Using the experimental results[8] we find order of magnitude agreement at $t=0$ but *complete failure* for other $t$ values because the cross sections are exponentially varying with different slopes. One is therefore inclined to dismiss the agreement of the predictions for the cross sections at $t=0$ as accidental. One is reluctant however to say the same of the agreement for the coupling constants, if only because of the success of similar methods for calculating the fourth order corrections to the $ee\gamma$ vertex.[6]

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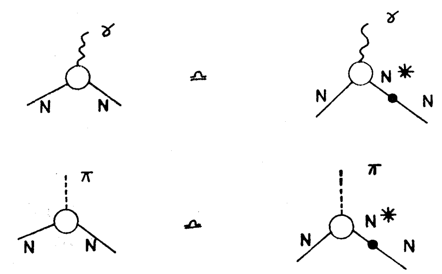

FIG. 1. The $N^*$ dominance approximation. The diagrams represent dispersion relation, not Feynman diagrams so all particles are on the mass shell.

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# Superconvergence and the $Y_0^*(1405)BP$ Coupling

S. N. BISWAS

*Centre for Advanced Study in Physics, University of Delhi, Delhi, India*

AND

SUBIR BOSE, K. DATTA, J. DHAR, AND R. P. SAXENA*

*Department of Physics and Astrophysics, University of Delhi, Delhi, India*

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We consider fixed-momentum-transfer dispersion relations for the process $\bar{K}+N\to\Sigma+\pi$. Under the assumption that the asymptotic behavior of the amplitude at high energies is governed by the Regge trajectories in the crossed $t$ channel $K+\pi\to\bar{\Sigma}+N$, we obtain a sum rule which allows us to estimate the $Y_0^*BP$ coupling. The calculated value agrees well with the estimate by Dalitz, Wong, and Rajasekharan, and that by Das and Mahanthappa.

## 1. INTRODUCTION

RECENTLY, sum rules involving parameters of strongly interacting particles have been derived within the framework of dispersion theory. In such derivations, assumptions are made about the asymptotic behavior of scattering amplitudes at high energy. Alfaro *et al.*[1] and Soloviev[2] have based these assumptions on considerations of unitarity and constancy of the diffraction peak. Sakita and Wali[3] consider a Regge-pole model in which the asymptotic behavior is given directly by the Regge trajectories of the crossed $t$ channel. Working within the framework of $SU(3)$ symmetry they obtain sum rules for $PB$ scattering corresponding to the exchange of a 27-plet in the $t$ channel. Our work is based on considerations similar to those used in Ref. 3; however, we only make use of the isospin invariance of the strong-interaction Hamiltonian. In particular, we consider here dispersion relations for the process $\bar{K}+N\to\Sigma+\pi$ to obtain the $Y_0^*(1405)BP$ coupling and compare our result with that of other authors.[4–6]

* Present address: Department of Physics, Syracuse University, Syracuse, N. Y.

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[2] L. D. Soloviev, JINR Report E-2343, Dubna, 1965 (unpublished).

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## 2. SUM RULE

For the process

$$\bar{K}+N\to\Sigma+\pi$$

let $p$ and $k$ be the 4-momenta of the incident nucleon and kaon and $p'$ and $k'$ those of the outgoing hyperon and pion. Then

$$p+k=p'+k'. \tag{1}$$

The $T$ matrix is used to define the invariant amplitudes

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in the usual fashion:

$$T_{\beta\alpha}=-A_{\beta\alpha}(\nu,t)+\tfrac{1}{2}\gamma\cdot(k+k')B_{\beta\alpha}(\nu,t)\,, \qquad (2)$$

where

$$\nu=p\cdot k/m_N,\quad t=(k+k')^2,$$

and $\alpha$ and $\beta$ refer to the isotopic spin of the outgoing hyperon and pion. $B_{\beta\alpha}$ ($A_{\beta\alpha}$) has the familiar isotopic-spin decomposition

$$B_{\beta\alpha}=\delta_{\beta\alpha}B^{(+)}+\tfrac{1}{2}[\tau_\beta,\tau_\alpha]B^{(-)}. \qquad (3)$$

