In our model there is no direct relationship between $K_1^0 \to \pi^0 e^+ e^-$ and $K^+ \to \pi^+ e^+ e^-$, unless some relation is assumed between f_1 and f_2.

(E) As we mentioned in the Introduction, the estimate of Lee and Wu[6] for the ratio $(K_1^0 \to \pi^+\pi^-\pi^0)_{CPC}/(K_2^0 \to \pi^+\pi^-\pi^0)$ is $\simeq 10^{-6}$. This was based on the fact that this ratio is proportional to $(kR)^{12}$ and that kR has been chosen to be $\frac{1}{3}$, corresponding to R, approximately equal to the pion Compton wavelength.[23] Our amplitude for $K_1^0 \to \pi^+\pi^-\pi^0$ obviously also behaves like "$(kR)^6$," as can be easily checked from Eq. (10). The value of the equivalent radius is however determined by the parameters of the model (like f_{VPP}, m_V). The result in Eq. (16) supports the choice of Lee and Wu in their rough estimate of the rate. It is of interest therefore to have a model also for the $K_1^0 \to (3\pi)_{I=2}$ amplitude, as this might turn out to be the dominant CP-conserving transition.[6]

(F) The strong cancellation occurring in Eq. (11) implies that electromagnetic corrections to the process could be significant. The reduction caused by the cancellation being of the order of 10^4, one expects in fact corrections as large as the matrix element itself. The simplest correction would be to take into account the mass difference $\mu_{\pi^+} - \mu_{\pi^0}$ and to allow for a small difference (e.g., a few per thousand) between $g_{\rho^0\pi^+\pi^-}$ and $g_{\rho^+\pi^+\pi^0}$. The effect of such corrections has been studied with a similar matrix element for the $\eta \to 3\pi$ C-nonconserving decay,[24] with the conclusion that the rate is increased by a factor of 1.5–2. As we do not know the accuracy of our SU_3 assumptions, there is no point in making detailed estimates. One should however keep in mind that the electromagnetic effect of isospin nonconservation alone, could change our numerical figure for $K_1^0 \to (\pi^+\pi^-\pi^0)_{I=0}$ by a factor of ~ 2.

[23] It should be remarked, however, that by using the same argumentation, N. Byers, S. W. MacDowell, and C. N. Yang [*High-Energy Physics and Elementary Particles* (International Atomic Energy Agency, Vienna, 1965)] predict a ratio

$$(K_1^0 \to \pi^+\pi^-\pi^0)_{CPC}/(K_2^0 \to \pi^+\pi^-\pi^0) \simeq 10^{-4}$$

which is much larger than our estimate. As these authors use $R \simeq 1/\mu$, i.e., the same value used by Lee and Wu, it seems to us that the discrepancy stems from a different interpretation for k.

[24] G. L. Shaw and D. Y. Wong, Phys. Rev. Letters **8**, 336 (1962); Y. Fujii and G. L. Shaw, Phys. Rev. **160**, 1551 (1967).

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Regge Theory of High-Energy Scattering with Relatively Large Momentum Transfer

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Functional equations for the Regge parameters $\alpha(t)$ and $\beta(t)$ are derived from the unitarity condition, which is valid for relatively large momentum transfer. By solving these equations we compute the high-energy cross section with relatively large momentum transfer up to two arbitrary periodic functions. The result agrees with Orear's fit except for the appearance of dips. We compare this result with high-energy experiments on p-p scattering with large momentum transfer, and some prediction is made concerning the position of the dips.

1. INTRODUCTION

A NUMBER of empirical formulas have been proposed[1,2] for the differential cross section of high-energy proton-proton scattering with large momentum ftranser. Among these the simplest is Orear's fit[2]

$$s\frac{d\sigma_{\rm el}}{d\Omega}(s,t) = A e^{(-ap\sin\theta)}, \qquad (1)$$

with $A = 595 \pm 135$ GeV^2 mb/sr and $1/a = 158 \pm 3$ MeV/c, which is true for relatively large momentum transfer, namely, for $|t| \gtrsim 1$ (GeV/c)^2. It is remarkable that Eq. (1) can cover measurements with a wide range of incident momenta—from 1.7 to 31.8 GeV/c. However, recent measurements made by Allaby *et al.*,[3] and also by Clyde *et al.*,[4] revealed a significant deviation from Orear's fit, although Eq. (1) can still reproduce the gross features of the elastic scattering. Another series of measurements of the elastic differential cross section for 90° center-of-mass (c.m.) scattering angle was performed by Akerlof *et al.*[5] for the range of incident momenta from 5.0 to 13.4 GeV/c. The plot of $\ln(d\sigma/dt)$ versus p^2 was fitted by two straight lines with a break at $p^2 = 3.4$(GeV/c)^2. In order to compare this measurement with Orear's formula, we plot the deviation Δ,

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[1] G. Cocconi *et al.*, Phys. Rev. Letters **11**, 499 (1963); S. Minami, T. A. Moss, and G. A. Armoadian, Nuovo Cimento **33**, 982 (1964); A. D. Krish, Phys. Rev. Letters **11**, 217 (1963); and Phys. Rev. **135**, 1456 (1964).

[2] J. Orear, Phys. Rev. Letters **12**, 112 (1964); and Phys. Letters **13**, 190 (1964).

[3] J. V. Allaby *et al.*, Phys. Letters **23**, 389 (1966).

[4] A. R. Clyde *et al.*, University of California Radiation Laboratory Report No. UCRL-11441 (unpublished); UCRL-16275 (unpublished).

[5] C. W. Akerlof *et al.*, Phys. Rev. Letters **17**, 1105 (1966).

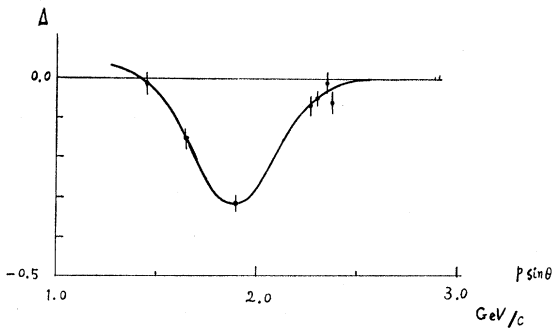

FIG. 1. Plot of $\Delta=\log_{10}(s d\sigma/d\Omega)-\log_{10}A-2.70p\sin\theta$ versus, $p\sin\theta$ GeV/c with $A=595\pm135$ GeV^2 mb/sr. Typical errors are shown on the graph. The data of Akerlof *et al.* (see Ref. 5) are used.

defined by

$$\Delta=\log_{10}\left\{\left(s\frac{d\sigma}{d\Omega}\right)\Big/\left(s\frac{d\sigma}{d\Omega}\right)_{\theta_{\text{rear}}}\right\}, \tag{2}$$

versus $p\sin\theta$ (in our case p itself) in Fig. 1. This quantity Δ has a dip around $p\approx1.8$ GeV/c.

