of papers I and II, have a complete set of coordinates.

Fortunately, none of our arguments concerning the dynamical triviality of the theory is affected by this difficulty. In particular, the expression (17) for $\theta^{\mu\nu}$ in terms of the current remains valid. Moreover, if we choose to think of our model as just a model, and not *the* Thirring model, the above comments are, of course, not germane.

The second point has to do with the Schwinger term. In papers I and II we were never forced to introduce such a term into the current algebra. It may well be that this was due to the fact that we never really tried to solve the dynamics. Recall that in the present model the Schwinger term was forced upon us when we found that without it the energy would not be positive. We have already stressed the point that the requirements of Lorentz invariance impose severe, and useful, constraints on the commutation relations between the various operators. Here we see that the requirement of positivity of the energy spectrum likewise imposes important constraints.

One should not jump to the conclusion, however, that the present results nullify the content of papers I and II where Schwinger terms were ignored. As long as we were talking about simple models such as nonrelativistic quantum mechanics or relativistic free fields, our previous results are formally correct. In the case of interacting relativistic theories, one might have to modify the assumed commutation relations in order to obtain a positive energy spectrum. However, it is not obvious how one is to know whether or not this is necessary until at least a partial solution of the dynamics is available.

In conclusion, it seems reasonable to say that, basically, we have learned two things: (i) why the Thirring model is solvable (essentially because the components of $\theta_{\mu\nu}$ form a closed algebra) and (ii) that positivity of the energy spectrum places important and, in the present case, explicit restrictions on a theory written in terms of currents.

PHYSICAL REVIEW VOLUME 165, NUMBER 5 25 JANUARY 1968

Poles and Resonances in η Production*

S. R. Deans† and W. G. Holladay

Department of Physics and Astronomy, Vanderbilt University, Nashville, Tennessee

(Received 11 September 1967)

Poles and resonances are used to find χ^2 fits to the data up to several hundred MeV above threshold for the processes $\pi^-+p\to n+\eta$ and $\gamma+p\to p+\eta$. Below center-of-mass energy $W=1700$ MeV, a consistent picture of these reactions is achieved with the following ingredients: a nucleon pole and $P_{11}(1400)$, $S_{11}(1570)$, and $D_{13}(1512)$ resonances for production by pions; a nucleon pole, a vector meson pole, and $S_{11}(1570)$ and $D_{13}(1512)$ resonances for photoproduction. Some $F_{15}(1688)$ also improves the fits. The value of the η-nucleon coupling constant, $g_\eta^2/4\pi$ is $\lesssim 0.002$ in both processes. Implications of these results for quark models and $SU(3)$ symmetry are discussed. Above $W=1700$, possible additional resonances are considered in η production by pions.

I. INTRODUCTION

Cross-section data are available on the process $\pi^-+p\to\eta+n$ up to several hundred MeV above threshold (pion laboratory kinetic energy $T_\pi=562$ MeV). Bulos *et al.*,[1] who presented results at nine energies up to $T_\pi=1151$ MeV (center-of-mass energy $W=1822$ MeV), found that the total cross section for the process $\pi^-+p\to\eta+n$ ($\eta\to 2\gamma$) rises rapidly to a peak of 1 mb at $T_\pi=650$ MeV and then begins a sharp but somewhat more moderate decline, going below 0.5 mb at $T_\pi=1151$ MeV. Bulos *et al.*[1] find little evidence for anisotropy in the angular distribution of the produced η's below $T_\pi=1003$ MeV, but above this energy a term at least linear in the cosine of the production angle is needed.

Richards *et al.*,[2] who studied the same reaction as Bulos *et al.*[1] at seven energies up to $T_\pi=1300$ MeV, confirmed the structure and magnitude of the total cross section for η production but with improved statistics found evidence for anisotropy in the angular distribution beginning at $T_\pi=655$ MeV.

Additional data on η production from the process $\pi^-+p\to\eta+n$ ($\eta\to$ all neutrals) are available from the experiments of Jones *et al.*[3] ($T_\pi\leq 589$ MeV) and of

* Supported in part by the National Science Foundation.

† Present address: Department of Physics, University of South Florida, Tampa, Fla.

[1] F. Bulos, R. E. Lanou, A. E. Pifer, A. M. Shapiro, M. Widgoff, R. Panvini, A. E. Brenner, C. A. Bordner, M. E. Law, E. E. Ronat, K. Strauch, J. J. Szymanski, P. Bastien, B. B. Brabson, Y. Eisenberg, B. T. Feld, V. K. Fischer, I. A. Pless, L. Rosenson, R. K. Yamamoto, G. Calvelli, L. Guerriero, G. A. Salandin, A. Tomasin, L. Ventura, C. Voci, and F. Waldner, Phys. Rev. Letters **13**, 486 (1964).

[2] W. Bruce Richards, Charles B. Chiu, R. D. Eandi, A. Carl Helmholz, R. W. Kenney, B. J. Moyer, J. A. Poirier, R. J. Cence, V. Z. Peterson, N. K. Sehgal, and V. J. Stenger, Phys. Rev. Letters **16**, 1221 (1966).

