Possibility of Second Sound in Turbulent Plasma*

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(Received 21 June 1967)

The propagation of energy density waves associated with the "gas" of acoustic plasma oscillations in a turbulent plasma is investigated. It is shown that a second-sound wave exists, and that it can grow exponentially under certain nonequilibrium conditions.

THE propagation of energy density waves, or second sound, has been observed in liquid He II [1] and solid He^4;[2] second sound has also been considered as a possible mode of propagation in piezoelectric materials following the onset of an acoustic wave instability.[3] Recently, Pines[4] has suggested that it may be likewise observed in turbulent gaseous plasmas under circumstances that the growing waves (acoustic waves, plasmons, etc.) collide more frequently with each other than with any other constituents of the plasma; a calculation of possible second-sound propagation which is based on the quasilinear theory has been carried out by Liperovskii and Tsytovich.[5]

In this paper we consider second-sound propagation in the quasistationary turbulent plasma formed when the electrons as a whole move with large drift velocity in a weakly ionized plasma. There then exists an appreciable enhancement over thermal background of a given group of acoustic waves; we use the new self-consistent theory of stationary turbulence[6] to show that in the acoustic wave "gas," an energy density wave can then propagate with a phase velocity s_2 which is somewhat slower than the velocity s of the unstable acoustic waves. This second sound may exhibit exponential growth under certain transient conditions; since the energy density couples directly to the electron drift velocity and temperature via the conservation laws of energy and momentum, a second-sound instability should be observed in the form of spontaneous current oscillations, a feature markedly different from the cases of ordinary density wave instabilities.

According to the self-consistent theory, a stationary state is brought about in the plasma by a balance between the emission of acoustic waves by the charged particles and their decay due to their mutual interactions, as may be clear from Eq. (8.6) in Ref. 6. We now define the energy density $\epsilon(k,\hat{k})$ contained in *both* $[\mathbf{k},\omega(\mathbf{k})]$ and $[-\mathbf{k},-\omega(\mathbf{k})]$ modes of the acoustic oscillations[7]; we thereby restrict the domain of $\mathbf{k}$ in a half space, $\mathbf{k}\cdot\mathbf{V}_d\geq 0$. The slowly varying space-time behavior of $\epsilon(k,\hat{k})$ is then governed by the following equation[8]:

$$\frac{\partial\epsilon(k,\hat{k})}{\partial t}+\nabla\cdot[s\hat{k}\epsilon(k,\hat{k})]$$
$$=\frac{m_-}{m_+\tau_-}\kappa T_-+\frac{m_-}{m_+\tau_-}\frac{\hat{k}\cdot\mathbf{V}_d-V_c}{s}\epsilon(k,\hat{k})$$
$$-\frac{sk}{4n^2(4\pi e)^2k_-^2}\epsilon(k,\hat{k})\left\{\int_{k_1}^{k}2kl^5\epsilon(l,\hat{k})dl+\int_{k}^{k_2}(k^2+l^2)l^4\epsilon(l,\hat{k})dl\right\}. \quad (1)$$

Here, $m_\pm$, $T_\pm$, and $\tau_\pm$ are the masses, temperatures, and relaxation times of ions and electrons, respectively; n is the average number density of the electrons; $\mathbf{V}_d$ and V_c are the drift velocity and its critical magnitude; $k_-=(4\pi ne^2/\kappa T_-)^{1/2}$ is the Debye wave number of the electrons; and $\hat{k}$ is the unit vector in the direction of the wave vector $\mathbf{k}$. The first term in the right-hand side of (1) represents the rate at which the acoustic waves are emitted by the random thermal motion of the electrons; the second term, the interaction of the waves with the charged particles; the last term, nonlinear coupling between the waves.

A detailed investigation[6] shows that when $\hat{k}\cdot\mathbf{V}_d-V_c>0$ the energy density spectrum in equilibrium exhibits a sharp peak around the lower wave number limit, $k_1\simeq 1/2s\tau_+$, and that the strength associated with this peak is proportional to $\hat{k}\cdot\mathbf{V}_d-V_c$; if we expand the equilibrium solution with respect to the plasma param-

* Supported in part by the U. S. Army Research Office (Durham) under Grant No. DA-31-124-ARO(D)-114.

