and

$$\mu^{(193)}=+0.1568(6)\mu_N, \tag{4.20}$$

where $\mu_N=5.0505\times10^{-24}$ erg G^{-1} is the nuclear magneton. Since the Knight shifts are identical for the two isotopes (except for possible hyperfine structure anomalies which are, however, quite negligible for the present purpose), the ratio of the two moments is given directly by (3.1) and (3.2).

$$\mu^{(193)}/\mu^{(191)}=+1.0890(1). \tag{4.21}$$

Finally, it is possible to correct the moments for core diamagnetism,[22] giving

$$\mu^{(191)}(\text{corr})=+0.1453(6)\mu_N, \tag{4.22}$$

and

$$\mu^{(193)}(\text{corr})=+0.1583(6)\mu_N. \tag{4.23}$$

Our results may be compared with the earlier optical hfs values obtained by Murakawa and Suwa[1] ($\mu^{(193)}=+0.17\pm0.03\mu_N$, $\mu^{(193)}/\mu^{(191)}=1.04\pm0.04$) and by v. Siemens[2] ($\mu^{(193)}=+0.2\pm0.1\mu_N$, $\mu^{(193)}/\mu^{(191)}=1.0\pm0.1$). Within the quoted uncertainties, the agreement is seen to be good.

[22] N. F. Ramsey, *Nuclear Moments* (John Wiley & Sons, Inc., New York, 1953), p. 86.

V. SUMMARY

The first observation of the ^{191}Ir and ^{193}Ir NMR has been reported. The experiments were performed on iridium metal at low temperatures and in high magnetic fields using high-power pulsed NMR techniques. An estimate of the Knight shift in iridium metal based on the observed spin-lattice relaxation rate of ^{193}Ir allowed the ground-state nuclear magnetic dipole moments of the two odd iridium isotopes to be determined with an uncertainty of about $\pm0.4\%$. The results of the present analysis provide evidence that the hyperfine interactions in iridium metal are dominated by the *s*-contact interaction. The results also suggest that the principal *d*-conduction electron contribution to the spin-lattice relaxation rate is associated with the electric quadrupole interaction.

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Nonlinear Excitation of Density Fluctuations in Anisotropic Semiconductors by Electromagnetic Radiation

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A nonlinear (parametric) mechanism for the excitation of electron-density fluctuations by a long-wavelength radiation field is discussed, for anisotropic many-valley semiconductors. Here the electron distribution function is given by many Fermi ellipsoids of revolution not parallel to one another. This gives rise to multimode density oscillations which can be excited nonlinearly by external electric fields. The threshold field for this phenomenon is calculated for realistic semiconductor models.

I. INTRODUCTION

WITH the development of high-intensity radiation sources in the microwave and optical spectrum, the study of nonlinear interactions of electromagnetic radiation with matter stimulates considerable theoretical and experimental interest.[1] One area of interest is the nonlinear excitation of plasma and ion-acoustic waves by a strong radiation field.[2–8] Here the ion-acoustic wave is driven by the plasma wave and the radiation field. Conversely, the plasma wave is driven by the ion-acoustic wave and the radiation field.

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The basic mechanism which couples two waves to excite a third one is the nonlinear interaction considered in this work. To illustrate our point, we use a simple physical picture. We consider a system of electrons and ions in which simultaneously we excite a plasma wave at $(\mathbf{k}, \omega_p)$ and an ion-acoustic wave at $(\mathbf{k}, \omega_{ia})$. Applying now a strong radiation field at $(\mathbf{k}_0, \omega_0)$ with $k_0 \ll k$, which couples strongly to the electrons, we produce electron-density fluctuations at wave vector $\mathbf{k}$ and frequencies $\omega_p \pm s\omega_0$, $s=0, \pm 1, \pm 2, \cdots$. If we choose, for example, $\omega_p - \omega_0 \approx \omega_{ia}$, we have created electron-density fluctuations coherent with the ion-acoustic wave and therefore provide a possible mechanism to enhance it. It is the different response to an electric field of the electrons and ions, because of their different charges and masses, which allows the growth of ion-acoustic waves, for example by the plasma wave and the external field.

This phenomenon can be observed for a system consisting of an electron gas embedded in an anisotropic lattice. We shall discuss, in this work, a degenerate many-valley semiconductor. The electron distribution function is described by many "Fermi ellipsoids" which are not parallel to one another. Under an applied electric field the electrons of the "different valleys" react as if they have different effective masses. Therefore their response to a radiation field is similar in a way to that of an electron-ion system, and we expect to excite density fluctuation in anisotropic degenerate semiconductors by radiation fields.

In Sec. II, we derive the basic equations which describe the excitation of density waves by the radiation field. In Sec. III, we solve for the growth rate of the excitation under some simplified approximations and calculate the threshold fields needed for real semiconductors in the microwave and infrared spectrum.