[3] W. G. Jones, D. M. Binnie, A. Duane, J. P. Horsey, D. C. Mason, J. A. Newth, I. U. Rahman, J. Walters, N. Horwitz, and P. Palit, Phys. Letters **23**, 597 (1966).

Hyman *et al.*[4] ($T_\pi \leq 677$ MeV). Earlier experiments[5] have also provided data on η production by pions.

Several authors[6] have studied the data on η production by pions with particular emphasis on the effect of the η channel on the S_{11} pion-nucleon scattering phase shift. Others have concentrated on fitting the η-production data itself with contributions from various poles and resonances. Altarelli, Bucella, and Gatto[7] find that a reasonable fit to the total cross section can be obtained with a nucleon pole and the $D_{13}(1512)$ resonance. The value of the η-nucleon coupling constant $g_\eta^2/4\pi$ resulting from their work is approximately 0.008. Srinivasan and Achuthan[8] have made additional calculations with the same model as Altarelli *et al.*[7] Their results for the differential cross section indicate that the model must be made more complex before agreement with the data can be obtained. Uchiyama-Campbell and Logan[9] analyzed the preliminary data from Ref. 2 fitting both the total and differential cross section (at $T_\pi = 592$, 655, 704, and 875 MeV) with three resonances, $S_{11}(1560)$, $P_{11}(1503)$, and $D_{13}(1531)$. Minami[10] also studied the preliminary data from Ref. 2 and showed that the effect of the D_{13} resonance could be comparable to or larger than that of the S_{11} in the neighborhood of the peak in the total cross section. Moss[11] has calculated the differential cross section with the nucleon pole and three resonances $S_{11}(1567)$, $P_{11}(1430)$, and $D_{13}(1512)$, using field theory with propagators modified by finite resonance widths, and he achieves excellent fits at $T_\pi = 592$, 655, and 704 MeV. The agreement is not so good at $T_\pi = 875$ MeV, as Uchiyama-Campbell and Logan[9] also found.

Our approach is similar in spirit to that contained in the papers discussed in the preceding paragraph. In this paper we go beyond the previous calculations in a number of respects. (1) We consider the effects of all known $T=\frac{1}{2}$ pion nucleon resonances with mass less than 2 BeV. (2) We assess quantitatively the importance of omitting and including various resonances. (3) We consider the data of Richards *et al.*[2] at higher energies, $T_\pi = 975$, 1117, and 1300 MeV. (4) We study pion production of η's jointly with their photoproduction to try to arrive at a consistent picture of both processes.

II. KINEMATICS

The essential kinematics of the process $\gamma + p \to \eta + p$ have been given in an earlier paper[12] and will not be repeated here. For the process $\pi^- + p \to \eta + n$ we shall include only those results which are essential in establishing notation. The four-momenta (three-momenta, energy, and mass) of the π, p, η, and n are, respectively, $k(\mathbf{k}, E_k, m_\pi)$, $p_1(\mathbf{p}_1, E_1, M_1)$, $q(\mathbf{q}, E_q, m)$, and $p_2(\mathbf{p}_2, E_2, M_2)$ in the center-of-mass (c.m.) system. The total c.m. energy is W, and T_π is the laboratory kinetic energy of the pion.

The differential cross section is given by

$$d\sigma/d\Omega = |a|^2 + |b|^2 \sin^2\Theta\,, \qquad (1)$$

where a and b are defined as in Ref. 13, and Θ is the c.m. production angle of the η,

$$\cos\Theta = \frac{\mathbf{k}\cdot\mathbf{q}}{|\mathbf{k}||\mathbf{q}|} = x\,. \qquad (2)$$

The polarization $\mathcal{P}$ is taken to be positive along the direction of

$$\hat{n} = \frac{\mathbf{k}\times\mathbf{q}}{|\mathbf{k}\times\mathbf{q}|}, \qquad (3)$$

and is given by

$$\mathcal{P}\frac{d\sigma}{d\Omega} = -2\,\mathrm{Im}(ab^*)\sin\Theta\,. \qquad (4)$$

The a and b amplitudes can be expanded in partial-wave amplitudes[13] $f_{l\pm}$, with orbital angular momentum l and total angular momentum $J = l \pm \frac{1}{2}$. We approximate a resonating partial wave by the Breit-Wigner formula

$$f_{l\pm} = (\sqrt{\tfrac{2}{3}})\frac{1}{2|\mathbf{k}|}\frac{(\Gamma_l{}^\pi \Gamma_l{}^\eta)^{1/2}}{W_r - W - i\Gamma/2}, \qquad (5)$$

where W_r is the mass of the resonance and $\sqrt{\frac{2}{3}}$ is an isotopic-spin factor. The partial widths are given by[14]

$$\Gamma_l{}^\pi = 2|\mathbf{k}|R v_l(|\mathbf{k}|R)\gamma_l{}^\pi \qquad (6)$$

---

[4] E. Hyman, W. Lee, J. Peoples, J. Schiff, C. Schultz, and S. Stein, Phys. Rev. **165**, 1437 (1968).

