Experimental Properties of Liquid He^3 near the Absolute Zero*

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Measurements of the coefficients of spin diffusion, thermal conductivity, and viscosity of liquid He^3 at very low temperatures and at pressures of 0.28 and 27.0 atm are discussed critically on the basis of a theory by Rice of transport in nearly ferromagnetic Fermi liquids. It is found that the temperature dependence of the thermal-conductivity coefficient is qualitatively similar to that predicted by Rice. However, neither the spin-diffusion coefficient nor the viscosity coefficient, as determined from ultrasonic attenuation measurements, is known precisely enough to give a good test of the Rice theory in the low-temperature limit. It is suggested on the basis of qualitative data that the anomalously low thermal boundary resistance between cerium magnesium nitrate and pure He^3 may result from the nearly persistent spin-fluctuation phenomenon.

I. INTRODUCTION

MEASUREMENTS[1] of the spin-diffusion coefficient D in pure liquid He^3 at low pressure showed that between 20 and 50 m°K D is proportional to T^{-2} within an experimental error which corresponds to the exponent 2 being known to $\pm 1\%$ in the above range. Although measurements[2] of the propagation of zero and first sound in pure He^3 at low pressure give a strong confirmation of Landau's ideas[3] in regard to pure He^3, we have regarded the above D measurements as strong evidence that in the above temperature range He^3 is a normal Fermi liquid. Subsequent measurements[4] of D to lower temperatures did not have the precision of the 1961 data.

Measurements of the heat capacity of the liquid[5,6] do not show a simple linear temperature dependence for the heat capacity below 50 m°K. Instead C/T, where C is the specific heat, continues to increase as T decreases. The limiting low-temperature behavior has not yet been determined experimentally. Although we pointed out in our 1963 paper[6] that the heat capacity was not linear in temperature, the problem did not receive much attention theoretically until Anderson[7] in 1965 showed that the existing experimental data at that time were not in disagreement with $C/T=A\ln(B/T)$. Anderson raised the question of whether He^3 was, in fact, a normal Fermi liquid. His question stimulated theoretical activity. Balian and Fredkin[8] postulated an anomalously long-range interaction between He^3 quasiparticles and zero-sound phonons to obtain $C/T=A[\ln(B/T)]^{1/2}$. According to Engelsberg and Platzman,[9] it is not possible to justify their postulate. Later Doniach and Engelsberg[10] found that the heat-capacity results could be fitted reasonably well on the basis of a theory in which nearly persistent spin fluctuations play a major role. The experimental basis for their calculation is that He^3 liquid at low T and low p has a susceptibility nine times greater than that of an ideal Fermi gas of the same density and hence may be classified as a "nearly ferromagnetic" Fermi liquid. Recent calculations of Amit, Kane, and Wagner[11] and also of Brenig and Mikeska[12] on the basis of theories in which spin fluctuations are of major importance, have obtained good agreement with experiments. Since our heat-capacity measurements will be discussed in Ref. 11, we will not review them here.

Recently, Rice[13] has written a very interesting series of papers on transport properties of nearly ferromagnetic Fermi liquids based on a model approximation in which persistent spin fluctuations in the He^3 serve to scatter bare He^3 atoms. He fitted his theoretical results to some of the experimental data and, with a rather reasonable choice of parameters, obtained a remarkable agreement with experiment. It is the purpose of the present paper to review experimental results on transport in liquid He^3 near the absolute zero obtained in our laboratory over the last several years and to compare them with the predictions of Rice's theory in order to discover to what extent his theory is suitable for describing the very low-temperature properties of liquid He^3.