II. DERIVATION OF THE BASIC EQUATIONS

For the purpose of this work, we will assume that the electron plasma in anisotropic degenerate semiconductors is made up of a set of N separate ellipsoids of revolution. The Hamiltonian of this model is given by

$$H_0=(2m)^{-1}\sum_{l=1}^{N}\int d\mathbf{x}\,\psi_l^{\dagger}(\mathbf{x})\,(-\nabla\cdot\mathfrak{y}_l\cdot\nabla)\psi_l(\mathbf{x})$$

$$+\tfrac{1}{2}e^2\sum_{l,l'}^{N}\int d\mathbf{x}d\mathbf{x}'\,\rho_l(\mathbf{x})\,|\,\mathbf{x}-\mathbf{x}'\,|^{-1}\rho_{l'}(\mathbf{x}'), \quad (1)$$

where

$$\rho_l(\mathbf{x})=\psi_l^{\dagger}(\mathbf{x})\psi_l(\mathbf{x}), \quad (2)$$

and the subscript l indicates the lth ellipsoid.[9]

The operators ψ_l, $\psi_l^{\dagger}$ are, respectively, the second-quantized destruction and creation operators obeying the usual commutation relations, and $\mathfrak{y}_l$ is the effective inverse mass tensor, measured in terms of the free-electron mass m. This way our model takes into account the anisotropy imposed on the electron-gas by the lattice. We also consider momentum and energy transfers, respectively, to be small compared with reciprocal lattice vectors and interband energy, so that our one-band model will describe our system.[10]

The coupling to the electromagnetic field[11] is obtained in the usual way by replacing ∇ with $\nabla-(e/c)\mathbf{A}$, where $\mathbf{A}$, the vector potential, represents the photon field. In the long-wavelength approximation (k_{photon} is small compared with any other k in our problem), we approximate the electromagnetic field by a homogeneous oscillatory electric field. The vector potential $\mathbf{A}$ is given in terms of the external electric field via $\mathbf{E}=-1/c(\partial\mathbf{A}/\partial t)$. Under the influence of the homogeneous field, our Hamiltonian, in the momentum representation, is given by

$$H=(2m)^{-1}\sum_{l=1}^{N}\sum_{\mathbf{p}}[\mathbf{p}-(e/c)\mathbf{A}]\cdot\mathfrak{y}_l\cdot[\mathbf{p}-(e/c)\mathbf{A}]a_{l,\mathbf{p}}^{\dagger}a_{l,\mathbf{p}}$$

$$+\tfrac{1}{2}\sum_{\mathbf{k}}\varphi_{\mathbf{k}}\sum_{ll'=1}^{N}\sum_{\mathbf{pp}'}a_{l,\mathbf{p}+\mathbf{k}}^{\dagger}a_{l',\mathbf{p}'-\mathbf{k}}^{\dagger}a_{l',\mathbf{p}'}a_{l,\mathbf{p}}. \quad (3)$$

Here $\varphi_{\mathbf{k}}=4\pi e^2/k^2$ represents the electron-electron interaction potential, and $a_{l,\mathbf{p}}a_{l,\mathbf{p}}^{\dagger}$ are, respectively, the destruction and creation operators of the state l, $\mathbf{p}$ obeying the usual commutation relations, where $\mathbf{p}$ is the canonical momentum.

We next calculate the equation of motion for the one-particle density matrix which determines the coupling of the external electric field to possible density fluctuations in our system. We define

$$F_l(\mathbf{p}+\mathbf{q},\mathbf{p},t)=\langle a_{l,\mathbf{p}}(t)\,a_{l,\mathbf{p}+\mathbf{q}}^{\dagger}(t)\rangle, \quad (4)$$

where $\langle\ \rangle$ means the usual thermodynamic average over all electron coordinates and $a_{l,\mathbf{p}}(t)=\exp(iHt)\,a_{l,\mathbf{p}}\exp(-iHt)$, etc. Using the Heisenberg picture, the change in time of the operator $a_{l,\mathbf{p}}(t)$ is given by

$$i(\partial/\partial t)\,a_{l,\mathbf{p}}=\exp(iHt)[H,a_{l,\mathbf{p}}]\exp(-iHt), \quad (5)$$

etc. We therefore arrive at the following coupled equations of motion for the density matrices given by

$$[i(\partial/\partial t)+\epsilon_l(\mathbf{p})-\epsilon_l(\mathbf{p}+\mathbf{k})]F_l(\mathbf{p}+\mathbf{k},\mathbf{p},t)$$

$$+\lambda_l\omega_0\cos\omega_0 t F_l(\mathbf{p}+\mathbf{k},\mathbf{p},t)-\sum_{\mathbf{k}'}\varphi_{\mathbf{k}'}\sum_{l'=1}^{N}N_{l'}(\mathbf{k}',t)$$

$$\times[F_l(\mathbf{p}+\mathbf{k}-\mathbf{k}',\mathbf{p},t)-F_l(\mathbf{p}+\mathbf{k},\mathbf{p}+\mathbf{k}',t)]=0, \quad (6)$$

where $l=1\cdots N$.