Proton-proton elastic scattering at high energy has been interpreted in the framework of the statistical model and also in Regge theory. The statistical models predict[6,7] an exponential decrease of the elastic differential cross section, with energy for large scattering angle

$$\frac{d\sigma_{\text{el}}}{d\Omega}(\theta\approx90°)\propto\exp(-s^c/a'). \tag{3}$$

From the numerical computation made by Fast, Hagedorn, and Jones,[7] c turns out to be $\frac{1}{2}$ and $a'\approx310$ MeV, which is not inconsistent with Eq. (1) for $\theta\approx90°$. However, it is difficult to explain the angular distribution of the elastic differential cross section from the statistical model alone. This is because when the collision complex, after having realized statistical equilibrium, decays into the original elastic channel in competition with decay processes into inelastic channels, the memory of the directions of the incident momenta is lost. Another characteristic feature of the statistical model, as pointed out by Ericson[8] in the analogy with low-energy nuclear reactions, is the fluctuation of the differential cross section. However, such a fluctuation was not observed[3] in high-energy proton-proton scattering, in contrast to the case of low-energy nuclear reactions. Thus, the simple statistical model must be modified[9] if high-energy scattering is to be understood in the framework of this model.

Application of Regge theory to high-energy scattering with large momentum transfer was recently made by Huang, Jones, and Teplitz,[10] and also by Anselm and Dyatlov.[11] Huang *et al.* interpreted the appearance of the break found by Akerlof *et al.*[5] in p-p scattering at $\theta=90°$ as a result of the branch point of the first Regge cut passing through the line $J=0$. Anselm *et al.* computed the differential cross section by summing the contribution of infinitely many Regge cuts, and they got a result which agrees qualitatively with Orear's fit [Eq. (1)]. It is well known that the Regge theory has been applied successfully to the analysis of the forward diffraction cone of high-energy scattering,[12] especially to N-N and K-N scattering where resonances in the direct channel are not present in practice. Thus it is desirable to make, if possible, the same type of analysis of high-energy scattering with large momentum transfer. In Regge theory, the high-energy scattering amplitude $T(s,t)$ is assumed to be the sum of the contributions of a few Regge trajectories in the crossed channels:

$$T(s,t)\sim\beta(t)s^{\alpha(t)}+\cdots. \tag{4}$$

Contrary to the case of small $|t|$, say $|t|\lesssim m^2$, where we can parametrize $\alpha(t)$ and $\beta(t)$ (for example by taking a few terms of the Taylor expansion), it is necessary to determine the functional form of $\alpha(t)$ and $\beta(t)$ in some way if we want to apply Regge theory to scattering with large momentum transfer, $m^2\lesssim|t|\ll s$. Finally, it is worthwhile to point out that the success of the analysis of the forward diffraction cone by the Regge hypothesis comes from the assumption that the Regge cuts do not exist, or at least that their contribution is very small. We have to reexamine this assumption when we apply Regge theory to scattering with large momentum transfer.

The organization of this paper is as follows. In Sec. 2, the idea for determining the functional form of $\alpha(t)$ and $\beta(t)$ is presented. In Sec. 3, functional equations for $\alpha(t)$ and $\beta(t)$ are derived. In Sec. 4, these equations are solved by changing them into difference equations, and their physical meaning is discussed. A semiempirical extension of the result to large-angle scattering is made, and then in Sec. 5 we compare our results with experiments and make some predictions concerning high-energy scattering with large momentum transfer.

2. ASSUMPTIONS AND PRESENTATION OF IDEAS

In the Regge hypothesis, the high-energy behavior of the elastic scattering amplitude $T(s,t)$ is given by Eq. (4), and it is usually assumed that the quasi-two-body (for example, $p+p\to p+N_\gamma^*$) scattering ampli-

[6] G. Fast and R. Hagedorn, Nuovo Cimento **27**, 208 (1963); R. Hagedorn, *ibid.* **35**, 216 (1965); A. Bialas and V. F. Weisskopf, *ibid.* **35**, 1211 (1965).

[7] G. Fast, R. Hagedorn and L. W. Jones, Nuovo Cimento **27**, 856 (1963).

[8] T. Ericson and T. Mayer–Kuckuk, CERN Report No. 66/TH 686 (unpublished); see also J. V. Allaby *et al.*, Phys. Letters **23**, 389 (1966).

[9] S. Machida, M. Namiki, and I. Ohba, Progr. Theoret. Phys. (Kyoto) **37**, 107 (1967).

[10] K. Huang, C. E. Jones, and V. L. Teplitz, Phys. Rev. Letters **18**, 146 (1967).

[11] A. A. Anselm and I. T. Dyatlov, Phys. Letters **24B**, 479 (1967).

[12] R. J. N. Phillips and W. Rarita, Phys. Rev. **139**, B1336 (1965). Further references will be found in their paper.

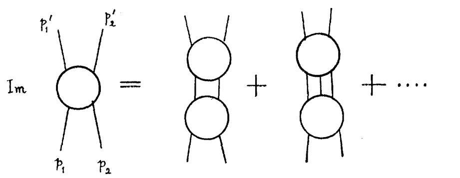

FIG. 2. The elastic unitarity condition on the scattering $p_1+p_2 \to p_1'+p_2$.

tudes $T_j(s,t)$ have the same form[13]:

$$T_j(s,t) \sim \beta_j(t) s^{\alpha(t)} + \cdots. \quad (5)$$

We do not yet have a convincing theory which determines the high-energy behavior of the production amplitudes (e.g., $p+p \to p+n+\pi^+$ with the correlated part subtracted). Since our aim is to derive restrictions on $\alpha(t)$ and $\beta(t)$ from the unitarity condition of the s channel, it is necessary to find a region of t where the contribution from the uncorrelated part of the production amplitudes is small, if possible. The unitarity condition of the elastic amplitudes of $p_1+p_2 \to p_1'+p_2'$ is[14] (see Fig. 2)

$$2\,\mathrm{Im}T(p_1',p_2';p_1,p_2) = -(2\pi)^4 \sum_{|\nu\rangle} \frac{\delta^{(4)}(p_1+p_2-p_\nu)}{N_\nu^2 V^n} \times T^*(\nu;p_1',p_2')T(\nu;p_1,p_2), \quad (6)$$

where n is the number of particles of the intermediate state $|\nu\rangle$, and the notation is same as that of Gasiorowicz.[13] Since for an n-particle intermediate state the sum is of the form

$$\int \cdots \int \frac{V d^3q_1}{(2\pi)^3} \cdots \frac{V d^3q_n}{(2\pi)^3},$$

it is more conveinent to rewrite the sum $\sum_{|\nu\rangle}$ as

$$-\frac{(2\pi)^4}{2} \sum_{|\nu\rangle} \frac{\delta^{(4)}(p_1+p_2-p_\nu)}{V^n} \to \sum_{n=2}^{\infty} \sum_r \int \cdots \int \rho\, d\gamma_1 \cdots d\gamma_{3n-4}, \quad (7)$$

where r stands for all the quantum numbers, other than linear momenta and particle number n, needed to specify the intermediate states.[15] Equation (6) becomes

$$\mathrm{Im}T(p_1',p_2';p_1,p_2) = \sum_{n=2}^{\infty} \sum_r \int \cdots \int \frac{\rho}{N_\nu^2} \times T^*(\nu;p_1',p_2')T(\nu;p_1,p_2)\, d\gamma_1 \cdots d\gamma_{3n-4}. \quad (8)$$