II. THEORETICAL PREDICTIONS

Perhaps the most interesting qualitative results of Rice's theory are expressed in the following three equations for the coefficients of spin diffusion D, thermal

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[1] A. C. Anderson, W. Reese, R. J. Sarwinski, and J. C. Wheatley, Phys. Rev. Letters 7, 220 (1961).
[2] W. R. Abel, A. C. Anderson, and J. C. Wheatley, Phys. Rev. Letters 17, 74 (1966).
[3] L. D. Landau, Zh. Eksperim. i Teor. Fiz. 32, 59 (1957) [English transl.: Soviet Phys.—JETP 6, 84 (1958)].
[4] W. R. Abel, A. C. Anderson, W. C. Black, and J. C. Wheatley, Physics 1, 337 (1965).
[5] W. R. Abel, A. C. Anderson, W. C. Black, and J C. Wheatley, Phys. Rev. 147, 111 (1966).
[6] A. C. Anderson, W. Reese, and J. C. Wheatley, Phys. Rev. 130, 495 (1963).
[7] P. W. Anderson, Physics 2, 1 (1965).
[8] R. Balian and D. R. Fredkin, Phys. Rev. Letters 15, 480 (1965).
[9] S. Engelsberg and P. M. Platzman, Phys. Rev. 148, 103 (1966).
[10] S. Doniach and S. Engelsberg, Phys. Rev. Letters 17, 750 (1966).
[11] D. J. Amit, J. W. Kane, and H. Wagner, Phys. Rev. Letters 19, 425 (1967).
[12] W. Brenig and H. J. Mikeska, Phys. Letters 24A, 332 (1967).
[13] M. J. Rice, Phys. Rev. 162, 189 (1967).

conductivity K, and viscosity η:

$$1/DT^2=\alpha_D-\beta_D T/\theta, \quad (1)$$

$$1/KT=\alpha_K-\beta_K T/\theta, \quad (2)$$

and

$$1/\eta T^2=\alpha_\eta-\beta_\eta(T/\theta)^3 \quad (3)$$

valid in the limit $T\to 0$. In these equations, α and β are quantities which depend on the transport property in question and θ is a characteristic temperature. As $T\to 0$, the quantities DT^2, KT, and ηT^2 approach constant values as predicted by Abrikosov and Khalatinkov[14] and by Hone[15] on the basis of the Landau theory of a normal Fermi liquid. For the viscosity, it is expected from Eq. (3) that over a substantial range of temperature ηT^2 is constant. However, according to the theory, Eqs. (1) and (2), both $(DT^2)^{-1}$ and $(KT)^{-1}$ decrease linearly with T at very low temperatures.

In order to make quantitative predictions, an approximation is made in which the Boltzmann equation for the single-particle distribution function is worked out for bare fermions which are scattered by an equilibrium distribution of spin fluctuations. The strength of the scattering is measured by a quantity $\bar{I}$ given by

$$\bar{I}=1-\chi_0/\chi, \quad (4)$$

where χ_0 is the susceptibility of an ideal gas of bare He^3 fermions having the same number density as the actual liquid and χ is the measured susceptibility. In terms of quantities tabulated[16] for pure He^3, one has

$$\chi_0/\chi=\tfrac{3}{2}T^*/T_F,$$

where T^* is the limiting effective magnetic temperature of the actual He^3 system and $T_F=p_F^2/2mk_B$, where p_F is the Fermi momentum corresponding to the actual number density, m is the bare He^3 particle mass, and k_B is Boltzmann's constant. To simplify the calculations an approximation to the spectral density function for the persistent spin fluctuations is made and a cutoff parameter $\bar{Q}$ introduced. $\bar{Q}$ is the ratio of the cutoff momentum for the spin fluctuations to the particle Fermi momentum and is related to the effective-mass ratio obtained from heat-capacity measurements by the equation

$$\frac{m^*}{m}=1+\frac{\bar{Q}^2}{2\pi}\frac{\bar{I}}{1-\bar{I}}.$$

In Rice's theory only the parameters $\bar{I}$ and $\bar{Q}$ are obtained from empirical data.

The parameters α and β in Eqs. (1)–(3) are given in terms of the quantities $\bar{I}$ and $\bar{Q}$ and other parameters such as the density of liquid He^3 and the mass of bare He^3 atoms. Formulas[13,17] for these quantities are given in the Appendix. A characteristic temperature θ is given by the equation

$$\theta=\frac{4}{\pi}\frac{1-\bar{I}}{\bar{I}}\bar{Q}T_F. \quad (5)$$

Computations of the parameters of Rice's theory on the basis of our experiments and an evaluation of the validity of the theory are made in the next section.