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[1] K. R. Atkins, *Liquid Helium* (Cambridge University Press, Cambridge, England, 1959).

[2] C. C. Ackerman, B. Bertman, H. A. Fairbank, and R. A. Guyer, Phys. Rev. Letters **16**, 789 (1966).

[3] H. Kroger, E. W. Prohofsky, and R. W. Damon, Phys. Rev. Letters **11**, 246 (1963); H. Kroger, E. W. Prohofsky, and H. R. Carleton, *ibid.* **12**, 555 (1964).

[4] D. Pines (private communication).

[5] V. A. Liperovskii and V. N. Tsytovich, Zh. Tekh. Fiz. **36**, 575 (1966) [English transl.: Soviet Phys.—Tech. Phys. **11**, 432 (1966)].

[6] S. Ichimaru and T. Nakano, preceding paper, Phys. Rev. **165**, 231 (1968).

[7] See footnote 21 of Ref. 6.

[8] For a collisionless plasma the first two terms in the right-hand side should be replaced by $(\pi m_-/2m_+)^{1/2}(sk)(\kappa T_-)-(\pi m_-/2m_+)^{1/2}\times[kV_c(k)-\mathbf{k}\cdot\mathbf{V}_d]\epsilon(k,\hat{k})$, where $V_c(k)$ represents the boundary curve between the growing and damped acoustic oscillations. For the validity of Eq. (1), the frequency and wave number associated with the space-time variation of $\epsilon(k,\hat{k})$ must be smaller than those frequencies and wave numbers which contain the bulk of the energy of the acoustic oscillations.

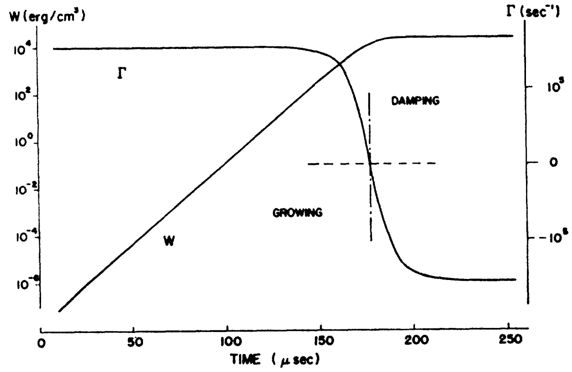

FIG. 1. Time development of the total energy density W in the acoustic plasma oscillations and the resulting change of the growth rate Γ for the associated second sound. For a neon plasma at 0.5 mm Hg, the following parameters are used: $n=10^{11}$ cm^{-3}, $T_-=50\ T_+=2\times10^4$ °K, $\tau_-=1.1\times10^{-9}$ sec, $\tau_+=0.79\times10^{-6}$ sec, $s=2.7\times10^5$ cm/sec, $V_c=1.45\times10^7$ cm/sec, $V_d=1.7\times10^7$ cm/sec.

eter, $g=k_-^3/n$, assuming $g\ll1$, we find that the leading contributions are

$$\epsilon(k,\hat{k})\cong\left[\epsilon_0 k_1\frac{\hat{k}\cdot\mathbf{V}_d-V_c}{s}\delta(k-k_1)+\epsilon_1(k)\right]\times\theta(\hat{k}\cdot\mathbf{V}_d-V_c),\quad(2)$$

where $\theta(x)$ is the unit step function; $\epsilon_1(k)$ is a slowly varying function almost independent of V_d; we estimate $\epsilon_1(k)/\epsilon_0\sim g^{1/2}$.

