Surface Impedance near the Cyclotron Edge for Helicons in Potassium*

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An extension of the theory of the anomalous skin effect given by Reuter and Sondheimer to include a magnetic field is used to calculate the surface impedance of a semi-infinite free-electron metal. The magnetic field and the helicon propagation are directed perpendicular to the metal surface. Parameters appropriate to potassium are used in the numerical calculations. Near the cyclotron absorption (Kjeldaas) edge the assumptions of perfectly specular ($p=1$) and completely diffuse ($p=0$) reflection of the electrons at the metal surface lead to significantly different results for the surface impedance. The theory is compared with Taylor's experimental work on potassium. The data for the surface reactance agree well with the theory for $p=0$; the data for the surface resistance do not show satisfactory agreement with the theory for $p=0$ or $p=1$. Possible explanations for this incomplete agreement are presented.

I. INTRODUCTION

SEVERAL theoretical papers[1–3] have recently considered the effect of helicon propagation on the surface impedance of metals near the cyclotron absorption edge. Although experimental work on metals in the absence of a helicon wave[4,5] indicates that the electron scattering at the surface of the metal is almost completely diffuse, the authors of these theoretical papers have assumed for reasons of simplicity that the electron scattering is perfectly specular. Therefore, numerical calculations of the magnetic-field dependence of the surface impedance near the cyclotron absorption edge have been done for the two limiting cases of completely diffuse and perfectly specular reflection of the electrons at the metal surface.

A helicon is a circularly polarized electromagnetic wave which may be self-sustained in an electron gas which is in a sufficiently strong magnetic field, that is, if $\omega_c\tau\gg1$, where ω_c is the electron cyclotron frequency and τ is the electron relaxation time. In a free-electron gas, the complex wave number q of a helicon with frequency ω such that $\omega\ll\omega_c\ll\omega_p{}^2/\omega$, where ω_p is the plasma frequency of the electron gas, is given by[6]

$$q^2=\omega\omega_p{}^2f_\pm/\omega_c c^2\,, \qquad (1)$$

where

$$f_\pm=\tfrac{3}{4}\int_0^\pi d\theta\,\sin^3\theta[(qv_F/\omega_c)\cos\theta\pm1+(i/\omega_c\tau)]^{-1}\,, \quad (2)$$

and v_F is the Fermi velocity of the electrons. The upper and lower signs correspond to left and right circularly polarized waves, respectively. For the model considered here, that is, a free-electron gas embedded in an isotropic background of positively charged ions, the right circularly polarized wave has a large attenuation.[1,6] For typical experimental conditions, the phase velocity of the helicon is several orders of magnitude smaller than v_F. Thus, an electron at the Fermi surface with velocity component v_z parallel to the magnetic field experiences, as a consequence of the Doppler effect, a circularly polarized electric field having an apparent frequency much larger than that of the helicon. If this apparent frequency $\omega+qv_z$ is equal to the electron cyclotron frequency ω_c, a Doppler-shifted cyclotron resonance occurs, and the electrons absorb energy from the helicon wave. The onset of absorption occurs at the magnetic field (sometimes called the Kjeldaas edge) such that[7]

$$\omega_c=\omega+qv_F\approx qv_F\,. \qquad (3)$$

In the absence of a magnetic field, the electric field in a semi-infinite free-electron metal produced by an electromagnetic wave incident upon the surface of the metal is described by an integrodifferential equation found by Reuter and Sondheimer.[8] The surface impedance of the metal is given by the solution to this equation evaluated at the surface. In deriving this equation, it is assumed that a fraction p of the electrons arriving at the surface is scattered specularly and that a fraction $(1-p)$ is scattered diffusely. In general, this equation has been solved only for the two limiting cases of perfectly specular reflection ($p=1$) and completely diffuse reflection ($p=0$). These two assumptions lead to identical expressions for the surface impedance in the classical limit defined by $l\ll\delta$, where $l=v_F\tau$ is the electron mean free path and $\delta=c/(2\pi\omega\sigma)^{1/2}$ is the classical penetration depth of an electromagnetic wave of frequency ω in a metal of conductivity σ. In the extreme anomalous region, defined by $l\gg\delta$, the assumption of specular reflection leads to a result for the surface im-

