Fluctuations of Energy Loss by Heavy Charged Particles in Thin Absorbers

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Significant fluctuations of energy loss are expected in certain cases of the passage of fast heavy charged particles through thin absorbers. When the number of particle-electron collisions in the upper collision-loss interval is small, the energy-loss distribution is asymmetric and is characterized by a broad peak around the most probable energy loss (which is significantly less than the mean energy loss) and by a high-energy-loss tail. Several theories predict the energy-loss distribution function, but previous experimental work is incomplete with respect to verification of theory over the whole significant range of the parameters involved. We have passed beams of 730- and 45-MeV protons, 910-MeV helium ions, and 370-MeV π^- mesons through lithium-drifted silicon *p-i-n* detectors and silicon *p-n* junction detectors with depletion layers from 0.0085 to 1.094 g/cm^2, and measured the resulting energy-loss distributions. Within the limits of experimental error, there is very good agreement between the measured energy-loss distributions and those predicted by Vavilov's theory, and good agreement on the value of the most probable energy loss.

I. INTRODUCTION

When a charged energetic particle passes through matter, it loses its energy by several competing processes. For fast heavy charged particles (i.e., particle mass$\gg$electron mass), the predominant mode of energy loss is that involving inelastic collisions with the electrons of the material, resulting in ionization and excitation of the atoms of the material. Because the collisions are discrete and random, statistical fluctuations are expected in the number of collisions.

In first approximation, the probability of energy loss ϵ in a single collision with an electron (the collision spectrum) is proportional to ϵ^{-2}. Thus collisions resulting in a large energy transfer to an electron are relatively infrequent in comparison with small-energy-transfer collisions. Although they are relatively infrequent, the large-energy-transfer collisions account for a significant proportion of the total energy loss. In a *thin* absorber (one in which the average total energy lost is very small compared with the kinetic energy of the particle), the probable number of large-energy-transfer collisions may be so small that the random statistical variations in this number are relatively large, and result in significant fluctuations in the energy lost in this mode, and thus fluctuations in the total energy loss occur.

Several existing theories of this phenomenon predict the probability distribution of energy loss occurring when a heavy charged particle passes through a thin absorber. Our purpose was to measure the energy-loss distribution experimentally over a wide range of the significant parameters, and compare results with the theoretical predictions.

II. REVIEW OF PREVIOUS WORK

A. Theoretical Studies

The theory of energy-loss fluctuations (often called energy-loss straggling) was first discussed by Flamm,[1] Bohr,[2] Williams,[3] and Livingston and Bethe.[4] Bohr showed that when the number of collisions in each collision-energy interval is large, the energy-loss probability distribution is Gaussian, with variance given by

$$\sigma^2 = 4\pi e^4 z^2 NZx, \tag{1}$$

where $e\equiv$electron charge; $z\equiv$particle charge number; $N\equiv$number of atoms/cm^3 of the absorber material; $Z\equiv$atomic number of the material; $x\equiv$absorber thickness. The condition for validity of this expression is equivalent to

$$\xi/\epsilon_{\max} \gg 1, \tag{2}$$

where

$$\xi \equiv 2\pi e^4 z^2 NZx/mv^2;$$

$m\equiv$electron mass; $v\equiv$particle velocity; and $\epsilon_{\max}$ is the maximum possible energy transfer in a heavy particle-electron collision, or (nonrelativistically)

$$\epsilon_{\max} = 2mv^2/(1-\beta^2),$$

where $\beta \equiv v/c$.

Landau,[5] in 1944, solved the opposite case, in which the number of collisions in the highest collision-energy interval is small, or equivalently,

$$\xi/\epsilon_{\max} \ll 1. \tag{3}$$

Here, the distribution of total energy losses is highly asymmetric, with a broad peak [FWHM (full width at half-maximum) $=3.98\xi$] around the most probable energy loss and a long tail corresponding to higher-energy losses. In such cases the most probable energy loss $\Delta_{\rm mp}$ is significantly less than the average energy loss $\Delta_{\rm av}$, and is given by

$$\Delta_{\rm mp} = \xi\{\ln[2mv^2\xi/I^2(1-\beta^2)] - \beta^2 + 0.37\}, \tag{4}$$

where $I\equiv$mean excitation potential of material.

