attraction and a not-too-short-range repulsion. In this case, as mentioned by the authors, the repulsion provides shielding to the higher-order branch cuts due to the attractive part. It is likely that if we consider the BS and N/D equations for such a case, the two results may not differ very significantly. However, for the present, it must remain as a hope that such repulsive forces exist in more than a few isolated cases. Similar remarks can be made about an interesting proposal by Chew[11] that long-range repulsive forces obtained from Regge exchanges may help in reducing the predicted widths of resonances or bound-state coupling constants from the N/D method. In the present work we have shown that with attractive single-particle-exchange potentials, the results of the BS equation are considerably better than those of the N/D equations even for the case of small binding. Of course, because of the practical difficulties of solution in realistic cases and the possibly strong model dependence of the results, one can hardly say at present that the BS equation will be a better tool for quantitative calculations than the S-matrix theory.

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ACKNOWLEDGMENTS

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Possible Existence of a Spin-2 Regge Recurrence of the $^1S_0$ p-p Pole (The Unbound Diproton)*

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A model is developed wherein the $^1D_2$ p-p partial-wave amplitude is coupled through N/D equations to a second, inelastic channel containing a proton and a second particle in a relative S wave. The mass of the second particle is "smeared out" according to an appropriate Breit-Wigner form around the (3,3) resonance at $1260-i120$ MeV. The phenomenological potential is adjusted to fit the elastic $^1D_2$ phase shift to 400 MeV and the existing imprecise data at 660 MeV. The model, when adjusted to fit experiment, predicts a pole on the second Riemann sheet at $400-i300$ MeV. This pole may be interpreted as a bound state in the inelastic channel and has quantum numbers ($I=1$, $B=2$, $J=2$) consistent with the spin-2 Regge recurrence of the pole in the $^1S_0$ amplitude, the "unbound" diproton. An alternative coupled-channel model is developed which fits the elastic data but which does not predict the pole. The high-energy predictions for elastic scattering which are derived from the second model are found to differ drastically from those of the first model and from the predictions of various theories for high-energy inelastic processes.

I. INTRODUCTION

An important aspect in the understanding of $p-p$ elastic scattering in an energy range from 400 to 700 MeV is the way in which the inelastic events are dominated by $N^*(3,3)$ production. This idea forms the basis for the well-known Mandelstam[1] model and has served as a foundation for the development of a variety of partial-wave representations for $p-p$ scattering in the energy range to 700 MeV.[2] The most important aspect of this model is the expected strong coupling between the $^1D_2$ $p-p$ elastic state and the S-wave components of the inelastic, $p-N^*(3,3)$, state.

Recently, Chew[3] has suggested that the very strong resemblance in singularity structure between the $p-p$, $^1D_2$ state and the $\pi-N$, D_{13} state may imply that in the $p-p$ system, as in the $\pi-N$ system,[4] a pole exists and is responsible for the enhancement in high-energy reaction cross section. In Sec. II we describe a coupled-channel N/D model which realistically produces the full singularity structure of the $^1D_2$ amplitude and compare the scattering to the D_{13}, $\pi-N$ system. In Sec. III the model is adjusted to fit the elastic $^1D_2$

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[2] There are numerous references on the use of the dominance of $N^*(3,3)$ production in high-energy p-p scattering, e.g., N. Hoshizaki and S. Machida, Progr. Theoret. Phys. (Kyoto) **29**, 49 (1963).

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scattering amplitude,[5–7] and we locate the pole predicted by our parameters. In Sec. IV we discuss the tentative nature of our prediction and develop a second coupled-channel model which fits the existing data nearly as well as the first, but which does not predict the pole. The high-energy predictions of the second model are shown to be at variance with various theoretical predictions for inelastic processes. Finally, in Sec. V, we draw conclusions and discuss some aspects of our results.