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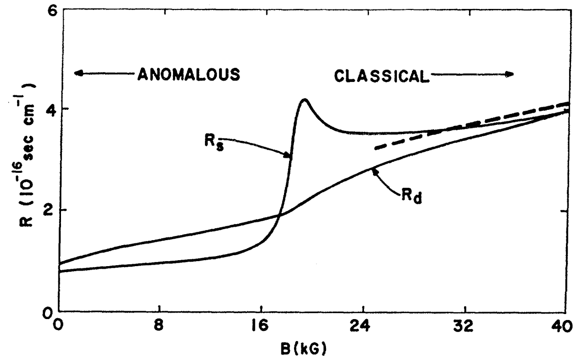

FIG. 1. The surface resistance R for a potassium slab with a magnetic field **B** applied perpendicular to the surface of the slab. The subscripts s and d refer to the assumptions of specular and diffuse electron surface scattering, respectively. A frequency of 10 Mc/sec and a relaxation time defined by $\omega_c(18)\tau=25$ were used. The dashed curve represents R in the classical limit.

pedance which differs from the result obtained for the assumption of diffuse reflection by the factor 8/9. In the intermediate region where $l\approx\delta$, explicit calculation for radio frequencies indicates that the ratio of the results obtained for $p=1$ to the results obtained for $p=0$ lies between 0.8 and 1.

Subsequent experimental measurements of the surface resistance on the metals Ag, Au, Sn, Cd, Pb, Al,[4] and Cu[4,5] as a function of temperature at microwave frequencies agree well with the theoretical results for $p=0$. Similar measurements[9] as a function of orientation on the semimetal Bi show better agreement with known data if the assumption of specular reflection is made. It is pointed out that in the semimetal Bi the wavelength of an electron on the Fermi surface is of the order of 100 atomic separations, whereas in metals this wavelength is of the order of one atomic separation. Thus, the surface roughness may not be so important in semimetals as in metals.

II. SURFACE IMPEDANCE

We consider a semi-infinite free-electron metal with its surface in the xy plane and the positive z axis directed toward the interior of the metal. An external magnetic field **B** is directed along the z axis. A rf electromagnetic field described by $E_\pm(z)e^{i\omega t}$ is normally incident on the surface of the metal, the upper and lower signs corresponding to left and right circularly polarized waves, respectively. The surface impedance $Z_\pm$ of the metal for these two circular polarizations is defined by

$$Z_\pm=-\frac{4\pi i\omega}{c^2}\left(\frac{E_\pm}{dE_\pm/dz}\right)_{z=0}. \qquad (4)$$

The integrodifferential equation obtained by Reuter and Sondheimer[8] to describe the electric field within the metal has been extended[10,11] to include the presence of the magnetic field **B** by replacing $a=\omega\tau$ by $a_\pm=(\omega\mp\omega_c)\tau$ in Eq. (15) of Ref. 8. The surface impedance $Z_\pm$ can then be written[12]

$$Z_{s\pm}=\frac{8i\omega}{c^2}\int_0^\infty dq\left[q^2+\frac{4\pi i\omega}{c^2}\sigma_\pm(q)\right]^{-1} \qquad (5)$$

for the assumption of perfectly specular reflection of the electrons at the surface, and as

$$Z_{d\pm}=\frac{4\pi^2 i\omega}{c^2}\left\{\int_0^\infty \ln\left[1+\frac{4\pi i\omega}{q^2c^2}\sigma_\pm(q)\right]dq\right\}^{-1} \qquad (6)$$

for the assumption of completely diffuse reflection of the electrons at the surface. The subscripts s and d refer to specular and diffuse reflection, respectively. The magnetoconductivity tensor $\sigma_\pm(q)$ is given by[1]

$$\sigma_\pm(q)=\sigma_{xx}\mp i\sigma_{xy}=\frac{e^2}{4\pi^2\hbar^2}\int\frac{m\tau v_\perp^2 dk_z}{1+i\tau(\omega\mp\omega_c-qv_z)}, \qquad (7)$$

where e is the electronic charge and m is the cyclotron effective mass taken to be the free-electron mass. The electronic velocity components parallel and perpendicular to the direction of **B** are given by v_z and $v_\perp$, respectively; the electron wave-vector component in the direction of **B** is denoted by k_z .The electron relaxation time τ is taken to be a constant over the Fermi surface. The polarization subscript on Z will be taken to be positive (left circular polarization), and will henceforth be dropped.