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[1] L. Flamm, Sitzber. Akad. Wiss. Wien, Math.-naturw. Kl. **123**, 1393 (1914); **124**, 597 (1915).

[2] N. Bohr, Phil. Mag. **30**, 581 (1915); Kgl. Danske Videnskab. Selskab Mat. Fys. Medd. **18**, No. 8 (1948).

[3] E. J. Williams, Proc. Roy. Soc. (London) **A125**, 420 (1929).

[4] M. S. Livingston and H. A. Bethe, Rev. Mod. Phys. **9**, 245 (1937).

[5] L. Landau, J. Phys. U.S.S.R. **8**, 201 (1944).

The further conditions for validity of the Landau theory are that $\xi \gg \epsilon_0$, where ϵ_0 is of the order of the mean binding energy of the atomic electrons (thus the theory breaks down in the limit of thinness of absorber), and that the collision spectrum be directly proportional to ϵ^{-2}.

In 1948, Symon[6] treated the cases intermediate between the Landau and Bohr theories by an approximation method, and Vavilov[7] treated the same problem exactly in 1957. Numerical evaluation of Vavilov's general solution yields a family of curves with dimensionless parameters β^2 and κ, where

$$\kappa \equiv \xi / \epsilon_{\max},$$

which effect a smooth transition between the Bohr and Landau distributions and include them as special cases. Note that the significant parameter κ may be evaluated by

$$\kappa = 0.150 s z^2 Z (1-\beta^2) / A \beta^4, \tag{5}$$

where $s \equiv$ absorber "thickness" in g/cm$^2 = \rho x$, $\rho \equiv$ density of material, $A \equiv$ atomic weight of material. Seltzer and Berger[8] provided a systematic and comprehensive tabulation of the Vavilov distribution in terms of the important parameters κ and β^2.

The problem of corrections to the energy-loss distribution functions due to resonance collision with atomic electrons (i.e., departure of the collision spectrum from ϵ^{-2} behavior) was discussed by Blunck *et al.*,[9] Rosenzweig,[10] Shulek, Golovin *et al.*,[11] and others.[12]

B. Experimental Work

Considerable experimental work has been done on the penetration of matter by protons, α particles, and mesons, and many experimenters have considered the problem of energy-loss fluctuations in thin absorbers. In general, workers with natural α particles[13] and low-energy protons[14] have found agreement with the Bohr theory with modifications by Livingston-Bethe. Workers studying resonance yields with 992-keV protons have noted straggling effects on yield curves, but their work has been outside the region of strict validity of the analytic theories.[15] With some exceptions, workers with cosmic-ray and accelerator mesons have found agreement with the Landau-Blunck theory,[16] as did workers with medium and high-energy protons.[17] The work of Van Putten and Vander Velde, Koch *et al.*, Miller *et al.*, and Labeyrie is of special interest for the present investigation because they showed the usefulness of semiconductor detectors for energy-loss measurements with fast charged particles. In particular, Miller *et al.* suggested that semiconductor detectors with uniform depletion layers offer the best means of evaluation of the fluctuation theories.

Gooding and Eisberg found agreement with Symon theory for 37-MeV protons.[18] Rosenzweig and Rossi did a detailed study of energy-loss fluctuations with 5.8-MeV α particles in a variable-thickness proportional counter, finding general agreement with Symon for κ values from 0.11 to 3.56, provided that substantial corrections were applied for the effects of electron binding and secondary electron escape from the detector.[19] Recently Galaktionov *et al.* found agreement with Landau for 600-MeV/c protons and pions,[20] but Grew[21] and Lander *et al.*[22] found broader distributions than expected for fast protons. Finally, Glass and Samsky found agreement with the Vavilov theory for protons of energy as low as 1 MeV in a proportional counter.[23]

In summary, there is good experimental evidence for the validity of the Bohr-Livingston-Bethe theory for natural α particles and low-energy protons, for which $\kappa \gg 1$. Similarly, there is much evidence for the validity of the Landau-Blunck theory for high-energy protons and mesons, when $\kappa \lesssim 0.01$. There has been only a small amount of unambiguous data, however, in the intermediate region of $0.01 \lesssim \kappa \lesssim 1$, where Symon's

[6] K. R. Symon, Ph.D. thesis, Harvard University, 1948 (unpublished); summary in B. Rossi, *High Energy Particles* (Prentice-Hall, Inc., Englewood Cliffs, N.J., 1952), p. 32.