(Isospin operators of the kaon and nucleon have not been distinguished.) The amplitudes $B^{(\pm)}$ are related to the eigenamplitudes of isospin by

$$B^{(+)}=(1/\sqrt{6})B^{(0)},$$
$$B^{(-)}=\tfrac{1}{2}B^{(1)}. \qquad (4)$$

It is well known that the two invariant amplitudes $A(\nu,t)$ and $B(\nu,t)$ have different asymptotic behavior in $\nu$ for fixed $t$. Thus, if

$$A(\nu,t)\underset{\nu\to\infty}{\longrightarrow}\nu^{\alpha(t)}\,,$$

then

$$B(\nu,t)\underset{\nu\to\infty}{\longrightarrow}\nu^{\alpha(t)-1}.$$

In a Regge-pole model $\alpha(t)$ refers to the dominant Regge trajectory in the crossed $t$ channel $K+\pi\to\bar{\Sigma}+N$. The isotopic-spin quantum numbers in this channel are $I=\frac{3}{2}$ and $I=\frac{1}{2}$. Since no mesons with $I=\frac{3}{2}$ have so far been observed experimentally, it is reasonable to assume that $\alpha_{I=3/2}(t)\leq 0$ ($t\leq 0$). We are therefore led to consider that combination of $s$-channel $B$ amplitudes which receive contributions only from $I=\frac{3}{2}$ amplitudes in the $t$ channel:

$$B_{3/2}(\nu,t)\equiv\sum_{i=0,1}C_{3/2,i}{}^{ts}B_s{}^i(\nu,t)\,. \qquad (5)$$

Here $B_s{}^i$ are the eigenamplitudes of isospin in the $s$ channel $\bar{K}+N\to\Sigma+\pi$ and $C_{3/2,i}{}^{ts}$ are the appropriate elements of the $s\to t$ isospin-crossing matrix for the system. We assume that the amplitudes $B_s{}^i(\nu,t)$ satisfy unsubtracted dispersion relations in $\nu$ for fixed $t$. Therefore

$$B_{3/2}(\nu,t)=\sum_{i=0,1}C_{3/2,i}{}^{ts}\int_{-\infty}^{\infty}\frac{\mathrm{Im}B_s{}^i(\nu',t)}{\nu'-\nu}d\nu'\,. \qquad (6)$$

The asymptotic behavior of $B_{3/2}(\nu,t)$ is given by the Regge-pole model:

$$\lim_{\substack{\nu\to\infty\\ t\text{ fixed }\leq 0}}B_{3/2}(\nu,t)\longrightarrow\nu^{\alpha_{3/2}(t)-1}. \qquad (7)$$

Hence we have the sum rule

$$\sum_{i=0,1}C_{3/2,i}{}^{ts}\int_{-\infty}^{\infty}\mathrm{Im}B_s{}^i(\nu,t)d\nu=0,\quad t\leq 0. \qquad (8)$$

We note, however, that

$$\mathrm{Im}B_s{}^i(-\nu,t)=C_{ij}{}^{su}\,\mathrm{Im}B_u{}^j(\nu,t). \qquad (9)$$

This leads to the nontrivial sum rule

$$\sum_{i=0,1}C_{3/2,i}{}^{ts}\int_0^{\infty}\mathrm{Im}B_s{}^i(\nu,t)d\nu+\sum_{j=1/2,3/2}C_{3/2,j}{}^{tu}\int_{-\infty}^{\infty}\times\mathrm{Im}B_u{}^2(\nu,t)d\nu=0,\quad t\leq 0. \qquad (10)$$

