Gauge Fields, Sources, and Electromagnetic Masses

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The hypothesis of strong-interaction gauge fields, with non-Abelian gauge invariance broken only by the 1^- particle mass terms, gives a natural source theory setting for the introduction of electromagnetic effects. The electromagnetic potential vector appears as a compensating field in the mass terms of the neutral 1^- particles. The resulting electromagnetic self-action is used to discuss mass displacements. The pion electromagnetic mass is computed in a number of ways—by direct calculation of various processes and by chiral methods, in two variants. The relationship of these approaches is established. A phenomenological modification of the chiral evaluation gives perfect agreement with the observed value. It is found, however, that the $(m_\pi/m_\rho)^2$ terms, which are neglected in this method, are not very small. Baryon electromagnetic mass splittings are described by a simple adaptation of gross mass-spectrum empirics. Agreement with the data is excellent.

THE great utility of phenomenological gauge-field descriptions of the 1^- and 1^+ mesons in connection with unitary and chiral transformations has become evident recently.[1] This raises again the question of the relative role of the photon gauge field. Previous discussions have used the language of quantum field theory.[2] The phenomenological orientation associated with the source concept[3] has produced a new situation, however. We shall give a simple solution of the problem. With its aid, we discuss in some detail the electromagnetic contributions to the mass difference between charged and neutral pions. There is also a brief treatment of baryon electromagnetic mass splittings.

GAUGE INVARIANCE I

Let us consider the isotopic spin gauge field associated with ρ. An illustrative phenomenological Lagrange function is that of the $\rho+\pi$ system,

$$\mathcal{L}=-\tfrac{1}{2}(D_\mu\pi)^2-\tfrac{1}{2}m_\pi{}^2\pi^2-\tfrac{1}{4}(\rho_{\mu\nu})^2-\tfrac{1}{2}m_\rho{}^2(\rho_\mu)^2,$$

where

$$D_\mu=\partial_\mu+g\rho_\mu\times=\partial_\mu-igt\cdot\rho_\mu$$

and

$$\rho_{\mu\nu}=\partial_\mu\rho_\nu-\partial_\nu\rho_\mu+g\rho_\mu\times\rho_\nu.$$

This Lagrange function is invariant under the infinitesimal isotopic gauge transformation

$$\delta\pi=-\delta\omega\times\pi,$$

$$\delta\rho_\mu=-\delta\omega\times\rho_\mu+(1/g)\partial_\mu\delta\omega,$$

with the exception of the ρ mass term,

$$\delta\mathcal{L}=-(m_\rho{}^2/g)\rho^\mu\cdot\partial_\mu\delta\omega.$$

This implies, incidentally, that

$$\partial_\mu\rho^\mu=0.$$

When $\delta\omega$ is directed along the third isotopic spin axis, the gauge transformation of $\rho_{\mu 3}$ is simply

$$\delta g\rho_{\mu3}=\partial_\mu\delta\omega.$$

We now recognize the possibility of realizing complete invariance under this transformation, through the compensating effect of the electromagnetic gauge transformation

$$\delta e_0A_\mu=\partial_\mu\delta\omega,$$

provided the ρ mass term is generalized to

$$-\tfrac{1}{2}m_\rho{}^2(\rho_{\mu1,2})^2-\tfrac{1}{2}m_\rho{}^2[\rho_{\mu3}-(e_0/g)A_\mu]^2.$$

It is an old idea to couple the photon directly to the unit spin mesons, as in $m_\rho{}^2(e_0/g)\rho^\mu{}_3A_\mu$, but questions of gauge invariance have produced uneasiness with this recipe[4] [rightly so, since the $(A_\mu)^2$ term is omitted]. In contrast, gauge invariance is our guide.

THE PARTICLES

The direct coupling between $\rho^\mu{}_3$ and A^μ means that these are not the fields associated with the particles ρ^0 and photon (γ). The relevant part of the Lagrange function is

$$\mathcal{L}_{\gamma\rho^0}=-\tfrac{1}{4}(F_{\mu\nu})^2-\tfrac{1}{4}(\rho_{\mu\nu3})^2-\tfrac{1}{4}m_\rho{}^2[\rho_{\mu3}-(e_0/g)A_\mu]^2,$$

with

$$F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu,\quad \rho_{\mu\nu3}=\partial_\mu\rho_{\nu3}-\partial_\nu\rho_{\mu3}.$$

The diagonalization transformation,[5]

$$\rho_{\mu3}=[1+(e_0/g)^2]^{-1/2}[\rho_\mu{}^{(0)}+(e_0/g)\gamma_\mu],$$

$$A_\mu=[1+(e_0/g)^2]^{-1/2}[\gamma_\mu-(e_0/g)\rho_\mu{}^{(0)}],$$

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[1] (a) S. Weinberg, Phys. Rev. Letters 18, 188 (1967); (b) J. Schwinger, Phys. Letters 24B, 473 (1967).