[7] P. V. Vavilov, Zh. Eksperim. i Teor. Fiz. **32**, 920 (1957) [English transl.: Soviet Phys.—JETP **5**, 749 (1957)].

[8] S. Seltzer and M. J. Berger, Natl. Acad. Sci.—Natl. Res. Council, Publ. **1133**, 187 (1964).

[9] O. Blunck and S. Leisegang, Z. Physik **128**, 500 (1950); O. Blunck and K. Westphal, *ibid.* **130**, 641 (1951).

[10] W. Rosenzweig, Phys. Rev. **115**, 1683 (1959).

[11] P. Shulek, B. M. Golovin, L. A. Kulyukina, S. V. Medved, and P. Pavlovitch, Yadern. Fiz. **4**, 564 (1966) [English transl.: Soviet J. Nucl. Phys. **4**, 400 (1967)].

[12] A more complete discussion of these and the following topics is given in H. D. Maccabee, Ph.D. thesis, Lawrence Radiation Laboratory Report No. UCRL-16931, 1966 (unpublished).

[13] P. T. Porter and J. I. Hopkins, Phys. Rev. **91**, 209 (1953); F. Demichelis, Nuovo Cimento **13**, 562 (1959); E. Rotondi and K. W. Geiger, Nucl. Instr. Methods **40**, 192 (1966); G. Fabri, J. Karolyi, and V. Svelto, *ibid.* **50**, 50 (1967).

[14] C. B. Madsen and P. Venkateswarlu, Phys. Rev. **74**, 1782 (1948); C, B. Madsen, Kgl. Danske Videnskab. Selskab Mat. Fys. Medd. **27**, No. 13 (1953); L. P. Nielsen, *ibid.* **33**, No. 6 (1961); H. K. Reynolds *et al.*, Phys. Rev. **92**, 742 (1953); A. B. Chilton *et al.*, *ibid.* **93**, 413 (1954).

[15] R. O. Bondelid and J. W. Butler, Phys. Rev. **130**, 1078 (1963); J. G. Skofronick *et al.*, *ibid.* **135**, A1429 (1964); A. L. Morsell, *ibid.* **135**, A1436 (1964).

[16] F. Bowen and F. X. Roser, Phys. Rev. **85**, 992 (1952); A. Hudson and R. Hofstadter, *ibid.* **88**, 589 (1952); T. E. Cranshaw, Progr. Nucl. Phys. **2**, 271 (1952); J. K. Parry, H. D. Rathgeber, and J. L. Rouse, Proc. Phys. Soc. (London) **A66**, 541 (1953); T. Bowen, Phys. Rev. **96**, 754 (1954); D. E. Palmatier, J. T. Meers, and C. M. Askey, *ibid.* **97**, 486 (1955); J. D. Van Putten and J. C. Vander Velde, IRE Trans. Nucl. Sci. **8**, 124 (1961); L. Labeyrie, in *Proceedings of the International School of Physics Enrico Fermi* (Academic Press Inc., New York, 1963), Course XIX, p. 187.

[17] L. Koch, J. Messier, and J. Valin, *Nuclear Electronics* (International Atomic Energy Agency, Vienna, 1962) Vol. 1, p. 465; G. L. Miller *et al.*, IRE Trans. Nucl. Sci. **8**, 73 (1961); G. L. Miller *et al.*, Nucl. Instr. Methods **20**, 303 (1963); G. J. Igo, D. D. Clark, and R. M. Eisberg, Phys. Rev. **89**, 879 (1953); G. J. Igo and R. M. Eisberg, Rev. Sci. Instr. **25**, 450 (1954).

[18] T. J. Gooding and R. M. Eisberg, Phys. Rev. **105**, 357 (1957).

[19] W. Rosenzweig and H. H. Rossi, Columbia Radiological Research Laboratories Report No. NYO-10716, 1963 (unpublished).

[20] Y. V. Galaktionov, F. A. Yech, and V. A. Lyubimov, Nucl. Instr. Methods **33**, 353 (1965).

[21] G. Grew, IEEE Trans. Nucl. Sci. **12**, 308 (1965).

[22] R. L. Lander *et al.*, Nucl. Instr. Methods **42**, 261 (1966).

[23] W. A. Glass and D. N. Samsky, Radiation Res. **32**, 138 (1967).

interpolation and Vavilov's exact expression are held to be valid. Our goal is to supply the needed data.

