PHYSICAL REVIEW VOLUME 165, NUMBER 5 25 JANUARY 1968

CP-Conserving Decay $K_1^0 \to \pi^+\pi^-\pi^0$†

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(Received 16 May 1967; revised manuscript received 5 September 1967)

The part of the weak Hamiltonian responsible for the CP-conserving decay $K_1^0 \to (3\pi)_{I=0}$ is constructed phenomenologically from three-meson-field products. With the aid of this Hamiltonian we also discuss the process $K^{+,0} \to \pi^{+,0}l^+l^-$ as well as the P-wave parts of various $K \to 3\pi$ decays. For the CP-conserving decay $K_1^0 \to (3\pi)_{I=0}$ we estimate a decay ratio, $K_1^0 \to 3\pi/K_2^0 \to 3\pi \simeq 10^{-7}$–$10^{-6}$.

I. INTRODUCTION

THE discovery[1] of CP violation in K_2^0 decay has increased the interest in the possible $K_1^0 \to \pi^+\pi^-\pi^0$ decay mode.[2] This decay can proceed as a CP-violating transition to a final state of S-wave pions, and is therefore not inhibited by angular momentum barrier penetration factors. The allowed total isotopic spin of the three-pion wave function is then $I=1$ (obtainable by a $\Delta I=\frac{1}{2}$ or $\frac{3}{2}$ transition) and $I=3$ (obtainable through $\Delta I=\frac{5}{2}$ or $\frac{7}{2}$). In addition, a $K_1^0 \to \pi^+\pi^-\pi^0$ CP-conserving transition can also occur, with the pions in P-wave angular momentum states. In this case, the symmetry restrictions allow for the isotopic spin of the three pions $I=0$ (for $\Delta I=\frac{1}{2}$) and $I=2$ (realizable through $\Delta I=\frac{3}{2}$ or $\frac{5}{2}$).

The experimental search[3] is still inconclusive. With a_1 and a_2 denoting the complex amplitudes for K_1^0, $K_2^0 \to \pi^+\pi^-\pi^0$, and $(a_1/a_2)=x+iy$, Anderson *et al.*[3] obtain $x=0.1_{-0.5}^{+0.4}$, $y=0.2_{-0.8}^{+0.9}$, when the $\Delta I=\frac{1}{2}$ rule is being used to calculate the $K_2^0 \to \pi^+\pi^-\pi^0$ lifetime. If no use is made of this constraint they obtain $x=-0.1_{-0.4}^{+0.5}$, $y=0.6\pm0.9$. One should remark that in the absence of final-state interactions, the $(K_1^0 \to 3\pi)_{CPV}$ amplitude is imaginary, compared to the real chosen $(K_2^0 \to 3\pi)_{CPC}$ amplitude. However, one knows that in practice the pion-pion interaction is not negligible. Nevertheless, when a_1 and a_2 are comparable in magnitude, Glashow and Weinberg[4] point out that CPT invariance and $\Delta I \leqslant \frac{3}{2}$ require the phase of their ratio to be close to $\frac{1}{2}\pi$, if the amplitudes a_1 and a_2 are completely symmetric.[5]

A mere detection of $K_1^0 \to 3\pi$ is evidently not yet sufficient to be attributed to a CP-violating transition and it is of obvious interest to have also estimates for the strength of the CP-conserving transition. This is all the more so as it is often supposed that this transition is negligible compared to the CP-violating part, owing to angular momentum barrier suppression.

The existing estimates for both types of $K_1^0 \to \pi^+\pi^-\pi^0$ amplitudes can be summarized as follows: By using the observed CP violation in $K_2^0 \to 2\pi$ decay as a characteristic strength, one expects[6] for the CP-violating decay $K_1^0 \to 3\pi$, a relative rate

$$\Gamma(K_1^0 \to \pi^+\pi^-\pi^0)_{CPV}/\Gamma(K_2^0 \to \pi^+\pi^-\pi^0)_{CPC} \simeq 10^{-5}.$$

Nonetheless, in some theoretical models[7] a figure as high as 1 is predicted for the above ratio. For the CP-conserving transitions, one finds from angular momentum and symmetry considerations that the amplitudes for $K_1^0 \to (3\pi)_{I=0}$ and $K_1^0 \to (3\pi)_{I=2}$ should behave like $(kR)^6$ and $(kR)^2$, respectively, where k is the average pion momentum and R is a characteristic radius of interaction. By assuming $kR \simeq \frac{1}{3}$, and that the ratio of the $(\Delta I=\frac{3}{2})/(\Delta I=\frac{1}{2})$ transition amplitudes for nonleptonic CP-conserving decays is a few percent, Lee and Wu[6] estimate

$$\Gamma[K_1^0 \to (3\pi)_{I=0}]_{CPC}:\ \Gamma[K_1^0 \to (3\pi)_{I=2}]_{CPC}:\ \Gamma[K_2^0 \to 3\pi]_{CPC} \simeq 10^{-6}:10^{-5}:1.$$

The authors cautiously warn, however, that "these estimated values may even be wrong by several orders of magnitude."