We now integrate (1) with respect to $\mathbf{k}$ from k_1 to k_2, an upper limit for the acoustic spectrum, and over the solid angle confined within the cone $\hat{k}\cdot\mathbf{V}_d-V_c>0$, assuming that the spectral distribution takes the form (2). We thus find an equation for the total energy density W of the acoustic fluctuations[9]:

$$\frac{\partial W}{\partial t}+\nabla\cdot[\hat{z}s_2W]=\frac{1}{8\pi^2}\left(\frac{2m_-}{3m_+\tau_-}\right)\frac{V_d-V_c}{V_d}(k_2^3-k_1^3)\kappa T_-+\frac{2m_-}{3m_+\tau_-}\frac{V_d-V_c}{s}W-\frac{2m_-}{3m_+\tau_-}\frac{V_d}{V_d-V_c}\frac{W^2}{W_0},\quad(3)$$

where $\hat{z}$ is the unit vector in the direction of $\mathbf{V}_d$; $W_0=n^2(4\pi e)^2m_-k_-^2/4\pi^2m_+s\tau_-k_1^5$; and

$$s_2=s(2V_d+V_c)/3V_d\quad(4)$$

will be seen to be the velocity of the second-sound propagation. Equation (3) may be looked upon as a hydrodynamic equation for an additional constituent of a plasma, the large-amplitude acoustic plasma oscillations.

In order to investigate the wave property of the system described by (3), let us consider a transient situation that an external electric field is suddenly applied to the plasma in such a way that the resulting drift velocity exceeds the critical value. The increase of the drift velocity may take place within a small time scale of order τ_-, while W will be built up rather slowly with a characteristic time of order $(m_+\tau_-/m_-)$. We may therefore separate W into two parts: a uniform, slowly varying part, $W(t)$; and an oscillatory part, $W'\exp[i(\mathbf{K}\cdot\mathbf{r}-\Omega t)]$. $W(t)$ develops in time according to (3), where the second term on the left-hand side vanishes because $W(t)$ represents a uniform distribution; its asymptotic solution at $t\to\infty$ is[10]

$$W(\infty)=W_0(V_d-V_c)^2/sV_d,\quad(5)$$

corresponding to a fully developed turbulent stationary state.

If we linearize Eq. (3) with respect to the oscillatory part, W', we obtain the dispersion relation for second sound; on writing $\Omega=\Omega_K+i\Gamma$, we find

$$\Omega_K=s_2\mathbf{K}\cdot\hat{z},\quad(6)$$

$$\Gamma=\frac{2m_-}{3m_+\tau_-}\left[1-\frac{2V_ds}{(V_d-V_c)^2}\frac{W(t)}{W_0}\right]\frac{V_d-V_c}{s}.\quad(7)$$

It follows that until $W(t)$ reaches half of its asymptotic value $W(\infty)$, Γ takes on a positive value, corresponding to a growing second-sound wave; when $W(t)$ exceeds that value, Γ becomes negative; in particular, the second sound is stable after the plasma reaches a turbulent stationary state.

As a numerical example, we have computed the time development of the growth and damping rate for second-sound propagation in a neon plasma, together with the total energy density in the acoustic oscillations[11]; our results are shown in Fig. 1. We notice that the initial period during which the second sound can grow persists for about 180 μsec in this example; it appears quite feasible to observe the second-sound propagation and the associated instability experimentally in a plasma.

The author wishes to thank Professor D. Pines and Professor K. Yamada for valuable discussions. He is also grateful to Professor M. Raether for calling his attention to Ref. 5.

[9] Equation (3) is valid only when $(V_d-V_c)/s\gg\epsilon_1/\epsilon_0\sim g^{1/2}$.

[10] In a fully developed turbulence, the first term in the right-hand side of (3) is a higher-order contribution with respect to g and thus negligible.

[11] In this computation, we have ignored for simplicity the effects of rising electron temperature resulting from the accumulation of W. The increase in electron temperature has a stabilizing effect on the acoustic plasma oscillations; it increases V_c, and acts to decrease V_d if the external electric field is kept unchanged. The essential feature of the second sound instability will not, however, be affected significantly by inclusion of these effects, since the instability is associated with the initial stage rather than the fully developed final stage of plasma turbulence.