The surface resistance R and the surface reactance X are defined by the relation $Z=R+iX$. Mention should be made concerning the phase of the surface reactance. Convention[8,13–15] seems to indicate that one defines $Z=R\pm iX$ in accordance with the phase $e^{\pm i\omega t}$ chosen to describe the external electromagnetic field although some authors[1,3] subscribe to different conventions.

The indicated integrations in Eqs. (5) and (6) were done numerically for parameters appropriate to potassium and for an impressed electromagnetic field of 10 Mc/sec. (The lattice constant was taken to be[16] 5.225 Å and an electron density of one free electron per atom was assumed.) The surface resistance and the surface reactance are shown as a function of magnetic field for a range of fields which includes the Kjeldaas edge in Figs. 1 and 2, respectively. The electron relaxation time is defined by $\omega_c\tau=25$ at $B=18$ kG. The dashed curve

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of Fig. 1 and the definition of regions will be discussed in a later paragraph. For $p=1$, a large peak in R_s occurs near the absorption edge. Also for $p=1$, the sample properties given by X_s change from inductive to capacitive at a magnetic field below the absorption edge[17]; near the absorption edge a maximum in the capacitive properties occurs. For $p=0$, an abrupt increase in R_d occurs near the absorption edge as one goes from lower to higher magnetic fields; X_d remains positive and decreases to zero near the absorption edge as the magnetic field is increased.

Reuter and Sondheimer define a generalized parameter [see Eq. (39) of Ref. 8]

$$\xi = i\alpha/(1+i\omega\tau)^3, \tag{8}$$

where $\alpha = \frac{3}{2}l^2/\delta^2$. Thus, when $\omega\tau \ll 1$, $|\xi|^{1/2}$ is proportional to the ratio of the electron mean free path to the classical penetration depth. Following Reuter and Sondheimer, the region, defined by $|\xi| \ll 1$ is called the classical region and the region $|\xi| \gg 1$ the anomalous region. In the presence of a magnetic field, ξ goes to

$$\xi' = i\alpha/[1+i(\omega-\omega_c)\tau]^3 \tag{9}$$

in accordance with the discussion above.[10,11] In the case we are considering, i.e., $\omega_c\tau \gg 1$ and $\omega \ll \omega_c$, $|\xi'|^{1/2}$ is proportional to the ratio of the distance traveled by an electron in one cyclotron period to the penetration depth (modified by the presence of the magnetic field) $\delta' = \delta(\omega_c\tau)^{1/2}$ (see Appendix). The classical and anomalous regions defined in Figs. 1 and 2 were chosen to conform with the notation of Reuter and Sondheimer, and hence are defined by $|\xi'| \ll 1$ and $|\xi'| \gg 1$, respectively. In these regions, approximate analytic solutions for the surface impedance may be found. In the anomalous region, the surface impedance is found to be independent of the magnetic field in the first approximation and is given by[8]

$$Z_d = (\sqrt{3}\pi\omega^2 l/c^4\sigma)^{1/3}(1+i\sqrt{3}) \tag{10}$$

and

$$Z_s = (8/9)Z_d. \tag{11}$$

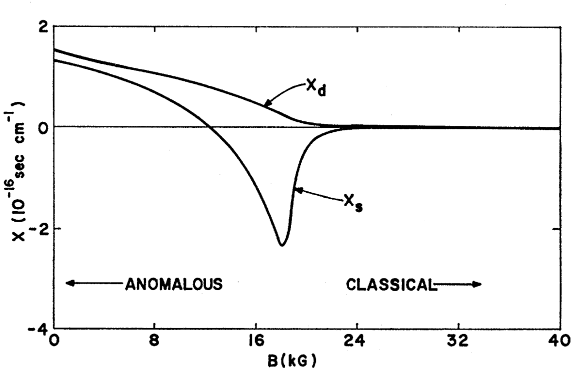

FIG. 2. The surface reactance X for a potassium slab under the conditions described in Fig. 1.