[9] E. I. Blaunt, in *Solid State Physics*, edited by F. Seitz and D. Turnbull (Academic Press Inc., New York, 1962), Vol. 13, p. 305.

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In arriving at Eq. (6), we have used the following approximation: Electron-electron correlations (collisions) have been neglected but the self-consistent field of the charged electrons has been taken into account. The kinetic energy $\epsilon_l(\mathbf{p})$ is defined by

$$\epsilon_l(\mathbf{p}) = (1/2m)\mathbf{p}\cdot\mathfrak{y}_l\cdot\mathbf{p}.$$

Also in Eq. (6), we used for the external field $\mathbf{E}$, $\mathbf{E}=\mathbf{E}_0\sin\omega_0 t$, and the density fluctuations $N_l(\mathbf{k}, t)$ is given by

$$N_l(\mathbf{k}, t) = \sum_{\mathbf{p}} F_l(\mathbf{p}+\mathbf{k}, \mathbf{p}, t), \tag{7}$$

where λ_l is defined by

$$\lambda_l = (e/m\omega_0^2)\tfrac{1}{2}(\mathbf{k}\cdot\mathfrak{y}_l\cdot\mathbf{E}_0+\mathbf{E}_0\cdot\mathfrak{y}_l\cdot\mathbf{k}). \tag{8}$$

It should be noted that the term proportional to A^2 commutes with the density operator and therefore does not affect Eq. (6). This can be simply understood, since this term depends on $\sum_{\mathbf{p}} a_{l,\mathbf{p}}{}^{\dagger} a_{l,\mathbf{p}} = N_l$, the average density of the lth ellipsoid, which is a constant of the motion.

In order to solve the N coupled equations given in Eq. (6), we use the following approximation:

$$F_l(\mathbf{p}+\mathbf{k}, \mathbf{p}, t) = f_l^{(0)}(\mathbf{p}) + f_l^{(1)}(\mathbf{p}, t)\delta_{\mathbf{k},0} + f_l(\mathbf{p}+\mathbf{k}, \mathbf{p}, t), \tag{9}$$

with

$$N_l(\mathbf{k}, t) = 0+0+n_l(\mathbf{k}, t). \tag{10}$$

Here $f_l^{(0)}(\mathbf{p})$ is the Fermi distribution function of electrons in the lth ellipsoid, $f_l^{(1)}(\mathbf{p})$ is the linear response to the external field, and $f_l(\mathbf{p}+\mathbf{k}, \mathbf{p}, t)$ represents the spatially dependent distribution function which leads to density fluctuations. From Eqs. (6) and (8), we obtain $[\partial f_l^{(1)}(\mathbf{p}, t)/\partial t]=0$ which means $f_l^{(1)}(\mathbf{p}t) = f_l^{(0)}(\mathbf{p})$ and indicates that in the linear approximation the electrons have the Fermi distribution function in terms of their canonical momentum. It is clear that the *velocity* distribution of the electron will depend on the oscillatory electric field since

$$m\mathbf{v} = [\mathbf{p}+(e/\omega_0)\mathbf{E}_0\cos\omega_0 t]\cdot\mathfrak{y}+\mathfrak{y}\cdot[\mathbf{p}+(e/\omega_0)\mathbf{E}_0\cos\omega_0 t].$$

The linearized equation of motion for f_l is given by

$$[i(\partial/\partial t)+\epsilon_l(\mathbf{p})-\epsilon_l(\mathbf{p}+\mathbf{k})]f_l(\mathbf{p}+\mathbf{k}, \mathbf{p}, t) + \lambda_l\omega_0\cos\omega_0 t f_l(\mathbf{p}+\mathbf{k}, \mathbf{p}, t) - \varphi_{\mathbf{k}}[\sum_{l'=1}^{N} n_l(\mathbf{k}, t)][f_l^{(0)}(\mathbf{p}) - f_l^{(0)}(\mathbf{p}+\mathbf{k})] = 0, \tag{11}$$

with

$$n_l(\mathbf{k}, t) = \sum_{\mathbf{p}} f_l(\mathbf{p}+\mathbf{k}, \mathbf{p}, t). \tag{12}$$

It follows that Eq. (11) does not give us the long time behavior of our system; however, its solutions predict the complex eigenvalues for the density fluctuations at resonance. We therefore are able to determine the condition for excitation of density fluctuation in our system driven by the homogeneous electric field. Our solution is equivalent to the random-phase approximation; looking at Eqs. (6) and (11), it is clear that in arriving at Eq. (11) we have considered in Eq. (6) only terms with wave number $\mathbf{k}'$, coherent with the wave number $\mathbf{k}$ of f_l, i.e., $\mathbf{k}'=\mathbf{k}$. To solve Eqs. (11) and (12), we first move to an accelerated frame:

$$f_l(\mathbf{p}+\mathbf{k}, \mathbf{p}, t) = \tilde{f}_l(\mathbf{p}+\mathbf{k}, \mathbf{p}, t)\exp(-i\lambda_l\sin\omega_0 t), \tag{13}$$

$$n_l(\mathbf{k}, t) = \rho_l(\mathbf{k}, t)\exp(-i\lambda_l\sin\omega_0 t). \tag{14}$$

