This expression can be put in the form

$$\frac{1}{4\pi\lambda\!\!\!^{-3}}B=\sum_l\{l(l+E/m)a_{l^-}-(l+1)(l+1-E/m)a_{l^+}\}. \tag{A16}$$

Representation of $a_l{}^\pm$ in the complex plane (Argand diagrams): Figure 12 gives a simple method to construct $\mathbf{a}_l$ in terms of δ and ρ. Conservation of probability imposes that $\mathbf{a}_l$ be inside the "unitary circle" centered at the point $(0,i/2)$ and with radius $\frac{1}{2}$.

Partial cross sections are given by

$$\sigma_{\rm tot}=\sigma_{\rm max}(1-\rho\cos2\delta)/2=\sigma_{\rm max}(OI),$$
$$\sigma_{\rm el}=\sigma_{\rm max}(1+\rho^2-2\rho\cos2\delta)/4=\sigma_{\rm max}(OA)^2,$$
$$\sigma_{\rm inel}=\sigma_{\rm max}(1-\rho^2)/4=\sigma_{\rm max}[1-4(CA)^2]/4,$$

with

$$\sigma_{\rm max}=4\pi\lambda\!\!\!^{-2}(J+\tfrac{1}{2}).$$

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Calculation of the Low-Energy *S*-Wave Pion-Pion Interaction from the K_{e4} Decay Using Charge Commutation Relations*

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By making use of an appropriate charge commutation relation, we have computed the low-energy pion-pion scattering length directly from the rate of K_{e4}-decay. The main differences from previous approaches are as follows: (a) Instead of using the current commutation relations (which have been extensively used with the soft-pion technique), we stick to the better-known charge commutation relations. This enables us, as in the Adler-Weisberger calculation, to avoid taking the usual soft-pion limit $k_\mu\to0$; instead we let only $k^2(=-m_\pi^2)\to0$, which is certainly a smaller extrapolation. (b) In contrast with previous calculations of K_{e4} decays, which utilized essentially the kaonic PCAC ($m_K\to0$), we use the pionic PCAC, which involves a much smaller extrapolation ($m_\pi\to0$). From the presently available K_{e4} decay rate, we have estimated the values of the $I=0$ and 2 *S*-wave pion-pion scattering length as $a_0\simeq0.18$ and $a_2\simeq-0.017$. Although our numerical results for a_0, a_2, and the K_{e4}-decay form factors turn out to be not very different from those obtained by using the current commutation relations with soft-pion techniques, we emphasize the important difference between the two approaches mentioned in (a) with respect to the extrapolation procedures.

THE K_{e4} decay has been known to be one of the best places to study the low-energy pion-pion interaction. At the moment, experiments do not allow a definite conclusion. They give for the $I=0$ *S*-wave pion-pion scattering length the value[1]

$$a_0=(0.6_{-0.5}{}^{+0.6})m_\pi{}^{-1}. \tag{1}$$

Theoretically, the strength of the low-energy pion-pion interaction is of crucial importance for judging the physical significance of a large number of calculations employing current algebra. The apparent success of the soft-pion emission technique based on the current commutation relations (CCR), which, for instance, relates the $K\to3\pi$ amplitude to the $K_1{}^0\to2\pi$ amplitude[2] and also the leptonic decay amplitudes of the *K* meson,[3,4] is hard to understand unless the value of a_0 is small. In fact, by using CCR and the hypothesis of partially conserved axial-vector current (PCAC) and by *assuming that the scattering length in question is small*, Weinberg,[5,6] for instance, did indeed obtain a small $I=0$ *S*-Wave π-π scattering length:

$$a_0=0.20m_\pi{}^{-1}. \tag{2}$$

However, the procedure used in deriving the value given by Eq. (2) is not entirely free from theoretical ambiguity, especially with respect to the off-mass-shell extrapolation procedure. It has been pointed out that one could obtain a family of solutions which contains

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[1] This value is presented by R. W. Birge *et al.*, in *Proceedings of the Thirteenth International Conference on High-Energy Physics, Berkeley, California, 1966* (University of California Press, Berkeley, Calif., 1967), p. 37; see also, R. W. Birge, R. P. Ely, G. Gidal, G. E. Kalmus, A. Kernan, W. M. Powell, U. Camerini, D. Cline, W. F. Fry, J. G. Gaido, D. Murphree, and C. J. Murphy, Phys. Rev. **139**, B1600 (1965).