In this article we present a model calculation for the $\Delta I=\frac{1}{2}$ CP-conserving amplitude $K_1^0 \to 3\pi$, which indicates that indeed

$$\Gamma[K_1^0 \to (3\pi)_{I=0}]_{CPC}/\Gamma[K_2^0 \to 3\pi]_{CPC}$$

could reasonably be as high as 10^{-6}. This is done by considering a weak Hamiltonian, from which the process of interest as well as the decays $K^{+,0} \to \pi^{+,0}l^+l^-$ (where l stands for lepton), and the P-wave parts in $(K_2^0 \to \pi^+\pi^-\pi^0)_{CPC}$ and $K^+ \to \pi\pi\pi$ can be calculated and related.

II. WEAK HAMILTONIAN

We are interested here in that part of the weak Hamiltonian which is responsible for nonleptonic CP-

† Based in part on a thesis by S. Eliezer submitted to the Senate of the Technion, Israel Institute of Technology, in partial fulfillment of the requirements for the degree of Master of Science.

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[1] J. H. Christenson, J. W. Cronin, V. L. Fitch, and R. Turlay, Phys. Rev. Letters **13**, 138 (1964).

[2] In our notation, K_1^0 and K_2^0 are the "nearly" eigenstates of CP with eigenvalues $(+1)$ and (-1), respectively, denoted also in the recent literature as K_S and K_L. We shall also use subscripts CPC and CPV to mark CP-conserving and CP-violating transitions, whenever necessary.

[3] J. A. Anderson, F. S. Crawford, Jr., R. L. Golden, D. Stern, T. O. Binford, and V. G. Lind, Phys. Rev. Letters **14**, 475 (1965); **15**, 645(E) (1965); **16**, 968(E) (1966); A. Engler (private communication); L. Behr *et al.*, Phys. Letters **22**, 540 (1966).

[4] S. L. Glashow and S. Weinberg, Phys. Rev. Letters **14**, 835 (1965).

[5] For additional discussion on these topics see M. K. Gaillard Nuovo Cimento **52**, 359 (1967).

[6] T. D. Lee and C. S. Wu, Ann. Rev. Nucl. Sci. **16**, 471 (1966).

[7] S. L. Glashow, Phys. Rev. Letters **14**, 35 (1965).

conserving meson decays with $\Delta Q=0$, $\Delta Y=\pm 1$. Moreover, as the $(K_1^0\to 3\pi)_{CPC}$ decay is also a parity-conserving one, we shall deal only with the parity-conserving part of the Hamiltonian.

As is well known, a current-current Hamiltonian with Cabibbo currents gives rise to $\Delta I=\frac{1}{2}$ as well as to $\Delta I=\frac{3}{2}$ transitions, and needs to be supplemented by some additional mechanism to enhance the $\Delta I=\frac{1}{2}$ part. We choose, therefore, to proceed by building directly an effective Hamiltonian containing the experimentally observed symmetries.

We shall assume henceforth that the relevant part of the Hamiltonian can be built in its effective form from three-meson fields. The part giving rise to the $K\to 3\pi$ decays with P-wave pions derives therefore from a product of one vector (V) and two pseudoscalar (P) fields. This is evidently a vector-dominance model, the decays proceeding through the sequence $P\to P+V\to P+P+P$.

There is now a vast amount of experimental evidence[8] that the CP-conserving nonleptonic (NL) transitions are either purely of $\Delta I=\frac{1}{2}$ type, or contain at most a few percent of $\Delta I=\frac{3}{2}$ transitions. We take the first point of view here, and our Hamiltonian is supposed to contain only $\Delta I=\frac{1}{2}$ transitions. The P and V are taken to be SU_3 multiplets, and then the most general form of our Hamiltonian, assumed to behave like a component of an SU_3 octet[9] and to induce $\Delta Y=\pm 1$, $\Delta Q=0$ transitions, will be

$$\begin{aligned}H_{PC}^{\mathrm{NL}}=Gm^2[&f_1\,\mathrm{Tr}(\lambda_6P'PV)+f_2\,\mathrm{Tr}(\lambda_6PVP')\\&+f_3\,\mathrm{Tr}(\lambda_6VP'P)+f_4\,\mathrm{Tr}(\lambda_6P'VP)+f_5\,\mathrm{Tr}(\lambda_6VPP')\\&+f_6\,\mathrm{Tr}(\lambda_6PP'V)+f_7\,\mathrm{Tr}(\lambda_6P)\,\mathrm{Tr}(P'V)\\&\quad+f_8\,\mathrm{Tr}(\lambda_6P')\,\mathrm{Tr}(PV)+f_9\,\mathrm{Tr}(\lambda_6V)\,\mathrm{Tr}(PP')]. \qquad (1)\end{aligned}$$

In writing this Hamiltonian we used P' for $\partial_\mu P$ and suppressed explicit Lorentz indices. P is taken to be the pseudoscalar octet, while V is the vector nonet, thus taking into account the ω-φ mixing. The usual assumption is taken that no terms proportional to $\mathrm{Tr}(V)$ appear in the Hamiltonian.[10] $G=10^{-5}m_p^{-2}$ is the weak-coupling constant, and m a characteristic mass, so that the f_i's are dimensionless.