[5] M. Chretien, F. Bulos, H. R. Crouch, R. E. Lanou, J. T. Massimo, A. M. Shapiro, J. A. Averel, C. A. Bordner, A. E. Brenner, A. R. Firth, M. E. Law, E. E. Ronat, K. Strauch, J. C. Street, J. J. Szymanski, A. Weinberg, B. Nelson, I. A. Pless, L. Rosenson, G. A. Salandin, R. K. Yamamoto, L. Guerriero, and F. Waldner, Phys. Rev. Letters **9**, 127 (1962); V. V. Barmin, A. G. Dolgolenko, Yu. S. Krestnikov, A. G. Meshkovskii, and V. A. Shebanov, Zh. Eksperim. i Teor. Fiz. **46**, 142 (1964) [English transl.: Soviet Phys.—JETP **19**, 102 (1964); M. Meer, R. Strand, R. Kraemer, L. Madansky, M. Nussbaum, A. Pevsner, C. Richardson, T. Toohig, M. Bloch, S. Orenstein, and T. Fields, in *Proceedings of the 1962 Annual International Conference on High-Energy Nuclear Physics at CERN*, edited by J. Prentki (CERN, Geneva, 1962), p. 103; E. Pauli, A. Muller, R. Barloutaud, L. Cardin, J. Meyer, M. Beneventano, G. Gialanella, L. Paoluzi, and R. Finzi, in *Proceedings of the Sienna International Conference on Elementary Particles and High-Energy Physics, 1963*, edited by G. Bernardini and G. P. Puppi (Societa Italiana di Fisica, Bologna, 1963), Vol. 1, p. 92; T. Toohig, R. Kraemer, L. Madansky, M. Meer, N. Nussbaum, A. Pevsner, C. Richardson, R. Strand, and M. Bloch, in *Proceedings of the 1962 Annual International Conference on High-Energy Nuclear Physics at CERN*, edited by J. Prentki (CERN, Geneva, 1962), p. 99.

[6] F. Uchiyama-Campbell, Phys. Letters **18**, 189 (1965); A. W. Hendry and R. G. Moorhouse, *ibid.* **18**, 171 (1965); P. N. Dobson, Phys. Rev. **146**, 1022 (1966); J. S. Ball, *ibid.* **149**, 1191 (1966).

[7] G. Altarelli, F. Buccella, and R. Gatto, Nuovo Cimento **35**, 331 (1965).

[8] S. K. Srinivasan and P. Achuthan, Nuovo Cimento **48**, 1124 (1967).

[9] F. Uchiyama-Campbell and R. K. Logan, Phys. Rev. **149**, 1220 (1966).

[10] S. Minami, Phys. Rev. **147**, 1123 (1966).

[11] T. A. Moss (to be published).

[12] S. R. Deans and W. G. Holladay, Phys. Rev. **161**, 1466 (1967).

[13] J. E. Rush and W. G. Holladay, Phys. Rev. **148**, 1444 (1966).

[14] J. M. Blatt and V. F. Weiskopf, *Theoretical Nuclear Physics* (John Wiley & Sons, Inc., New York, 1952).

TABLE I. Contributions of resonances to the a and b amplitudes.

Resonance, Γ_r (MeV)	Amplitude a	b
$S_{11}(1570)$, 130	f_{0+}	0
$S_{11}'(1700)$, 240	f_{0+}'	0
$P_{11}(1400)$, 210	$x\,f_{1-}$	f_{1-}
$D_{13}(1518)$, 100	$(3x^2-1)\,f_{2-}$	$3x\,f_{2-}$
$D_{13}'(1700)$, 25	$(3x^2-1)\,f_{2-}'$	$3x\,f_{2-}'$
$D_{15}(1688)$, 100	$\frac{3}{2}(3x^2-1)\,f_{2+}$	$-3x\,f_{2+}$
$F_{15}(1688)$, 100	$\frac{3}{2}(5x^3-3x)\,f_{3-}$	$\frac{3}{2}(5x^2-1)\,f_{3-}$
$G_{17}(2190)$, 200	$\frac{1}{2}(35x^4-30x^2+3)\,f_{4-}$	$\frac{1}{2}(35x^3-15x)\,f_{4-}$

and

$$\Gamma_l^\eta=2|\mathbf{q}|Rv_l(|\mathbf{q}|R)\gamma_l^\eta, \qquad (7)$$

and the total width Γ is given in Ref. 12. For convenience we define

$$(\sqrt{\tfrac{2}{3}})(\gamma_l^\pi\gamma_l^\eta)^{1/2}=\gamma(L_{2T2J}) \qquad (8)$$

which are adjustable parameters in units of MeV that can be either positive or negative. The interaction radius R is taken to be 1F.

III. POLES AND RESONANCES

The poles and resonances that presumably make contributions in the process $\gamma+p\to\eta+p$ have been discussed in Ref. 12. For the process $\pi^-+p\to\eta+n$, it appears that perhaps the only important pole contribution to the production amplitudes at low energy is the nucleon pole. Vector-meson exchange is forbidden by charge conservation in the case of ω and ϕ and by G-parity conservation in the case of ρ. The exchange of a pseudoscalar particle is ruled out since the vertex with three pseudoscalar particles vanishes. A_2 exchange is possible, but we shall neglect it in the present calculations both because of its rather high mass and since it would presumably give essentially a constant background which we can obtain by the nucleon pole.