In particular, for the case of forward scattering, i.e., for $t=0$, the unitarity condition becomes

$$\mathrm{Im}T(p_1,p_2;p_1,p_2) = \sum_{n=2}^{\infty} \sum_r \int \cdots \int d\gamma_1 \cdots d\gamma_{3n-4} \times \frac{\rho}{N_\nu^2} |T(\nu;p_1,p_2)|^2 \quad (9)$$

and

$$\mathrm{Im}T(p_1,p_2;p_1 t p_2) \propto \sum_{n=2}^{\infty} \sum_r \sigma_{n,r}(s) = \sigma_{\mathrm{tot}}(s), \quad (10)$$

where $\sigma_{n,r}(s)$ is the cross section of $p_1+p_2 \to |n,r\rangle$. Since we know experimentally that for 30 GeV/$c > p_{\mathrm{inc}} > 10$ GeV/c the ratio[16] $\sigma_{\mathrm{el}}(s)/\sigma_{\mathrm{tot}}(s)$ for p-p scattering is 0.23, the contribution of the quasi-two-body intermediate states (including the elastic state) to $\mathrm{Im}R(s,t=0)$ is larger than 23%. However, the contribution of the uncorrelated multiparticle intermediate states is not necessarily small for the case of $t=0$ (in the worst case it may become as large as 77%). Now let us examine how the ratio of the contributions of the quasi-two-body states and the multiparticle intermediate states to $\mathrm{Im}R(s,t)$ changes when $|t|$ increases from zero.

Although the integrand of Eq. (9) is positive definite, for $t=0$ the integrand of Eq. (8) is complex in general and assumes various phases when we perform the integration. Thus, we can expect that for finite value of t the cancellation becomes more appreciable as the number of particles in the intermediate state, namely, the number of integration variables, increases when we perform the integration of Eq. (8).[17] Therefore, if the momentum transfer becomes larger than a critical value, say $|t_c|$, the contributions of the quasi-two-body intermediate states to $\mathrm{Im}T(s,t)$ of Eq. (8) exceeds 50%. From these considerations, it is reasonable to adopt the following assumtpions:

Assumption 1. For $|t|>|t_c|$ the unitarity condition becomes

$$\mathrm{Im}T(s,t) = \frac{p}{\sqrt{s}} \sum_{j=\text{two-body}} \int \frac{d\Omega'}{4\pi} \times T_j(s,t')T_j^*(s,t'') + R(s,t), \quad (11)$$

where $T_j(s,t')$ are the amplitudes of the quasi-two-body (including the elastic) scattering, and t, t', and t'' are related by Eq. (15). $R(s,t)$ is the sum of the contributions of all the intermediate states other than those of the quasi-two-body intermediate states; it is as-

[13] S. Gasiorowicz, Fortschr. Physik 8, 665 (1960).

[14] In this paper, we suppress the spins of all the particles for the sake of simplicity.

[15] r distinguishes different states of the same particle number n, since, for example, we have states $p+n+\pi^+$, $p+\Lambda+K^+$, $\cdots$ even for the case $n=3$.

[16] O. Czyzewski *et al.*, Phys. Letters 15, 188 (1965).

[17] In fact, the contribution of the uncorrelated jet intermediate states is known to decrease as $C\exp(ct)$ for $\theta^2 \ll 1$ [L. Van Hove, Rev. Mod. Phys. 36, 665 (1964)], whereas, as we shall see later, the contribution of the two-body intermediate states behaves as $C'\exp(-c'\sqrt{-t})$. Thus, as $|t|$ increases, the contribution of the latter eventually exceeds that of the former.

sumed to be less than the first term of the right-hand side of Eq. (11) in magnitude and to decrease faster than the first term. The next assumption concerns the high-energy behavior of the quasi-two-body scattering amplitudes.

Assumption 2. The asymptotic behavior of the quasi-two-body scattering amplitudes $T_j(s,t)$ is determined by a small number of Regge trajectories of the crossed channels, namely,

$$T_j(s,t) \sim \beta_j(t)(\ln s)^N s^{\alpha(t)} + \cdots . \tag{12}$$

In Eq. (12), although $\alpha(t)$ is common, the residues $\beta_j(t)$ depend on reaction (j) of $p_1 + p_2 \to |j\rangle$. For generality, we have placed a factor $(\ln s)^N$ in Eq. (12), where N is a non-negative integer to be specified later. $N=0$ corresponds to the Regge pole of the crossed channel, whereas $N=1, 2 \cdots$ correspond to Regge dipole quadrupole, and so on. Thus N indicates the type of the Regge trajectory.

Our idea is very simple. By substituting the asymptotic form of Eq. (12) into both sides of the unitarity condition given in Eq. (11) which is valid for $|t| > |t_c|$, we compare the power of s and also its coefficient. It is sufficient to assume

$$\left| \frac{p}{\sqrt{s}} \sum_j \int \frac{d\Omega'}{4\pi} T_j(s,t') T_j^*(s,t'') \right| > |R(s,t)| \tag{13}$$

for $|t| > |t_c|$ and $s \gg m^2$, in order to derive the functional equation of $\alpha(t)$, since the power of s of the right-hand side of Eq. (11) is determined by the first term and the existence of $R(s,t)$ does not alter the power. On the other hand, in order to obtain a functional equation for $\beta_j(t)$ by comparing the coefficient of $s^{\alpha(t)}$ on both sides in Eq. (11), it is necessary to assume that $R(s,t)$ is negligible compared to the right-hand side of Eq. (13). Thus in this case, we have to take $|t_c|$ a little larger.

Finally, it is worthwhile to give a different explanation of our way of determining the Regge parameters $\alpha(t)$, $\beta_j(t)$, and N in Eq. (12). Suppose that the Regge-pole trajectory $l=\alpha(t)$ is a straight line with $\alpha(0)=1$, namely,

$$\alpha(t) = 1 + \alpha'(0)t ; \tag{14}$$

then using the unitarity condition of the s channel by iteration, as is well known, there appears a Regge cut with a branch point at $l = 1 + \frac{1}{2}\alpha'(0)t$, which does not coincide with the original Regge trajectory of Eq. (14) (see Fig. 3).[18] Our problem is to determine from what curve $l=\alpha(t)$ and from what type N of Regge trajectory defined in Eq. (12) we should start, in order for the induced Regge singularity to be the same as the original one in type as well as position. In the next section, we shall find that $N=1$, namely the Regge dipole,[19] satisfies the requirement we have made.[20]

3. EQUATIONS FOR $\alpha(t)$ AND $\beta_j(t)$

In this section we study high-energy proton-proton scattering with finite momentum transfer, namely $|t_c| < |t| \ll s$. Thus, the scattering angle θ is infinitesimal; $|\theta| \ll 1$, the case of large scattering angle, will be treated in Sec. 5. The variables t, t', and t'' appearing in the unitarity condition given in Eq. (11) are related by

$$\cos\theta'' = \cos\theta \cos\theta' + \sin\theta \sin\theta' \cos\phi' , \tag{15}$$

where the θ's and t's are connected by

$$t = -2p^2(1-\cos\theta) , \quad \text{etc.} \tag{16}$$

Let us compute the integral appearing on the right-hand side of Eq. (11) for large s:

$$I_j(s,\eta_t) = (\ln s)^{2N} \int \frac{d\Omega'}{4\pi} \beta_j(t')\beta_j^*(t'') s^{\alpha(t')+\alpha(t'')} . \tag{17}$$