We therefore obtain for $\tilde{f}_l$ the equation of motion as

$$(i(\partial/\partial t)+\epsilon_l(\mathbf{p})-\epsilon_l(\mathbf{p}+\mathbf{k}))\tilde{f}_l(\mathbf{p}+\mathbf{k}, \mathbf{p}, t) - \varphi_{\mathbf{k}}(f_l^{(0)}(\mathbf{p})-f_l^{(0)}(\mathbf{p}+\mathbf{k}))\rho_l(\mathbf{k}, t) - \varphi_{\mathbf{k}}(f_l^{(0)}(\mathbf{p})-f_l^{(0)}(\mathbf{p}+\mathbf{k})) \times \sum_{l'=1}^{N}{}' \rho_{l'}(\mathbf{k}, t)\exp(-i\lambda_{l'l}\sin\omega_0 t) = 0, \tag{15}$$

where

$$\lambda_{l'l} = \lambda_{l'} - \lambda_l \tag{16}$$

and $\sum_{l'}{}'$ means sum over $l'\neq l$.

We next write

$$\exp(-i\lambda_{l'l}\sin\omega_0 t) = \sum_{s=-\infty}^{+\infty} J_s(\lambda_{ll'})\exp(-is\omega_0 t). \tag{17}$$

The solution of Eq. (15) for the density fluctuation in the frequency representation is given by

$$\epsilon_l(\mathbf{k}, \omega+n\omega_0)\rho_l(\mathbf{k}, \omega+n\omega_0) + [\epsilon_l(\mathbf{k}, \omega+n\omega_0)-1] \times \sum_{l'=1}^{N}{}' \sum_{s=-\infty}^{+\infty} J_{s-n}(\lambda_{l'l})\rho_{l'}(\omega+s\omega_0) = 0. \tag{18}$$

In Eq. (18), $\epsilon_l(\mathbf{k}\omega)$ is given by

$$\epsilon_l(\mathbf{k}, \omega) = 1-\varphi_{\mathbf{k}}\sum_{\mathbf{p}} \frac{f_l^{(0)}(\mathbf{p}+\mathbf{k})-f_l^{(0)}(\mathbf{p})}{\epsilon_l(\mathbf{p}+\mathbf{k})-\epsilon_l(\mathbf{p})-\omega-i\eta} = 1-\varphi_{\mathbf{k}}Q_l(\mathbf{k}, \omega), \tag{19}$$

and $f_l^{(0)}(\mathbf{p})$ is explicitly given by

$$f_l^{(0)}(\mathbf{p}) = (\exp\{-\beta[\epsilon_l(\mathbf{p})-\mu]\}+1)^{-1}. \tag{20}$$

In order to understand our result [Eq. (18)] we first go to the limit $\mathbf{E}_0=0$. Here $\lambda_{l'l}=0$, therefore $J_{s-n}(0)=\delta_{sn}$, and Eq. (18) reduces to the well-known dispersion relation for density fluctuations, namely,

$$1-\varphi_{\mathbf{k}}\sum_{l} Q_l(\mathbf{k}, \omega) = 0. \tag{21}$$

This same result is obtained if all mass tensors $\mathfrak{y}_l$ are equal which implies again $\lambda_{l'l}=0$. We obtain therefore the expected result that a homogeneous field does not couple to density fluctuations of charged particles with the same charge and mass. The effective coupling

in our case is given from Eqs. (16) and (8) as

$$(1/m\omega_0)\tfrac{1}{2}(\mathbf{k}\cdot\mathbf{v}_{l'l}\cdot\mathbf{E}_0+\mathbf{E}_0\cdot\mathbf{v}_{l'l}\cdot\mathbf{k}), \tag{22}$$

where

$$\mathbf{v}_{l'l}=\mathbf{v}_{l'}-\mathbf{v}_l, \tag{23}$$

which indicates different effective mass for the electrons in the lth and l'th ellipsoids depending of course on the relative directions of $\mathbf{k}$ and $\mathbf{E}_0$. If we now adopt the model of electrons of different masses, the physical mechanism for density excitation via the field is clear; for two ellipsoid models, it is the difference between the oscillatory motion of the light and heavy electrons in the applied field which couples density fluctuations of the heavy electrons at frequency ω to density fluctuation of the light electrons at frequencies $\omega+s\omega_0$, where $s=0, \pm1, \pm2, \cdots$, and conversely.