[2] M. Suzuki, Phys. Rev. **144**, 1154 (1966); C. G. Callan and S. B. Treiman, Phys. Rev. Letters **16**, 153 (1966); Y. Hara and Y. Nambu, *ibid.* **16**, 875 (1966); L. J. Clavelli, Phys. Rev. **160**, 1384 (1967); D. K. Elias and J. C. Taylor, Nuovo Cimento **44A** 518 (1966); H. D. Abarbarnel, Phys. Rev. **153**, 1547 (1967).

[3] C. G. Callan and S. B. Treiman (Ref. 2); M. Suzuki, Phys. Rev. Letters **16**, 212 (1966); V. S. Mathur, S. Okubo, and L. K. Pandit, *ibid.* **16**, 371 (1966); L. J. Clavelli, Phys. Rev. **154**, 1509 (1967).

[4] S. Weinberg, Phys. Rev. Letters **17**, 336 (1966); **18**, 1178(E) (1967).

[5] S. Weinberg, Phys. Rev. Letters **17**, 616 (1966); see also, Y. Tomozawa, Nuovo Cimento **46A**, 707 (1966); A. P. Balachandran, M. Gundzik, and F. Nicodemi, *ibid.* **44A**, 1257 (1966); N. Khuri, Phys. Rev. **153**, 1477 (1967).

[6] F. T. Meiere, Phys. Rev. **159**, 1462 (1967); F. T. Meiere and M. Sugawara, *ibid.* **153**, 1702, (1967); **153**, 1709 (1967).

not only a small value of a_0 consistent with Eq. (2), but also a much larger value.[7,8] Moreover, there have been many indirect experiments which favor a rather large scattering length of the order $a_0 \simeq m_\pi^{-1}$. Therefore, to obtain more insight into the problem, it seems highly desirable to attack it by a different approach.

In this paper we wish to discuss the K_{e4}-decay form factors and the S-wave π-π scattering length. The main differences from previous approaches are as follows:

(a) Instead of using the current commutation relations, we employ the better-known *charge commutation relations*. This enables us, as in the case of the Adler-Weisberger calculation, to avoid taking a limit $k_\mu \to 0$ (k_μ is the pion four-momentum); instead we let only $k^2 \to 0$.

(b) In contrast with previous calculations of K_{e4} decay, which utilized essentially the kaonic PCAC ($m_K \to 0$), we try to use the well-known pionic PCAC, which involves a much smaller extrapolation ($m_\pi \to 0$).

We write the relevant part of the weak interaction as follows:

$$(G_V/\sqrt{2}) \sin\theta_A A_\mu{}^{K^+} \bar{e}\gamma_\mu(1+\gamma_5)\nu. \qquad (3)$$

Here $G_V = 1.02 \times 10^{-5} m_p{}^{-2}$ is the weak-coupling constant and θ_A the axial-vector Cabibbo angle. $A_\mu{}^{K^+}$ is the $\Delta S=1$, $|\Delta \mathbf{I}| = \frac{1}{2}$ axial-vector current. As usual, we neglect the contribution of the vector current to the K_{e4} decay. The axial-vector K_{e4} form factors F_1, F_2, and F_3 are defined by

$$\begin{aligned}\langle \pi^+(p^+)\pi^-(p^-)|A_\mu{}^{K^+}(0)\sin\theta_A|K^-(k)\rangle &= i(8p_0{}^+p_0{}^-k_0)^{-1/2}(1/m_K)[(p^++p^-)_\mu F_1 \\ &\quad + (p^+-p^-)_\mu F_2 + (k-p^+-p^-)_\mu F_3], \qquad (4)\end{aligned}$$

where k_μ and $p_\mu{}^+$, $p_\mu{}^-$ are the four-momenta of the K meson and π mesons. The form factors F_i are functions of the variables

$$s = -(p^++p^-)^2, \quad t = -(p^+-k)^2, \quad u = -(p^--k)^2.$$

We assume that the dependence of F_i on s, t, and u is not very rapid.[9]