If we change the integration variables (θ',ϕ') to (η_t',ϕ'), where

$$\eta_t' = -\sqrt{t'} , \quad \text{etc.}, \tag{18}$$

then

$$I_j(s,\eta_t) = (\ln s)^{2N} \frac{1}{\pi} \int_{-\pi}^{\pi} d\phi' \times \int_0^\infty \eta_t' d\eta_t' \bar{\beta}_j(\eta_t') \bar{\beta}_j^*(\eta_t'') s^{\bar{\alpha}(\eta_t')+\bar{\alpha}(\eta_t'')-1} , \tag{19}$$

[18] When we compute the higher-order iterations assuming $\alpha(t) = 1 + \alpha'(0)t$, a series of the Regge cuts appears with branch points at $l = 1 + (1/n)\alpha'(0)t$ with $n = 2, 3, 4, \cdots$, which may accumulate at $l=1$. However, if we make the requirement on $\alpha(t)$, instead of assuming $l = g(t)$ is a straight line, that the Regge trajectory obtained from the first-order iteration is the same as the zeroth-order Regge trajectory in position and type, then the additional Regge trajectory or cut does not appear when we compute the higher-order iterations.

[19] T. Sawada, Nuovo Cimento **48**, 534 (1967); and also TUEP Report, TUEP-67-4 (unpublished).

[20] This conclusion is correct, as we shall see later, if

$$\sum_{j=\text{quasi-two-body}} \beta_j(t/4)\beta_j^*(t/4)$$

converges for $|t| > |t_c|$. This fact seems to indicate that some or all of the Regge trajectories of mesons may be Regge dipoles rather than Regge (mono) poles. Experimentally, it is not yet clear. However, the proton form factor [T. Janssens, R. Hofstadter, E. B. Hughes, and M. R. Yearian, Phys. Rev. **142**, 922 (1966); W. Bartel *et al.*, Phys. Rev. Letters **17**, 618 (1966)] is well fitted by $F(q^2) = (1+q^2/0.71)^{-2}$, which is easily understood if the vector mesons are Regge dipoles. On the other hand, the usual three-poles fit [L. H. Chan, K. W. Chen, J. R. Dunning, Jr., N. F. Ramsey, and J. K. Walker, Phys. Rev. **141**, 4 (1966); W. Albrect *et al.*, Phys. Rev. Letters **18**, 1014 (1967); M. Goitein *et al.*, *ibid.* **68**, 1016, 1018 (1967)] $F(q^2) = \sum_i c_i(m_i^2+q^2)^{-1}$ with $\sum_i c_i m_i^{-2} = 1$ does not work very well except when we add a large background contribution. Moreover, the total cross section grows with energy as $\sigma_{\text{tot}}(s) \sim C \ln(s/s_0)$ if the leading Regge dipole trajectory has the property $\alpha(0)=1$, contrary to the constant behavior of the total cross section for the case of the Regge pole. We cannot yet determine which is actually the case from cosmic-ray experiments. However, a recent measurement of the inelastic cross section of $p + C^{12}$ with incident proton energy from 10 to 10^3 GeV, made by Grigorov *et al.* using the earth satellites "Proton 1" and "Proton 2," revealed that the cross section increases slowly with energy and can be fitted by $\ln(s/s_0)$ or by a small power of s. References to this experiment and also other experiments in the cosmic-ray energy region can be found in Barashenkov's [Fortschr. Physik **14**, 741 (1966)] report.

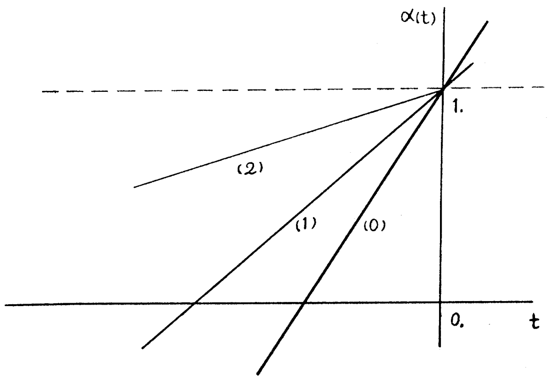

FIG. 3. Regge-pole trajectory (0) and the branch point of the first and second Regge cuts.

where $\bar{\alpha}(\eta_t)$ is the same as $\alpha(t)$ except for the argument; it is defined by

$$\bar{\alpha}(\eta_t)\equiv\alpha(t)\,, \tag{20}$$

and $\bar{\beta}(\eta_t)$ is similarly defined. Remembering Eq. (15), η_t'' is related to η_t, η_t', and ϕ' by

$$\eta_t''^2=\eta_t^2+\eta_t'^2-2\eta_t\eta_t'\cos\phi'\,. \tag{21}$$

It is more convenient to introduce Cartesian coordinates (x',y'), in place of the polar coordinates (η_t', ϕ'):

$$x'=\eta_t'\cos\phi'-\tfrac{1}{2}\eta_t\,,\qquad y'=\eta_t'\sin\phi'\,. \tag{22}$$

The shift of the origin is made in order to make the new integration variables symmetric with respect to η_t' and η_t''. Then Eq. (19) becomes

$$I_j(s,\eta_t)=(\ln s)^{2N}\frac{1}{\pi}\int\int_{-\infty}^{\infty}dx'dy'\bar{\beta}_j(\eta_t')\bar{\beta}_j^*(\eta_t'')s^{\lambda(x',y';\eta_t)}\,, \tag{23}$$

with

$$\lambda(x',y';\eta_t)=\bar{\alpha}(\eta_t')+\bar{\alpha}(\eta_t'')-1\,. \tag{24}$$

In Fig. 4, we draw as an example the graph of $\lambda(x',y';\eta_t)$ on the (x',y') plane for the case of $\alpha(t)=1+\alpha'(0)t$. From Eq. (24), it is evident that $\lambda(x',y';\eta_t)$ is symmetric with respect to the x' axis and y' axis, namely

$$\lambda(x',-y';\eta_t)=\lambda(x',y';\eta_t) \tag{25}$$

and

$$\lambda(-x',y';\eta_t)=\lambda(x',y';\eta_t)\,. \tag{26}$$

If $d\alpha(t)/dt>0$, which means $d\bar{\alpha}(\eta_t)/d\eta_t<0$, then $\lambda(x',y';\eta_t)$ becomes a maximum for $y'=0$:

$$\lambda(x',0;\eta_t)\geqq\lambda(x',y';\eta_t)\,. \tag{27}$$

Considering Eqs. (26) and (27), at the origin, we see that $\lambda(x',y';\eta_t)$ becomes maximal or has a saddle point. In fact, for the case of $\alpha(t)=1+\alpha'(0)t$, as we can see in Fig. 4, $\lambda(x',y';\eta_t)$ has a peak at the origin.