III. EXPERIMENTAL METHOD

Our experimental method basically consists of passing a beam of fast heavy charged particles through a silicon semiconductor detector and measuring the energy losses in the detector. Semiconductor detectors are used because of their superior energy resolution for this purpose, the linearity of their response, and their high stopping power. In a given experiment, the detector is mounted in a plane normal to the beam axis and bias voltage is applied. The charge pulses formed due to ionization and excitation in the detector are amplified and sorted (individually) in a multichannel pulse-height analyzer (abbreviated PHA). Information from the PHA is then printed out in the form of counts per channel versus channel number. This information is processed to yield a plot of relative probability versus pulse height, which is proportional to the energy loss in the detector.

We have used beams of 730-MeV protons, 910-MeV α particles (He^{+2} ions), and 370-MeV negative pions from the 184-in. synchrocyclotron at the Lawrence Radiation Laboratory, Berkeley, and 45.3-MeV protons from the Berkeley 88-in. isochronous (sector-focused) cyclotron, thus bracketing a range of β^2 from 0.09 to 0.92.

Until recently, semiconductor detectors were limited in sensitive thickness to fractions of a millimeter. The development of the lithium-drifting process has solved this problem, permitting fully compensated depletion layers upwards of 5 mm in thickness. In addition, lithium-drifted *p-i-n* junction detectors are generally more uniform in depletion-layer thickness than the thinner *p-n* detectors. These advances in technology have made possible reliable detectors with good enough resolution to measure energy-loss distributions accurately, and with sufficient thickness to explore the intermediate range of the parameter κ for intermediate-energy particles. We have used lithium-drifted silicon detectors of sensitive thicknesses between roughly 0.5 and 5 mm, as described by Goulding.[24] When thinner detectors were needed for work with lower-energy particles, we used silicon diffused *p-n* junction detectors with depletion layers between roughly 0.04 and 0.25 mm. Since the density of the silicon used is 2.33 g/cm^3, we have been able to cover more than two orders of magnitude in thickness, from 0.0085 to 1.094 g/cm^2. Since the values of β^2 available have bracketed nearly an order of magnitude, we have thus been able to explore nearly three orders in κ, from $\kappa=0.0029$ to $\kappa=2.23$.

In order to take advantage of the good resolution, low noise, linearity, and fast pulse characteristics of the semiconductor detectors, an amplification system with similar characteristics is necessary. We have used preamplifiers and linear amplifier systems designed in this Laboratory for such applications.[25]

In the course of preliminary measurements,[26,27] we found a significant proportion of detected pulses to be anomalously small, because of particles that pass through the circumference of the sensitive intrinsic area of the detector. To avoid this difficulty, we used an auxiliary detector with smaller sensitive area (aligned directly behind the analyzing detector) as a coincidence gate on the main detector, thus eliminating pulses due to particles passing through the outer edge of the main detector.

The calibration of channel number to energy loss is done by a standard method of spectrometry. The detector is exposed to radiation from a standard source with a known energy spectrum, and the channel numbers of peaks in the output pulse-height spectrum are correlated with the energies of known peaks in the input spectrum. Linear interpolation or extrapolation and the use of a calibrated pulse generator yields the energies corresponding to all other channel numbers. Sources used for calibration included ^{207}Bi, ^{241}Am, ^{57}Co, and ^{212}Po.

Precise calibration of the sensitive thickness of the detector (i.e., the absorber thickness) is more difficult. A fairly accurate determination for the thicker detectors is done by exposing them to a spectrum of α particles with ranges of the order of the detector thickness. The maximum energy lost in the detector (i.e., the cutoff of the measured spectrum) corresponds to an α particle whose range is exactly equal to the sensitive thickness. This method is accurate within $\pm 2\%$.[28] Another method used is subtraction of the measured dead-layer thickness from the known overall thickness, and cross-checking by exposing the detector to penetrating α particles with a well-known energy-loss spectrum; the measured most probable energy loss is also a measure of the sensitive thickness of the detector.

IV. RESULTS

A. Comparison with Theory

The experimental data, in the form of counts per channel versus channel number, are processed with the calibration information to yield a plot of counts per energy-loss interval versus energy loss. For comparison, the Vavilov theoretical distribution is numerically evaluated by a computer code which uses the pertinent initial parameters of the experiment as input information. This code, developed by Seltzer and Berger and modified by Heckman and Brady,[29] takes the

[24] F. S. Goulding, Nucl. Instr. Methods **43**, 1 (1966).