[17] For plane-polarized waves, X_s will remain positive near the Kjeldaas edge for $\omega_c\tau \lesssim 50$. See Ref. 19.

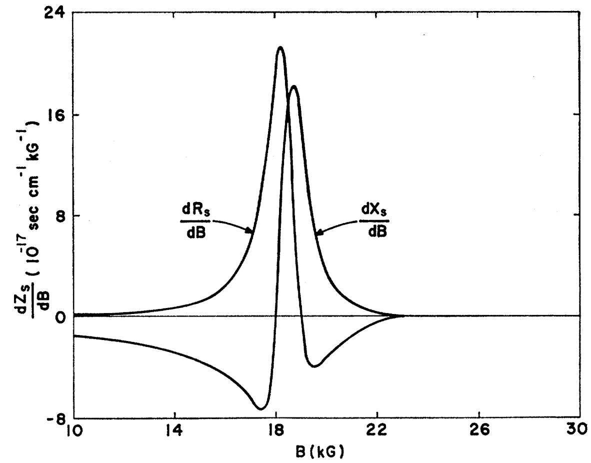

FIG. 3. Derivatives of the surface resistance and the surface reactance with respect to the applied magnetic field B for a potassium slab in which the electron scattering at the surface is perfectly specular. The conditions are those described in Fig. 1.

For the numerical constants used in Figs. 1 and 2, $Z_d = 0.864(1+i\sqrt{3})\times 10^{-16}$ sec/cm. In the classical region the surface impedance (for $p=0$ or 1) is given by (see Appendix)

$$Z = Z_s = Z_d = (4\pi\omega\omega_c\tau/c^2\sigma)^{1/2}[1+i(2\omega_c\tau)^{-1}]. \tag{12}$$

The resistive part of Z is shown by the dashed curve of Fig. 1. The reactive part is essentially zero throughout the classical region of Fig. 2.

Experimentally, the derivative of the surface impedance with respect to magnetic field is observed. Figure 3 shows the derivatives of the surface resistance and the surface reactance for the assumption of specular reflection and for the conditions of Fig. 1. Figure 4 shows similar quantities for the assumption of diffuse

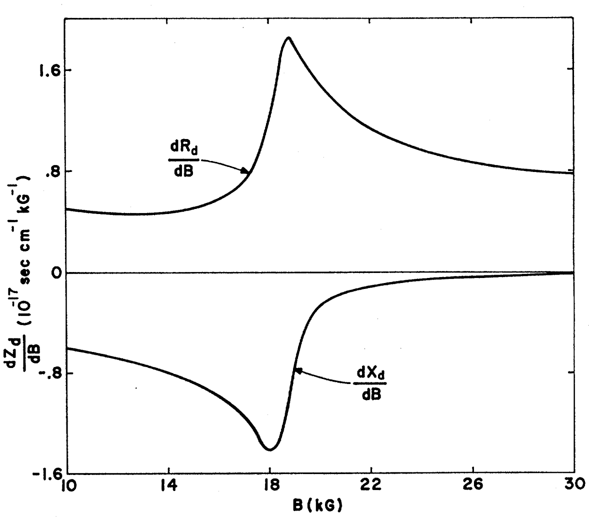

FIG. 4. Derivatives of the surface resistance and the surface reactance with respect to the applied magnetic field **B** for a potassium slab in which the electron scattering at the surface is completely diffuse. The conditions are those described in Fig. 1.

reflection under the same conditions. It should be noted that the vertical scale of Fig. 3 is an order of magnitude larger than the vertical scale of Fig. 4. The curves shown in Fig. 3 are, of course, similar to those displayed in Figs. 1–5 of Ref. 1 since both sets of curves were obtained from the same equations. The shape of the dR_s/dB curve (Fig. 3) agrees with that observed by Taylor,[18] and the minimum of the curve occurs at a magnetic field which agrees well with experiment, although, as is pointed out in Refs. 1 and 2, the peak and the midpoint between the peak and minimum occur at magnetic fields considerably larger than those observed. The shape and the position of the minimum of the dX_d/dB curve (Fig. 4) are in agreement with the observations of Taylor.[19]