There are at least six $T=\frac{1}{2}$ resonances that may contribute to the process $\pi^-+p\to\eta+n$ for c.m. energies up to $W\simeq1700$ MeV. They have been discussed in Ref. 12. Table I shows these resonance contributions to the a and b amplitudes. We also show two other possible resonance contributions which are considered in a later section. The various partial-wave amplitudes which appear in Table I are found by use of Eq. (5)–(8); explicitly,

$$f_{l\pm}=T[v_l(|\mathbf{k}|R)v_l(|\mathbf{q}|R)]^{1/2}\gamma(L_{2T2J}), \qquad (9)$$

where,

$$T=\frac{R}{W_r-W-i\Gamma/2}\left(\frac{|\mathbf{q}|}{|\mathbf{k}|}\right)^{1/2}. \qquad (10)$$

The barrier-penetration factors v_l are listed in Ref. 14 for $l=0$, 1, 2, 3, and v_4 is given by

$$v_4(z)=z^8/(11\,025+1575z^2+135z^4+10z^6+z^8). \qquad (11)$$

The nucleon pole contributions to the A and B amplitudes defined in Ref. 15 are

$$A=\frac{M_1-M_2}{2}\frac{\sqrt{2}g_\pi g_\eta}{M_2^2-s}+\frac{M_1-M_2}{2}\frac{\sqrt{2}g_\pi g_\eta}{M_1^2-u}, \qquad (12)$$

$$B=\frac{\sqrt{2}g_\pi g_\eta}{M_2^2-s}-\frac{\sqrt{2}g_\pi g_\eta}{M_1^2-u}. \qquad (13)$$

The pole contributions to the a and b amplitudes are found by

$$a=G+H\cos\Theta, \qquad (14)$$

$$b=H, \qquad (15)$$

where

$$G=\frac{1}{8\pi W}\left(\frac{|\mathbf{q}|}{|\mathbf{k}|}\right)^{1/2}[(E_1+M_1)(E_2+M_2)]^{1/2}\times\left[A+\left(\frac{M_1+M_2}{2}-W\right)B\right], \qquad (16)$$

and

$$H=\frac{-1}{8\pi W}\left(\frac{|\mathbf{q}|}{|\mathbf{k}|}\right)^{1/2}[(E_1-M_1)(E_2-M_2)]^{1/2}\times\left[A+\left(\frac{M_1+M_2}{2}+W\right)B\right]. \qquad (17)$$

We have presented the equations which are needed to calculate the differential cross section and polarization for η production in the process $\pi^-+p\to\eta+n$. Various combinations of resonances and pole terms can be considered. In the following sections we compare certain pole and resonance combinations with the data.

IV. TREATMENT OF THE DATA

In view of the different angular distributions reported in Refs. 1 and 2, there is no point in fitting both sets of data, and we have chosen to use only the data from Ref. 2. We have not used the data from Refs. 3 and 4 in the fitting, since these data (all in the backward direction) appear to be, for the most part, in agreement with the data which we have used.

Richards *et al.*[2] observe only the 2γ decay mode of the η and thus they give a "partial" differential cross section which we convert to a differential cross section by use of the branching ratio

$$R_{\gamma\gamma}=\frac{\eta\to2\gamma}{\eta\to\text{all modes}}. \qquad (18)$$

In the fitting we have used only $R_{\gamma\gamma}=0.38$; however, if another ratio is desired, all that needs to be done is to make an appropriate change of scale for $d\sigma/d\Omega$. One

[15] W. G. Holladay, Phys. Rev. 138, B1348 (1965).

TABLE II. Values of the parameters for η production by pions. These are the results for $T_\pi \leq 875$ MeV. The units of $\gamma(L_{2T2J})$ are MeV.

Parameter omitted	$g_\eta/(4\pi)^{1/2}$	$\gamma(F_{15})$	$\gamma(D_{15})$	$\gamma(D_{13})$	$\gamma(P_{11})$	$\gamma(S_{11}')$	$\gamma(S_{11})$	χ^2	χ^2/N
None (Model A)	0.101	−8.84	0.02	14.33	16.47	2.80	9.80	31.0	0.94
g_η		−5.26	0.01	8.36	12.36	3.44	12.12	58.7	1.17
$\gamma(F_{15})$	0.111		0.01	17.44	16.37	4.02	8.46	42.7	1.13
$\gamma(D_{15})$	0.102	−9.03		14.59	16.44	2.68	9.76	31.0	0.91
$\gamma(D_{13})$	0.079	−11.21	0.03		24.52	2.94	10.40	68.5	2.02
$\gamma(P_{11})$	0.097	−8.45	0.01	12.15		3.94	10.00	69.1	2.04
$\gamma(S_{11}')$	0.115	−11.34	0.01	14.85	16.74		10.40	31.6	0.93
$\gamma(S_{11})$	0.066	−2.29	0.77	10.60	52.92	4.32		167.7	4.93

TABLE III. Values of the parameters for η production by pions. These are the results for $T_\pi \leq 975$ MeV. The units of $\gamma(L_{2T2J})$ are MeV.