[25] F. S. Goulding and D. Landis, Natl. Acad. Sci.—Natl. Res. Council Publ. 61 (1964); **1184**, 124 (1964).

[26] H. D. Maccabee and M. R. Raju, Nucl. Instr. Methods **37**, 176 (1965).

[27] H. D. Maccabee, M. R. Raju, and C. A. Tobias, IEEE Trans. Nucl. Sci. **13**, 176 (1966).

[28] M. R. Raju, H. Aceto, and C. Richman, Nucl. Instr. Methods **37**, 152 (1965).

[29] M. J. Berger (private communication); V. Brady (private communication).

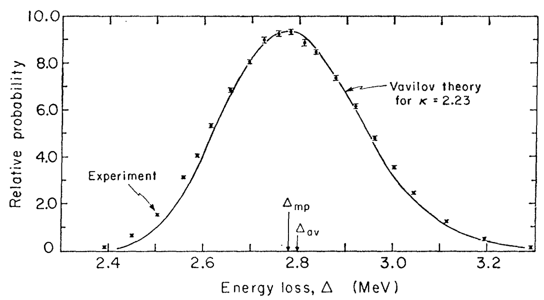

FIG. 1. Energy-loss distribution of 45.3-MeV protons in 0.265-g/cm² silicon, $\kappa=2.23$.

particle mass, velocity, and charge, the thickness s, the mean excitation potential I, and A/Z of the absorber, and computes $\epsilon_{\max}$, ξ, κ, and Δ_{av}, as well as the theoretical energy-loss probability distribution in tabular form. For silicon absorber data, we used $A/Z=2.0064$, $s=(2.33\text{ g/cm}^3)\times x$, where x is our best value of the depleted thickness, and $I_{\text{Si}}=176$ eV. This value for the mean excitation potential was obtained by multiplying $(Z_{\text{Si}}/Z_{\text{Al}})$ times I_{Al}, where $I_{\text{Al}}=163$ eV, the accepted value for aluminum.

Figures 1 through 6 show a representative selection of our experimentally measured energy-loss distributions compared with the Vavilov theoretical predictions (solid line); the ordinate normalization is such that the maximum theoretical probability corresponds to the maximum number of counts per channel. Vertical error bars are shown on the experimental points corresponding to one standard deviation $\approx N^{1/2}$, where $N=$ the number of counts in the energy interval. Although horizontal error bars are not shown, it should be understood that the accuracy of the experimental energy losses is approximately $\pm 2\%$, owing to uncertainties in the depletion thickness, the channel-number-to-energy calibration, etc. For convenience the calculated values of Δ_{mp} (the most probable energy loss), Δ_{av} (the mean energy loss, as computed by the standard

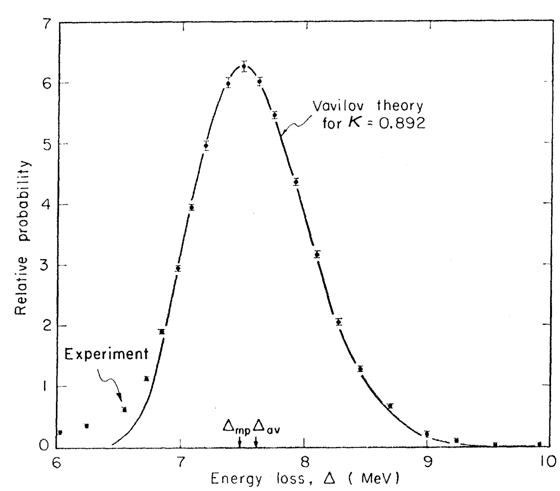

FIG. 2. Energy-loss distribution of 895-MeV helium ions in 0.560-g/cm² silicon, $\kappa=0.892$.

