Line Shapes for the Radio-Frequency Size Effect in Potassium*

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We have measured the rf size effect in potassium using unilateral excitation of surface currents. Line shapes for both the surface reactance and resistance derivatives are obtained as a function of H/H_0, where H_0 is the value of the magnetic field at which the central orbit on the potassium Fermi surface just spans the specimen thickness. This value of H_0 has been determined independently from known Fermi-surface dimensions and the measured sample thickness. The experimental lines are compared with the line shapes calculated from Kaner and Falko's theory.

I. INTRODUCTION

THE rf impedance of thin metallic plates exhibits detailed structure as a function of magnetic field applied parallel to the surface of the specimen.[1] These anomalies in the surface resistance and reactance are associated with the cutting off of electronic orbits when their diameters become larger than the specimen thickness. The effect has been observed and studied in a great many metals.

Kaner and Falko[2] (KF) have made an attempt to calculate the line shapes for the impedance anomalies expected for the case of the spherical Fermi surface. Their model specifically examines the case where the rf currents are excited on one surface of the parallel-plate specimen only. With some essential approximations they obtain expressions for the impedance anomalies in terms of integrals over the Fermi surface and Fourier components of the electric field in the metal. Kaner and Falko have evaluated the necessary integrals only for a very approximate form of the electric field in the skin layer.

The present experiments represent an extension of our earlier work in potassium.[3,4] The experimental arrangement is modified here so as to produce unilateral excitation of the rf currents. We use a flat spiral coil placed on one of the surfaces of the specimen, rather than the conventional rectangular coil wrapped about the specimen.[5] The experimentally observed line shapes for potassium are then compared with those calculated from the KF formalism in a reasonably consistent manner. The expressions for the line shapes are integrated numerically, using parameters appropriate to our experiments. Even though the calculated and experimental line shapes are substantially in agreement, there appear some essential discrepancies.

In Sec. II, we give experimental details and results relevant to this study. Section III contains a brief summary of the KF model, together with a discussion of our calculations based on this model. In Sec. IV, we compare experiment and theory and comment on possible reasons for the disagreement.

II. EXPERIMENTAL ASPECTS AND RESULTS

The experiments measure the derivative with respect to magnetic field of both the surface resistance (dR/dH) and reactance (dX/dH) of the specimen. A description of the apparatus and techniques of sample preparation is contained in Ref. 3. There are only two significant differences, pertaining to the coil geometry and sample preparation.

The coil of the tank circuit in the present studies is in the form of a flat spiral placed in contact with one surface of the disk-shaped sample. The sample diameter is about twice that of the coil. This arrangement surves to excite circularly polarized rf currents in one surface of the specimen only.

Because the theory specifically applies to the case of linearly polarized currents in only one of the sample surfaces, we found it necessary to check whether the observed line shapes depended on polarization. For this purpose we have made samples where sectors of the disk are of different thicknesses. The surface in contact with the coil is flat, while the other surface has small sectors (centered with respect to the spiral coil) of nearly twice the thickness of the remaining sample. The image currents in these sectors are then substantially linear. The signals due to the two parts of the sample occur at very different fields and are readily identified. We note that the line shape observed with linearly polarized currents is identical with that observed for the circular polarization.

The electron mean free path in our potassium samples at 4.2°K was determined directly from the amplitude dependence of signals due to the limiting point electrons in a field tipped relative to the sample surface (see

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[1] V. F. Gantmakher, Zh. Eksperim. i Teor. Fiz. 43, 345 (1962) [English transl.: Soviet Phys.—JETP 16, 247 (1963)].
[2] E. A. Kaner and V. L. Falko, Zh. Eksperim i Teor. Fiz. 51, 586 (1966) [English transl.: Soviet Phys.—JETP 24, 392 (1967)].
[3] J. F. Koch and T. K. Wagner, Phys. Rev. 151, 467 (1966).
[4] J. F. Koch and T. K. Wagner, in *Proceedings of the Tenth International Conference on Low-Temperature Physics* (Publishing House Viniti, Moscow, 1967), Vol. III.
[5] The rectangular coil wrapped about the specimen produces currents on both surfaces. An attempt had been made in Ref. 3 (p. 470) to produce unilateral excitation by mounting the sample on a copper plate and winding the coil around both. Judging by the very different line shape that we observe in the present experiments, we conclude that the copper plate-sample combination never really gave us proper one-sided excitation of rf current.