TABLE I. Parameters derived from experiment for liquid He^3.

p (atm)	m^*/m	T^* (°K)	p_F (g cm/sec)	T_F (°K)
0.28	3.0	0.36_1	8.31×10^{-20}	5.00
27.0	5.6	0.19_7	9.24×10^{-20}	6.17

III. COMPARISON OF EXPERIMENT AND THEORY

Although reasonably accurate results for the limiting nuclear susceptibility are available as a function of pressure, the heat capacity has been measured to very low temperatures only for pressures of 0.28 and 27.0 atm. Our comparison with theory will therefore be limited to these two pressures, or to pressures near them.

In Table I we list relevant quantities obtained from experimental data. The values of m^*/m are obtained from the experimental data by assuming, in the spirit of the present paper, a limiting behavior of C/T similar to that predicted by the theories of Doniach and Engelsberg,[10] of Amit *et al.*,[11] and of Brenig and Mikeska.[12] In view of the possibilities for making calibrational and other errors and, as well, of the sort of reproducibility of results over the past several years in our laboratory and elsewhere, it seems unlikely that the absolute values of either m^*/m or T^* are known to an accuracy of better than $\pm5\%$. Such an estimate of error should be taken as reasonable though not overly conservative. Using these values, some of the parameters of Rice's theory were computed. These are given in Table II.

We first present the experimental data on spin diffusion. All of the data obtained in our laboratory on spin diffusion at low pressure are shown in Fig. 1. Data used are given by Abel, Anderson, Black, and Wheatley,[4] Anderson, Reese, and Wheatley,[18] Anderson, Reese,

TABLE II. Characteristic parameters of the Rice transport theory. See text for definitions of these quantities.

p atm	$\bar{I}$	$\bar{Q}$	θ (°K)	α_D/β_D	α_K/β_K	$(\alpha_D/\beta_D)\theta$ (°K)	$(\alpha_K/\beta_K)\theta$ (°K)
0.28	0.892	1.24	0.96	0.39	0.24	0.37	0.23
27.0	0.952	1.20	0.48	0.40	0.24	0.19	0.11

[14] A. A. Abrikosov and I. M. Khalatinkov, in *Reports on Progress in Physics* (The Physical Society, London, 1959), Vol. 22, p. 329.

[15] D. Hone, Phys. Rev. **121**, 669 (1961).

[16] J. C. Wheatley, in *Quantum Fluids*, edited by D. F. Brewer (North-Holland Publishing Company, Amsterdam, 1966), p. 183. See Table 3 for a list of T^* versus pressure.

[17] M. J. Rice (private communication). The formulas in the Appendix are somewhat different from those given in Ref. 13.

[18] A. C. Anderson, W. Reese, and J. C. Wheatley, Phys. Rev. **127**, 671 (1962).

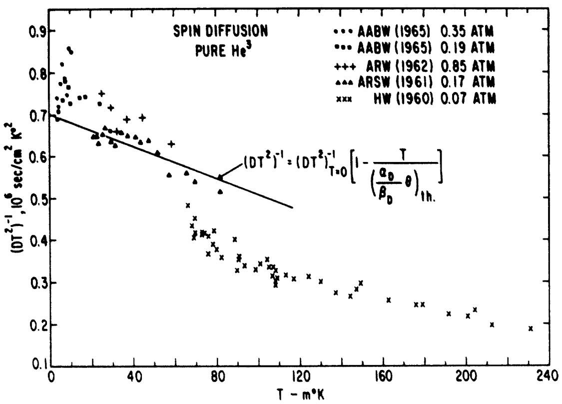

FIG. 1. Spin diffusion in pure liquid He[3] at low pressure. AABW (1965) is Ref. 4; ARW (1962) is Ref. 18; ARSW (1961) is Ref. 1; and HW (1960) is Ref. 19. The solid line shows the dependence on temperature of $(DT^2)^{-1}$ expected from Rice's theory if the value of $(DT^2)^{-1}$ at $T=0$ is chosen to agree with a reasonable extrapolation of the ARSW (1961) data.