III. CALCULATION OF THE GROWTH RATE

In order to extract useful information from Eq. (18), let us limit ourselves to the two ellipsoid systems. We also shall omit all coupling other than between the two possible plasma modes given by the linear solution. We therefore are interested in the mode coupling of density fluctuations at frequencies ω and $\omega-\omega_0$. We obtain from Eq. (18) four coupled equations in which the momentum $\mathbf{k}$ has been omitted for simplicity and $\lambda=\lambda_1-\lambda_2$:

$$\begin{aligned}
&\epsilon_1(\omega)\rho_1(\omega)\\
&\quad+[\epsilon_1(\omega)-1][J_0(\lambda)\rho_2(\omega)-J_1(\lambda)\rho_2(\omega-\omega_0)]=0,\\
&\epsilon_1(\omega-\omega_0)\rho_1(\omega-\omega_0)\\
&\quad+[\epsilon_1(\omega-\omega_0)-1][J_0(\lambda)\rho_2(\omega-\omega_0)+J_1(\lambda)\rho_2(\omega)]=0,\\
&\epsilon_2(\omega)\rho_2(\omega)\\
&\quad+[\epsilon_2(\omega)-1][J_0(\lambda)\rho_1(\omega)+J_1(\lambda)\rho_1(\omega-\omega_0)]=0,\\
&\epsilon_2(\omega-\omega_0)\rho_2(\omega-\omega_0)\\
&\quad+[\epsilon_2(\omega-\omega_0)-1][J_0(\lambda)\rho_1(\omega-\omega_0)-J_1(\lambda)\rho_1(\omega)]=0.
\end{aligned} \tag{24}$$

The solution of Eq. (24) is straightforward, and after much algebra we obtain the nonlinear dispersion relations

$$\begin{aligned}
&\left(\epsilon(\omega)-\epsilon_1(\omega)\epsilon_2(\omega)\frac{J_0^2(\lambda)+J_1^2(\lambda)-1}{J_0^2(\lambda)+J_1^2(\lambda)}\right)\\
&\times\left(\epsilon(\omega-\omega_0)-\epsilon_1(\omega-\omega_0)\epsilon_2(\omega-\omega_0)\frac{J_0^2(\lambda)+J_1^2(\lambda)-1}{J_0^2(\lambda)+J_1^2(\lambda)}\right)\\
&+\left(\frac{J_1(\lambda)}{J_0^2(\lambda)+J_1^2(\lambda)}\right)^2[\epsilon_2(\omega)-\epsilon_2(\omega-\omega_0)]\\
&\times[\epsilon_1(\omega)-\epsilon_1(\omega-\omega_0)]=0,
\end{aligned} \tag{25}$$

where

$$\epsilon(\omega)=\epsilon_1(\omega)+\epsilon_2(\omega)-1.$$

Here $\epsilon(\omega)$ is the total dielectric function of the system in the absence of the external field. In laboratory situations, λ would be less than unity, and thus the shift of the resonance frequencies will be small and proportional to λ^2. Therefore the approximate equation for the growth rate is given by

$$\begin{aligned}
&(\omega^2-\omega_L^2)[(\omega-\omega_0)^2-\omega_H^2]\\
&=\frac{J_1^2[\epsilon_2(\omega)-\epsilon_2(\omega-\omega_0)][\epsilon_1(\omega)-\epsilon_1(\omega-\omega_0)]}{[\partial\epsilon(\omega)/\partial\omega^2]_{\omega=\omega_L}[\partial\epsilon(\omega-\omega_0)/\partial(\omega-\omega_0)^2]_{\omega-\omega_0=\omega_H}},
\end{aligned} \tag{26}$$

where ω_L and ω_H are, respectively, the resonance frequencies given by

$$\epsilon(\omega_L)=0, \qquad \epsilon(\omega_H)=0. \tag{27}$$

We next evaluate $Q_1(\omega)$ and $Q_2(\omega)$, respectively, using Eqs. (18) and (19). First we define the matrix $[\Lambda]$ which diagonalizes the reciprocal mass tensor, i.e.,[11]

$$[\Lambda][\mu][\Lambda]^{-1}=[\lambda_i\delta_{ij}], \tag{28}$$

where λ_i are the eigenvalues of $[\mu]$. It is then easy to show that if we choose the coordinate (x, y, z) in the direction of the three principal axes of the ellipsoid we obtain

$$Q(k_x, k_y, k_z, \omega)=(\lambda_1\lambda_2\lambda_3)^{-1/2}Q(\lambda_1^{1/2}k_x, \lambda_2^{1/2}k_y, \lambda_3^{1/2}k_3, \omega). \tag{29}$$