The requirements of C conservation (equivalent to CP conservation in our case) and correct Bose statistic properties imposed on (1) give the following relations:

$$\begin{aligned}f_1=-f_5,\quad f_2=-f_4,\quad f_3=-f_6,\\ f_7=f_8=f_9=0,\quad f_1=f_3.\end{aligned}\qquad (2)$$

We are therefore left with two independent amplitudes in the Hamiltonian:

$$\begin{aligned}H_{PC}^{\mathrm{NL}}=Gm^2\{&f_1\,\mathrm{Tr}([P',P]\{V,\lambda_6\})\\&+f_2(\mathrm{Tr}(\lambda_6PVP')-\mathrm{Tr}(\lambda_6P'VP))\}. \qquad (3)\end{aligned}$$

Written explicitly in terms of the various meson fields, our Hamiltonian has the following form:

$$\begin{aligned}H_{PC}^{\mathrm{NL}}=Gm^2\{&K_\mu^{*+}(\sqrt{2}f_1[\pi^0,\partial^\mu\pi^-]+(1/\sqrt{2})(f_2-f_1)[K_1^0,\partial^\mu K^-]-(2/\sqrt{6})f_2[\eta,\partial^\mu\pi^-]\\&-(1/\sqrt{2})(f_1+f_2)[K_2^0,\partial^\mu K^-])+K_\mu^{*-}(\sqrt{2}f_1[\pi^+,\partial^\mu\pi^0]+(1/\sqrt{2})(f_2-f_1)[K_1^0,\partial^\mu K^+]-(2/\sqrt{6})f_2[\eta,\partial^\mu\pi^+]\\&+(1/\sqrt{2})(f_1+f_2)[K_2^0,\partial^\mu K^+])+(K_2^{*0})_\mu(\sqrt{2}f_1[\pi^+,\partial^\mu\pi^-]+\sqrt{2}f_1[K^+,\partial^\mu K^-])+(K_1^{*0})_\mu(2/\sqrt{6})f_2[\pi^0,\partial^\mu\eta]\\&+\rho_\mu^0((1/\sqrt{2})(f_2-f_1)[\pi^0,\partial^\mu K_1^0]+(1/\sqrt{2})(f_1+f_2)[K^-,\partial^\mu\pi^+]+(1/\sqrt{2})(\sqrt{3}f_1-(f_2/\sqrt{3})[\eta,\partial^\mu K_1^0]\\&+(1/\sqrt{2})(f_1+f_2)[\pi^-,\partial^\mu K^+])+\rho_\mu^+((1/\sqrt{2})(f_2-f_1)[\pi^-,\partial^\mu K_1^0]+(1/\sqrt{2})(f_1+f_2)[\pi^-,\partial^\mu K_2^0]\\&+(1/\sqrt{2})(f_1+f_2)[\pi^0,\partial^\mu K^-]+(1/\sqrt{2})(\sqrt{3}f_1-(1/\sqrt{3})f_2)[\eta,\partial^\mu K^-])+\rho_\mu^-((1/\sqrt{2})(f_2-f_1)[\pi^+,\partial^\mu K_1^0]\\&-(1/\sqrt{2})(f_1+f_2)[\pi^+,\partial^\mu K_2^0]-(1/\sqrt{2})(f_1+f_2)[\pi^0,\partial^\mu K^+]-(1/\sqrt{2})(\sqrt{3}f_1-(1/\sqrt{3})f_2)[\eta,\partial^\mu K^+])\\&+\varphi_\mu^0(f_1[\pi^-,\partial^\mu K^+]-f_1[\pi^+,\partial^\mu K^-]-f_1[\pi^0,\partial^\mu K_1^0]+(\sqrt{3}f_1-(2/\sqrt{3})f_2)[\eta,\partial^\mu K_1^0])+\omega_\mu^0((1/\sqrt{2})(f_2-f_1)[\pi^-,\partial^\mu K^+]\\&\qquad-(1/\sqrt{2})(f_2-f_1)[\pi^+,\partial^\mu K^-]-(1/\sqrt{2})(f_2-f_1)[\pi^0,\partial^\mu K_1^0]-(1/\sqrt{2})(\sqrt{3}f_1-(1/\sqrt{3})f_2)[\eta,\partial^\mu K_1^0])\}. \qquad (4)\end{aligned}$$

In the expression (4), K_1^0 and K_2^0 are defined as eigenstates of CP with eigenvalues $(+1)$ and (-1), respectively, while K_1^{0*} and K_2^{*0} are eigenstates of CP with eigenvalues (-1) and $(+1)$, respectively. f_1 and f_2 are real by the requirement of CPT invariance.

From the Hamiltonian (4), various weak processes can be calculated. The most straightforward ones are the weak nonleptonic decays of vector mesons into two pseudoscalar mesons. Relations among such transitions can be obtained from (4). They are, however, of no practical interest at this stage, and we shall return to this in the last section.

By considering the strong coupling of vector mesons to two pseudoscalar mesons and also their electromagnetic coupling, the Hamiltonian (4) can be used to calculate the processes $K_1^0\to\pi^+\pi^-\pi^0$ (CP conserving) and $K^+\to\pi^+e^+e^-$, $K^+\to\pi^+\mu^+\mu^-$ [as well as $K_1^0\to\pi^0l^+l^-$ and the P-wave parts of $(K_2^0\to\pi^+\pi^-\pi^0)_{CPC}$ and $K^+\to 3\pi$]. By using the experimental upper limit for $K^+\to\pi^+l^+l^-$, as well as theoretical considerations, some meaningful conclusions can be drawn for the expected rate of $K_1^0\to(\pi^+\pi^-\pi^0)_{I=0}$.