Parameter omitted	$g_\eta/(4\pi)^{1/2}$	$\gamma(F_{15})$	$\gamma(D_{15})$	$\gamma(D_{13})$	$\gamma(P_{11})$	$\gamma(S_{11}')$	$\gamma(S_{11})$	χ^2	χ^2/N
None (Model B)	0.050	−3.53	−0.88	10.47	23.46	6.73	9.58	80	1.86
g_η		−3.03	−1.04	8.48	19.37	5.60	10.94	96	2.18
$\gamma(F_{15})$	0.051		−1.13	11.52	22.08	6.72	9.58	89	2.02
$\gamma(D_{15})$	0.055	−4.36		8.65	25.85	6.85	9.39	81	1.84
$\gamma(D_{13})$	0.042	−5.56	−0.34		28.32	6.22	9.73	117	2.66
$\gamma(P_{11})$	0.105	−8.63	0.60	−18.77		5.68	7.37	147	3.34
$\gamma(S_{11}')$	0.057	−4.53	−1.13	11.92	26.19		11.59	119	2.70
$\gamma(S_{11})$	0.064	−1.59	−0.10	13.88	44.28	7.18		231	5.25

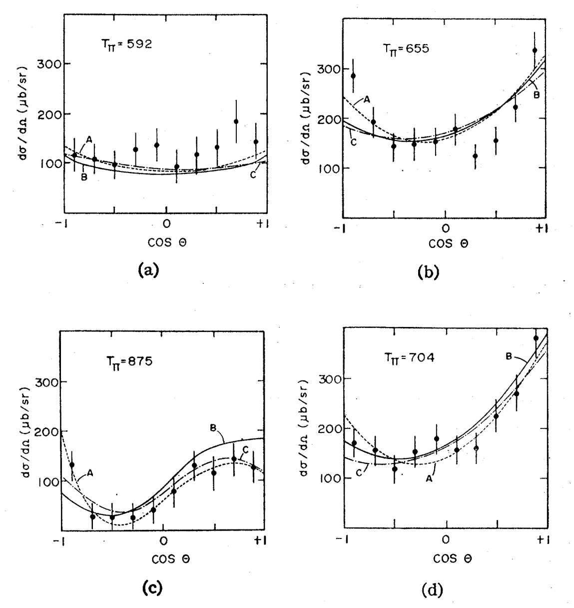
other adjustment of the data is made. The data shown in Fig. 3 of the paper by Richards *et al.*[2] represent the angular distribution of the bisector of the angle between the two γ rays. Richards *et al.*[2] show a best fit to the bisector distribution (dashed line) and a calculated η differential cross section (solid line). In order to obtain a differential cross-section datum point we shift each bisector point by the distance between the dashed and solid lines. One can imagine that the bisector points are rigidly attached to their best-fit curve (dashed line) and then the dashed line is moved vertically so as to coincide with the calculated differential cross section (solid curve). We take these shifted values of the data to be the actual differential-cross-section data for $\pi^- + p \rightarrow \eta + n$. As it turns out, this correction is not very large since the dashed and solid curves are not widely separated.

V. COMPARISON WITH EXPERIMENT

In making a comparison with the data the parameters g_η and $\gamma(L_{2T\,2J})$ are allowed to vary until χ^2 is minimized, where

$$\chi^2 = \sum_{\text{data}} \left(\frac{d\sigma/d\Omega(\text{theory}) - d\sigma/d\Omega(\text{experiment})}{\text{experimental error}} \right)^2 .$$

We also calculate χ^2/N where N is the number of data points minus the number of adjustable parameters. Tables II (and III) show the results obtained by optimizing χ^2 for the data through $T_\pi = 875$ MeV and $T_\pi = 975$ MeV. We shall refer to the first row of Table II



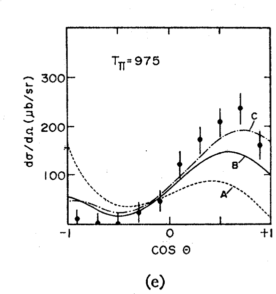
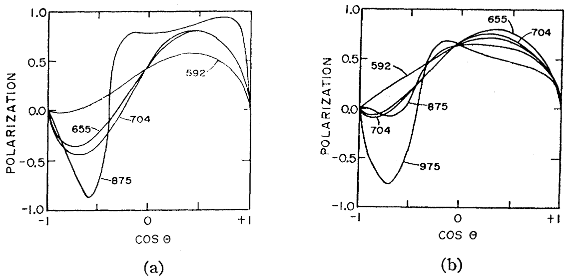


FIG. 1. (a) The differential cross section for models A, B, and C at $T_\pi = 592$ MeV. The data in Figs. (a)–(e) are from Ref. 2. (b) The differential cross section for models A, B, and C at $T_\pi = 655$ MeV. (c) The differential cross section for models A, B, and C at $T_\pi = 704$ MeV. (d) The differential cross section for models A, B, and C at $T_\pi = 875$ MeV. (e) The differential cross section for models A, B, and C at $T_\pi = 975$ MeV.