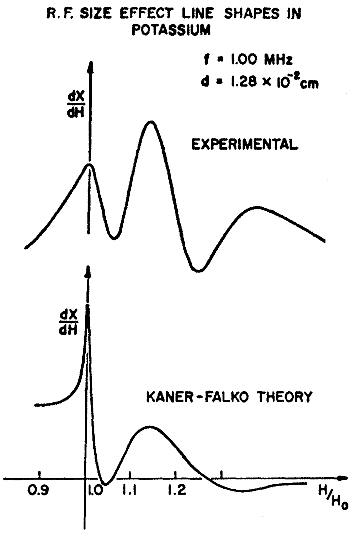

FIG. 1. Experimental and theoretical line shapes of the dX/dH anomaly for the rf size effect in potassium. The mean free path in the sample was 6×10^{-3} cm. This value is used in the calculation. (Note: the experimental signal plotted is actually $-dX/dH$. It has been inverted to agree with the line shape given by Kaner and Falko.)

Ref. 3). From these measurements we obtain a mean free path $l=6\times10^{-3}$ cm. This value is used in the calculations.

The dX/dH and dR/dH signals that we observe are shown in the upper tracings of Figs. 1 and 2, respectively. They are plotted versus H/H_0, where H_0 represents the cutoff field for the central orbit about the spherical Fermi surface. The value of H_0 is determined independently from the measured thickness of the sample and the known dimensions of the potassium Fermi surface. In the experiments using the spiral-coil geometry, we noted that the signal amplitude was substantially less than for the case of the rectangular coil wrapped about the specimen. This is to be expected,

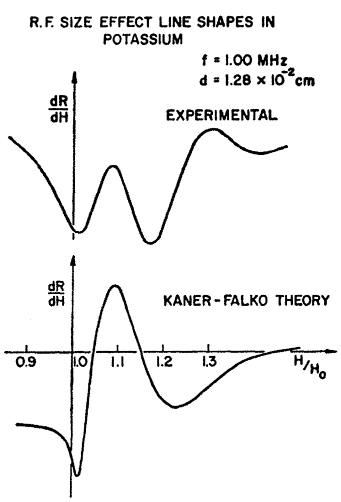

FIG. 2. Experimental and theoretical line shapes of the dR/dH anomaly for the rf size effect in potassium.

because the size-effect signal for the spiral coil is due to electrons that have completed at least one cyclotron orbit, whereas for the rectangular coil, electrons giving rise to a signal need only travel from one surface to the other in half a cyclotron orbit. Because for our potassium samples, l is only about $\frac{1}{2}$ the specimen thickness d ($d=1.28\times10^{-2}$ cm) the exponential damping is noticeably increased for the spiral coil geometry.

III. CALCULATION OF LINE SHAPES

The KF theory considers the impedance anomalies for the case of a parallel plate of thickness d, with the rf current linearly polarized and confined to one surface of the specimen. For spherical Fermi-surface geometry they obtain an expression for the derivative of the impedance as

$$Z'=\left(\frac{4\omega}{c^2}\right)^2\frac{4\pi e^2}{h^3\nu[E_y'(0)]^2}\int_0^{p_{x\max}}dp_x m v_\perp^2\theta(d-2R)B^2, \quad (1)$$

where

$$B=2\int_0^\infty dk\,\mathcal{E}_y(k)\sin(k)(d-R)J_1(kR). \quad (2)$$

In these equations ν, p, m, and $v_\perp$ are, respectively, the the collision frequency, momentum, effective mass, and velocity in the plane perpendicular to the magnetic field, for electron on the Fermi surface. θ is the unit step function and R is the radius of the cyclotron orbits. $E_y'(0)$ represents the derivative of the electric field $E(z)$, at the surface of the sample. The dependence of Z' on the magnetic field is contained in $R(H)$ and $\mathcal{E}_y(k,H)$. The geometry is such that the field is parallel to the surface along the x direction, the rf currents are along the y direction in the surface, and the z direction is along the inward normal. The Fourier components of the electric field, $\mathcal{E}_y(k)$, are given by KF as

$$\mathcal{E}_y(k)=\frac{-2E_y'(0)}{k^2-i4\pi(\omega/c^2)\sigma(k)}, \quad (3)$$

with

$$\sigma(k)=(3Ne^2/8m\nu R_0)(1/|k|)\begin{cases}\times 1 & \text{if } d>2R_0,\\ \times(1/\pi)(2\theta_0-\sin 2\theta_0) & \text{if } d<2R_0.\end{cases} \quad (4)$$

Here R_0 is the maximum value of the cyclotron radius on the Fermi surface and $\theta_0=\sin^{-1}(d/2R_0)$.