[8] N. Cabibbo, in *Proceedings of the Thirteenth Annual International Conference on High-Energy Physics, Berkeley, 1966* (University of California Press, Berkeley, 1967).

[9] M. Gell-Mann, Phys. Rev. Letters **12**, 155 (1964); B. W. Lee, *Lectures in Theoretical Physics* (University of Colorado Press, Boulder, 1964), Vol. VIIB, p. 51.

[10] S. Okubo, Phys. Letters **5**, 165 (1963); S. L. Glashow and R. H. Sokolow, Phys. Rev. Letters **15**, 329 (1965).

III. CALCULATIONS

We consider first the decay $K^+ \to \pi^+ l^+ l^-$. This type of weak decay, involving an electromagnetically induced neutral lepton current, has been discussed by several authors in recent yeas.[11–14] Its descrption involves the knowledge of the $\langle K^+ | J_\mu | \pi^+ \rangle$ weak electromagnetic transition. Assuming vector-meson dominance of the electromagnetic current, we calculate this decay by using the Hamiltonian (4) and the direct transition vector meson $\leftrightarrow \gamma$.[15] The two relevant Feynman diagrams are summarized in Fig. 1(a). ϕ_8 is the appropriate combination of the ω and φ fields which has octet transformation properties under SU_3 and thus couples to the photon (the isoscalar part), i.e.,

$$\phi_8 = (1/\sqrt{3})\omega + \sqrt{2}/\sqrt{3})\varphi .$$

The strength of the vector-meson–photon transition is taken as $f_{\rho\gamma} = e/f_\rho$, $f_{\phi_8\gamma} = e/f_{\phi_8}$, with a gauge-invariant vertex function

$$\tfrac{1}{2} e f_{V\gamma} (p_\mu{}^{(V)} \epsilon_\nu{}^{(V)} - p_\nu{}^{(V)} \epsilon_\mu{}^{(V)}) (p_{(\gamma)}{}^\mu \epsilon_{(\gamma)}{}^\nu - p_{(\gamma)}{}^\nu \epsilon_{(\gamma)}{}^\mu) .$$

The matrix element for the $K^+ \to \pi^+ l^+ l^-$ transition is given then by

$$M = \sqrt{2} e^2 G m^2 (f_1 + f_2) \times (f_{\rho\gamma} + (1/\sqrt{3}) f_{\phi_8\gamma}) \frac{p_\mu{}^{(K)} \bar{u}(p_-) \gamma^\mu v(p_+)}{(p_+ + p_-)^2 - m_V{}^2} , \quad (5)$$

where $p^{(K)}$, p_+, p_- are the four-momenta of K^+, l^+, l^-, and we have simplified to $m_\rho = m_{\phi_8} = m_V$.

By using SU_3 we now relate $f_{\phi_8} = \sqrt{3} f_\rho$. For f_ρ we use the value recently obtained experimentally[16] by measuring the branching ratio for ρ^0 decay into lepton pairs. The observed ratio $(\rho \to \mu^+\mu^- / \rho \to \text{all}) = 0.44_{-0.07}{}^{+0.16}$ gives $(f_\rho{}^2/4\pi) = 2.5$, in very good agreement[17] with the equality $f_\rho = g_{\rho\pi\pi}$ predicted by the ρ dominance of the pion form factor.

[11] N. Cabibbo and E. Ferrari, Nuovo Cimento **18**, 928 (1960).
[12] L. B. Okun' and A. Rudik, Zh. Eksperim. i Teor. Fiz. **39**, 600 (1960) [English transl.: Soviet Phys.—JETP **12**, 422 (1961)].
[13] M. Baker and S. L. Glashow, Nuovo Cimento **25**, 857 (1962).
[14] M. A. Baqi Bég, Phys. Rev. **132**, 426 (1963).
[15] M. Gell-Mann and F. Zachariasen, Phys. Rev. **124**, 953 (1961); see also N. Kroll, T. D. Lee, and B. Zumino, *ibid.* **157**, 1376 (1967); P. Singer, Phys. Rev. Letters **12**, 524 (1964).
[16] R. A. Zdanis, L. Madansky, R. W. Kraemer, S. Herzbach, and R. Strand, Phys. Rev. Letters **14**, 721 (1965); J. K. de Pagter, J. I. Friedman, G. Glass, R. C. Chase, M. Gettner, E. von Goeler, R. Weinstein, and A. M. Boyarski, *ibid.* **16**, 35 (1966); see also S. D. Drell, in *Proceedings of the Thirteenth Annual International Conference on High-Energy Physics, Berkeley, 1966* (University of California Press, Berkeley, 1967), for the corrected value we quoted.
[17] J. J. Sakurai, Phys. Rev. Letters **17**, 1021 (1966).