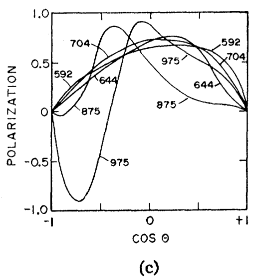


FIG. 2. (a) Polarization in the direction $\mathbf{k}\times\mathbf{q}$ for model A as a function of $\cos\theta$ at four energies. (b) Polarization in the direction $\mathbf{k}\times\mathbf{q}$ for model B as a function of $\cos\theta$ at five energies. (c) Polarization in the direction $\mathbf{k}\times\mathbf{q}$ for model C as a function of $\cos\theta$ at five energies.

as model A and the first row of Table III as model B. We treated all seven parameters as variables in these models. In each of the following rows we excluded one parameter. This was done to assess the relative importance of the various resonances and the pole. Figures 1(a) through 1(e) show models A, B, and C along with the adjusted differential-cross-section data from Ref. 2. Figures 2(a), (b), and (c) show the polarization curves for various energies. The total cross-section curves are shown in Fig. 3.

There are several important conclusions that one can deduce from these results. We were unable to obtain reasonable fits with $g_\eta^2/4\pi$ significantly larger than 0.01. Apparently the P_{11}, S_{11}, and D_{13} resonances are essential (see Tables II and III), and the F_{15} seems to be of some importance at $T_\pi=875$ and 975 MeV. If the F_{15} is omitted at these energies, the trend is not right in the forward direction. Slight improvement could be obtained by allowing the masses and widths of the resonances to undergo slight changes. For example, one might lower the mass of the $S_{11}(1570)$ in order to obtain a better fit to the 592-MeV data. It is not likely that any new information would be obtained by the additional complication of varying the masses and widths of the resonances so we have held them constant.[16] The total cross sections for models A and B do not fit both the 875- and 975-MeV points. The implication here is that the model must be made even more complex.

In an attempt to fit the data at both 875 and 975 MeV as well as at the lower energies, we added another D_{13} resonance (which we denote by D_{13}'). The best fit was obtained for $W_r=1700$ MeV and $\Gamma_r=25$ MeV.

TABLE IV. Model C parameters. The units of $\gamma(L_{2T2J})$ are MeV.

Parameters	Model C
$g_\eta/(4\pi)^{1/2}$	0.077
$\gamma(F_{15})$	−3.59
$\gamma(D_{15})$	−0.71
$\gamma(D_{13})$	8.13
$\gamma(P_{11})$	33.22
$\gamma(S_{11}')$	4.55
$\gamma(S_{11})$	8.83
$\gamma(D_{13}')$	1.54
χ^2	61.7
χ^2/N	1.47

There is very little experimental evidence for such a resonance; however, it is not ruled out,[17] and in the Dalitz quark model[18] such a resonance could be given a $\{8\}^4P_J$ classification. In Table IV we list the values of the parameters for the case where the D_{13}' is included. This model is denoted by model C in the figures. The total cross section for this model (Fig. 3) has a second peak which occurs between 875 and 975 MeV. The actual existence of such a peak could be tested experimentally. The three models A, B, and C predict somewhat different values for the polarization of the recoil nucleon as Figs. 2(a), (b), and (c) show.

VI. η PHOTOPRODUCTION

It is possible to obtain a consistent picture of η production by both pions and photons and to incorporate certain quark-model and unitary-symmetry predictions. Moorhouse[17] has shown that within the framework of a nonrelativistic quark model it is not possible for the D_{15}, D_{13}', or S_{11}' to be photoproduced, if these resonances belong to an $\{8\}\,^4P_J$. The available η-photoproduction data (used in Ref. 12) are insensitive to the presence or absence of these resonances as well as the P_{11}. The absence of the P_{11} is consistent with its being a member of a $\{\overline{10}\}$ representation of $SU(3)$.[19] Although the existence of a low-lying $\{\overline{10}\}$ representation and the

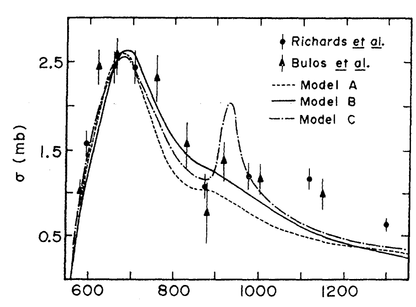


FIG. 3. The total cross section for models A, B, and C.

[16] A. H. Rosenfeld, A. Barbaro-Galtieri, W. J. Podolsky, L. R. Price, P. Soding, C. G. Wohl, M. Roos, and W. J. Willis, Rev. Mod. Phys. 39, 1 (1967).

[17] R. G. Moorhouse, Phys. Rev. Letters 16, 772 (1966).

[18] R. H. Dalitz, in *High Energy Physics*, edited by C. DeWitt and M. Jacob (Gordon and Breach, Science Publishers, Inc., New York, 1965).

