Phenomenological Regge Trajectory of the ρ Meson*

RONALD AARON AND MICHAEL T. VAUGHN
Northeastern University, Boston, Massachusetts
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A phenomenological model of the Regge trajectory of the ρ meson, based on a dispersion relation for the trajectory function and a parametrized absorptive part, is constructed and compared with existing data for $t\leq 0$, and with possibly observed recurrences. A linear term, which may or may not be entirely due to inelastic channels, is required to explain the data.

THERE has been speculation recently that there may be infinite sequences of Regge recurrences of the fundamental particles.[1] The trajectories which have been traced to the highest energies are those of the N and the N^*; these seem to be rising linearly with S to the present limits of experiment.[2,3] Do boson trajectories exhibit the same behavior? The experimental situation is less clear—several broad, high mass peaks have been observed,[4–6] which may well contain unresolved structure, but spin and parity assignments are not known with certainty. The data show one striking feature—the widths of the higher mass resonances increase slowly, if at all, with increasing mass, and the missing-mass spectrometer (MMS) data[6] suggest that, in fact, the widths decrease rather sharply with increasing mass. In this work, we present a phenomenological model, admittedly somewhat crude, of the ρ-meson trajectory (for which the negative t behavior is best known,[7]) based on an assumed dispersion relation for the trajectory function $\alpha(t)$. We find:

(i) The 3^- recurrence of the ρ is most naturally identified with *one* of the R peaks observed in the MMS experiments.[6]

(ii) The 5^- recurrence of the ρ is most naturally identified with the $U(2380)$.[5,6]

(iii) If these identifications are correct, then the linear rise of the trajectory, coupled with the experimental limits on the widths of these states, forces the dispersion relation to contain a term linear in t.

(iv) Moreover, the model gives a natural and quantitatively accurate explanation of the curvature of the trajectory for negative t found by Hohler *et al.*[7]

We start with the assumed dispersion relation for the $I=1$ π-π trajectory function $\alpha(\nu)$ $[\nu=\frac{1}{4}t-\mu^2]$

$$\alpha(\nu)=A\nu+B+\frac{1}{\pi}\int_0^\infty \frac{\mathrm{Im}\alpha(\nu')}{\nu'-\nu}d\nu', \qquad (1)$$

where $A\geq 0$, B are constants. $\mathrm{Im}\alpha(\nu)$ is required to be positive on the physical cut; it has singularities at channel thresholds, with known threshold behavior at two-particle thresholds,[8] and is expected to be a rather smooth function of ν apart from these kinematical singularities.

We first parametrize $\mathrm{Im}\alpha(\nu)$ according to

$$\mathrm{Im}\alpha(\nu)=\lambda\nu\left(\frac{\nu}{\nu+1}\right)^{\xi}\left(\frac{a^2}{\nu+a^2}\right)\left(\frac{b^2}{\nu+b^2}\right), \qquad (2)$$

with λ, a^2, b^2 as parameters, and

$$\xi=\alpha(0)-\tfrac{1}{2}$$

to give correct behavior at the π-π threshold.[8] (The effect of inelastic thresholds will be discussed below.) For fixed a^2, b^2 we adjust the parameters A, B, λ to fit the mass and width of the ρ (several values of mass and width were used, with quite similar results), and the intercept α_0 of the trajectory at $t=0$, given by[7]

$$\alpha_0=0.58\pm0.01.$$

We then require that a^2, b^2 be such that we obtain a reasonable value for the slope α_0' of the trajectory at $t=0$[9] *and* a reasonable fit to the over-all behavior of the trajectory for negative t found by Hohler *et al.*[7]

We compute the masses and widths of the higher spin recurrences, with typical results shown in Table

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[2] V. Barger and D. Cline, Phys. Rev. Letters **16**, 913 (1966).

[3] Additional evidence that the trajectories continue to rise is provided by the analysis of the polarization in π^-p charge-exchange scattering by B. Desai, D. Gregorich, and R. Ramachandran, Phys. Rev. Letters **18**, 565 (1967). An alternative explanation of these data in terms of Regge cuts has been given by V. de Lany, D. Gross, I. Muzinich, and V. L. Teplitz, Phys. Rev. Letters **18**, 148 (1967).

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[5] R. Abrams, R. Cool, G. Giacomelli, T. Kycia, B. Leontić, K. K. Li, and D. Michael, Phys. Rev. Letters **18**, 1209 (1967).

[6] M. Focacci, W. Kienzle, B. Levrat, B. Maglić, and M. Martin, Phys. Rev. Letters **17**, 890 (1966).

[7] G. Höhler, J. Baacke, H. Schaile, and P. Sondergger, Phys. Letters **20**, 79 (1966); C. B. Chiu, R. J. N. Phillips, and W. Rarita, Phys. Rev. **153**, 1485 (1967).