In order to utilize *pionic* PCAC, we make use of the following charge commutator[10]:

$$-\tfrac{1}{2}A_{K^+} = [V_{K^+}, A_{\pi^0}]. \qquad (5)$$

Here the vector and axial-vector currents are denoted by $V_\mu{}^{K^+}$ and $A_\mu{}^{\pi^0}$, $A_\mu{}^{K^+}$, respectively. They are normalized so that in a quark model we would have, e.g.,

$$V_\mu{}^{K^+}(x) = i\bar{q}(\lambda_4 + i\lambda_5/2)\gamma_\mu q, \quad A_\mu{}^{\pi^0}(x) = i\bar{q}(\lambda_3/2)\gamma_5\gamma_\mu q$$

and

$$A_\mu{}^{K^+}(x) = i\bar{q}(\lambda_4 + i\lambda_5/2)\gamma_5\gamma_\mu q.$$

The space integrals of $V_0{}^{K^+}(\mathbf{x},0)$, $A_0{}^{\pi^0}(\mathbf{x},0)$, and $A_0{}^{K^+}(\mathbf{x},0)$ have been denoted by V_{K^+}, A_{π^0}, and A_{K^+}, respectively.

We now take the matrix element of Eq. (5) between the two-pion state and the K^--meson state. We choose to work in the Lorentz frame in which

$$\mathbf{p}^+ = \mathbf{p}^- = \mathbf{k}/2 \text{ and take the limit } |\mathbf{k}| = \infty. \qquad (6)$$

We note that in the limit of $SU(3)$ symmetry the operator V_{K^+} is an $SU(3)$ generator so that it only connects those states which belong to the same irreducible $SU(3)$ representation. Therefore, if we assume that $SU(3)$ symmetry works fairly well, we may neglect the nondiagonal matrix elements of V_{K^+} since they are at least of first order in the $SU(3)$ symmetry-breaking interaction. We note that we are making this approximation in the limit (6). Then from (5), we obtain

$$\begin{aligned}(-\tfrac{1}{2})\langle \pi^+(p^+)\pi^-(p^-)|A_{K^+}|K^-(k)\rangle &= \textstyle\sum_n \langle \pi^+(p^+)\pi^-(p^-)|V_{K^+}|n\rangle\langle n|A_{\pi^0}|K^-(k)\rangle \\ &\quad - \langle \pi^+(p^+)\pi^-(p^-)|A_{\pi^0}|\pi^0\rangle\langle \pi^0|V_{K^+}|K^-(k)\rangle. \qquad (7)\end{aligned}$$

On inserting Eq. (4), the left-hand side of Eq. (7) becomes

$$\begin{aligned}\text{left-hand side} &= (-\tfrac{1}{2})(2\pi)^3\delta^3(\mathbf{p}^+ + \mathbf{p}^- - \mathbf{k}) \\ &\quad \times \frac{1}{(8p_0{}^+p_0{}^-k_0)^{1/2}}\left(\frac{1}{m_K}\right)\frac{1}{\sin\theta_A}[(p_0{}^+ + p_0{}^-)F_1 \\ &\quad + (p_0{}^+ - p_0{}^-)F_2 + (k_0 - p_0{}^+ - p_0{}^-)F_3]. \qquad (8)\end{aligned}$$

In our limit (6), the form factors $F_i(s,t,u)$ are evaluated at the point[11] $s_0 = 4m_\pi{}^2$, $t_0 = u_0 = (m_K{}^2/2) - m_\pi{}^2$. We first study the configuration in which the final two pions are in the symmetric $I=0$ state.[9] Then in Eq. (8) only the F_1 and F_3 terms survive, and on the right-hand side of Eq. (7) the intermediate states n correspond to the $I=\frac{1}{2}$ two-pseudoscalar-meson states which belong to a symmetric octet and a **27** representation of the $SU(3)$ group. Explicitly, the states n will consist of the $\bar{K}^0\pi^-$, $K^-\pi^0$, and $K^-\eta^0$ states.