Sarwinski, and Wheatley,[1] and Hart and Wheatley.[19] It should not be assumed that data from one experiment can be extrapolated or interpolated into those from another. It is unfortunate, but five different low pressures are represented. It is known that $(DT^2)^{-1}$ decreases with increasing pressure.[18] Some evidence for this effect is present in Fig. 1. However, measurements of the spin-diffusion coefficient by the method of spin echoes[20,21] are quite susceptible to systematic errors; the variations in $(DT^2)^{-1}$ from experiment to experiment are most likely traced to this cause. In the measurements, we photographed two successive echoes with amplitude ratio R and time separation t when the He[3] sample, gyromagnetic ratio γ, is subjected to a magnetic field gradient G. The spin-diffusion coefficient D is related to these quantities by the equation $\ln R = \gamma^2 G^2 D t^3 / 12$. Hence errors in the field gradient enter quadratically and those in the time calibration of the oscillograph sweep enter cubically. Moreover, in the quantity DT^2, error in the temperature scale enters quadratically. It is now known that in our first work[19] an experimental defect in the diffusion cell led to a temperature-dependent gradient which caused the low-T data to be inaccurate. It is possible that a temperature-dependent G also affected some of our later work[4] though such an effect was probably absent in our most definitive experiments.[1,18] The later experiments[4] were performed mainly as part of a search for a possible superfluid transition in pure He[3]; agreement with $(DT^2)^{-1}=$ constant was considered at that time to be satisfactory. A straight line is plotted on Fig. 1 showing the proposed temperature dependence of $(DT^2)^{-1}$ using the value of $\alpha_D\theta/\beta_D$ computed from Rice's theory and listed in Table II. The value of α_D was adjusted to fit reasonably with the data.[1] It is clear that the data are not sufficiently accurate and precise or extensive, in terms of temperature range at the same pressure in the same experiment, to provide a definitive test of the Rice theory. Comparison of $\alpha_D\theta/\beta_D$ and $\alpha_K\theta/\beta_K$, Table II, shows that the larger effect is to be expected in the thermal conductivity.

In Fig. 2 we show results[18] for $(DT^2)^{-1}$ versus T for a pressure of 28.2 atm, which is sufficiently close to 27.0 atm to make comparison with theory reasonable. The experiments were not carried to a sufficiently low temperature to allow the temperature dependence of $(DT^2)^{-1}$ to be convincingly demonstrated. Also shown on Fig. 2 is a line showing the expected dependence of $(DT^2)^{-1}$ on the basis of Rice's theory and the value of $\alpha_D\theta/\beta_D$ given in Table II for a pressure of 27.0 atm. If $(DT^2)^{-1}$ does in fact vary linearly with T at low T, it seems unlikely that the experimental value of $\alpha_D\theta/\beta_D$ will be as large as that given in Table II.

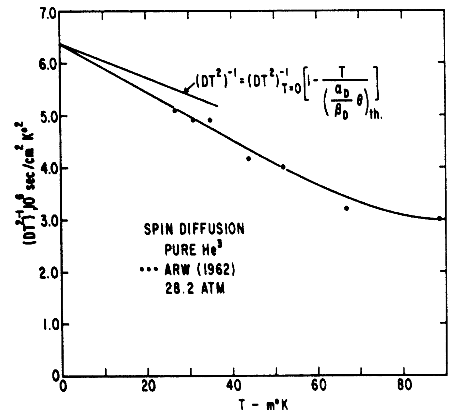

FIG. 2. Spin diffusion in pure liquid He[3] at a pressure near the melting pressure. ARW (1962) is Ref. 18. The line labeled by an equation shows the dependence on temperature of $(DT^2)^{-1}$ expected from Rice's theory if the value of $(DT^2)^{-1}$ at $T=0$ is chosen to agree with a reasonable extrapolation of the experimental data

In Fig. 3 we show all of our results on $(KT)^{-1}$ versus T at low pressure. The results of three different experiments[22–24] are shown. In $(KT)^{-1}$, systematic errors in the temperature scale do not enter. Of course this quantity is plotted against T, but a few percent error in the T scale cannot have any substantial effect on our conclusions. It is likely that the principal systematic errors in $(KT)^{-1}$ arise from uncertainties in geometry. We feel that of the three experiments shown the last two[23,24] have the most well-determined geometry. Hence in interpolating between the data of Ref. 23 and the higher-T data, we have chosen the data of Ref. 24 to represent $(KT)^{-1}$ better at higher T. In regard to the

[19] H. R. Hart, Jr., and J. C. Wheatley, Phys. Rev. Letters 4, 3 (1960).