[2] J. Schwinger, Rev. Mod. Phys. 36, 609 (1964).

[3] J. Schwinger, Phys. Rev. 152, 1219 (1966); 158, 1391 (1967).

[4] For a recent quantum field theory discussion, see N. Kroll, T. D. Lee, and B. Zumino, Phys. Rev. 157, 1376 (1967); also T. D. Lee, S. Weinberg, and B. Zumino, Phys. Rev. Letters 18, 1029 (1967).

[5] These transformations also appear in Ref. 2. Something similar occurs in Appendix B of the first paper mentioned in Ref. 4. I became aware of this Appendix only after the present paper was written, when the published article was distributed. Although it does not have the background of the non-Abelian gauge treatment of ρ, one can recognize in Eq. (B8) the counterpart of our mass-term prescription.

gives

$$\mathcal{L}_{\gamma\rho^0}=-\tfrac{1}{4}(\gamma_{\mu\nu})^2-\tfrac{1}{4}(\rho^{(0)}{}_{\mu\nu})^2-\tfrac{1}{2}m_{\rho^0}{}^2(\rho^{(0)}{}_{\mu})^2,$$
$$m_{\rho^0}{}^2=[1+(e_0/g)^2]m_\rho{}^2.$$

If we include a photon-source term, written as $e_0J^\mu A_\mu$, the latter becomes

$$e_0J^\mu A_\mu=eJ^\mu[\gamma_\mu-(e_0/g)\rho_\mu{}^{(0)}],$$

where

$$e=e_0[1+(e_0/g)^2]^{-1/2}$$

is identified as the physical unit of charge. This is also evident from the relation

$$g\rho_{\mu3}=e\gamma_\mu+(g^2-e^2)^{1/2}\rho_\mu{}^{(0)}.$$

To *first* order in e, and in terms of the physical ρ field $(\rho_{\mu1,2},\rho_\mu{}^{(0)})$, we have

$$D_\mu=\partial_\mu-igt\cdot\rho_\mu-iet_3\gamma_\mu,$$

which exhibits et_3 as the absolute charge matrix of unit isotopic spin.

The simple diagonalization transformation gives a first indication of ρ-meson electromagnetic mass splittings

$$m_{\rho^0}{}^2=[1-(e/g)^2]^{-1}m_\rho{}^2,$$

while maintaining the masslessness of the photon. It is interesting to apply perturbation methods to these questions. We do not distinguish here between e_0 and e. The coupling term $(em_\rho{}^2/g)\rho^\mu{}_3A_\mu$ displays $(em_\rho{}^2/g)\rho^\mu{}_3$ as an effective photon source. The self-coupling of that source supplies an additional term in the action w:

$$\delta w_\rho=\tfrac{1}{2}(em_\rho{}^2/g)^2\int(dx)(dx')\rho^\mu{}_3(x)D_+(x-x')\rho_{\mu3}(x').$$

The insertion of the unperturbed ρ field gives

$$\int(dx')D_+(x-x')\rho_{\mu3}(x')=-(1/m_\rho{}^2)\rho_{\mu3}(x),$$

which implies the mass displacement

$$\delta m_{\rho^0}{}^2\simeq(e/g)^2m_\rho{}^2.$$

The same coupling term displays $(em_\rho{}^2/g)A_\mu$ as an effective ρ^0 source. That produces the additional action term

$$\delta w_A=\tfrac{1}{2}(em_\rho{}^2/g)^2\int(dx)(dx')A^\mu(x)\left(g_{\mu\nu}-\frac{1}{m_\rho{}^2}\partial_\mu\partial_\nu\right)\times\Delta_+(x-x')_\rho A^\nu(x').$$

It is not gauge-invariant. But the gauge variance is cancelled by that of the quadratic A term in $\mathcal{L}$, $-\tfrac{1}{2}(em_\rho/g)^2(A_\mu)^2$. The use of the Lorentz gauge, $\partial_\mu A^\mu=0$, and reference to the unperturbed photon field,

$$\int(dx')\Delta_+(x-x')_\rho A^\nu(x')=(1/m_\rho{}^2)A^\nu(x),$$

results in complete cancellation of the quadratic A terms.