II. COUPLED-CHANNEL N/D FOR $^1D_2$, p-p SCATTERING

The low-energy $^1D_2$ singularity structure is shown in Fig. 1; it consists of four important discontinuities. First is the force or potential cut covering the region ($-\infty$ to -10 MeV). (We shall use a lab kinetic-energy variable T which is related to the momentum variable k by $k^2=\frac{1}{2}MT$, where M=nucleon mass.) Second is the elastic unitarity cut covering the region (0 to ∞). Third is a cut associated with one-pion production covering the region (280 MeV to ∞). And finally, there is a cut associated with $N^*(3,3)$ production ($650-i140$ MeV to ∞). The position of the last branch point corresponds to the mass or the N^* having the value $1238-120i$ MeV. A nearly identical singularity structure characterizes the D_{13} state for $\pi-N$ scattering and can be obtained by replacing the incident proton with a pion. In this latter system (D_{13}), an absorptive resonance occurs around 611 MeV; the resonance can be predicted by a coupled channel ($\pi-N$, D wave and $\pi-N^*$, S wave) model in which a phenomenological force pole is adjusted to fit the elastic scattering data. We shall develop a similar model to explain the elastic $p-p$ scattering data and examine the consequent predictions for resonance type behavior.

We start with and N/D representation for the scattering amplitude M

$$M=\rho^{1/2}ND^{-1}\rho^{1/2},$$

$$\rho=\begin{pmatrix}\rho_1 & 0\\ 0 & \rho_2\end{pmatrix}. \tag{2.1}$$

The potential V will be given by a sum of two poles located at T_1 and T_2:

$$V\equiv V_1+V_2=\frac{1}{T-T_1}\begin{pmatrix}\gamma_- & \gamma_0\\ \gamma_0 & \gamma_+\end{pmatrix}+\frac{1}{T-T_2}\begin{pmatrix}\gamma_{2-} & \gamma_{20}\\ \gamma_{20} & \gamma_{2+}\end{pmatrix}. \tag{2.2}$$

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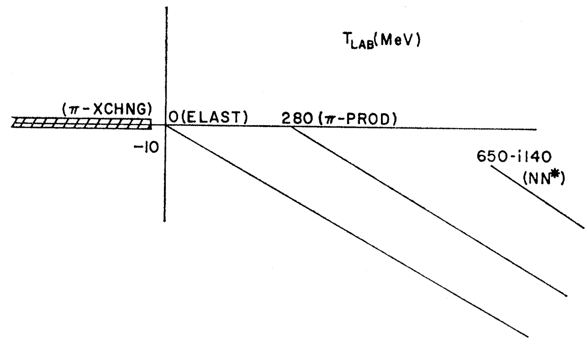

FIG. 1. Singularity structure in the T plane near the threshold for pion production for the $^1D_2$ partial-wave amplitude. We have illustrated, from left to right on the figure, the important singularities that arise from pion exchange, elastic unitarity, pion production, and N^* production.

We have here associated a $-$ subscript with the elastic ($pp\to pp$) channel, a $+$ subscript with the elastic ($pN^*\to pN^*$) channel, and a 0 subscript with the inelastic coupling of ($pN^*\leftrightarrow pp$). The "potential" V is, of course, symmetric in the channel indices.

Unitarity gives us the familiar integral relationship of D to N:

$$D_{ij}=\delta_{ij}-\frac{T-T_1}{\pi}\int_{T_i^{\text{th}}}^{\infty}\frac{\rho_i(T')N_{ij}(T')}{(T'-T)}dT'. \tag{2.3}$$

T_i^{th} is the laboratory kinetic energy in MeV at which channel i (pp or pN^*) first becomes kinematically allowed.

D has been normalized to 1 at T_1. The ρ's appearing in Eq. (2.3) are phase-space factors for the two channels. For the first (elastic) channel, we take a D-wave phase-space factor

$$\rho_1(T)=\frac{T^2}{(1+2/T)^{-1/2}},\quad T_1^{\text{th}}=0. \tag{2.4}$$

Equation (2.4) and all subsequent equations employ energies and masses expressed in nucleon mass units (938 MeV).