[20] E. L. Hahn, Phys. Rev. 80, 580 (1950).

[21] H. Y. Carr and E. M. Purcell, Phys. Rev. 94, 630 (1954).

[22] A. C. Anderson, G. L. Salinger, and J. C. Wheatley, Phys. Rev. Letters 6, 443 (1961).

[23] W. R. Abel, R. T. Johnson, J. C. Wheatley, and W. Zimmermann, Phys. Rev. Letters 18, 737 (1967).

[24] A. C. Anderson, J. I. Connolly, O. E. Vilches, and J. C. Wheatley, Phys. Rev. 147, 86 (1966).

data of Ref. 23, we find that there is a rather consistent picture for the data between 5 and 30 m°K but that for $T<5$ m°K the data points lie consistently above the extrapolation of the higher-T data of this experiment. These points are particularly susceptible to errors in the assumption that the Kelvin and magnetic temperatures are the same, and also to errors arising from lack of thermal equilibrium. Hence we feel that they should not be heavily weighted. The results shown on Fig. 3 strongly support the temperature dependence of Eq. (2) as predicted in Rice's theory. The dependence of $(KT)^{-1}$ on T given by the value of $\alpha_K\theta/\beta_K$ for 0.28 atm (Table II) is also shown on Fig. 3. The experimental value of $\alpha_K\theta/\beta_K$ is less than that derived from the theory using our experimental parameters.

In Fig. 4 are shown the only data available[24] on the low-temperature thermal conductivity of pure He^3 at low temperatures and higher pressures. The data do not extend to sufficiently low T to give the Rice theory an adequate test. On Fig. 4, we have extrapolated the data to $T=0$, using a slope corresponding to the value of $\alpha_K\theta/\beta_K$ for He^3 at 27.0 atm given in Table II. Careful

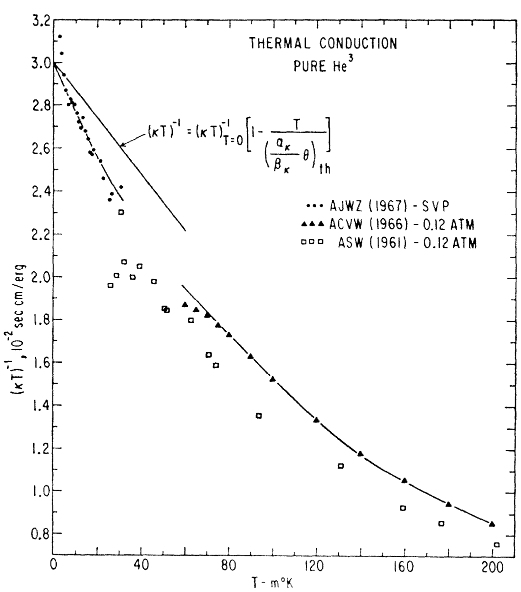

FIG. 3. Thermal conduction in pure He^3 at low pressure. AJWZ (1967) is Ref. 23; ACVW (1966) is Ref. 24; and ASW (1961) is Ref. 22. The line labeled by an equation shows the dependence on temperature of $(KT)^{-1}$ expected from Rice's theory if the value of $(KT)^{-1}$ at $T=0$ is chosen to agree with a reasonable extrapolation of the AJWZ (1961) data.

examination of the figure shows that the value of α_K given by the intercept is probably lower than that which would be observed in an experiment conducted to lower T. Hence it is again probable that in the event of the validity of Rice's theory the quantity $\alpha_K\theta/\beta_K$ will be less than that given in Table II.