The electromagnetic vector potential can also be eliminated, in the manner of the perturbation discussion, but without approximation. The electromagnetic field equations are

$$-\partial^2A_\mu=(m_\rho{}^2e_0/g)[\rho_{\mu3}-(e_0/g)A_\mu].$$

They are solved by

$$A_\mu(x)=(m_\rho{}^2e_0/g)\int(dx')D_+'(x-x')\rho_{\mu3}(x'),$$

where D_+' is the propagation function associated with the mass $(e_0/g)m_\rho$. [Reader: Do not too hastily conclude that this is a predicted photon mass.] The electromagnetic terms in w are

$$w_e=\int(dx)\{-\tfrac{1}{2}(\partial_\mu A_\nu)^2-\tfrac{1}{2}m_\rho{}^2[-2(e_0/g)\rho^\mu{}_3A_\mu+(e_0/g)^2(A_\mu)^2]\}$$
$$=\tfrac{1}{2}(m_\rho{}^2e_0/g)^2\int(dx)(dx')\rho^\mu{}_3(x)D_+'(x-x')\rho_{\mu3}(x').$$

The implied field equations for $\rho^\mu{}_3$, referring to noninteracting particles, are then

$$-\partial^2\rho^\mu{}_3(x)+\partial^\mu\partial_\nu\rho^\nu{}_3(x)+m_\rho{}^2\rho^\mu{}_3(x)=(m_\rho{}^2e_0/g)^2\int(dx')D_+'(x-x')\rho^\mu{}_3(x').$$

The condition for natural oscillations with unit spin is

$$p^2+m_\rho{}^2=(m_\rho{}^2e_0/g)^2[p^2+(e_0/g)^2m_\rho{}^2]^{-1}$$

or

$$p^2\{p^2+[1+(e_0/g)^2]m_\rho{}^2\}=0.$$

Here again are the masses of the photon and of ρ^0. Both are in evidence since $\rho^\mu{}_3$ is a mixture of the fields of the two particles. If we approximate D_+' by D_+ we lose the ability to describe the photon and must restrict application to ρ^0.

Let us use the latter framework to construct Green's function for ρ^0. The momentum form of the field equations, including a source term, is

$$[p^2+m_\rho{}^2-([e_0/g]m_\rho{}^2)^2(1/p^2)]\rho^\mu{}_3(p)-p^\mu p_\nu\rho^\nu{}_3(p)=J^\mu{}_3(p).$$

This differs from the usual version only through the appearance of an effective, momentum-dependent mass. Accordingly,

$$G_{\mu\nu}(p)_{\rho^0}=[p^2+m_\rho{}^2-([e_0/g]m_\rho{}^2)^2(1/p^2)]^{-1}\times\{g_{\mu\nu}+[m_\rho{}^2-([e_0/g]m_\rho{}^2)^2(1/p^2)]^{-1}p_\mu p_\nu\}.$$

We are particularly interested in the charge-dependent part of this function. It is

$$G_{\mu\nu}(p)_{\rho^0}-G_{\mu\nu}(p)_{\rho}=\delta G_{\mu\nu}(p)_{\rho}$$

$$\cong\left(\frac{e}{g}\right)^2\frac{1}{p^2}\left(\frac{m_\rho^2}{p^2+m_\rho^2}\right)^2\left[g_{\mu\nu}+\frac{p_\mu p_\nu}{m_\rho^2}\right]+\left(\frac{e}{g}\right)^2\frac{1}{p^2}\frac{1}{p^2+m_\rho^2}p_\mu p_\nu.$$

This can also be obtained through the perturbation solution

$$\delta G_{\mu\nu}(p)_\rho=G_\mu{}^\lambda(p)_\rho[(e/g)m_\rho^2]^2(1/p^2)G_{\lambda\nu}(p)_\rho$$

$$=\left(\frac{e}{g}m_\rho^2\right)^2\frac{1}{p^2}\frac{1}{(p^2+m_\rho^2)^2}\times\left[g_{\mu\nu}+\frac{2}{m_\rho^2}p_\mu p_\nu+\frac{p^2}{m_\rho^2}\frac{1}{m_\rho^2}p_\mu p_\nu\right].$$

To the extent that $\delta G_{\mu\nu}(p)_\rho$ enters in gauge-invariant combinations, it is equivalent to

$$g^2\delta G_{\mu\nu}(p)_\rho=g_{\mu\nu}(e^2/p^2)[m_\rho^2/(p^2+m_\rho^2)]^2,$$

which can be interpreted as photon exchange, modified by the ρ form factor, $m_\rho^2/(p^2+m_\rho^2)$, at both emission and absorption.