Essential approximations in the KF theory are the following. The integration for the surface current is such as to include contributions due only to electronic orbits that never collide with the surfaces. This is expected to be valid in the limit $\omega_c/\nu\gg1$, because electrons that do not collide at the surfaces will traverse the skin layer many times, greatly enhancing their contribution to the currents. The integral is evaluated in this limit. Diffuse scattering of electrons at the sample surface is assumed. The expressions for impedance are such as to account solely for the geometrical cutoff in the current, and specifically ignore the anomalous penetration of electromagnetic field (i.e., the effect on the impedance of the surface at $z=0$ due to the fact that currents and fields are set up in the second surface at $z=d$). Expression for $\mathcal{E}_y(k)$ used in evaluating Z' are valid specifically only outside the region of magnetic field for the size-effect anomaly. If $d>2R_0$, then $\mathcal{E}_y(k)$ is the expression appropriate for a semi-infinite sample. The approximation amounts essentially to the small signal limit, where it is assumed that the cutoff phenomenon does not greatly modify the electric fields in the surface. The equation for Z' really represents a derivative with respect to sample thickness d, rather than with respect to field H. This is valid for relatively narrow size-effect lines, i.e., $\Delta H/H_0 \ll 1$, where ΔH is the width of the line.

Kaner and Falko have evaluated the line shapes only in a very approximate fashion, using a simple exponential approximation for $E(z)$, rather than the reasonably consistent formalism that they have developed. To make a direct comparison with our experiments, we have numerically integrated the necessary equations to evaluate both dR/dH and dX/dH. The integration over k in Eq. (2) is terminated for values $k>10(4\pi\omega/c^2) \times |k|\sigma(k))^{1/3}$.

In addition to the appropriate Fermi-surface dimensions, the only parameters in the calculation are the experimental frequency ω, and collision frequency ν. Values for these are taken from the experiments. The amplitudes of the calculated curves are arbitrarily adjusted to appear similar to the experimental traces. The resulting size-effect lines are plotted in the lower portions of the two adjoining figures.

IV. DISCUSSION AND CONCLUSION

A superficial comparison of experimental and theoretical line shapes shows a reasonable similarity between these. The agreement of theory and experiment is more than just barely recognizable.

As points of disagreement we note that for both dX/dH and dR/dH, the structure in the immediate vicinity of $H/H_0=1.0$ is both larger in amplitude and narrower for the calculated curves. The well-defined high-field peak in the experiments does not appear in the calculations.

The discrepancies in amplitude and width that we note at $H/H_0=1$, we attribute to the fact that ω_c/ν is on the order of unity for the experiments, whereas the calculation is valid in the limit of large ω_c/ν. We expect that the change in conductivity due to boundary scattering will not be as great for an electron that can barely make it once around the orbit, as for one that completes many revolutions. One is also tempted to wonder about the diffuse-scattering assumption in the theory. When the field is very near $H/H_0=1$, electrons will be coming into the surface at glancing angles, with a finite chance of being specularly reflected. The disagreement in the high-field portion of the lines can be ascribed to two possible reasons. The replacement of the derivative with respect to thickness for the field derivative may not be valid. More importantly, however, the anomalous-penetration effects, ignored in the theory, would be expected to dominate the field dependence away from where the geometrical cutting off is important. At fields above $H/H_0=1.2$, where the orbits are significantly smaller than the sample thickness, cutting off occurs only for electrons far down in the skin layer, and should not greatly affect the impedance.

We conclude that theory and experiment are sufficiently in agreement to allow a meaningful comparison. The areas of disagreement, noted above, serve to point out possible shortcomings in the theory. In any case, the experimental curves should be a good proving ground for a theory of line shapes.