Finally, in Fig. 5 we give data[2] on the sound attenuation in pure He^3 at low pressure and in the classical

TABLE III. Comparison of experimental and theoretical values for the characteristic temperature θ.

p (atm)	θ_{theory} (°K)	$\theta_{D\text{expt.}}$ (°K)	$\theta_{K\text{expt.}}$ (°K)
0.28	0.98	0–0.8	0.54
27.0	0.48	0.35	...

viscous region. The measured attenuation coefficients are divided by $8\pi^2\nu^2/3\rho c_1^3$, where ν is frequency, ρ is mass density, and c_1 is hydrodynamic sound velocity, in order that they represent viscosity. In the measurements, we measure not only a viscous attenuation, dependent on ν^2, but also a "geometrical" attenuation, due to imperfect alignment. We assume that the geometrical attenuation is independent of T, an assumption which should be valid as long as the sound wavelength is independent of T. The attenuation results shown in Fig. 5 already have some geometrical attenuation subtracted. According to Rice's theory, Eq. (3), the viscosity should be quite well represented by $\eta T^2=$ constant over a rather broad temperature range. The data shown in Fig. 5 for both frequencies conform quite well on the whole to this law, but the presence of the geometrical attenuation precludes any possibility of making, on the basis of our ultrasonic attenuation data, a careful study of the temperature dependence of the viscosity coefficient at higher temperatures. Other methods such as those of Betts, Keen, and Wilks[25] or of Hall and Thompson[26] would be more appropriate to test Rice's theory, though we feel that the experiments performed up to now are not sufficiently precise nor carried to a sufficiently low temperature to make a convincing case. The slopes of the curves in Fig. 5 yield a value of ηT^2 for pure He^3 at 0.32 atm of 2.1×10^{-6} poise $(K°)^2$. It

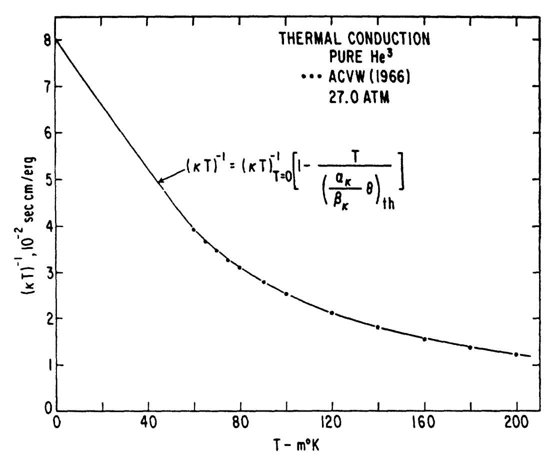

FIG. 4. Thermal conduction in pure He^3 at a pressure near the melting pressure. ACVW (1966) is Ref. 24. The extrapolation of $(KT)^{-1}$ to $T=0$ is by means of the Rice theory with $(KT)^{-1}$ at $T=0$ adjusted so that the data are correctly predicted.

[25] D. S. Betts, B. E. Keen, and J. Wilks, Proc. Roy. Soc. (London) **A289**, 34 (1965).
[26] H. E. Hall and K. Thompson (to be published).

TABLE IV. Comparison of the zero-temperature experimental limits for the quantities $(DT^2)^{-1}$, $(KT)^{-1}$, and $(\eta T^2)^{-1}$ with these quantities as calculated from the Rice theory and from the Landau theory using empirical data on heat capacity, sound velocity, and susceptibility.

p (atm)	$(DT^2)^{-1}$ – 10^6 sec/cm² (K°)² Expt.	Rice	Landau	$(KT)^{-1}$ – 10^{-2} sec cm/erg Expt.	Rice	Landau	$(\eta T^2)^{-1}$ – 10^6 cm²/dyne sec (K°)² Expt.	Rice	Landau
0.28	0.7	2	0.2	3	8	1.3	0.5	1.7	0.4
27.0	6	18	1.6	>8	25	5	...	4.0	1.5

is not possible to give an accurate estimate of the error in this value for ηT^2, though it seems unlikely that it could be in error by more than 15%.

Figures 1–4 may be used to estimate an empirical value for θ in the cases of spin diffusion and thermal conductivity. These values are given in Table III. The experimental values are less than the theoretical ones.