In general, we can write the scattering amplitude $T(s,t)$, using the Regge spectral function $\rho(\lambda,t)$, in the form

$$T(s,t)=\int_{-\Lambda}^{\infty}\rho(\lambda,t)s^{\lambda}d\lambda+O(s^{-\Lambda})\,, \tag{28}$$

which is convenient for studying the high-energy behavior of the amplitude. In Eq. (28),

$$\rho(\lambda,t)=0\quad\text{for}\quad\lambda>\Lambda_{\max}(t)\,. \tag{29}$$

A Regge pole corresponds to $\beta(t)\delta(\lambda-\alpha(t))$ for the spectral function and a Regge dipole corresponds to a δ' function, whereas the finite spectral function $\rho(\lambda,t)$ yields a Regge cut. It is an easy matter to change Eq. (23) into the form of Eq. (28) and compute the spectral function,

$$\rho_j(\lambda,t)=\frac{1}{\pi}\int_{\lambda=\lambda(x',y';\eta_t)}dL'\bar{\beta}_j(\eta_t')\bar{\beta}_j^*(\eta_t'')\times\left[\frac{\partial\lambda(x',y';\eta_t)}{\partial n}\right]^{-1}, \tag{30}$$

where the integration is performed on the curves of $\lambda=\lambda(x',y';\eta_t)$ and the derivative $\partial/\partial n$ is to be made normal to the curves. When we fix t, $\rho(\lambda,t)$ is a continuous function of λ unless $\partial\lambda/\partial n$ vanishes on the curve of integration. The vanishing actually occurs at the peak and at the saddle point of $\lambda(x',y';\eta_t)$, namely, at

$$\frac{\partial\lambda(x',y';\eta_t)}{\partial x'}=0\quad\text{and}\quad\frac{\partial\lambda(x',y';\eta_t)}{\partial y'}=0\,. \tag{31}$$

If (x_0', y_0') is one of the solutions of Eq. (31) for given η_t, then at the corresponding value of λ, i.e., at $\lambda=\lambda(x_0',y_0';\eta_t)$, there appears a finite discontinuity in $\rho_j(\lambda,t)$. Since we know that $\lambda(x',y';\eta_t)$ has a peak or a saddle point at $(x',y')=(0,0)$, let us compute the corre-

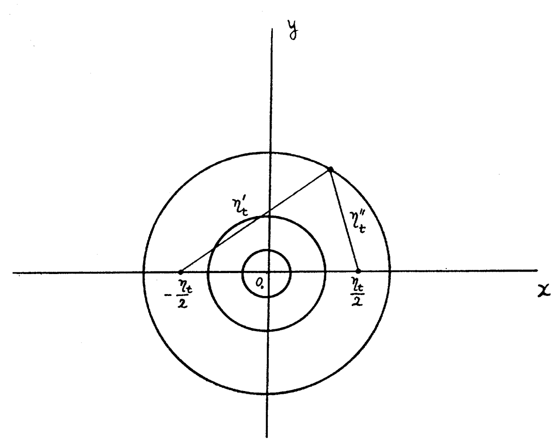

FIG. 4. Graph of $\lambda(x',y';\eta_t)$ on the (x',y') plane, for the case of the linear trajectory $\alpha(t)=\alpha(0)+\alpha'(0)t$. The scales of the x' and y' axis are different.

sponding discontinuity of the spectral function $\rho_j(\lambda,t)$. At the origin $(x',y')=(0,0)$,

$$\lambda=\lambda_0\equiv 2\bar{\alpha}(\eta_t/2)-1\,, \tag{32}$$

since $\eta_t'=\eta_t''=\eta_t/2$. If we consider the case where $\lambda(x',y';\eta_t)$ has a peak at $(x',y')=(0,0)$, it is evident that for a small positive number ϵ, the expression

$$\lambda(x',y';\eta_t)=\lambda_0-\epsilon \tag{33}$$

gives a small ellipse in the neighborhood of the origin. In fact, expanding $\lambda(x',y';\eta_t)$ up to second order of x' and y', Eq. (33) becomes

$$[2\bar{\alpha}'(\eta_t/2)/\eta_t]y^2+\bar{\alpha}''(\eta_t/2)x^2=-\epsilon\,. \tag{34}$$

Since $\bar{\alpha}'(\eta_t)<0$, Eq. (34) gives an ellipse for $\bar{\alpha}''(\eta_t)<0$ which is the case of interest. If we make the line integration on this ellipse, Eq. (30) is

$$\rho_j^0(\lambda,t)=|\bar{\beta}_j(\eta_t/2)|^2[(2/\eta_t)\bar{\alpha}'(\eta_t/2)\bar{\alpha}''(\eta_t/2)]^{-\frac{1}{2}} \tag{35}$$

and does not depend on ϵ. Thus

$$\lim_{\lambda\to\lambda_0-0}\rho_j(\lambda,t)-\lim_{\lambda\to\lambda_0+0}\rho_j(\lambda,t)=|\bar{\beta}_j(\eta_t/2)|^2 f(\eta_t/2)\,, \tag{36}$$

where $f(\eta_t/2)$ is a factor which is independent of j:

$$f(\eta_t/2)=[(2/\eta_t)\bar{\alpha}'(\eta_t/2)\bar{\alpha}''(\eta_t/2)]^{-\frac{1}{2}}. \tag{37}$$

In general, there may exist other discontinuities in $\rho_j(\lambda,t)$, although for the case shown in Fig. 4 $\rho_j(\lambda,t)$ has only one discontinuity which we have already computed. The first term of the right-hand side of Eq. (11) can be written as

$$\frac{p}{\sqrt{s}}\sum_j I_j(s,\eta_t)=(\ln s)^{2N}\frac{p}{\sqrt{s}}\int_{-\Lambda}^{\infty}\rho(\lambda,t)s^{\lambda}d\lambda+O(s^{-\Lambda})\,, \tag{38}$$

where

$$\rho(\lambda,t)=\textstyle\sum_j\rho_j(\lambda,t)\,. \tag{39}$$

The discontinuity of $\rho(\lambda,t)$ at $\lambda=\lambda_0$ is

$$\lim_{\lambda\to\lambda_0-0}\rho(\lambda,t)-\lim_{\lambda\to\lambda_0+0}\rho(\lambda,t)=f(\eta_t/2)\sum_j|\bar{\beta}_j(\eta_t/2)|^2\,, \tag{40}$$

and thus the discontinuity at $\lambda=\lambda_0$ does not disappear when we make the summation over j. On the other hand, since the phases of other possible discontinuities depend on j, $\rho(\lambda,t)$ does not necessarily have the same discontinuities.