[19] H. J. Lipkin, Phys. Letters 12, 154 (1964).

TABLE V. Photoproduction parameters. The units of $\gamma^{E,M}(L_{2T2J})$ and $\gamma(L_{2T2J})$ are MeV.

Pole and resonance parameters for η photoproduction[a]		Resonance parameters calculated by means of Eq. (24) for η production by pions	
$g_\eta/(4\pi)^{1/2}$	0.049	$g_\eta/(4\pi)^{1/2}$	0.049
$G_\omega{}^V/4\pi$	0.035		
$G_\omega{}^T/4\pi$	0.599		
$\gamma^E(F_{15})$	−0.218		
		$\gamma(F_{15})$	±4.08
$\gamma^M(F_{15})$	0.345		
$\gamma^E(D_{13})$	0.284		
		$\gamma(D_{13})$	±8.73
$\gamma^M(D_{13})$	0.825		
$\gamma^E(S_{11})$	1.04	$\gamma(S_{11})$	±10.4
χ^2	37.5		
χ^2/N	1.04		

[a] Very little change is found by replacing ω by ρ.

present quark model are themselves incompatible, nevertheless we try a model of η photoproduction which omits the P_{11}, D_{15}, S_{11}', and D_{13}' resonances but includes a nucleon pole (with $g_\eta{}^2/4\pi \lesssim 0.01$), a vector meson pole, and $S_{11}(1570)$, $D_{13}(1512)$, and $F_{15}(1688)$ resonances. In Ref. 12 this particular combination is not considered. The results obtained by minimizing χ^2 for this model are satisfactory and are shown in Table V and Figs. 4, 5, and 6.

VII. SCALED PARAMETERS

An over-all consistent picture of the two processes $\gamma+p\rightarrow\eta+p$ and $\pi^-+p\rightarrow\eta+n$ can be found by scaling the resonance parameters in the following manner. Let us write the total cross section in the vicinity of a resonance, which contributes to both η-production reactions as

$$\sigma_\gamma=\sigma_\gamma(\text{res})+\sigma_\gamma(\text{B}) \tag{19}$$

for η photoproduction, and

$$\sigma_\pi=\sigma_\pi(\text{res})+\sigma_\pi(\text{B}) \tag{20}$$

for η production by pions. $\sigma(\text{res})$ and $\sigma(\text{B})$ represent the resonant contributions and background contribution,

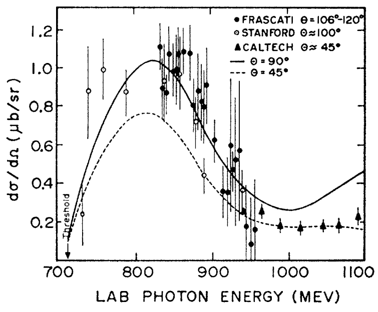


FIG. 4. The differential cross section for η photoproduction as a function of energy for $\theta=45°$ and $90°$ as given by the Model discussed in Sec. VI.

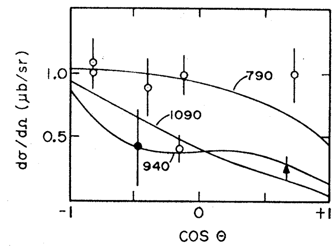


FIG. 5. The differential cross section for η photoproduction at three energies as given by the model discussed in Sec. VI. The upper set of data points were taken at $E_\gamma=790$ MeV and the lower set at $E_\gamma=940$ MeV. There is also a solid triangle data point at $E_\gamma=1090$ MeV almost coincident with the one shown.

respectively. We form the ratio

$$\frac{\sigma_\pi}{\sigma_\gamma}=\frac{\sigma_\pi(\text{res})+\sigma_\pi(\text{B})}{\sigma_\gamma(\text{res})+\sigma_\gamma(\text{B})}. \tag{21}$$

Actually we do not know exactly how much of either cross section comes from the separate parts $\sigma(\text{res})$ and $\sigma(\text{B})$. Nevertheless we can write $\sigma_\pi(\text{B})=\beta\sigma_\pi(\text{res})$, where β is some energy-dependent quantity. If we assume that β' applies to photoproduction, then we can write $\sigma_\gamma(\text{B})=\beta'\sigma_\gamma(\text{res})$ and Eq. (21) becomes

$$\frac{\sigma_\pi}{\sigma_\gamma}=\frac{(1+\beta)\sigma_\pi(\text{res})}{(1+\beta')\sigma_\gamma(\text{res})}. \tag{22}$$

If $\beta\approx\beta'$ or if both β and β' are much smaller than 1, then the ratio $(1+\beta)/(1+\beta')$ can be approximated by unity and we have

$$\frac{\sigma_\pi}{\sigma_\gamma}\approx\frac{\sigma_\pi(\text{res})}{\sigma_\gamma(\text{res})}. \tag{23}$$