For the diagonal matrix elements of V_K which we retain in Eq. (7), we use the $SU(3)$ value as mentioned before. In the last term of the right-hand side of Eq. (7), we have

$$\langle \pi^0(p^0)|V_{K^+}|K^-(k)\rangle = (2\pi)^3\delta^3(\mathbf{k}-\mathbf{p})(1/\sqrt{2}). \qquad (9)$$

For the matrix elements $\langle \pi^+(p^+)\pi^-(p^-)|V_{K^+}|n\rangle$, only the terms $n = \bar{K}^0\pi^-$ and $K^-\pi^0$ contribute in the sym-

[7] J. Sucher and C. H. Woo, Phys. Rev. Letters **18**, 723 (1967).

[8] J. Iliopoulos, CERN Report (unpublished); A. Donnachie, CERN Report (unpublished).

[9] N. Van Hiew, Zh. Eksperim. i Teor. Fiz. **44**, 162 (1963) [English transl.: Soviet Phys.—JETP **17**, 113 (1963)], shows that within an error of $\simeq 10\%$ one could assume that the form factors are only functions of s. If we assume this, the form factors F_1, F_3, and F_2 receive contributions only from the $I=0$ and $I=1$ final two-pion states, respectively.

[10] The usefulness of this procedure and the validity of the approximation adopted in this paper have been discussed by S. Matsuda and S. Oneda, Phys. Rev. **158**, 1594 (1967).

[11] Since the phase-space factor for K_{e4} decay has a rather sharp maximum near $s = 4m_\pi{}^2$, the approximation of replacing $F(s,t,u)$ by $F(s_0,t_0,u_0)$ is good. The chosen values of t_0 and u_0 are also adequate.

metry limit. It is not easy to make an accurate estimate of the effect of $SU(3)$ symmetry breaking on the values of the diagonal elements of V_K that we have retained. However, we note that from the K_{e3} decay it is known[12] that the experimental value of $\langle\pi^0(p^0)|V_{K^+}|K^-(k)\rangle$ at $|\mathbf{p}^0|=|\mathbf{k}|=\infty$ (i.e., at zero momentum transfer) is very close (within, say, 5%) to the $SU(3)$ value given by (9). This probably gives a reasonable estimate of the error made in neglecting $SU(3)$ symmetry breaking. Note that we need only the values of these matrix elements taken in the limit (6) where the mass differences between the pseudoscalar mesons can be neglected.

We now use pionic PCAC, i.e.,

$$\partial_\mu A_\mu{}^{\pi^0}(x)=(1/\sqrt{2})C_\pi\phi_\pi{}^0(x)\,, \qquad (10)$$

for the matrix elements involving A_{π^0} on right-hand side of Eq. (7). We obtain in our limit (6)

$$\langle\pi^+(p^+)\pi^-(p^-)|A_{\pi^0}|\pi^0(p^0)\rangle$$
$$=\left(\frac{1}{\sqrt{2}}\right)\left(\frac{C_\pi}{m_\pi{}^2}\right)\frac{1}{i(p_0{}^++p_0{}^--p_0{}^0)}$$
$$\times\langle\pi^+(p^+)\pi^-(p^-)|J_{\pi^0}(0)|\pi^0(p^0)\rangle, \qquad (11)$$

where

$$(\Box-m_\pi{}^2)\phi_{\pi^0}(x)=-J_{\pi^0}(x)\,.$$

The right-hand side of Eq. (11) thus involves the desired pion-pion scattering amplitude with only one of the pions off the mass shell ($m_\pi\to 0$). Our limit (6) allows us to relate this amplitude directly to a_0.

We also have to evaluate the other A_{π^0} term which becomes, using PCAC, proportional to $\langle n|J_{\pi^0}|K^-\rangle$. This term, if it survives, corresponds to the $K\pi$-$K\pi$ or $K\eta$-$K\pi$ (which does not appear in the symmetry limit) scattering amplitudes *with only one of the pions off the mass shell* ($m_\pi\to 0$).