For the second channel we choose an unequal-mass S-wave phase-space factor where the effective mass comes from that part of the Breit-Wigner spectrum for which the second channel is "open";

$$\rho_2(T)=\frac{1}{W}\int_{1+\mu}^{M_u}\frac{dM'}{(M'-M^*)^2+(\Gamma/2)^2}\times\frac{[(3-M'^2+2T)^2-4M'^2]^{1/2}}{(2+T)},$$
$$M_\mu=(4+2T)^{1/2}-1, \tag{2.5}$$

$T_2^{\text{th}}=2\mu+\frac{1}{2}\mu^2$ (pion production threshold), where

$$\mu=\text{pion mass } (0.144),$$

$$M^*=(3,3)\text{ mass } (1.32),$$

$$\Gamma=(3,3)\text{ width } (0.128),$$

$$W=\int_{1+\mu}^{\infty}\frac{dM'}{(M'-M^*)^2+(\frac{1}{2}\Gamma)^2}.$$

We can now express N in terms of the potential parameters V_1, V_2, and the D function as evaluated at T_2;

$$N(T)=V_1+V_2D(T_2). \tag{2.6}$$

The dispersion integrals in Eq. (2.3) which involve the D-wave phase-space factor (2.4) will diverge unless

$$N_{12},N_{11}\xrightarrow[T\to\infty]{}\frac{1}{T^2}. \tag{2.7}$$

We accomplish this by an appropriate choice of residue parameters at the second pole. This will require specification of γ_{2-} and γ_{20}. γ_{2+} will be determined to simplify the expression for N. The residue factors at the second pole will be determined to satisfy

$$\begin{aligned}&\text{(a)}\quad \gamma_{2-}D_-(T_2)+\gamma_{20}D_{0+}(T_2)=-\gamma_-,\\&\text{(b)}\quad \gamma_{2-}D_{0-}(T_2)+\gamma_{20}D_+(T_2)=-\gamma_0,\\&\text{(c)}\quad \gamma_{20}D_{0-}(T_2)+\gamma_{2+}D_+(T_2)=0,\end{aligned} \tag{2.8}$$

where $D(T)$ has the implied structure

$$D(T)=\begin{pmatrix}D_-(T) & D_{0-}(T)\\ D_{0+}(T) & D_+(T)\end{pmatrix}. \tag{2.9}$$

Given the conditions (2.8), we can write N explicitly as

$$\text{N}=\begin{bmatrix}\dfrac{\gamma_-(T_1-T_2)}{(T-T_1)(T-T_2)} & \dfrac{\gamma_0(T_1-T_2)}{(T-T_1)(T-T_2)}\\[2ex] \dfrac{\gamma_0}{T-T_1}+\dfrac{\alpha}{T-T_2} & \dfrac{\gamma_+}{T-T_1}\end{bmatrix}, \tag{2.10}$$

where

$$\alpha=\gamma_{20}D_-(T_2)+\gamma_{2+}D_{0+}(T_2).$$

It is convenient now to define the following dispersion functions:

$$\begin{aligned}&\text{(a)}\quad f(T)\equiv-\frac{(T-T_1)(T_1-T_2)}{\pi}\times\int_0^{\infty}\frac{\rho_1(T')dT'}{(T'-T)(T'-T_2)(T'-T_1)^2},\\&\text{(b)}\quad \eta_1(T)\equiv-\frac{T-T_1}{\pi}\int_{T_2^{\text{th}}}^{\infty}\frac{\rho_2(T')dT'}{(T'-T)(T'-T_1)^2},\\&\text{(c)}\quad \eta_2(T)\equiv-\frac{T-T_1}{\pi}\int_{T_2^{\text{th}}}^{\infty}\frac{\rho_2(T')dT'}{(T'-T)(T'-T_1)(T'-T_2)},\\&\text{(d)}\quad f_2\equiv f(T_2),\\&\text{(e)}\quad \eta_{12}\equiv\eta_1(T_2),\\&\text{(f)}\quad \eta_2(T)=\frac{T-T_1}{T-T_2}[\eta_1(T)-\eta_{12}].\end{aligned} \tag{2.11}$$