[8] A. O. Barut and D. E. Zwanziger, Phys. Rev. **127**, 974 (1962); A. O. Barut, *ibid.* **128**, 1959 (1962).

[9] From Ref. 7, $\alpha_0'=1.0\pm0.1$ $(\mathrm{GeV}/c)^{-2}$.

TABLE I. Masses and $\pi^-\pi$ partial decay widths for (inMeV) of the Regge recurrences of the ρ meson various values of the parameters in Eqs. (1) and (2) of the text. α_0 is the value of the ρ trajectory at $t=0$, α_0' the slope at $t=0$, and A is the coefficient of the linear term in Eq. (1) in pion mass units. m_ρ is the mass of the ρ, Γ_ρ its width. χ^2 is the value of χ^2 for a fit of the trajectory for $t\leq 0$ to the empirical trajectory determined by Hohler *et al.* (Ref. 7).

$m_\rho=760$ MeV, $\Gamma_\rho=130$ MeV.	$\alpha_0=0.57$			$\alpha_0=0.58$			$\alpha_0=0.59$	
a^2	2.0	1.5	5.0	2.0	1.5	1.5	1.5	1.5
b^2	5.0	15.0	10.0	2.5	5.0	10.0	2.0	3.0
α_0' [$(\mathrm{GeV}/c)^{-2}$]	1.02	0.96	0.90	1.07	1.02	0.96	1.12	1.06
α_0'	0.079	0.075	0.070	0.084	0.080	0.075	0.0875	0.083
A	0.065	0.0625	0.061	0.066	0.064	0.062	0.066	0.064
B	0.56	0.56	0.56	0.57	0.57	0.57	0.57	0.58
m_3	1739	1769	1795	1725	1745	1773	1718	1738
Γ_3	17	24	26	15	16	20	14	15
m_5	2330	2372	2408	2312	2340	2379	2302	2332
Γ_5	8	11.5	12	6.5	7	9	6	6
m_7	2800	2850	2893	2777	2812	2859	2767	2803
Γ_7	4.5	7	7.5	4	4.3	5.6	3.5	3.7
χ^2	14.4	12.7	12.9	13.9	13.0	13.6	15.0	15.2

$m_\rho=780$ MeV, $\Gamma_\rho=140$ MeV.	$\alpha_0=0.55$		$\alpha_0=0.57$		$\alpha_0=0.58$		$\alpha_0=0.59$	
a^2	3.5	15	2.5	3.0	1.5	3.0	1.75	1.75
b^2	25	50	3.5	5.0	3.5	4.0	1.75	2.5
α_0' [$(\mathrm{GeV}/c)^{-2}$]	0.90	0.83	1.02	0.96	1.06	0.96	1.12	1.06
α_0'	0.070	0.064	0.079	0.075	0.083	0.075	0.087	0.083
A	0.060	0.056	0.063	0.061	0.063	0.060	0.064	0.063
B	0.52	0.47	0.55	0.56	0.56	0.57	0.56	0.57
m_3	1817	1887	1767	1790	1761	1798	1750	1768
Γ_3	37	80	18	19	16	18	15	16
m_5	2433	2531	2366	2398	2360	2411	2345	2369
Γ_5	19	48	8	9	7	8	6	7
m_7	2920	···	2842	2880	2834	2897	2817	2847
Γ_7	12	···	4.5	5.2	4.2	4.7	3.7	3.9
χ^2	18.9	17.0	13.0	12.1	12.2	13.6	13.6	14.5

I.[10,11] As can be seen, the 3^- and 5^- recurrences can be identified with the R peak and the $U(2380)$,[6] although the 3^- mass is probably not accurately given, for reasons explained below. The parameter A is significantly different from zero; the values obtained correspond to an asymptotic slope of the trajectory about 15–20% smaller than the slope at $t=0$. We remark that our results are rather sensitive to the value of α_0, as can be seen from the parameters shown in Table I for $\alpha_0=0.55$. We thus rely heavily on the validity of the phenomenological analysis of Ref. 7.

Inelastic channels certainly give important contributions to the total widths of the recurrences, but we do not expect, in general, substantial shifts in the masses of the recurrences, since channels above and below the mass of the recurrence contribute with opposite sign to the mass shift, and the magnitudes of these contributions are not expected to be large in any event.[12] The 3^- recurrence is exceptional; here we have taken into account explicitly the π-π channel, but the neglected channels lie mostly above the mass of the 3^-, and tend to lower the mass from the value given here, perhaps by as much as 50 MeV or so.[13]

[10] χ^2 is the χ^2 for a fit to the "empirical" trajectory determined by Hohler *et al.* (Ref. 7); there are 14–15 degrees of freedom (the fit is not sensitive to a^2 and b^2 separately), and the fit is significantly better than a straight-line fit.