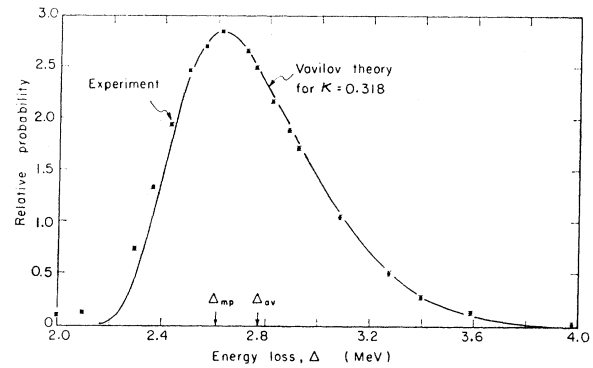

FIG. 3. Energy-loss distribution of 910-MeV heluim ions in 0.206-g/cm² silicon, $\kappa=0.318$.

Bethe-Bloch formula for dE/dx), and the value of κ are shown on each figure.

Figure 1 shows the energy-loss distribution of 45.3-MeV protons in 0.265-g/cm² silicon, $\kappa=2.23$. Note the general Gaussian shape, with slight asymmetry, and note that Δ_{mp} is slightly less than Δ_{av}. There is good agreement between theory and experiment on the value of Δ_{mp} and the shape of the curve, with a mild deviation on the low-energy-loss side. Figure 2 shows the distribution for 895-MeV α particles in 0.560-g/cm² Si, $\kappa=0.892$. Figure 3 shows the energy-loss distribution for 910-MeV alphas in 0.206-g/cm² Si, $\kappa=0.318$. Note, in general, the very good agreement between theory and experiment for these higher intermediate cases where κ is of the order of unity. Note also the increase in the asymmetry with decreasing κ, the increasing tendency toward a high-energy-loss tail, and the decrease of Δ_{mp} relative to Δ_{av}.

Figure 4 shows the distribution for 895-MeV α particles in 0.057-g/cm² Si, $\kappa=0.0908$; the distribution for 730-MeV protons in 0.413-g/cm² Si, $\kappa=0.021$; is shown in Fig. 5. Note the pronounced asymmetry of the distributions in these cases of lower intermediate values of κ $(0.01<\kappa\ll 1)$, the growth of the high-energy-loss tail, and the marked shrinkage of Δ_{mp} relative to Δ_{av}. Agreement between theory and experiment is good in these cases, with a continuing small deviation on the low-energy-loss shoulder. Figure 6 shows the energy-loss distribution for 730-MeV protons

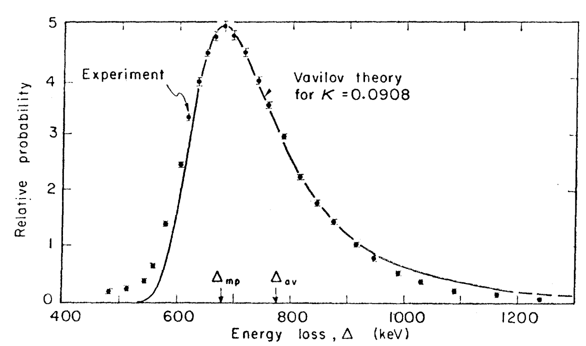

FIG. 4. Energy-loss distribution of 895-MeV helium ions in 0.057-g/cm² silicon, $\kappa=0.0908$.

in 0.108-g/cm² Si, $\kappa=0.0055$. This is typical of very small κ ($\kappa \lesssim 0.01$), where the Vavilov theory reduces to the Landau distribution, with a very long high-energy-loss tail. In this case, however, the experimental distribution is broadened by resolution effects in the detection system.

B. Resolution of System

The resolution of our system for measuring energy-loss spectra is limited by two main effects: statistical fluctuations in the number of hole-electron pairs due to a fixed energy loss in the detector, and electrical noise in the detector-amplifier system. Normally, one could evaluate the root-mean-square fluctuations $\langle n\rangle$ in the number of pairs produced by

$$\langle n\rangle=\left(\frac{\text{energy absorbed in detector}}{\text{mean energy per hole-electron pair}}\right)^{1/2}.$$

For the case of interest here, this formula yields

$$\langle n\rangle=(160\text{ keV}/3.66\text{ eV})^{1/2}=210\text{ pairs}.$$

This corresponds to a fluctuation of only 0.77 keV, and is clearly negligible. In actuality, the effect is even smaller than this, because the hole-electron pair production process is not statistically independent of the thermal and vibrational energy-loss modes in the semiconductor; this phenomenon is usually expressed by introducing the Fano factor in the rms fluctuation formula.