The D function can now be written explicitly in terms of these dispersion functions as

$$D=\begin{pmatrix}1+\gamma_-f(T) & \gamma_0f(T)\\ \gamma_0\eta_1(T)+\sigma\eta_2(T) & 1+\gamma_+\eta_1(T)\end{pmatrix}, \tag{2.12}$$

where

$$\sigma=-\frac{\gamma_0}{1+\gamma_+\eta_{12}}.$$

It will be useful to invert the order of integration for η_1 (mass and energy) and to express η_1 by

$$\eta_1(T)\equiv\frac{1}{W}\int_{1+\mu}^{\infty}\frac{dM'}{(M'-M^*)^2+(\frac{1}{2}\Gamma)^2}\beta(T,M'), \tag{2.13}$$

where

$$\beta(T,M')=-\frac{(T-T_1)}{\pi}\times\int_{\frac{1}{2}(M'^2+2M'-3)}^{\infty}\frac{dT'[(3-M'^2+2T')^2-4M'^2]^{1/2}}{(2+T')(T'-T)(T'-T_1)^2}.$$

The expressions for $f(T)$ and for $\beta(T,M')$ can be evaluated analytically, leaving only the integral over M' [Eq. (2.13)] to be done numerically. The elastic scattering amplitude can now be calculated at any energy T, and is a function of the coupling parameters γ_+, γ_-, γ_0 and the pole positions T_1, T_2. The "experimental" parameters (δ,η) can be extracted from the scattering amplitude through

$$M_e=\frac{1}{2i}(\eta e^{2i\delta}-1)=\rho_1(T)(ND^{-1})_{11}. \tag{2.14}$$

The model just described contains the full singularity structure of the amplitude as depicted in Fig. 1.

III. FIT TO THE DATA

Experimentally, the $^1D_2$ p-p partial-wave amplitude is well determined below about 400 MeV and we fit to the phase shifts of Arndt and MacGregor,[5] which are in general agreement with other analyses. The high-energy (above 400 MeV) data are very sparse and imprecise. We shall rely on the 660-MeV analysis of Azhgirey *et al.*[7] We anticipated that the force pole should be located somewhat below the pion exchange threshold (-10 MeV) and that the cutoff pole at T_2 should be somewhere below T_1. The coupling parameters γ_-, γ_+, γ_0 were adjusted through an automatic search procedure to produce a least-squares fit to the experimental quantities δ and η. The pole positions T_1, T_2 were not automatically adjusted but were varied for a "best" fit commensurate with common sense. We determined that the force pole could be taken between -10 and -50 MeV and the cutoff pole could be spaced between 10 and 50 MeV below the first pole to obtain good and qualitatively similar fits. The fit for $(T_1,T_2) = (-15$ MeV, -25 MeV) is shown in Fig. 2 and is, in fact, quite good. The coupling constants for this fit are $(\gamma_-, \gamma_0, \gamma_+ = 32690, 66260, 1053)$. The strong dip in the determinant of the real part of the D function around 600 MeV [Fig. 2(c)] suggests the existence of a pole (0 in det D) on a nearby unphysical sheet which may be located by analytic continuation of det D. If is found in this fit to be at $400-i300$ MeV on the second Riemann sheet of the first and second cuts (from 0 and 280 MeV), and on the top sheet of the last, complex branch cut (from $650-i140$ MeV).