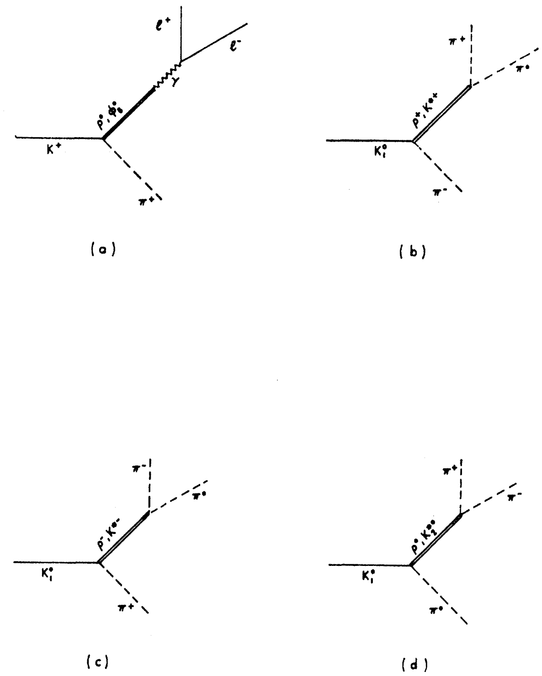

FIG. 1. Feynman diagrams (a) for the process $K^+ \to \pi^+ l^- l^+$ and (b), (c), (d) for CP-conserving part of the process $K_1{}^0 \to \pi^+\pi^-\pi^0$.

The decay width is found to be

$$\Gamma(K^+ \to \pi^+ l^+ l^-) = \frac{4G^2 m^4 (f_1 + f_2)^2 \alpha^2}{27\pi^2 (f_\rho{}^2/4\pi)} m_K I , \quad (6)$$

with

$$\omega_{\max} = (1/2m_K)(m_K{}^2 + \mu^2 - 4m_l{}^2) ,$$

$$I = \int_\mu^{\omega_{\max}} \frac{d\omega (\omega^2 - \mu^2)^{3/2}}{(m_K{}^2 + \mu^2 - m_V{}^2 - 2m_K\omega)^2} \times \left(1 + \frac{2m_l{}^2}{m_K{}^2 + \mu^2 - 2m_K\omega}\right) \times \left(1 - \frac{4m_l{}^2}{m_K{}^2 + \mu^2 - 2m_K\omega}\right)^{1/2} , \quad (7)$$

where m_K, μ, m_l, are respectively, the kaon, pion, and lepton mass and α is the fine-structure constant. For m_V we use the ρ mass in the numerical calculations.

From (6) and (7) we obtain

$$\Gamma(K^+ \to \pi^+ \mu^+ \mu^-) / \Gamma(K^+ \to \pi^+ e^+ e^-) = 0.2 . \quad (8)$$

So far the K^+ decays to neutral leptonic pairs have not been positively identified, and only upper limits have been established experimentally. These are: $(K^+ \to \pi^+ e^+ e^-)/(K^+ \to \text{all}) < 1.1 \times 10^{-6}$ (Ref. 18) and

[18] U. Camerini, D. Cline, W. F. Fry, and W. M. Powell, Phys. Rev. Letters **13**, 318 (1964).

$(K^+\to\pi^+\mu^+\mu^-)/(K^+\to \text{all})<3\times10^{-6}$.[19] By using the limit on the electronic decay, which is the more stringent one, one obtains from (6)

$$G^2m^4(f_1+f_2)^2<1.8\times10^{-13}. \quad (9)$$

We now turn to calculations of the CP-conserving decay $K_1^0\to\pi^+\pi^-\pi^0$. This mode can be obtained from (4) by considering the sequence $P\to V+P\to P+P+P$ with an intermediate vector meson. The strong coupling of vector mesons to pseudoscalar mesons is used in its SU_3 invariant form, namely with an effective Lagrangian of the form $L_{\text{eff}}=f_{VPP}\,\text{Tr}(V_\mu[P,\partial^\mu P])$.

The diagrams contributing to this decay are given in Figs. 1(b), (c), and (d). In addition to the diagrams with the weak vertex preceding the strong one [e.g., $K_1^0\to(\rho^+)+\pi^-\to\pi^+\pi^0\pi^-$], one has to include also the diagrams with the reversed order [e.g., $K_1^0\to(K^{*+})+\pi^-\to\pi^+\pi^0\pi^-$]. We shall again assume that $m_{K^*}=m_\rho=m_V$ in order to simplify calculations, and keeping in mind that this hardly affects our numerical estimate. Then the matrix element for $(K_1^0\to\pi^+\pi^-\pi^0)_{CPC}$ is

$$M=\sqrt{2}Gm^2f_{VPP}(f_2-2f_1)\left\{\frac{(p_K+q_0)(q_+-q_-)}{(p_K-q_0)^2-m_V^2}+\frac{(p_K+q_+)(q_--q_0)}{(p_K-q_+)^2-m_V^2}+\frac{(p_K+q_-)(q_0-q_+)}{(p_K-q_-)^2-m_V^2}\right\}. \quad (10)$$

The decay rate is given by

$$\Gamma(K_1^0\to\pi^+\pi^-\pi^0)=\frac{3G^2m^4(f_2-2f_1)^2(f_{VPP}^2/4\pi)}{8\pi^2m_K}|J-L|, \quad (11)$$

with

$$J=\frac{4m_K^2}{3}\int_\mu^{\omega_{\max}}\frac{d\omega[\varphi(\omega)]^3}{(m_K^2+\mu^2-2m_K\omega-m_V^2)^2},$$

$$L=\int_\mu^{\omega_{\max}}\frac{d\omega\ \varphi(\omega)\chi(\omega)}{m_V^2-m_K^2-\mu^2+2m_K\omega}\times\left\{2-\frac{\lambda(\omega)}{m_K\varphi(\omega)}\ln\frac{\lambda(\omega)+m_K\varphi(\omega)}{\lambda(\omega)-m_K\varphi(\omega)}\right\}, \quad (12a)$$