The ratio on the left can be taken from experiment and the ratio on the right can be written in terms of the adjustable parameters $\gamma(L_{2T2J})$ and $\gamma^{E,M}(L_{2T2J})$. The result is

$$\frac{[\gamma(L_{2T2J})]^2}{[\gamma^E(L_{2T2J})]^2+[\gamma^M(L_{2T2J})]^2}\approx 100. \tag{24}$$

The calculated values of the parameters for η production by pions are given in Table V. The entries in the

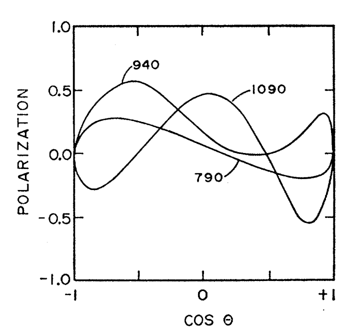


FIG. 6. Predicted polarization of the recoil proton in η photoproduction in the direction $\mathbf{k}\times\mathbf{q}$ at three energies for the model discussed in Sec. VI.

last column of this table should be compared with the values obtained in Tables II–IV. If one chooses the appropriate sign from Table V, the agreement is better than might have been anticipated. It would be interesting to know whether the scaling procedure used here could be applied with success to other processes, for example Λ production by pions and photons.

VIII. η PRODUCTION BY PIONS AT HIGHER ENERGY

It is clear from Fig. 3 that none of the models gives large enough values of the total cross sections above $T_\pi = 1$ BeV. We attempted to fit the data in this region along with the lower energy data by including possible effects of the $G_{17}(2190)$ resonance.[16] We found that it was necessary to make the resonance parameter $\gamma(G_{17})$ rather large in order to obtain even a fair fit. As a result of the large value of $\gamma(G_{17})$ the total cross section in the neighborhood of $W = 2190$ MeV has a huge bump (more than five times as high as the total cross section in the vicinity of the first peak). The data point by Barmin *et al.*[5] at $W = 2482$ MeV does not show evidence of any such behavior. They find $\sigma = 0.21 \pm 0.19$ mb and a model with the strong G_{17} enhancement predicts $\sigma \simeq 4$ mb. Therefore we conclude that it is not possible to explain the data from $T_\pi = 1$ to 1.3 BeV by the use of the $G_{17}(2190)$ resonance.

IX. SUMMARY AND CONCLUSIONS

Several of the established isotopic spin-$\frac{1}{2}$ resonances are needed for an acceptable χ^2 fit to the data for the process $\pi^- + p \to n + \eta$. For laboratory pion kinetic energy T_π up to 875 MeV (1684 MeV in c.m.), the $S_{11}(1570)$, $D_{13}(1518)$, and $P_{11}(1400)$ resonances are essential. The $F_{15}(1688)$ provides some improvement to the fits. Above this energy additional contributions appear to be needed. The $G_{17}(2190)$ and a possible $D_{13}'(1700)$ are considered. The former is found to contribute insignificantly with T_π below 1300 MeV. The $D_{13}'(1700)$ can improve the fit in the 900–1000-MeV region and leads to a peak in the total cross section which could be tested experimentally.

There are various other ways to try to improve the fit in the 1- to 1.3-BeV region. First, one might try A_2 exchange. The effect of this presumably would be to give more background contribution at higher energies. This same effect might be obtained by including nucleon resonances in the u channel. Another possible way is to try various unknown resonances in the direct channel. It is also possible that the simple pole-and-resonance model breaks down at these higher energies and an entirely different approach must be used.

A comparison is made between photoproduction and pion production of η particles, and a scaling procedure is developed which gives some information about the parameters of pion production (photoproduction) if the parameters for photoproduction (pion production) are known. With this procedure we find that the data available on pion and photoproduction of η's below 1700-MeV c.m. energy can be fit with the following ingredients: a nucleon pole, a vector meson pole, an $S_{11}(1570)$ and $D_{13}(1512)$ resonance for photoproduction; a nucleon pole, a $P_{11}(1400)$, $S_{11}(1570)$, and $D_{13}(1512)$ resonance for pionproduction. The value of the η-nucleon coupling constant $g_\eta^2/4\pi$ is $\lesssim 0.002$ in both processes, leading to a D/F ratio of 3/1 in pure $SU(3)$ coupling of the lowest-mass baryon octet to the pseudoscalar meson octet. The inclusion of some $F_{15}(1688)$ is consistent with this scaling procedure, but it does not appear to play a very significant role in either process, a fact consistent with its membership in an $SU(3)$ octet coupled to the lowest-mass pseudoscalar meson octet with a D/F ratio also of about 3/1.

The indication that the $S_{11}(1700)$, the $D_{15}(1688)$, and the $D_{13}'(1700)$ suggested above do not contribute to η photoproduction is consistent with an $\{8\}^4P_J$ assignment of these states in Dalitz's quark model. Further, the apparent dispensibility of the $P_{11}(1400)$ in this process (with the proton as target) is consistent with its membership in an $SU(3)$ $\{\overline{10}\}$ representation. Unfortunately, Dalitz's quark model and the existence of a low-lying $\{\overline{10}\}$ representation are themselves incompatible, but more data on η photoproduction may change the inferences drawn above about the assignments of these states.