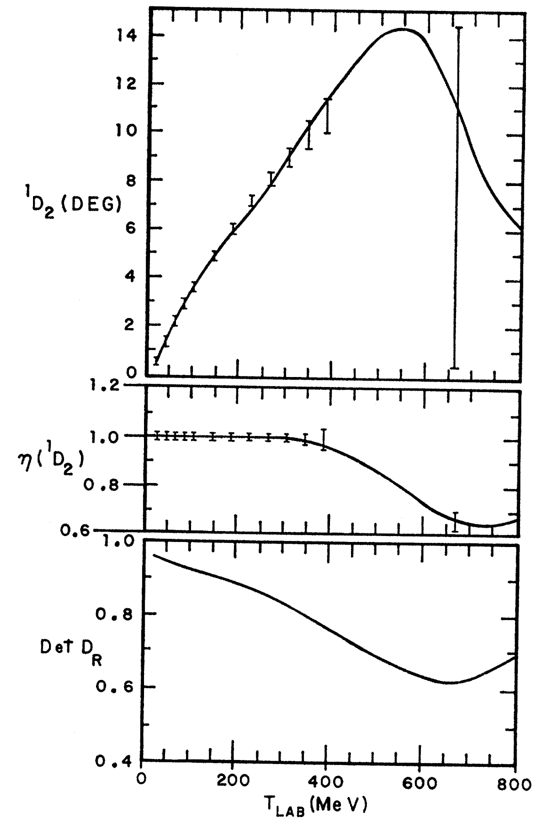

FIG. 2. Model fit to $^1D_2$ partial-wave amplitude.

IV. ALTERNATIVE DESCRIPTION

Experiments and a number of semitheoretical models for pion production in $p-p$ scattering suggest that the branch cut starting at 280 MeV may be relatively unimportant, and so an alternative model was developed in which the middle cut was totally ignored. This was accomplished by considering a two-channel problem in which the inelastic channel contains a proton and a second, real mass, particle in a relative S wave. The scattering amplitude can be solved for in closed, analytic form; as a matter of fact, the dispersion function which must be determined is simply $\beta(T,M')$ defined by Eq. (2.13). The analytic form is then evaluated at the complex mass appropriate to the (3,3) resonance, $1238-i120$ MeV. As in the previously defined model, the resultant amplitude is a function of $(\gamma_-, \gamma_0, \gamma_+, T_1, T_2)$, the "force" parameters. A difficulty arises when comparing this model to experiment; it is *not* implicitly unitary below pion production threshold and cannot, therefore, be characterized by a single real phase δ, assuming $\eta=1$. The dilemma is resolved by introducing artificial data points on η below 300 MeV which can be used to keep the amplitude as unitary ($\eta \sim 1$) as desired. Figure 3 depicts a fit using the second formalism and taking $(T_1,T_2) = (-15,-25)$. In this fit the low-energy absorption parameter η is fixed at 1 ± 0.01. Parameter studies on T_1 and T_2 revealed about the same dependence of fit which was described in Sec. III. Figure 3 shows that the existing experimental data can be fairly well accommodated by the second formalism and the energy dependence of det D [Fig. 3(c)] is not such as to suggest a nearby pole. As a matter of fact, the sign of the force in the inelastic channel is minus, precluding the existence of a bound state in that channel.

The two models described differ drastically in their high-energy predictions, as can be seen by comparing Figs. 2 and 3, and our present experimental understanding of the $^1D_2$ partial-wave amplitude between 400 and 800 MeV is sufficiently poor that a clean experimental rejection of either model is impossible. Two theoretical models for pion production, the Mandelstam[1] and the Amaldi[8] peripheral models, give definite predictions for the $^1D_2$ absorption parameter η. These predictions (which are quite similar in the two models) were used to complement the existing low-energy data and the two formalisms were adjusted to fit the resulting combination. Figure 4 illustrates the "best" fits

[8] U. Amaldi, Jr., R. Biancastelli, and S. Francaviglia, Nuovo Cimento 47, 85 (1967).