Since $C_{3/2,0}{}^{ts}=1/\sqrt{6}$, $C_{3/2,1}{}^{ts}=\frac{1}{2}$, $C_{3/2,1/2}{}^{tu}=\frac{2}{3}$, $C_{3/2,3/2}{}^{tu}=\frac{1}{3}$, we have, for $t=0$,

$$\int_0^{\infty}d\nu[(1/\sqrt{6})\mathrm{Im}B_s{}^0(\nu,t)+\tfrac{1}{2}B_s{}^1(\nu,0)]+\int_0^{\infty}[\tfrac{2}{3}\mathrm{Im}B_u{}^{1/2}(\nu,0)+\tfrac{1}{3}\mathrm{Im}B_u{}^{3/2}(\nu,0)]d\nu=0. \qquad (11)$$

### 3. EVALUATION OF THE COUPLING CONSTANT

To evaluate the $Y_0^*BP$ coupling we saturate the dispersion integrals in (11) with the $\Lambda$, $\Sigma$, $N$, and $N^*(1236)$ poles below threshold and the $Y_0^*(1405)$, $Y_1^*(1382)$ resonances above threshold. We obtain

$$B_s{}^0(s,0)=(\sqrt{6})\frac{G_{\Lambda\Sigma\pi}G_{\Lambda NK}}{s-m_\Lambda{}^2+i\epsilon}+(\sqrt{6})\frac{G_{Y_0^*\Sigma\pi}G_{Y_0^*NK}}{s-m_{Y_0^*}{}^2+i\epsilon}, \qquad (12)$$

$$B_s{}^{(1)}(s,0)=2\frac{G_{\Sigma\Sigma\pi}G_{\Sigma NK}}{s-m_\Sigma{}^2+i\epsilon}+2\frac{G_{Y_1^*\Sigma\pi}G_{Y_1^*NK}}{s-m_{Y_1^*}{}^2+i\epsilon}a(s_1m_{Y_1^*}{}^2)\,,$$

where (13)

$$a(s,m_{Y_1^*}{}^2)=[\tfrac{1}{2}(\mu^2+m_K{}^2)-\tfrac{1}{3}\{\mu^2-(m_N+m_\Sigma)(m_{Y_1^*}+m_\Sigma)\} \qquad (14)$$
$$-(1/6s)\{(s-m_N{}^2+m_K{}^2)(m_{Y_1^*}m_\Sigma+s)-(s-m_\Sigma{}^2+\mu^2)(m_{Y_1^*}m_N-m_N{}^2+m_K{}^2)\}],$$

$$B_u{}^{1/2}(s,0)=3\frac{G_{NN\pi}G_{N\Sigma K}}{s-(\mu^2+m_K{}^2+m_\Sigma{}^2)-i\epsilon}, \qquad (15)$$

$$B_u{}^{3/2}(s,0)=2\frac{G_{N^*N\pi}G_{N^*\Sigma K}a(\mu,m_{N^*}{}^2)}{s-(m_N{}^2+m_\Sigma{}^2-\mu^2-m_K{}^2+m_{N^*}{}^2)-i\epsilon}. \qquad (16)$$

Here $s$ and $u$ are the usual Mandelstam variables; $s=m_K{}^2+m_N{}^2+2\nu m_N$ and $u=m_\Sigma{}^2+u^2-2\nu m_N-t$. Use of (12) to (16) in (11) results in the following equation:

$$G_{Y_0^*NK}G_{Y_0^*\Sigma\pi}=-G_{\Lambda\Sigma\pi}G_{\Lambda NK}-G_{\Sigma\Sigma\pi}G_{\Sigma NK}-G_{Y_1^*\Sigma\pi}G_{Y_1^*NK}a(m_{Y_1^*}{}^2,m_{Y_1^*}{}^2)+2G_{NN\pi}G_{N\Sigma K}+\tfrac{2}{3}G_{N^*N\pi}G_{N^*\Sigma K}\times a(m_{N^*}{}^2,m_{N^*}{}^2). \qquad (17)$$