The factor $(\lambda_1\lambda_2\lambda_3)^{-1/2}$ is merely a volume factor which comes into a redefinition of the electron average density. In the long-wavelength limit $q/q_F\ll1$, Q_l is given by

$$\begin{aligned}
Q_l(x)&=-\tfrac{3}{4}(n/\epsilon_F)\\
&\times[x_l\ln|(1-x_l)/(1+x_l)|+2+i\pi x_l\theta(x_l-1)]
\end{aligned} \tag{30}$$

and

$$x_l=\omega/k_lv_F. \tag{31}$$

The Fermi velocity v_F is defined by $\frac{1}{2}mv_F^2=\epsilon_F$ and k_l is defined by $k_l=(\lambda_1^{1/2}k_x, \lambda_2^{1/2}k_y, \lambda_3^{1/2}k_z)$. We next calculate the dielectric function for a special case of two ellipses in the x-y plane at right angles, where $\mathbf{k}$ is in the x direction. The inverse matrices of the two ellipsoids are defined as

$$[\mu_1]=\begin{pmatrix} b & 0 & 0\\ 0 & a & 0\\ 0 & 0 & a\end{pmatrix}, \qquad [\mu_2]=\begin{pmatrix} a & 0 & 0\\ 0 & b & 0\\ 0 & 0 & a\end{pmatrix}, \tag{32}$$

and the dielectric functions are

$$\begin{aligned}
\epsilon_1&=1+\tfrac{3}{2}(\omega_p^2/v_F^2k_1^2)\\
&\times[x_1\ln|(1-x_1)/(1+x_1)|+2+i\pi x_1\theta(x_1-1)],
\end{aligned} \tag{33}$$

and

$$\epsilon_2=1+\tfrac{3}{2}(\Omega_p^2/v_F^2k_2^2)\times[x_2\ln|(1-x_2)/(1+x_2)|+2+i\pi x_2\theta(x_2-1)]. \quad (34)$$

Here specifically $\omega_p^2=4\pi e^2nb/m$, $\Omega_p^2=4\pi e^2na/m$, $k_1=b^{1/2}k$, $k_2=a^{1/2}k$, and x_1 and x_2 are, respectively, given by ω/k_1v_F, ω/k_2v_F.

Substituting Eqs. (33) and (34) into Eq. (26) is not very illuminating. In order to simplify further the analysis, we shall use a power-series expansion for ϵ_1 and ϵ_2 under the following approximations. We assume that

$$\omega/k_1v_F<1,\ \omega/k_2v_F>1,\ (\omega-\omega_0)/k_1v_F>1,\ (\omega-\omega_0)/k_2v_F\gg1$$

and thus obtain

$$\epsilon_1(\omega)=1+3\omega_p^2/5v_F^2k_1^2, \quad (35)$$

$$\epsilon_2(\omega)=1-\Omega_p^2/\omega^2-\tfrac{3}{5}(\Omega_p^2/\omega^4)v_F^2k_2^2, \quad (36)$$

$$\epsilon_1(\omega-\omega_0)=1-[\omega_p^2/(\omega-\omega_0)^2]-\tfrac{3}{5}[\omega_p^2/(\omega-\omega_0)^4]v_F^2k_1^2, \quad (37)$$

$$\epsilon_2(\omega-\omega_0)=1-\Omega_p^2/(\omega-\omega_0)^2. \quad (38)$$

Here we have neglected the imaginary part of ϵ_1, ϵ_2 and consider later the decay rate of the excited modes by defining a phenomenological collision frequency which takes into account the Landau damping as well as the collisions with ions and phonons in the lattice. The behavior of our system under the conditions stated above is well-known; the electrons of ellipsoid 1 statically screen those of ellipsoid 2 and cause the lower eigenmode to be an acoustic excitation having the dispersion relation $\omega^2=c^2k^2$. The electrons in ellipsoid 1 are responsible for a plasmalike excitation slightly modified by electrons of ellipsoid 2. The solution of Eqs. (26) and (27) is given, after some algebra, to dominate contributions only:

$$(\omega^2-\omega_L^2)[(\omega-\omega_0)^2-\omega_H^2]=J_1^2(\lambda)\omega_p^2\Omega_p^2, \quad (39)$$

where

$$\omega_L^2\cong(\Omega_p^2/3\omega_p^2)5v_F^2k_1^2 \quad (40)$$

and

$$\omega_H^2\cong\omega_p^2+\Omega_p^2+\tfrac{3}{5}v_F^2k_1^2/\omega_p^2. \quad (41)$$

We next solve for the growth rate, and write

$$\omega=\omega_L+\gamma,$$
$$\omega_0=\omega_L+\omega_H, \quad (42)$$

and obtain for γ, in the case $\omega_H\gg\gamma$, the equation, up to dominant terms,

$$\gamma^2(2\omega_L+\gamma)=-J_1^2(\lambda)\omega_p^2\Omega_p^2/2\omega_H. \quad (43)$$

For the simple case $\gamma<2\omega_L$, we obtain

$$|\gamma_0|=J_1(\lambda)\omega_p\Omega_p/(4\omega_H\omega_L)^{1/2}. \quad (44)$$