Explicitly evaluating the second term of the right-hand side of Eq. (7) by substituting the values given by Eqs. (9) and (11), taking the limit (6), and comparing with Eq. (8), we obtain

$$F_1(s_0,t_0,u_0)=-m_K\sin\theta_A\left(\frac{C_\pi}{m_\pi{}^2}\right)$$
$$\times\left[(K\pi-K\pi)-\left(\frac{1}{3m_\pi{}^2}\right)(32\pi)\left(\frac{a_0}{3}\right)m_\pi\right], \qquad (12)$$

$$F_1(s_0,t_0.u_0)=2F_3(s_0,t_0,u_0)\,. \qquad (13)$$

Equation (13) coincides with the result of Weinberg[4] at the point (s_0,t_0,u_0). In (12) we have symbolically written the contribution of the first term of the right-hand side of Eq. (7) as the $(K\pi$-$K\pi)$ term. We claim that on the right-hand side of (12) this term is relatively unimportant, and its contribution is less than 20%. In our approach the terms involving the $K\pi$-$K\pi$ and $K\pi$-$K\eta$ scattering amplitudes contain the kinematical factors $1/(2m_\pi{}^2+m_K{}^2)$ and $1/(2m_\eta{}^2+m_K{}^2)$, respectively, in contrast to the factor $\frac{1}{3}m_\pi{}^2$ which appears for the $\pi\pi$-$\pi\pi$ term in (12). Therefore, if other factors are comparable, the one-pion intermediate state will be the most important one, as will be explicity shown later.

We now wish to estimate F_2. We put the final two pions in an $I=1$ antisymmetric state. On the left-hand side of Eq. (7) only the F_2 term now survives,[9] while on the right-hand side the first term involves (using PCAC) terms of the form $\langle n|J_{\pi^0}|K^-\rangle$, which are the antisymmetric K-π scattering amplitudes with one of the pions off the mass shell ($m_\pi\to 0$). The second term does not give a contribution. We evaluate the K-π scattering amplitudes by assuming that vector mesons dominate the whole process. By comparing the coefficients of the factor $(p_0{}^+-p_0{}^-)$ in both sides of Eq. (7), we obtain, again in the same limit (6),

$$F_2(s_0,t_0,u_0)=m_K\sin\theta_A\left(\frac{C_\pi}{m_\pi{}^2}\right)\left\{\left[\frac{4}{m_{K^*}{}^2-2(m_K{}^2+m_\pi{}^2)}\right.\right.$$
$$\left.\left.+\frac{2}{m_{K^*}{}^2-(\frac{1}{2}m_K{}^2-m_\pi{}^2)}\right]G_{K^{*0}K^0\pi^0}{}^2-\frac{G_{\rho^0\pi^+\pi^-}{}^2}{m_\rho{}^2+\frac{1}{2}m_K{}^2}\right\}, \qquad (14)$$

where $G_{K^{*0}K^0\pi^0}$ and $G_{\rho^0\pi^+\pi^-}$ are the coupling constants for the $K^{*0}\to K^0+\pi^0$ and $\rho^0\to\pi^++\pi^-$ decays. Using $\Gamma(K^*\to\text{all})=50$ MeV, $\Gamma(\rho^0\to\pi^++\pi^-)\simeq 129$ MeV, and the value of C_π determined from the Goldberger-Treiman relation, we obtain

$$F_2=1.67\,. \qquad (15)$$

The K_{e4} decay rate can be expressed[4] as (assuming $a_0\simeq 0$)

$$\Gamma(K_{e4})=[1.67F_1{}^2+0.32F_2{}^2]\times 10^3\ \text{sec}^{-1}, \qquad (16)$$

which shows that the determination of F_1 from this rate does not require a very precise value of F_2 if $|F_1|\simeq|F_2|$ as indicated by experiment.[1] If we use the above estimate of F_2, and use $\Gamma(K_{e4}{}^+)=(3.1\pm 0.65)\times 10^3$ sec^{-1} from (16), we obtain

$$|F_1|=1.14\,. \qquad (17)$$

If we now use (12) and assume that $\theta_V=\theta_A$ with $\cos\theta_V=0.978$, we obtain from (17), neglecting the $(K\pi$-$K\pi)$ term,

$$|a_0|=0.16\,. \qquad (18)$$

We now make an estimate of the effect of the neglected $(K\pi$-$K\pi)$ term. Instead of using the scattering length a_0, we compute the right-hand side of Eq. (7) by assuming that the *effective* S-wave pseudoscalar-meson–pseudoscalar-meson *low-energy* interaction can be reasonably approximated by the $SU(3)$-invariant interaction

$$H=4\pi\lambda[\pi\cdot\pi+2\bar{K}\cdot K+\eta\eta]^2. \qquad (19)$$

[12] S. Oneda and J. Sucher, Phys. Rev. Letters 15, 927 (1965); *ibid.* 15, 1049(E) (1965).