Not only the temperature dependence but also the limiting value at $T=0$ for $(DT^2)^{-1}$, $(KT)^{-1}$, and $(\eta T^2)^{-1}$ are given by Rice's theory as evaluated using our susceptibility and heat-capacity data. The transport coefficients may also be evaluated from measurements of these and other experimental properties of pure He^3 using the Landau Fermi liquid theory.[16] In Table IV, we list the values of these quantities as determined experimentally, as evaluated using the Rice theory, and as evaluated using the Landau theory. In all cases for which there are measurements, the Rice theory predicts values of $(DT^2)^{-1}$, $(KT)^{-1}$, and $(\eta T^2)^{-1}$ which are too high. However, Rice has strongly stressed[13,17] that his approximations do tend to overestimate the limiting low-temperature values for these quantities, as observed here. The calculation based on the Landau theory is also approximate. Although the absolute values of $(DT^2)^{-1}$, $(KT)^{-1}$, and $(\eta T^2)^{-1}$ are not accurately predicted by the theories, the effect of pressure is predicted rather well, as may be seen in Table V.

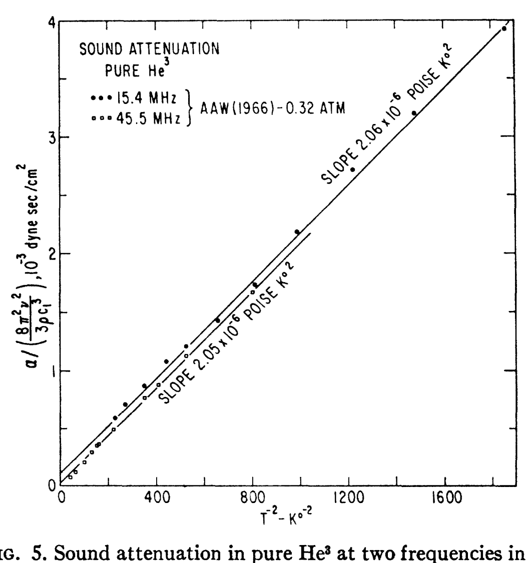

FIG. 5. Sound attenuation in pure He^3 at two frequencies in the hydrodynamic region. The attenuation coefficient α is divided by $8\pi^2\nu^2/3\rho c_1^3$, so that the ordinate represents a viscosity. The intercepts of the straight lines represent geometric attenuation.

Besides our measurements of the transport quantities and heat capacity, we have other qualitative information which may bear on the matter of spin fluctuations in pure He^3. This is concerned with heat transfer from cerium magnesium nitrate (CMN) to helium.[16,27,28]

Before our work on dilute solutions of He^3 in He^4, we had observed[16,27] an anomalously low thermal boundary resistance between pure liquid He^3 and a powder of CMN. In our first attempt[16] to rationalize this behavior without invoking any new mechanism for heat transfer from solid to liquid, we speculated that perhaps the anomalously low boundary resistance to the liquid was a result of a large area of contact produced by many cracks and fissures in the CMN which are made effective by the very high thermal conductivity of the He^3. Our subsequent attempt[29] to cool dilute solutions of He^3 in He^4, using the same CMN powder, showed a high effective thermal boundary resistance of uncertain temperature dependence though of magnitude not unreasonable on the basis of earlier measurements[30] of the thermal boundary resistance between solids and helium. Our most recent measurements of the thermal conductivity of dilute solutions of He^3 in He^4 [23] have shown that the thermal conductivity of these dilute solutions at a given temperature is rather similar to that for pure He^3. Hence our original hypothesis of an anomalously high effective area appears to be unwarranted. On the other hand, granted the validity of the physical picture of Doniach and Engelsberg,[10] of Amit *et al.*,[11] of Brenig

TABLE V. Comparison of experiment with the Rice and Landau theories for the effect of pressure on the limiting low-temperature values of $(DT^2)^{-1}$ and $(KT)^{-1}$.

	Expt.	Rice	Landau
$(DT^2)^{-1}$ (0.28 atm) / $(DT^2)^{-1}$ (27.0 atm)	0.12	0.11	0.12
$(KT)^{-1}$ (0.28 atm) / $(KT)^{-1}$(27.0 atm.)	<0.4	0.32	0.26

[27] W. R. Abel, A. C. Anderson, W. C. Black, and J. C. Wheatley, Phys. Rev. Letters **16**, 273 (1966).