Let us assume that the summation of the right-hand side of Eq. (40) converges for some range of η_t. If $N=0$, Eq. (38) gives the high-energy behavior which corresponds to a Regge cut, whereas the left-hand side of Eq. (11) behaves as $\mathrm{Im}\beta(t)s^{\alpha(t)}$ for large s; this is a contradiction. It is not difficult to restore the consistency if we remember an identity:

$$(\ln s)\int_{-\Lambda}^{\infty}\rho(\lambda,t)s^{\lambda}d\lambda=-\int_{-\Lambda}^{\infty}\frac{d}{d\lambda}\rho(\lambda,t)s^{\lambda}d\lambda+O(s^{-\Lambda})\,, \tag{41}$$

where $\rho(\lambda,t)=0$ for $\lambda>\Lambda_{\max}$ is used. The unitarity condition of Eq. (11) becomes, for large s,

$$(\ln s)^N\,\mathrm{Im}\beta(t)s^{\alpha(t)}=(\ln s)^{2N-1}\times\frac{1}{2}\int_{-\Lambda}^{\infty}\left(-\frac{d}{d\lambda}\right)\rho(\lambda,t)s^{\lambda}d\lambda+R(s,t)+O(s^{-\Lambda})\,. \tag{42}$$

The discontinuity of $\rho(\lambda,t)$ becomes a δ function in $-d\rho(\lambda,t)/d\lambda$, and the left-hand side of Eq. (42) can be written as

$$(\ln s)^N\,\mathrm{Im}\beta(t)s^{\alpha(t)}=(\ln s)^N\int_{-\Lambda}^{\infty}\mathrm{Im}\beta(t)\delta(\lambda-\alpha(t))s^{\lambda}d\lambda\,. \tag{42'}$$

Since $-d\rho(\lambda,t)/d\lambda$ definitely has a δ function at $\lambda=\lambda_0$, although the existence of other possible δ functions is not clear, let us identify the power of $(\ln s)$ and the position of the δ function at $\lambda=\lambda_0$ and its coefficient in Eq. (42). We obtain[21,22]

$$N=2N-1\,, \tag{43}$$

$$\bar{\alpha}(\eta_t)=2\bar{\alpha}(\eta_t/2)-1\,, \tag{44}$$

and

$$\mathrm{Im}\bar{\beta}(\eta_t)=\tfrac{1}{2}f(\eta_t/2)\sum_j|\bar{\beta}_j(\eta_t/2)|^2\,, \tag{45}$$

where $f(\eta_t/2)$ is defined in Eq. (37). In order to derive Eqs. (43) and (44), it is not necessary that $R(s,t)$ of Eq. (42) be small but it is sufficient to satisfy the condition of Eq. (13). On the other hand, we assumed that $R(s,t)$ is negligible in order to obtain Eq. (45). If the high-energy behavior of the elastic scattering amplitude $T(s,t)$ is determined by a few Regge trajectories and not by the Regge cuts, then $\rho(\lambda,t)$ in Eq. (39) is expected to be the sum of step functions as shown in Fig. 5. Equation (43) implies the leading Regge trajectory is a Regge dipole and has a contribution of the form of $(\ln s)\beta(t)s^{\alpha(t)}$, as explained in Sec. 2. In the next section we shall solve Eqs. (44) and (45).

4. SOLUTION OF THE FUNCTIONAL EQUATIONS FOR $\alpha(t)$ AND $\beta(t)$

In order to solve Eqs. (44) and (45), we introduce a new variable x defined by

$$x+1=\ln\eta_t/\ln 2\,, \tag{46}$$

[21] If we consider Eq. (12), the solution $N=1$ corresponds to a Regge dipole. The dipole nature of the leading trajectory [T. Sawada, Nuovo Cimento **51**, 208 (1967)] is expected, when we adopt the point of view of "nuclear democracy" and consider the high-energy scattering of a "composite particle."

[22] Although we derived Eq. (43) at $t<-|t_c|$, we may expect that some of the meson trajectories are dipole also at $t>0$ and have dipole resonances [R. J. Eden and P. V. Landshoff, Phys. Rev. **136**, B1817 (1964); J. S. Bell and C. J. Goebel, *ibid.* **138**, B1198 (1965); M. Goldberger and K. Watson, *ibid.* **136**, B1472 (1964); E. P. Wigner, *ibid.* **98**, 145 (1955)]. It is worthwhile to point out that even if the Regge trajectory is a pure dipole, in the s plane the amplitude in general has dipole plus single pole for fixed l. We shall discuss elsewhere the shape of the meson resonances.

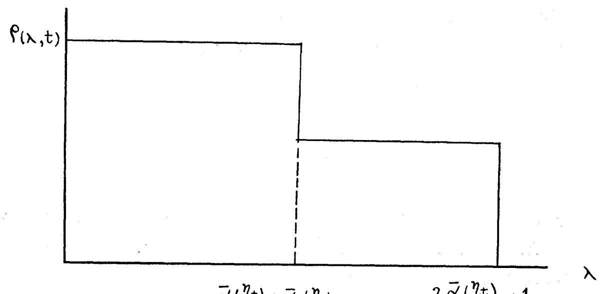

FIG. 5. Graph of the Regge spectral function $\rho(\lambda,t)$ versus λ.

and write $\bar{\alpha}(\eta_t)$ and $\bar{\beta}(\eta_t)$ as $\tilde{\alpha}(x)$ and $\tilde{\beta}(x)$, using the new variable x:

$$\tilde{\alpha}(x)=\bar{\alpha}(\eta_t/2)\,,\quad \text{etc.} \tag{47}$$

Then Eq. (44) is reduced to a difference equation of first order:

$$\tilde{\alpha}(x+1)-2\tilde{\alpha}(x)=-1\,. \tag{48}$$

The general solution of Eq. (48) is

$$\tilde{\alpha}(x)=1-2^x T(x)\,, \tag{49}$$

where $T(x)$ is an arbitrary periodic function with

$$T(x+1)=T(x)\,. \tag{50}$$

Thus we obtain[23]

$$\bar{\alpha}(\eta_t)=1-\eta_t T(\ln\eta_t/\ln 2) \tag{51}$$

or

$$\alpha(t)=1-(\sqrt{-t})T(\ln|t|/\ln 4)\,. \tag{52}$$

Let us now turn to the equation for $\beta(t)$. Since Eq. (45) contains too many unknown functions $\bar{\beta}_j(\eta_t)$, we have to solve a set of equations. However, in this note we introduce a phenomenological parameter function $\sigma(\eta_t)$ which is essentially an inverse function of elasticity. Equation (45) becomes

$$\operatorname{Im}\bar{\beta}(\eta_t e=\tfrac{1}{2}f(\eta_t/2)\,|\bar{\beta}(\eta_t/2)|^2\sigma(\eta_t/2)\,, \tag{53}$$

with

$$\sigma(\eta_t/2)\geq 1\,, \tag{54}$$

and later we shall assume that the η_t dependence of $\sigma(\eta_t/2)$ is weak. Separating the signature factor from $\bar{\beta}(\eta_t)$ by

$$\bar{\beta}(\eta_t)=-\frac{e^{-i\pi\bar{\alpha}(\eta_t)}+1}{\sin\pi\bar{\alpha}(\eta_t)}b(\eta_t)\,, \tag{55}$$

where $b(\eta_t)$ is a real function at least for some range of η_t, Eq. (53) becomes

$$b(\eta_t)=b^2(\eta_t/2)\frac{1+\cos\pi\bar{\alpha}(\eta_t/2)}{\sin^2\pi\bar{\alpha}(\eta_t/2)}f(\eta_t/2)\sigma(\eta_t/2)\,. \tag{56}$$

[23] Since our functional equation given in Eq. (44) is true only at $t<-|t_c|$, the solution of $\alpha(t)$ is also true only in this limited range of t. Therefore the appearance of the singularity at $t=0$ is not surprising, because the contribution from the uncorrelated multiparticle intermediate states to the unitarity condition given in Eq. (9) is not negligible in the neighborhood of $t=0$.