We then can determine the value of λ by using (17). The magnitude of the ($K\pi$-$K\pi$) term in (12) will then be reasonably fixed. We then again use (12) to determine a_0. In this way we find

$$|a_0|=0.18\,, \tag{20}$$

which is not very different from (18), as mentioned before. We may remark that we can also derive information about the $I=2$ S-wave pion-pion scattering length a_2. If we restrict, in Eq. (7), the final two-pion states to the symmetric ones instead of the $I=0$ state, we obtain, corresponding to (12),

$$F_1(s_0,t_0,u_0)=-m_K\sin\theta_A\left(\frac{C_\pi}{m_\pi^2}\right)\left[(K\pi-K\pi)-\left(\frac{1}{3m_\pi^2}\right)(32\pi)\left(\frac{a_0-a_2}{3}\right)m_\pi\right]. \tag{21}$$

If we use the same procedure and the same value of λ for the estimate of the ($K\pi$-$K\pi$) term, we obtain a_0 and a_2 from (12) and (21). Choosing the sign $F_1>0$, we obtain

$$a_0=0.18\,,\quad a_2=-0.017\,. \tag{22}$$

We regard these values of a_0 and a_2 as a reasonable estimate in our approach.[13] The above calculation is based on Eq. (16), where a_0 is taken to be zero. However, a calculation taking the final-state pion-pion interaction into account[14] changes the result (19) only by a negligibly small amount. We therefore conclude that the K_{e4}-decay rate indicates rather unambiguously[15] a small value for the $I=0$ pion-pion scattering length over the low-energy range of s values running from $4m_\pi^2$ to about $7.5\ m_\pi^2$.

We have avoided the use of kaon PCAC which demands the off-mass-shell extrapolation $m_K\rightarrow 0$. For the present problem this extrapolation does not appear to be a good approximation.[16] Using CCR and PCAC, Callan and Treiman[2] and Weinberg[4] estimated from the current K_{e3}-decay data, $F_1=F_2=0.97$. If we choose the sign of a_0 as positive as given by (2), we obtain in our approach $F_1=1.14$ and $F_2=1.67$. This predicts, for instance,

$$\frac{\Gamma(K^+\rightarrow\pi^0+\pi^0+e+\nu)}{\Gamma(K^+\rightarrow\pi^++\pi^-+e+\nu)}=0.32\,.$$

In view of the approximations of both approaches and the errors in experiments, one may argue that the results of both types of calculations agree rather well. It will be interesting to compute F_1 by applying a dynamical model. This will be discussed elsewhere.

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[13] If we determine the value of λ by using Eq. (7) for the symmetric two-pion final states instead of the $I=0$ state, we obtain $a_0=0.20$, $a_2=-0.04$. The value of a_2 is more sensitive to the estimate of the ($K\pi$-$K\pi$) term, in contrast to that of a_0.

[14] We have used Table I and Eq. (12) of N. Cabibbo and A. Maksymowicz, Phys. Rev. **137**, B438 (1965).

[15] The value of $|a|$ given by Eq. (22) is not sensitive to the value of F_2. If we take, for instance, $F_1=F_2>0$, as suggested in Ref. 4, we obtain $a_0=0.20$, $a_2=-0.02$.

[16] For the dispersion-theoretic treatment with the use of kaonic PCAC, see B. Sakita, M. Kato, and E. McCliment (unpublished); N. Van Hiew, Ref. 9; C. Kacser, P. Singer, and T. N. Truong, Phys. Rev. **137**, B1605 (1965); **139**, AB5(E) (1965); J. Iliopoulos, Nuovo Cimento **38**, 907 (1965); **39**, 413(E) (1965). If we use kaonic PCAC for the left-hand side of Eq. (7) and use interaction (19) for the estimate of the off-shell $K\pi$-$K\pi$ scattering (where one of the kaons has zero mass), we still obtain a small value of a_0, but it has an opposite sign to the one we obtained.