III. DISCUSSION

On several occasions it has been found that the distinction between the assumptions of specular and diffuse reflection of the electrons at the surface has led to widely divergent theoretical results. For example, it was noted[15,20] that under certain conditions the calculated optical absorptivitiy of metals for the assumption of completely diffuse reflection of the electrons at the surface will be several orders of magnitude larger than that calculated for the assumption of specular reflection. Experiments[21] on copper appear to indicate order of magnitude agreement with the theoretical results for diffuse reflection.

Here, however, we have a situation in which one portion of the experiment seems to be in agreement with the theoretical result for diffuse reflection while the other portion does not agree at all with the theoretical result for diffuse reflection and shows only qualitative agreement with the theoretical result for specular reflection. One might conclude that neither the assumption of completely diffuse nor the assumption of perfectly specular reflection of the electrons at the surface is physically correct. The true situation may require that one take p in the range $0<p<1$ rather than taking only the limiting cases of $p=0$ or 1. Also, it has been pointed out[2] that it is very difficult experimentally to separate the surface resistance from the surface reactance. Finally, it might be noted that the position of the minimum of the dX/dB curve observed by Taylor[18] has been atrributed by Overhauser and Rodriguez[1] to the hypothesis that the electronic ground state of potassium possesses a spin-density wave; however, if the assumption of completely diffuse scattering of the electrons at the surface is physically correct, and if the experimental data for dX/dB are correctly described by the theoretical calculations of this paper, then the experimental position of this minimum is in agreement with the predictions of this paper based on the free-electron model.

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APPENDIX

The model is that considered in the text, namely a semi-infinite free-electron metal with its surface in the xy plane and positive z axis directed toward the interior of the metal. An external magnetic field $\mathbf{B}$ is directed along the z axis and a left circularly polarized electromagnetic field described by $E(z)e^{i\omega t}$ is propagating in the z direction. Maxwell's equations take the form

$$dH/dz=(\omega/c)E-(4\pi i/c)J\,,\quad dE/dz=-(\omega/c)H\,, \tag{A1}$$

where $J(z)$ represents the left circularly polarized component of the current density. The time-dependent factor has been omitted. Elimination of H between these equations gives

$$d^2E/dz^2+(\omega^2/c^2)E=(4\pi i\omega/c^2)J\,. \tag{A2}$$

In the classical (local) approximation, the current density becomes

$$J=\sigma E[1+i(\omega-\omega_c)\tau]^{-1}\,, \tag{A3}$$

where σ is the conductivity for static electric fields. From Eqs. (A2) and (A3) one obtains (neglecting the displacement current)

$$E(z)=E(0)\exp\{-(1+i)z/\delta[1+i(\omega-\omega_c)\tau]^{1/2}\}\,, \tag{A4}$$

where $\delta=c/(2\pi\omega\sigma)^{1/2}$ is the classical penetration depth. In the presence of a magnetic field such that $\omega_c\tau\gg1$ and $\omega\ll\omega_c$, the penetration depth is modified to $\delta'=\delta(\omega_c\tau)^{1/2}$. Insertion of Eq. (A4) in Eq. (4) of the text defining Z yields

$$Z=\{(2\pi\omega/c^2\sigma)[1+i(\omega-\omega_c)\tau]\}^{1/2}(1+i)\,. \tag{A5}$$

Upon extracting the square root and making the approximation $\omega\ll\omega_c$, this becomes

$$Z=(2\pi\omega/c^2\sigma)^{1/2}\{[\omega_c\tau+(1+\omega_c^2\tau^2)^{1/2}]^{1/2} +i[-\omega_c\tau+(1+\omega_c^2\tau^2)^{1/2}]^{1/2}\}\,. \tag{A6}$$

In the limit $\omega_c\tau\gg1$, this reduces to Eq. (12) of the text.