Thus, using the variable x defined in Eq. (46), we find

$$\ln\tilde{b}(x+1)-2\ln\tilde{b}(x)=K(x)\,, \tag{57}$$

where $\tilde{b}(x)$ is defined as before by

$$\tilde{b}(x)=b(\eta_t/2) \tag{58}$$

and

$$K(x)=\ln\left[\frac{1+\cos\pi\bar{\alpha}(\eta_t/2)}{\sin^2\pi\bar{\alpha}(\eta_t/2)}f(\eta_t/2)\sigma(\eta_t/2)\right]; \tag{59}$$

we regard $K(x)$ as a known function. Equation (57) is a typical inhomogeneous difference equation of first order, and the general solution is the sum of one particular solution of Eq. (57) and the general solutions of the homogeneous equation. The result is

$$\ln\tilde{b}(x)=2^x R(x)-\sum_{n=0}^{\infty}\frac{1}{2^{n+1}}K(x+n)\,, \tag{60}$$

and $R(x)$ is an arbitrary periodic function with

$$R(x+1)=R(x)\,. \tag{61}$$

From Eqs. (60) and (59), the residue $b(\eta_t)$ is

$$b(\eta_t)=e^{\eta_t R(\ln\eta_t/\ln 2)}\times\prod_{n=0}^{\infty}\left[\frac{\sin^2\pi\bar{\alpha}(2^n\eta_t)}{1+\cos\pi\bar{\alpha}(2^n\eta_t)}\frac{1}{f(2^n\eta_t)}\frac{1}{\sigma(2^n\eta_t)}\right]^{1/2^{n+1}}\,, \tag{62}$$

and the convergence of the product is evident, since the inside of the cubic bracket of Eq. (62) is bounded. The cross section is related to the residue $\beta(t)$ by

$$s\frac{d\sigma}{d\Omega}(s,t)=(\ln s)^2|\beta(t)|^2 s^{2\alpha(t)}\,, \tag{63}$$

in the high-energy limit. Since we have obtained the solution of $\bar{\alpha}(\eta_t)$ in Eq. (51), $f(\eta_t)$ defined in Eq. (37) can be written as

$$f(\eta_t)=\eta_t\{\tilde{U}(x)\}^{-\frac{1}{2}}\,, \tag{64}$$

where

$$\tilde{U}(x)=\frac{1}{\ln 2}\left\{T(x)+\frac{1}{\ln 2}T'(x)\right\}\left\{T'(x)+\frac{1}{\ln 2}T''(x)\right\}\,, \tag{65}$$

and thus $\tilde{U}(x)$ is also a periodic function;

$$\tilde{U}(x+1)=\tilde{U}(x)\,.$$

If we substitute Eq. (60) into Eq. (63), considering Eqs. (59) and (64), we finally obtain

$$\ln\left[\frac{1}{(\ln s)^2}\frac{2}{s}\eta_t^2\frac{d\sigma}{d\Omega}\right]=2\eta_t\left[R\left(\frac{\ln\eta_t}{\ln 2}\right)+(\ln s)T\left(\frac{\ln\eta_t}{\ln 2}\right)\right]+\sum_{n=1}^{\infty}\frac{1}{2^n}\ln\left|\frac{\sin^2\pi\bar{\alpha}(2^n\eta_t)}{1+\cos\pi\bar{\alpha}(2^n\eta_t)}\right|+\ln\left|\tilde{U}\left(\frac{\ln\eta_t}{\ln 2}\right)\right|-\sum_{n=0}^{\infty}\frac{1}{2^n}\ln\sigma(2^n\eta_t)\,, \tag{66}$$

for $\eta_t > \sqrt{|t_c|}$. We determined the differential cross section up to two arbitrary functions $R(x)$ and $T(x)$; therefore, we can in principle predict the t dependence of the cross section in the high-energy limit, when $d\sigma/d\Omega$ is given in the range of one period, for example, in the range $|t_c| < |t| \leq 4|t_c|$, as input information. We need such information on $d\sigma/d\Omega$ for two different energy values in order to fix the arbitrary periodic functions $R(x)$ and $T(x)$. Concerning the last term of the right-hand side of Eq. (66), if $\sigma(\eta_t)$, the inverse of the elasticity, is a constant $\bar{\sigma}$, then the last term assumes the simple form:

$$-\sum_{n=0}^{\infty} \frac{1}{2^n} \ln\sigma(2^n\eta_t) \to -2\ln\bar{\sigma}. \qquad (67)$$

Let us examine the meaning of our final result, Eq. (66), term by term. The first term on the right-hand side, $2\eta_t[R+(\ln s)T]$, determines the main features of the cross section, since the second and the third terms are oscillating functions and give a finite modification. This first term $2\eta_t[R+(\ln s)T]$ is the sum of the "shrinking" and "nonshrinking" terms. If the $R(x)$ and $T(x)$ are bounded periodic functions, then globally $2\eta_t[R+(\ln s)T]$ is a linear function of η_t, namely of $\sqrt{t}$. There exist two different types of "dip" in $\ln[\eta_t^2 d\sigma/d\Omega]$. One occurs in the term $\ln|\tilde{U}(\ln\eta_t/\ln 2)|$, since $\tilde{U}(x)$ defined in Eq. (66) is a product of two periodic functions $\{T(x)+(1/\ln 2)T'(x)\}$ and $\{T'(x)+(1/\ln 2)T(x)''\}$, and the latter factor is a derivative of a periodic function; therefore $\tilde{U}(x)$ has at least two zeros in one period. At the point where $\tilde{U}(x)$ vanishes, we must observe a dip. It is worthwhile to point out that this type of dip appears in a periodic way with respect to $\ln\eta_t/\ln 2$, since $\ln|\tilde{U}(x)|$ is a periodic function. If we consider the case where the slope of the Regge trajectory is very small, we can observe only this type of dip, since in this limit Eq. (66) becomes

$$\ln\left[\frac{1}{(\ln s)^2}\frac{2}{s}\eta_t^2\frac{d\sigma}{d\Omega}\right] = 2\eta_t\left[R\left(\frac{\ln\eta_t}{\ln 2}\right)+(\ln s)T\left(\frac{\ln\eta_t}{\ln 2}\right)\right] + \ln 2 + \ln|\tilde{U}(\ln\eta_t/\ln 2)| - 2\ln\bar{\sigma}. \qquad (68)$$

Another type of dip comes from the term

$$V(\eta_t) = \sum_{n=1}^{\infty} \frac{1}{2^n} \ln\left|\frac{\sin^2\pi\bar{\alpha}(2^n\eta_t)}{1+\cos\pi\bar{\alpha}(2^n\eta_t)}\right| \qquad (69)$$

on the right-hand side of Eq. (66). For even integer values of $\bar{\alpha}(2^n\eta_t)$ with $n=1, 2, 3, \cdots$, the argument of the logarithm function of Eq. (69) vanishes and thus $\ln[\eta_t^2 d\sigma/d\Omega]$ has dips. However, this type of dip is not easy to observe if the slope of the Regge trajectory is relatively small. First of all, we shall show that we can observe only the dips with small values of n in Eq. (69). Suppose that the error of the measurement of the cross section is $\Delta\sigma$ and the nth term of Eq. (69) has a dip at $\eta_t = \eta_t(n)$; then in order to observe the dip, we have to make a measurement of the cross section inside the region

$$|\eta_t - \eta_t(n)| \leq (1/\pi a_0)\exp[-2^{n-1}\Delta\sigma], \qquad (70)$$

which very rapidly becomes narrower with increasing n. In Eq. (70), a_0 is the average value of $T(x)$ and will be defined in Eq. (71). Thus we cannot observe this type of dips except for the case of small n in Eq. (69). In particular, if the slope of the Regge trajectory is small, $\bar{\alpha}(2^n\eta_t)$ goes through even integers at large value of $2^n\eta_t$. Since we can observe only the dips with small n, the dips which we can see occur at large value of momentum transfer η_t. Thus this type of dip is difficult to observe when the slope of the Regge trajectory is small.