$$\lambda(\omega)=\mu^2-m_V^2+m_K\omega,$$

$$\chi(\omega)=2(-m_V^2+\mu^2-m_K^2+4m_K\omega),$$

$$\varphi(\omega)=(\omega^2-\mu^2)^{1/2}\left(1-\frac{4\mu^2}{m_K^2+\mu^2-2m_K\omega}\right)^{1/2},$$

$$\omega_{\max}=(1/2m_K)(m_K^2-3\mu^2).$$

In performing numerical calculations we use $m_V=m_\rho$, and for f_{VPP} which is related to $g_{\rho\pi\pi}$ by $f_{VPP}=\frac{1}{2}g_{\rho\pi\pi}$ from the definition of the strong-interaction Lagrangian, we use the experimentally determined $g_{\rho\pi\pi}^2/4\pi=2.4$,

Then we obtain

$$J=3.51239\ \text{MeV}^2,\quad L=3.51261\ \text{MeV}^2. \quad (12b)$$

An enormous reduction occurs, therefore, in the rate for $K_1^0\to\pi^+\pi^-\pi^0$, due to the totally antisymmetric form of the matrix element (10). The difference $|J-L|$ is smaller by a factor $\sim10^4$ than J, which represents the contribution to the rate from an individual term in (10). Such a strong cancellation could induce gross numerical errors. Our computation is performed with the necessary accuracy, the numerical error in $|J-L|$ being less than 10%.

For an additional check, we calculated $\Gamma(K_1^0\to\pi^+\pi^-\pi^0)$ with a nonrelativistic approximation to Eq. (10). To this end, we combine the three terms in (10) to obtain

$$M\propto\frac{(q_0\cdot q_--q_-\cdot q_+)(q_0\cdot q_+-q_-\cdot q_+)(q_0\cdot q_--q_0\cdot q_+)}{[m_V^2-(p_K-q_0)^2][m_V^2-(p_K-q_-)^2][m_V^2-(p_K-q_+)^2]},$$

and we neglect the kinetic energies of pions in the denominator, i.e., $m_V^2-(p_K-q_\pi)^2\simeq m_V^2-(m_K-\mu)^2$. The rate is now obtained with the technique of Barrett *et al.*,[20] and we verified in this manner the correctness of the result given in Eqs. (11) and (12).

IV. DISCUSSION AND CONCLUSIONS

(A) The dependence of $K^+\to\pi^+l^+l^-$ and $K_1^0\to\pi^+\pi^-\pi^0$ on the coefficients f_1 and f_2 exhibited in Eqs. (6) and (11) does not allow us to directly deduce either one from the experimental knowledge of the other. However, we believe that some meaningful considerations can be made on the basis of these equations.

For $K^+\to\pi^+e^+e^-$, indeed only the upper limit of 1.1×10^{-6} for its nonoccurrence is known. The various theoretical calculations[11–14,21] of this process, which treat it as electromagnetically induced, predict for it a ratio very close to this limit, namely, between 10^{-7} and 10^{-6}. Combining the experimental as well as this theoretical information, one expects therefore from Eqs. (6) and (9) that

$$G^2m^4(f_1+f_2)^2\lesssim10^{-14}\text{–}1.8\times10^{-13}. \quad (13)$$

[19] U. Camerini, D. Cline, G. Gidal, G. Kalmus, and A. Kernan, Nuovo Cimento **37**, 1795 (1965).
[20] B. Barrett, M. Jacob, M. Nauenberg, and T. N. Troung, Phys. Rev. **141**, 1342 (1966).
[21] K. Tanaka, Phys. Rev. **151**, 1203 (1966).

A first estimate for the $(K_1{}^0 \to \pi^+\pi^-\pi^0)_{CPC}$ decay mode can now be obtained by assuming

$$(f_1+f_2)^2 \simeq (f_2-2f_1)^2. \quad (14)$$

By using Eq. (13) and (14) one obtains from Eq. (11)

$$\Gamma(K_1{}^0 \to \pi^+\pi^-\pi^0)_{CPC} \lesssim 8.6\times10^{-2}\text{–}1.6 \text{ sec}^{-1}, \quad (15)$$

which gives

$$\Gamma(K_1{}^0 \to \pi^+\pi^-\pi^0)_{CPC}/\Gamma(K_2{}^0 \to \pi^+\pi^-\pi^0) \lesssim 0.37\times10^{-7}\text{–}0.68\times10^{-6}. \quad (16)$$

One could try various possibilities for the ratio of f_1, f_2, such as $f_1 \ll f_2$, $f_2 \ll f_1$, $f_2 \approx f_1$. None of these possible choices changes appreciably the result given in Eq. (16), the maximal change being less than one order of magnitude. The only obvious exceptions are $f_1 \approx -f_2$ and $f_1 \approx \frac{1}{2}(f_2)$. For the first case the $K^+ \to \pi^+ l^+ l^-$ would be strongly depressed, while for the second one this would happen to $(K_1{}^0 \to \pi^+\pi^-\pi^0)_{CPC}$. However, we consider these to be possible either by a "Clebsch-Gordan accident" or because of the assumption of a higher symmetry. Although both cases are possible in principle, they do not relate to the main purpose of our work. Our model is intended to take into account the reduction of $K_1{}^0 \to \pi^+\pi^-\pi^0$, due to angular momentum barriers, by use of a specific model as this is generally considered to be the reason for the inhibition of this decay mode. Moreover, our Hamiltonian contains only those symmetries which are on a more solid ground. Therefore, the possible above-mentioned "accidents" bear no relevance to the problem under study, and, on the basis of the calculations and considerations we described, one expects a ratio $(K_1{}^0 \to \pi^+\pi^-\pi^0)_{CPC}/(K_2{}^0 \to \pi^+\pi^-\pi^0)$ of the order of 10^{-7}–10^{-6}.