The resolution limitation introduced by electrical noise is much more serious in our case. In particular, there is noise due to detector leakage current (primarily due to thermal excitation), and there are shot noise and flicker-effect noise (due to plate-current fluctuations in the preamplifier), both of which increase directly as the total input capacitance,[24] which is dominated by the detector capacity (30 *pF* in the case of Fig. 6). In experiments in which the resolution was critical, the detector was cooled to liquid nitrogen temperature to reduce leakage-current noise. System resolution was measured by recording the output pulse

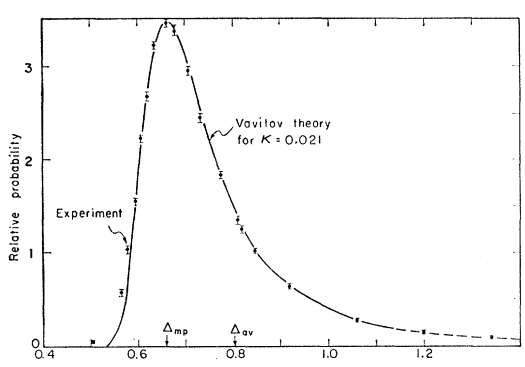

FIG. 5. Energy-loss distribution of 730-MeV protons in 0.413-g/cm² silicon, $\kappa=0.021$.

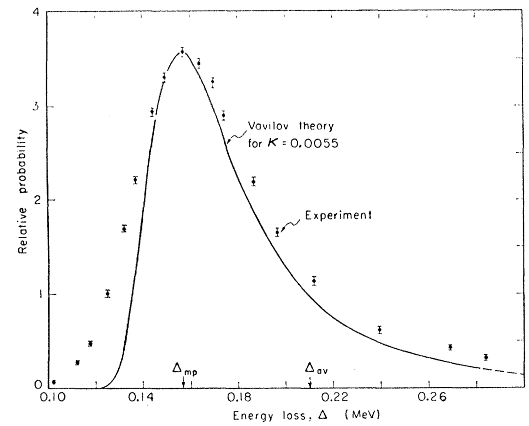

FIG. 6. Energy-loss distribution of 730-MeV protons in 0.108-g/cm² silicon, $\kappa=0.0055$.

spectrum due to an essentially monoenergetic input, e.g., from a pulse generator or a radioactive source.

The effect of system resolution on our measurements of the energy-loss distribution can be calculated by folding in the resolution spectrum with the "actual" energy-loss spectrum, yielding the measured spectrum. For practical purposes we assume the resolution spectrum may be represented by a Gaussian (a good approximation), and that the *peak* in the actual spectrum may be represented by a Gaussian (a fair approximation), and fold in the spectral widths by quadrature. Specifically,

$$(\text{FWHM})^2_{\text{measured}}=(\text{FWHM})^2_{\text{actual}}+(\text{FWHM})^2_{\text{resolution}}.$$

When this method was applied to the data of Fig. 6, the measured width was 59.6 keV and the resolution 35 keV, indicating an actual width of 48.2 keV. This is in good agreement with the theoretical width of 48.4 keV. The resolution correction is negligible in the other cases shown, since the resolution widths (20 to 30 keV) are small compared with the measured widths.

C. Sources of Error

Previous calculations have shown that only negligibly small errors are introduced by nuclear interactions, angular and energy spread of the incident beam, multiple scattering, bremsstrahlung, Cerenkov radiation, and sensitive thickness nonuniformity.[12] We have mentioned the errors introduced in the measurements owing to such factors as uncertainties in the channel-number–to–energy calibration due to small system nonlinearities, electronic drift (e.g., in amplifier gain), dead-layer effects, and depletion-layer thickness uncertainty. The combined effect of these errors is probably less than ±2%, and would be expressed by a shift in the whole distribution to the left or right on the energy scale in a given case, not by the small deviation noted on the low-energy-loss shoulders.

