V. S. Verebryusov, V. G. Kirillov-Ugryumov, A. K. Ponosov, and F. M. Sergeev, Zh. Eksperim. i Teor. Fiz. **46**, 1220 (1964) [English transl.: Soviet Phys.—JETP **19**, 826 (1964)].

[10] P. I. Fedotov, Yadern. Fiz. **2**, 466 (1965) [English transl.: Soviet J. Nucl. Phys. **2**, 335 (1966)].

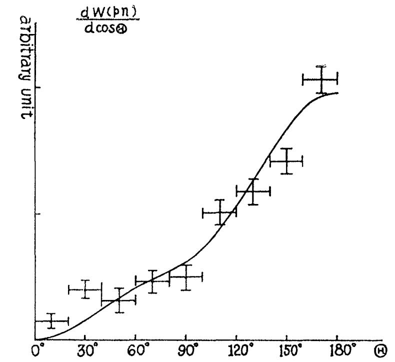

FIG. 2. Distributions of the absorption rate with respect to the projection of the opening angle between the emitted nucleons.

The parameter set PI (see Table III) makes the theoretical value of the ratio R_π fit Fedotov's result, and the parameter set PII fits the result of Ozaki *et al.* By using these parameters, the ratio $R(pp;180°/90°)$ can be calculated. With the parameter sets SI, PII, and DI.

$$R^{\text{theor}}(pp;\, 180°/90°)=1.49.$$

The experimental result obtained by Ozaki *et al.* is

$$R^{\text{expt}}(pp;\, 180°/90°)=12.5.$$

The theoretical value is about 10 times smaller than the experimental value. With the parameter sets, SI, PI, and DI,

$$R^{\text{theor}}(pp;\, 180°/90°)=1.86.$$

The distribution of the absorption rate with respect to the projection of the opening angle between the emitted neutrons is shown in Fig. 2. The theoretical curve is normalized arbitrarily. The experimental values in Fig. 2 were measured by Demidov *et al.*[9] The results obtained by our theory are satisfactory. We can thus conclude that the absorption of stopped π^- mesons in carbon is mainly through the two-nucleon absorption mechanism. As can be seen in Fig. 2 there is a peak near 30°. This peak may be due to the knock-on of the other neutrons in the nucleus by the proton which absorbed the π^- meson. Fig. 3 shows the correlation functions for various parameter sets. The dashed curves were obtained by Akaishi *et al.*[7] with the Hamada-Johnston potential in the case of $k=1.30$ F^{-1} for nuclear matter. Our curves for S and D states are similar to those obtained by them. However, the slope of our curve for P state is smoother than theirs. The total absorption rate by (pn) pairs is

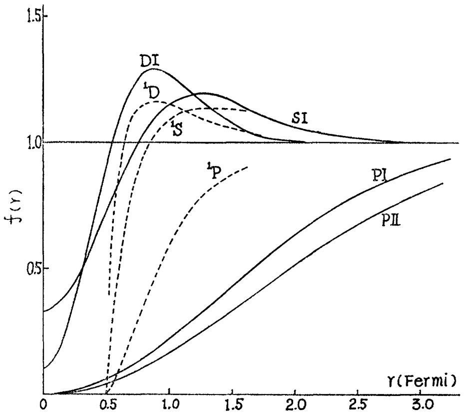

FIG. 3. The correlation functions for various states. The dashed curves were obtained by Akaishi *et al.* with the Hamada-Johnston potential in the case of $k=1.30$ for the nuclear matter.

$$W(pn)=1.38\times10^{15}\ \text{sec}^{-1}.$$

The experimental value obtained by Huguenin[11] is

$$1.34\leq W(pn)\leq 1.7$$

in units of 10^{15} sec^{-1}. Our result is in excellent agreement with the experimental value. The results obtained in I are $W(pn)=1.54\times10^{16}$ sec^{-1}. This value is 10 times as large as the experimental value.

Taking into account antisymmetrization of the nuclear wave functions in Eqs. (3) and (4), the values of the branching ratios will scarcely change. However, the theoretical value of the total absorption rate may change somewhat. Furthermore, this value may possibly change with consideration of the rescattering of the s-wave pions.

ACKNOWLEDGMENTS

The author would like to express his gratitude to Professor H. Yukawa and the members of the Research Institute for Fundamental Physics for the hospitality extended to him, and to Professor M. Kobayasi for encouragement.

[11] P. Huguenin, Z. Physik **167**, 416 (1962).