However, for the more general case, we have to solve

$$\Gamma^2(1+\Gamma)=-\gamma_0^2/4\omega_L^2, \quad (45)$$

where $\Gamma=\gamma/2\omega_L$. The solution for $\Gamma>1$ result in a frequency shift as well as a growth rate and is given by

$$\tilde{\gamma}_0=(-2)^{1/3}\gamma_0^{2/3}\omega_L^{1/3}. \quad (46)$$

In order to see the dependence of the threshold field $(E_0)_{\rm th}$ on the physical parameters of the system, we use Eqs. (44), (16), and (8) and also the well-known result (see Appendix) that the effective collision frequency of our system is given by $(\nu_H\nu_L)^{1/2}$, where ν_H and ν_L are, respectively, the collision frequencies of the two linear modes. We therefore obtain for the threshold field

$$\frac{1}{4}\frac{e(E_0)_{\rm th}}{\epsilon_\infty^{1/2}(\omega_p+\Omega_p)^2}\left(\frac{k_1}{m_1}-\frac{k_2}{m_2}\right)\frac{\omega_p\Omega_p}{(\omega_H\omega_L)^{1/2}}=(\nu_L\nu_H)^{1/2}. \quad (47)$$

In Eq. (47), $k_1=kb^{1/2}$, $k_2=ka^{1/2}$, $m_1=m/b^{1/2}$, and $m_2=m/a^{1/2}$. Also ω_L and ω_H are defined respectively by Eqs. (40) and (41). In our case $k_1/m_1>k_2/m_2$ and also $\omega_p>\Omega_p>\omega_L$, we obtain the approximate result

$$\tfrac{1}{4}(ek_1/\epsilon_\infty^{1/2}m_1\omega_p^2)(\Omega_p/\omega_L)(E_0)_{\rm th}=(\omega_H\tau_H)^{-1/2}(\omega_L\tau_L)^{-1/2}, \quad (48)$$

which is given in terms of the system parameters as

$$(e/m_1)(E_0)_{\rm th}=4(3)^{1/4}\epsilon_\infty^{1/2}(m_2/m_1)^{1/4}(k_0/k_1)^{1/2}v_F(\nu_L\nu_H)^{1/2}. \quad (49)$$

Here the wave number k_0 is given by $k_0=\omega_p/v_F$. For real semiconductors b/a can be varied between 5 (Si) and 20 (Ge). For example in germanium and lead telluride, the anisotropy of the lattice enforces an angle different from 90° between the ellipsoids. This does not change quantitatively the result for the growth rate by any substantial factor. We therefore calculate the threshold field with our simplified model of two perpendicular ellipsoids. For heavily doped semiconductors (degenerate) with $n\sim10^{16}$–10^{17} with m as the free-electron mass and $b=12$, $a=0.6$ (the parameters for Ge) we obtain $\omega_p\approx10^{13}$, $\Omega_p\approx2\times10^{12}$, $k_{FT}\approx10^6$, and $\omega_L\approx5\times10^{10}$ for $k_1\approx5\times10^5$. Using Eq. (48) the threshold field given by

$$(E_0)_{\rm th}({\rm V/cm})\approx2\times10^4(\omega_H\tau_H)^{-1/2}(\omega_L\tau_L)^{-1/2}, \quad (50)$$

which indicates reasonably low threshold field even for strongly damped systems. As for the microwave region we need dopings of 10^{13}–5×10^{14} impurities per cm^3 since by lowering the sample temperature some of the electrons in the conduction band would "freeze." This

may present an additional difficulty in preparing the sample. However, we have the advantage that in the microwave regime where $\omega_p \sim 10^{11}$, the coupling of the electrons to the radiation field is more effective [see Eq. (47)]. Calculations of the growth rate in the microwave regime result in threshold field of $\sim$300 V/cm even for heavily damped density oscillations.

In conclusion, we have calculated the nonlinear excitation of density fluctuation driven by long-wavelength radiation field. The threshold field was determined and estimated for the infrared and microwave bands. It is concluded that observations of the phenomena either by measuring the electron density fluctuations or by measuring the change in the absorption fields $E_0 > (E_0)_{\text{th}}$ are plausible experimentally.