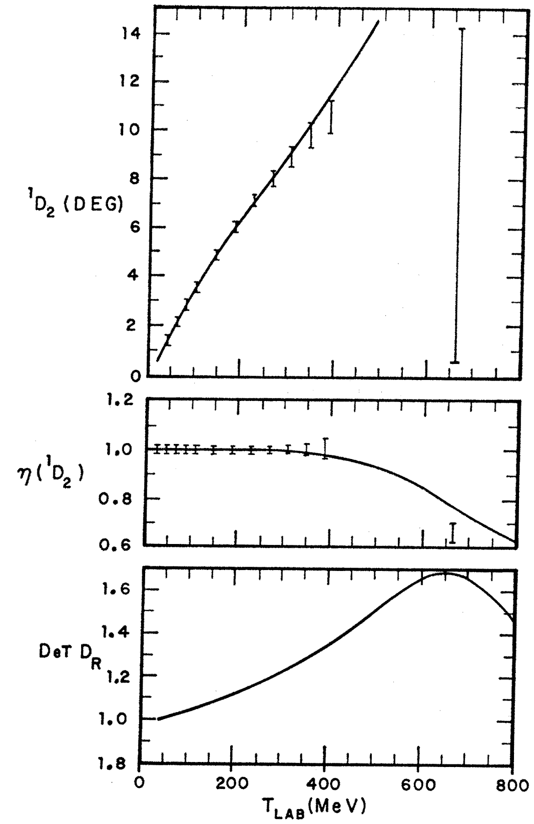

FIG. 3. Alternative description of $^1D_2$ scattering.

from each formalism to the high-energy predictions of the Amaldi model (the dashed curve in Fig. 4). The phase shifts were allowed to vary freely above 400 MeV in these fits. Both models fit the elastic phases quite

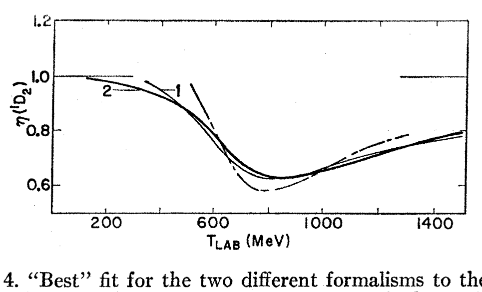

FIG. 4. "Best" fit for the two different formalisms to the predictions of the Amaldi peripheral model. Curve 1 is the prediction of formalism 1 and curve 2 is from the second model. The dashed curve is the Amaldi prediction above 500 MeV and is 1 (the unitarity limit) below 300 MeV.

well, but the second formalism (curve 2 of Fig. 4) gave strongly nonunitary ($\eta<1$) predictions above 150 MeV and thus failed in fitting the "data". The result was a clean rejection of the second model (on its inability to describe the data), and model predictions from the first formalism which are essentially those discussed in Sec. III and depicted in Fig. 2. The pole, as before, is predicted at $400-i300$ MeV. The $N\Delta$ coupling constant (γ_+) from the fit depicted for formalism 2 in Fig. 4 was positive (attractive force) and there is some slight evidence that a nearby pole may exist.

V. CONCLUSIONS AND DISCUSSION

We have attempted, in Sec. IV, to illustrate the speculative nature of our prediction which results from the very imprecise nature of our understanding of the $^1D_2$ partial-wave amplitude above the pion production threshold. We believe, however, that the basic simplicity and credibility of the model, combined with its ability to fit the existing data very well and to match predictions of well-founded theoretical models for pion production to about 1 BeV, suggests that the pole prediction may stand and be clarified as subsequent experiments and analyses clarify our understanding of the $^1D_2$ partial-wave amplitude in this energy region. Despite the rather large distance between the pole and the physical scattering region, its effect is notably manifested in a marked structure (energy dependence) of the $^1D_2$ phase parameters. We might introduce a note of caution to those attempting phenomenological analysis of N-N or of other strongly interacting scattering systems; in order to obtain proper "structure" it may be necessary to work with energy-dependent forms which contain the full singularity structure of the partial-wave amplitudes they are to simulate.

The location of the pole in the complex plane suggests its interpretation as being a bound state in the N^*p (S-wave) channel, and its quantum numbers are those of a spin-2 Regge recurrence of the $^1S_0$ p-p pole (the unbound diproton).

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