5. SEMIEMPIRICAL EXTENSION TO LARGE-ANGLE SCATTERING

Since our results given in Eq. (66) concerning the differential cross section of high-energy scattering with $\eta_t > |t_c|^{\frac{1}{2}}$ contains two arbitrary periodic functions $T(x)$ and $R(x)$, we need, as input information, $d\sigma/d\Omega$ in the range of two periods in order to determine $T(x)$ and $R(x)$. However, in practice we do not know values of $d\sigma/d\Omega$ sufficiently precise to determine $T(x)$ and $R(x)$. Therefore, we introduce an additional assumption that $T(x)$ and $R(x)$ are continuous periodic functions which do not oscillate violently. In other words, when we make a Fourier expansion of the periodic functions $T(x)$ and $R(x)$,

$$T(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos 2\pi nx + \sum_{n=1}^{\infty} b_n \sin 2\pi nx \qquad (71)$$

and

$$R(x) = c_0 + \sum_{n=1}^{\infty} c_n \cos 2\pi nx + \sum_{n=1}^{\infty} d_n \sin 2\pi nx, \qquad (72)$$

we assume that the lower harmonics dominate, namely,

$$|a_0| \gg |a_1|, |b_1| \gg |a_2|, |b_2| \gg \cdots. \qquad (73)$$

The same relation is assumed also for the c_i and d_i. If we retain only the lowest term of the Fourier series, then Eq. (66) is

$$\ln\left[\frac{1}{(\ln s)^2}\frac{2}{s}\eta_t^2\frac{d\sigma}{d\Omega}\right] = 2\eta_t\{c_0 + a_0\ln s\} + W(\eta_t), \qquad (74)$$

where

$$W(\eta_t) = \sum_{n=1}^{\infty} \frac{1}{2^{n-1}} \ln|\sqrt{2}\cos(\tfrac{1}{2}\pi a_0\eta_t)| + \ln|b_1\cos 2\pi x - a_1\sin 2\pi x| + \text{const}. \qquad (75)$$

In Eq. (74), $W(\eta_t)$ is almost constant except for the dips.

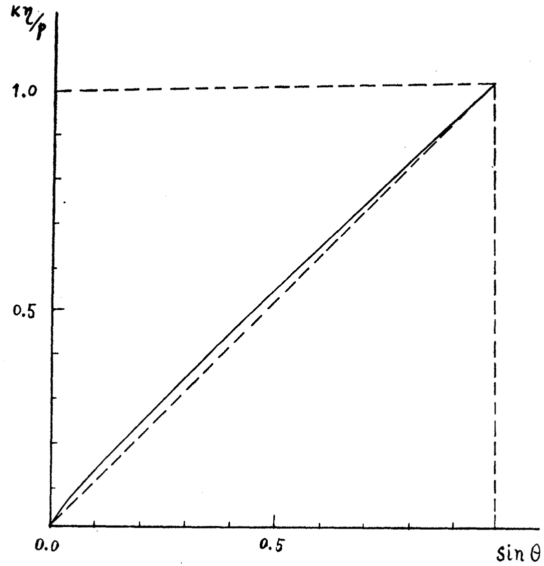

FIG. 6. Plot of $\kappa\eta/p$ versus $\sin\theta$ with $\kappa=1.21$.

Before we apply Eq. (66) to the analysis of high-energy scattering at large momentum transfer, we have to extend the formula, since Eq. (66) was derived in the region of $|t_c|<|t|\ll s$, where the scattering angle θ is small, while the momentum transfer η_t is finite. However, in practice most of the data for scattering with large momentum transfer comes at present from experiments on relatively large-angle scattering. Except for the logarithmic dependence, the first term of the right-hand side of Eq. (74) coincides with Orear's fit given in Eq. (1) barring large scattering angle, since

$$\eta_t=p\sin\theta\,[(1+\cos\theta)/2]^{-\frac{1}{2}}. \qquad (76)$$

When we consider proton-proton scattering, it is evident that Eq. (66) is not applicable to the cross section in the neighborhood of $\theta=90°$, since $d\sigma/d\Omega$ must be symmetric with respect to $\theta=90°$, whereas the right-hand side of Eq. (66) is a decreasing function of η_t. Therefore, it is necessary to make a semiempirical extension in such a way that $d\sigma/d\Omega$ becomes symmetric concerning the exchange of t and u. If we consider that $\sqrt{-t}$ is a more basic variable than t in our case, the simplest way to do so is to introduce a new variable η by

$$\eta=(\sqrt{-t})+(\sqrt{-u})-2p, \qquad (77)$$

which reduces to η_t for $\theta\to 0$ and to $\sqrt{-u}$ for $\theta\to\pi$. We replace η_t with η in Eqs. (66), (68), and (74). The new variable η can also be written as

$$\eta=2p(\sin\tfrac{1}{2}\theta+\cos\tfrac{1}{2}\theta-1). \qquad (78)$$

In terms of this variable η, Eq. (74) is

$$\ln\left[\frac{1}{(\ln s)^2}\frac{2}{s}\eta^2\frac{d\sigma}{d\Omega}\right]=2\eta\{c_0+a_0\ln s\}+W(\eta). \qquad (79)$$

This is very close to Orear's fit plus the dips of $W(\eta)$, since η and $\kappa^{-1}p\sin\theta$ are nearly the same for $\kappa=1.21$ as shown in Fig. 6.

Figure 1 shows the existence of a dip at $p^2=3.4$ $(\text{GeV}/c)^2$ for $\theta=90°$, which corresponds to $\eta=1.5$ GeV/c. Therefore, if this dip comes from the term $\ln|\tilde{U}(\ln\eta/\ln 2)|$ of Eq. (66), which is the case when the slope of the Regge trajectory is small, then we can predict that the second dip must occur at $\eta=3.0$ GeV/c, since $\tilde{U}(x)$ is a periodic function. The second dip at $\eta=3.0$ GeV/c corresponds to $p^2=13.6(\text{GeV}/c)^2$ for $\theta=90°$ scattering. Since $U(x)$ has at least two zeros in one period, we can expect another dip between the two dips at $\eta=1.5$ and 3.0 GeV/c. A more detailed analysis of large-angle scattering using Eq. (66) will be published elsewhere.

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