For the right-hand side of (17) we use the following values of the various coupling constants[7–10]:

$$G_{NN\pi}=(15)^{1/2}\equiv g,\quad G_{\Sigma\Sigma\pi}=2fg,\quad G_{\Sigma NK}=(1-2f)g,$$
$$G_{\Lambda\Sigma\pi}=(2/\sqrt{3})(1-f)g,\quad G_{\Lambda NK}=-(1/\sqrt{3})(1+2f)g,$$

with $f=0.4$, and

$$G_{N^*N\pi}=-G_{N^*\Sigma K}=4.4\ \text{BeV}^{-1},$$
$$G_{Y_1^*\Sigma\pi}=-G_{Y_1^*NK}=1.81\ \text{BeV}^{-1}.$$

This gives

$$G_{Y_0^*\Sigma\pi}G_{Y_0^*NK}=0.4. \tag{18}$$

[7] The coupling constants are normalized as follows: $G_{\pi NN}{}^2=g_{\pi NN}{}^2/4\pi=15$.

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$$G^2=3\Gamma\frac{m_{B^*}}{m_B}\frac{1}{p^3}\frac{1}{(p_0/m_B+1)},$$

which follows from the interaction used to calculate the matrix elements (14) and (16). [See S. Gasiorowicz, *Elementary Particle Physics* (John Wiley & Sons, Inc., New York, 1966), p. 310].

## 4. DISCUSSION

The value of the $Y_0^*BP$ coupling obtained in Eq. (18) compares reasonably well with the estimate of Dalitz, Wong, and Rajesekharan[4] who obtain for it a value of 0.5, and of Das and Mahanthappa[5] who obtain a value of 0.4. As has been remarked by Dalitz *et al.*, this is a rather large coupling for an *s*-wave interaction. However, Dalitz *et al.* obtain their value from a coupled-channel *K*-matrix theory with $SU(3)$ symmetry and kinematic mass breaking; Das and Mahantappa's estimate results from a coupled-channel $N/D$ solution. Our calculation is based on entirely different considerations of high-energy behavior of an inelastic amplitude in a Regge-pole model. The closeness of the various estimates suggests that the $Y_0^*BP$ coupling is indeed comparatively large.

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# Application of Analyticity Properties to the Numerical Solution of the Bethe-Salpeter Equation*

Richard W. Haymaker†
*Lawrence Radiation Laboratory, University of California, Berkeley, California*
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A new method of calculating phase shifts from the Bethe-Salpeter equation is presented. The differential equation is solved below threshold by using a variational method, and then the scattering amplitude is continued to the physical-scattering region, using Padé approximants. The singularity structure of the Bethe-Salpeter partial-wave amplitude in its off-shell variables was studied to find the nearby singularities that could most strongly affect the continuation.

## I. INTRODUCTION

The recent calculations of phase shifts for the Bethe-Salpeter equation[1] in the ladder approximation have demonstrated the practical use of the equation for the study of two-body scattering amplitudes. Schwartz and Zemach[2] used the Schwinger variational principle based on the integral equation which yielded a rapidly convergent sequence of approximations to the phase shifts. A mesh-point solution has also been achieved which in addition was applicable to the three-particle inelastic region.[3] As more difficult problems of higher dimensionality are attempted (e.g., the three-body problem), the disadvantages of both methods become apparent. The Schwinger method becomes increasingly difficult to set up, and the mesh-point method may require prohibitively large matrices.

A comparison of the bound-state calculation of Schwartz[4] with the calculation of phase shifts by Schwartz and Zemach[2] shows that less sophisticated methods suffice to solve the former problem. The reason is that the boundary conditions on the wave function can be easily imposed, and thus the differential equation can be used. In this paper, we present a method of computing phase shifts by calculating the scattering amplitude below elastic threshold and continuing it to the scattering region. By calculating below threshold we avoid the problems of solving a singular integral

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† Present address: Department of Physics, University of California, Santa Barbara, Calif.

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