[11] The Γ_J are computed from

$$\Gamma_J=\frac{2}{(\nu_J+1)^{1/2}}\frac{\mathrm{Im}\alpha(\nu_J)}{\mathrm{Re}\alpha'(\nu_J)}.$$

Since we have included in $\mathrm{Im}\alpha(\nu)$ only the contribution from the $\pi^-\pi$ cut, we interpret the Γ_J as partial widths for decay into the $\pi^-\pi$ channel.

[12] The point is that $\mathrm{Re}\alpha(\nu)$ is modified by a principal-value integral over the neglected terms in $\mathrm{Im}\alpha(\nu)$. Provided these neglected terms are relatively smooth for some distance on either side of a resonance position ν_R, the modification of $\mathrm{Re}\alpha(\nu_R)$, and hence of the mass of the resonance, is expected to be small.

[13] We estimated the effects of inelastic channels in the following way: We add to $\mathrm{Im}\alpha(\nu)$ a term of the form of Eq. (2), with ν replaced on the right-hand side by $\nu-\nu_R$, where ν_R is an inelastic threshold. The appearance of new parameters λ', a'^2, b'^2, ξ' makes detailed quantitative studies impossible, but so long as the contribution of the additional term of $\mathrm{Im}\alpha(\nu)$ to the total $\alpha(\nu)$ is limited by requiring the total widths of the 3^- and 5^- recurrences to be less than 100–150 MeV, then the effect on the masses, *and on the parameter A*, is rather small—A is decreased by at most about 4%, and the masses of the recurrences are shifted by less than 50 MeV (the partial widths for decay into $\pi^-\pi$ are essentially unchanged)—for inelastic thresholds corresponding to $K\bar{K}$, $\rho\rho$, and $N\bar{N}$ channels. These channels are chosen to illustrate the effects of inelastic thresholds with various locations relative to the masses of the recurrences. It would be surprising if effects due to channels with yet higher thresholds (e.g., $N^*\bar{N}^*$) were appreciably larger. See also Ref. 17 below.

We are thus inclined to identify the 3^- recurrence with either the R_1 peak or the R_2 peak of the MMS experiment, but cannot distinguish between the two possibilities.[14]

The appearance of the linear term in Eq. (1) is made necessary by the relatively narrow width of the ρ meson. This narrow width has always been difficult to explain in models based on elastic unitarity in the π-π channel,[15] but some authors have claimed that including sufficiently many inelastic channels can explain the width.[16] However, if the interactions in the channels considered are sufficiently strong to produce a narrow ρ, the same interactions should lead to much larger widths than have been observed for the recurrences (if the experimental peaks are indeed the recurrences), since these channels are open at the higher masses of the recurrences.[17]

It may be that the successive narrow widths are nonetheless the result of a conspiracy of a large number of channels extending to very high energies, and the ρ trajectory eventually turns around. Then Eq. (1) with $A \neq 0$ may still provide a useful phenomenological representation of the trajectory at energies presently studied, with the effects of higher energy channels contained in the parameters A and B. However, the bounds of Ref. 17 apply to a general channel, and we believe that the alternative possibility, i.e., that the ρ trajectory continues to rise indefinitely, with asymptotic slope as a new fundamental constant, must be considered seriously.

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[14] The partial width given here for decay into $\pi^-\pi$ is consistent with the experimental branching ratio $(R \to 1 \text{ charged})/(R \to >3$ charged) for both R_1 and R_2. We remark here that the ρ trajectory may have a daughter [D. Z. Freedman and J. M. Wang, Phys. Rev. Letters **18**, 863 (1967), and earlier works cited therein] which passes through 1 at a mass near the mass of the 3^- recurtence, but with a mass difference of the order of 50 MeV, say. This possibility is worth careful experimental study.

[15] See, for example, L. F. Cook and C. E. Jones, Phys. Rev. **144**, 1165 (1966); or P. D. B. Collins and V. L. Teplitz, *ibid.* **140**, B663 (1965).

[16] See, for example, J. S. Ball and M. Parkinson (to be published).

[17] An interesting possibility, suggested to the authors by O. W. Greenberg, is that contributiions to Imα an order of magnitude larger than those described here might come from quark-antiquark channels (if quarks exist). However, if the contribution from such channels is written

$$\alpha_Q(\nu) = \frac{1}{\pi}\int_{\nu_Q}^{\infty} \frac{\mathrm{Im}\alpha_Q(\nu')}{\nu'-\nu} d\nu'$$

($\nu_Q = m_Q^2 - 1$), then, assuming $\mathrm{Im}\alpha_Q(\nu) > 0$, we have at $t=0$,

$$\alpha_Q > m_Q^2 \alpha_Q'.$$

If the trajectory approaches a finite limit at ∞ (so that $A=0$ when all contributions to Imα are correctly included), then m_Q^2 cannot be too large. (For example, if the finite limit is -1, then we require $m_Q \lesssim 1.5$ BeV.)