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APPENDIX A

In this work we have calculated the threshold field for the excitation of the two density fluctuation modes in anisotropic semiconductors by a radiation field. To show that the effective collision frequency used in Eq. (47) is $(\nu_L \nu_H)^{1/2}$, we begin with our nonlinear dispersion formula [Eq. (25)], for $\lambda < 1$ and rewrite it in the form

$$\epsilon(\omega_L)\epsilon(\omega_H) + \chi(\omega_L, \omega_H) = 0. \tag{A1}$$

Here

$$\chi(\omega_L, \omega_H) = J_1^2(\lambda)[\epsilon_2(\omega_L) - \epsilon_2(\omega_H)][\epsilon_1(\omega_L) - \epsilon_1(\omega_H)], \tag{A2}$$

and $\epsilon(\omega)$ is the total dielectric function of the electron plasma including the phenomenological frequency-dependent collision frequency which generally is different for ω_L and ω_H. The nonlinear effective dielectric functions can therefore be defined as

$$\epsilon_{NL}(\omega_L) = \epsilon(\omega_L) + \chi(\omega_L \omega_H)/\epsilon(\omega_H), \tag{A3}$$

$$\epsilon_{NL}(\omega_H) = \epsilon(\omega_H) + \chi(\omega_L \omega_H)/\epsilon(\omega_L). \tag{A4}$$

The threshold is given by

$$\text{Im}\epsilon_{NL}(\omega_L) = \text{Im}\epsilon(\omega_L) - \frac{\chi(\omega_L \omega_H)\ \text{Im}\epsilon(\omega_H)}{[\text{Re}\epsilon(\omega_H)]^2 + [\text{Im}\epsilon(\omega_H)]^2} = 0, \tag{A5}$$

$$\text{Im}\epsilon_{NL}(\omega_H) = \text{Im}\epsilon(\omega_H) - \frac{\chi(\omega_L \omega_H)\ \text{Im}\epsilon(\omega_L)}{[\text{Re}\epsilon(\omega_L)]^2 + [\text{Im}\epsilon(\omega_L)]^2} = 0. \tag{A6}$$

The optimum condition occurs at resonance, when $\text{Re}\epsilon(\omega_L) = \text{Re}\epsilon(\omega_H) = 0$. We therefore obtain from Eqs. (A5) and (A6)

$$\text{Im}\epsilon(\omega_L)\ \text{Im}\epsilon(\omega_H) - \chi(\omega_L \omega_H) = 0. \tag{A7}$$

This is equivalent to our condition given in Eq. (47).

APPENDIX B

The feasibility of the particular experiments discussed in this communication depends largely on the lifetimes of the excited density modes. We would like to discuss particular examples in which the mode lifetimes are long enough such that (a) our two-modes approximation is valid and (b) the threshold field is relatively small. To understand condition (a), we must go back to our exact solution [Eq. (18)]. There we find that, in general, density fluctuations occur at frequencies ω_2, $|\omega_1| = \omega_0 - \omega_2$, $|\omega_1'| = \omega_0 + \omega_2$ and at higher frequencies of approximately $2\omega_p$, $3\omega_p$, etc., which we do not consider. However, as one can easily check, the mode at ω_1' can only exist if the mode at ω_1 is excited. Therefore the mode at ω_1' can only increase the threshold field. In order to eliminate the mode at ω_1' and thus keep the threshold field low, we need a resonant mode at ω_1 such that $2\tau_1\omega_2 \gtrsim 1$. Our result for E_{th}, stemming from the two-mode approximation, will not be applicable for $2\tau_1\omega_2 \ll 1$, and we have to extend our analysis, in this case, to include all three modes.

In choosing the particular materials for the experiments, it is desirable to have $2\tau_1\omega_2 \gtrsim 1$ for low threshold field. We first consider lead telluride [PbTe] which has an electron concentration larger than 10^{17} cm^{-3} and thus is a suitable material for the infrared region. Mobility[11] measurements give $\mu \gtrsim 3\times10^8$ (cgs) for liquid-helium temperatures, and $\mu \approx 3\times10^6$ (cgs) for liquid-nitrogen temperatures, for electron densities such that $\omega_p \approx 2\times10^{13}$ [for $T \sim 10$°K one can estimate that $\mu \approx (3\text{–}6)\times10^7$ (cgs)]. Using the parameter of PbTe, one easily obtains for low temperatures $\tau_1 \approx 5\times10^{-11}$. (Here we have used the relation $\tau_e = \mu m^*/e$ and $\tau_{\text{collective}} = 2\tau_e$.) The mode frequencies are $\omega_1 \approx 2\times10^{13}$ and $\omega_2 \approx 10^{11}$, and we are in the region of applicability of our two mode analysis.

For the microwave region, we take advantage of the high mobility of Germanium at very low concentration. For GeSb at low temperatures[12] the mobility $\mu \approx 1.5\times10^8$ cgs), and the "plasma" lifetime is $\tau \approx 2\times10^{-10}$. The plasma mode $\omega_1 \approx 10^{11}$ and the acoustic mode $\omega_2 \approx 3\times10^9$. (For low electron concentrations, the electron distribution may be Maxwellian, and we have to make sure that $k < k_D$ in order to eliminate Landau damping.) Here again, we check $2\tau_1\omega_2 \approx 1$ and conclude that our estimate of the threshold power is valid (up to a factor of 2).

[12] S. H. Koenig, R. D. Brown III, and W. Schillinger, Phys. Rev. **128**, 1668 (1962).