[28] J. C. Wheatley, Ann. Acad. Sci. Fennicae Ser. A VI, No. **210**, 15 (1966).

[29] A. C. Anderson, W. R. Roach, R. J. Sarwinski, and J. C. Wheatley, Phys. Rev. Letters **16**, 263 (1966).

[30] A. C. Anderson, J. I. Connolly, and J. C. Wheatley, Phys. Rev. **135**, A910 (1964).

and Mikeska,[12] and of Rice[13] to describe pure He^3, spin fluctuations play a major role in the properties of pure He^3. They will not in the case of the dilute solutions. Hence it seems more reasonable to attribute the anomalously low thermal boundary resistance between CMN and He^3 to the special property of pure He^3 that spin fluctuations are important and to the special property of CMN (as opposed to copper, for example) that its heat capacity results from spin interactions. Energy may be transferred directly from the CMN spin system to the spin fluctuations in the He^3. Experiments to check this possibility by comparing directly the heat transfer from CMN to either pure He^3 or to He^3 in dilute solutions are particularly straightforward to perform in a device similar to one we have already described.[31] Work along these lines should give qualitatively interesting results and is contemplated.

IV. CONCLUSIONS

The recent results on thermal conductivity in low-pressure He^3 provide the most striking confirmation of the temperature dependence for a transport property suggested by Rice. Examination of the available experimental evidence for the thermal conductivity at higher pressure and also for the coefficients of spin diffusion and viscosity shows that the available data are inadequate to provide a convincing test of the Rice theory in regard to the very-low-temperature behavior of these transport properties. It is particularly important that new accurate measurements of the spin-diffusion coefficient be made at constant pressure and over a temperature range extending from 5 or 10 m°K to about 100 m°K.

Although Rice's theory predicts well the pressure dependence of the transport properties, the absolute values of the transport properties are predicted high. Rice[13] emphasized strongly that his approximations would lead to such an error.

[31] O. E. Vilches and J. C. Wheatley, Phys. Letters 24A, 440 (1967).

The spin-fluctuation theory of Doniach and Engelsberg[10] has not yet been modified to predict or to use as a parameter the velocity of sound in pure He^3. In view of the other successes of the theory, further theoretical work in which nonspin interactions are considered should be undertaken.

ACKNOWLEDGMENT

We wish to acknowledge a fruitful correspondence with M. J. Rice.

APPENDIX

Formulas[13,17] for the quantities α and β are as follows:

$$\alpha_D=\frac{3\pi W_D(0)\bar{I}^2\bar{Q}p_F^2}{4k_B\hbar T_F^3(1-\bar{I})^3}, \tag{A1}$$

$$\alpha_K=\frac{27}{8\pi}\frac{W_K(0)\bar{I}^2\bar{Q}\hbar^2}{k_B^2T_F^2p_F(1-\bar{I})^2}, \tag{A2}$$

and

$$\alpha_\eta=\frac{15\pi^3}{8}\frac{W_\eta(0)\bar{I}^2\bar{Q}^3\hbar^2}{T_F^2p_F^3(1-\bar{I})^2}. \tag{A3}$$

In these expressions one has

$$W_D(0)=J_{(2)}{}^0(1-\tfrac{1}{12}\bar{Q}^2), \tag{A4}$$

$$W_K(0)=J_{(4)}{}^0(1-\bar{Q}^2/18)+\tfrac{1}{9}\pi^2\bar{Q}^2J_{(2)}{}^0, \tag{A5}$$

and

$$W_\eta(0)=J_{(2)}{}^0(1-3\bar{Q}^2/20), \tag{A6}$$

where

$$J_{(n)}{}^0=n!Z(n), \tag{A7}$$

and $Z(n)$ is the Riemann Zeta function.

The ratios α/β are given by the following expressions:

$$\alpha_D/\beta_D=J_{(2)}{}^0(1-\tfrac{1}{12}\bar{Q}^2)/J_{(3)}{}^0, \tag{A8}$$

$$\alpha_K/\beta_K=[J_{(4)}{}^0(1-\bar{Q}^2/18)+\tfrac{1}{9}\pi^2\bar{Q}^2J_{(2)}{}^0]/J_{(5)}{}^0. \tag{A9}$$