Consider now the effects that *could* result in the observed low-energy-loss deviation. Shulek, Golovin,

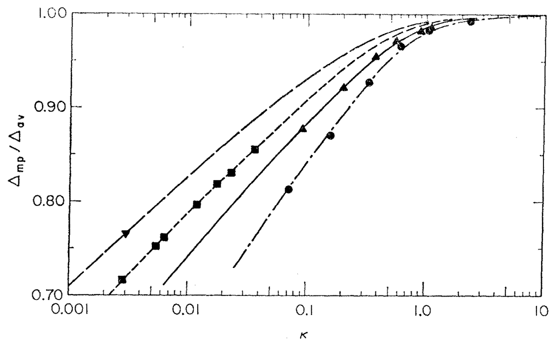

FIG. 7. Plot of Δ_{mp}/Δ_{av} as a function of κ for silicon, with β^2 as a parameter. Experimental points: ●, protons, $\beta^2=0.09$; ▲, helium ions, $\beta^2=0.35$; ■, protons, $\beta^2=0.68$; ▼, pions, $\beta^2=0.92$. Theoretical lines: — - —, $\beta^2=0.09$; ——, $\beta^2=0.35$; - - - -, $\beta^2=0.68$; — —, $\beta^2=0.92$.

et al. have shown that in certain cases, the effect of resonance collisions with bound atomic electrons could be such a deviation. In order to evaluate this effect in the worst case shown, we estimate the Blunck parameter b^2 for 730-MeV protons in 0.108-g/cm² Si (when $b^2 \ll 3$, resonance broadening is negligible):

$$b^2 \approx (\Delta_{av})(Z^{4/3}) \times 20 \text{ eV}/\xi^2$$

$$\approx (0.21\times10^6 \text{ eV})(33) 20 \text{ eV}/(0.012\times10^6 \text{ eV})^2 \approx 1.$$

The value of the parameter is below the threshold where resonance effects become important (though not far below). The contribution to the deviation due to resonance collisions is probably small.

Another physical effect which could cause an increase in the number of very-low-energy-loss traversals is the phenomenon of "channeling." There is reason to believe that channeling effects are relatively unimportant in our experiments. First, the angular definition of our most collimated beam is about 0.2°, which is large compared with the definitions required in channeling experiments. Moreover, in the fabrication of our silicon detectors, the crystals are purposely sliced a few degrees off the ⟨111⟩ plane in order to prevent channeling when particles are incident normal to the detector face. In addition, the likelihood of channeling is greatly decreased for incident energies greater than 10 MeV per nucleon.

Finally, we consider the effect of secondary electron entry and escape from the detector. When a fast particle imparts a large energy in a collision with an electron, the resulting knock-on electron or "δ ray" can have a considerable range of its own, and, in certain cases, can escape the detector. Similarly, secondary electrons from collisions in the materials in front of the detector can enter and pass through the detector. For example, the maximum δ-ray energy for a 730-MeV proton is 2.19 MeV, which corresponds to a range of 4.5 mm in Si. Now it should be understood that the Vavilov theory predicts the distribution of energy *losses* by the particles, including energy lost to penetrating secondaries, while the experiment measures only the energy *absorbed* by the detector. As Gooding and Eisberg and others have pointed out, the measured quantity will be equal to the theoretically predicted energy loss only if there is no *net* transfer of energy into or out of the detector. The qualitative effect of δ-ray entry and escape is probably to shift events from the ultrahigh-energy-loss tail to the very-low-energy-loss shoulder.[12] This effect becomes negligible when detector thickness is much larger than the maximum δ-ray range, as for the slower particles in thicker detectors, but could account for the deviations observed for 730-MeV protons in thinner detectors.

V. SUMMARY

Several theories predict the fluctuations of energy loss for heavy charged particles in thin absorbers. The Bohr theory, with modifications by Livingston and Bethe, predicts a symmetric Gaussian distribution around the mean energy loss, and is valid for $\kappa \gg 1$. The Landau theory, with modifications by Blunck and Leisegang, is valid for $\kappa \lesssim 0.01$ and predicts a broad asymmetric distribution with a peak around the most probable energy loss and a long high-energy-loss tail. For the intermediate region of $0.01 \lesssim \kappa \lesssim 1$, Symon has given an approximate and Vavilov an exact theory to predict the energy-loss distributions, which form a smooth transition between the Landau and Bohr distributions, but experimental data for establishing the validity of theory in this region are not plentiful.

We have measured the energy-loss distributions of fast protons, pions, and helium ions in silicon detectors of various thicknesses, covering a range of κ from 0.0029 to 2.23. We find very good agreement with the Vavilov theoretical distributions. Figure 7 summarizes our data on the measured most probable energy loss Δ_{mp}, as compared with the Vavilov theoretical values; the agreement is satisfactory.

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