(B) The Hamiltonian we have suggested can also be used to calculate the P-wave contribution to the decay modes $K_2{}^0 \to \pi^+\pi^-\pi^0$, $K^+ \to \pi^+\pi^+\pi^-$, and $K^+ \to \pi^+\pi^0\pi^0$. For the K^+ decays, we obtain for the amplitudes to P-wave decays

$$A(K^+ \to \pi^+\pi^+\pi^-)_{P\text{ waves}} = -A(K^+ \to \pi^+\pi^0\pi^0)_{P\text{ waves}}. \quad (17)$$

This is of course consistent with the isotopic-spin wave function, which describes three pions in a state $I=1$, where a pair of pions has also $I^{(1,2)}=1$, namely

$$|I^{(1,2)},I,I_3\rangle = |1,1,1\rangle = \tfrac{1}{2}\{|\pi^+\pi^0\pi^0\rangle - |\pi^0\pi^+\pi^0\rangle - |\pi^+\pi^-\pi^+\rangle + |\pi^-\pi^+\pi^+\rangle\}. \quad (18)$$

For the contribution of the P wave to the total decay rate we obtain

$$\Gamma(K^+ \to \pi^+\pi^+\pi^-)_{P\text{ waves}} = \frac{G^2 m^4 (f_2+2f_1)^2 (f_{VPP}{}^2/4\pi)}{8\pi^2 m_K} J, \quad (19)$$

where J is given in Eq. (12). By again reasoning along the same lines as before for the coefficients f_1, f_2, we obtain using $(f_1+f_2)^2 \simeq (f_2+2f_1)^2$ and Eqs. (12b), (13),

$$\Gamma(K^+ \to \pi^+\pi^+\pi^-)_{P\text{ waves}}/\Gamma(K^+ \to \pi^+\pi^+\pi^-)_{\text{exp}} \lesssim 0.54\times10^{-3}\text{–}0.96\times10^{-2}, \quad (20)$$

while for the same ratio in τ' decay the ratio is 4 times as large. This seems to us a very reasonable number for the P-wave contribution.

For the P-wave contribution in $K_2{}^0 \to \pi^+\pi^-\pi^0$ we obtain

$$\Gamma(K_2{}^0 \to \pi^+\pi^-\pi^0)_{P\text{ waves}} = \frac{G^2 m^4 (f_2+2f_1)^2 (f_{VPP}{}^2/4\pi)}{8\pi^2 m_K}(2J+L), \quad (21)$$

with J and L given in (12).

We can compare our result with a recent estimate of Truong[22] for the P-wave contribution in $K_2{}^0 \to \pi^+\pi^-\pi^0$ decay. Truong uses for the $(K_2{}^0 \to \pi^+\pi^-\pi^0)_{CPC}$ amplitude the expression $M = \lambda + h_1 r \cos\theta$, and using the experimental data for the π^0 energy spectrum he obtains $h_1/\lambda = -0.4$. In performing the fit he includes an S-wave π-π interaction in the $I=0$ state with scattering length $a_0 = 1.5\hbar/\mu c$. From Truong's result one can calculate with his amplitude the ratio of the

$$\Gamma(K_2{}^0 \to \pi^+\pi^-\pi^0)_{P\text{ waves}}/\Gamma(K_2{}^0 \to \pi^+\pi^-\pi^0)_{S\text{ waves}},$$

thus obtaining 4.4×10^{-2}. If we use this ratio in combination with our result (21), we get

$$G^2 m^4 (f_2+2f_1)^2 = 3.8\times10^{-13},$$

in very good agreement with our considerations. Hence, we can conclude that our Hamiltonian gives quite reasonable results for the P-wave contributions to K decays.

(C) As we have already mentioned, the Hamiltonian (4) can be used to calculate nonleptonic weak decays of vector mesons. Relations between amplitudes can be obtained from it. To give an example:

$$A(\rho^+ \to K_1{}^0\pi^+) + A(\rho^+ \to K_2{}^0\pi^+) = \sqrt{3}A(K^{*+} \to \eta\pi^+), \quad (22)$$

as well as many other similar relations which can easily be written down. Unfortunately, there does not seem to be any sensible way to check such relations experimentally in the foreseeable future.

(D) The decay $K_1{}^0 \to \pi^0 l^+ l^-$ can also be calculated in our model, and it turns out to be proportional to $(f_2-2f_1)^2$, and hence directly related to the $(K_1{}^0 \to \pi^+\pi^-\pi^0)_{CPC}$ decay, although the basic contributing diagrams are quite different. The decay rate for this mode comes out as given in Eq. (6), with $(f_2-2f_1)^2$ replacing $(f_1+f_2)^2$. If we again consider $(f_2-2f_1)^2 \simeq (f_1+f_2)^2$, then the experimental upper limit for $K^+ \to \pi^+ e^+ e^-$ combined with our model gives

$$\Gamma(K_1{}^0 \to \pi^0 e^+ e^-) | \Gamma(K_1{}^0 \to \text{all}) < 7\times10^{-9}. \quad (23)$$

[22] T. N. Truong, Phys. Rev. Letters **17**, 153 (1966).

In our model there is no direct relationship between $K_1^0 \to \pi^0 e^+e^-$ and $K^+ \to \pi^+ e^+ e^-$, unless some relation is assumed between f_1 and f_2.

(E) As we mentioned in the Introduction, the estimate of Lee and Wu[6] for the ratio $(K_1^0 \to \pi^+\pi^-\pi^0)_{CPC}/(K_2^0 \to \pi^+\pi^-\pi^0)$ is $\simeq 10^{-6}$. This was based on the fact that this ratio is proportional to $(kR)^{12}$ and that kR has been chosen to be $\frac{1}{3}$, corresponding to R, approximately equal to the pion Compton wavelength.[23] Our amplitude for $K_1^0 \to \pi^+\pi^-\pi^0$ obviously also behaves like "$(kR)^6$," as can be easily checked from Eq. (10). The value of the equivalent radius is however determined by the parameters of the model (like f_{VPP}, m_V). The result in Eq. (16) supports the choice of Lee and Wu in their rough estimate of the rate. It is of interest therefore to have a model also for the $K_1^0 \to (3\pi)_{I=2}$ amplitude, as this might turn out to be the dominant CP-conserving transition.[6]

(F) The strong cancellation occurring in Eq. (11) implies that electromagnetic corrections to the process could be significant. The reduction caused by the cancellation being of the order of 10^4, one expects in fact corrections as large as the matrix element itself. The simplest correction would be to take into account the mass difference $\mu_{\pi^+} - \mu_{\pi^0}$ and to allow for a small difference (e.g., a few per thousand) between $g_{\rho^0\pi^+\pi^-}$ and $g_{\rho^+\pi^+\pi^0}$. The effect of such corrections has been studied with a similar matrix element for the $\eta \to 3\pi$ C-nonconserving decay,[24] with the conclusion that the rate is increased by a factor of 1.5–2. As we do not know the accuracy of our SU_3 assumptions, there is no point in making detailed estimates. One should however keep in mind that the electromagnetic effect of isospin nonconservation alone, could change our numerical figure for $K_1^0 \to (\pi^+\pi^-\pi^0)_{I=0}$ by a factor of ~ 2.

[23] It should be remarked, however, that by using the same argumentation, N. Byers, S. W. MacDowell, and C. N. Yang [*High-Energy Physics and Elementary Particles* (International Atomic Energy Agency, Vienna, 1965)] predict a ratio

$$(K_1^0 \to \pi^+\pi^-\pi^0)_{CPC}/(K_2^0 \to \pi^+\pi^-\pi^0) \simeq 10^{-4}$$

which is much larger than our estimate. As these authors use $R \simeq 1/\mu$, i.e., the same value used by Lee and Wu, it seems to us that the discrepancy stems from a different interpretation for k.

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PHYSICAL REVIEW VOLUME 165, NUMBER 5 25 JANUARY 1968

Regge Theory of High-Energy Scattering with Relatively Large Momentum Transfer

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Functional equations for the Regge parameters $\alpha(t)$ and $\beta(t)$ are derived from the unitarity condition, which is valid for relatively large momentum transfer. By solving these equations we compute the high-energy cross section with relatively large momentum transfer up to two arbitrary periodic functions. The result agrees with Orear's fit except for the appearance of dips. We compare this result with high-energy experiments on p-p scattering with large momentum transfer, and some prediction is made concerning the position of the dips.

1. INTRODUCTION

A NUMBER of empirical formulas have been proposed[1,2] for the differential cross section of high-energy proton-proton scattering with large momentum ftranser. Among these the simplest is Orear's fit[2]

$$s\frac{d\sigma_{\rm el}}{d\Omega}(s,t) = Ae^{(-ap\sin\theta)}, \qquad (1)$$

with $A = 595 \pm 135$ GeV^2 mb/sr and $1/a = 158 \pm 3$ MeV/c, which is true for relatively large momentum transfer, namely, for $|t| \gtrsim 1$ (GeV/c)^2. It is remarkable that Eq. (1) can cover measurements with a wide range of incident momenta—from 1.7 to 31.8 GeV/c. However, recent measurements made by Allaby *et al.*,[3] and also by Clyde *et al.*,[4] revealed a significant deviation from Orear's fit, although Eq. (1) can still reproduce the gross features of the elastic scattering. Another series of measurements of the elastic differential cross section for 90° center-of-mass (c.m.) scattering angle was performed by Akerlof *et al.*[5] for the range of incident momenta from 5.0 to 13.4 GeV/c. The plot of $\ln(d\sigma/dt)$ versus p^2 was fitted by two straight lines with a break at $p^2 = 3.4$(GeV/c)^2. In order to compare this measurement with Orear's formula, we plot the deviation Δ,

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[3] J. V. Allaby *et al.*, Phys. Letters **23**, 389 (1966).

[4] A. R. Clyde *et al.*, University of California Radiation Laboratory Report No. UCRL-11441 (unpublished); UCRL-16275 (unpublished).

[5] C. W. Akerlof *et al.*, Phys. Rev. Letters **17**, 1105 (1966).