SOURCE THEORY

Before discussing electromagnetic masses, it is necessary to recognize the relation provided by source theory[3] between processes involving two real particles and processes that refer to a virtual particle. The vacuum amplitude that represents an arbitrary number of noninteracting particles, of any type, is of the form

$$\langle 0_+|0_-\rangle^s=\exp\left[i\tfrac{1}{2}\int(dx)(dx')S(x)G_+(x-x')S(x')\right].$$

If two partial sources S_1 and S_2 are made explicit, their multiplicative contribution appears as

$$\exp\left[i\int(dx)(dx')S_1(x)G_+(x-x')S_2(x')+i\int(dx)S_1(x)\chi(x)+i\int(dx)\chi(x)S_2(x)\right],$$

where $\chi(x)$ is the field

$$\chi(x)=\int(dx)G_+(x-x')S(x').$$

The vacuum amplitude terms that are bilinear in S_1 and S_2 are combined in

$$-\int(dx)(dx')S_1(x)[\chi(x)\chi(x')-iG_+(x-x')]S_2(x').$$

Accordingly, the two types of processes are related by the correspondence

$$\chi(x)\chi(x')\rightarrow -iG_+(x-x').$$

π ELECTROMAGNETIC MASS

The electromagnetic modification of the ρ^0 propagation function introduces new processes associated with ρ exchange. They are superimposed upon the phenomenological framework which already incorporates the physics of strong interactions. This implies changes of phenomenological parameters, including particle masses. The simplest example is the displacement of the charged pion mass, arising from the ρ-π coupling mechanism. The relevant Lagrangian terms are

$$\mathcal{L}_{\pi\rho}=g\rho^\mu(\partial_\mu\pi\times\phi-\pi\times\partial_\mu\phi)-\tfrac{1}{2}g^2(\rho_\mu\times\phi)^2,$$

in which we have distinguished between the π field of interest, designated as ϕ, and the π field that will describe an exchanged particle. We shall also use simplifications associated with the small (mass)^2 ratio $(m_\pi/m_\rho)^2\simeq 1/30$ and therefore discard the $\partial_\mu\phi$ term. It would contribute as $(\partial_\mu\phi)^2\rightarrow -m_\pi^2\phi^2$.

The electromagnetically induced ρ-exchange contribution of the first term in $\mathcal{L}_{\pi\rho}$ is

$$\delta w_\pi^{(1)}=\tfrac{1}{2}g^2\int(dx)(dx')(\partial_\mu\pi\times\phi)_3(x)\delta G^{\mu\nu}(x-x')_\rho\times(\partial_\nu\pi\times\phi)_3(x')\rightarrow\tfrac{1}{2}g^2(1/i)\int(dx)(dx')\phi(x)\cdot\phi(x')\times\delta G^{\mu\nu}(x-x')_\rho\partial_\mu\partial_\nu'\Delta_+(x-x')_\pi,$$

where the second step introduces a virtual pion. In the latter form, ϕ refers to charged pions only. On adding the direct contribution of the second term in $\mathcal{L}_{\pi\rho}$, while continuing to regard $(m_\pi/m_\rho)^2$ as very small, we get

$$\delta w_\pi'=-\tfrac{1}{2}g^2\int dx(\phi(x))^2(1/i)\times\int\frac{(dp)}{(2\pi)^4}\delta G^{\mu\nu}(p)_\rho\left[g_{\mu\nu}-\frac{p_\mu p_\nu}{p^2}\right].$$

This identifies the (mass)^2 displacement contribution $(m_{\pi^\pm}{}^2-m_{\pi^0}{}^2)$:

$$\delta m_\pi{}^{2\prime}=(1/i)\int\frac{(dp)}{(2\pi)^4}g^2\delta G^{\mu\nu}(p)_\rho\left[g_{\mu\nu}-\frac{p_\mu p_\nu}{p^2}\right]$$
$$=3e^2(1/i)\int\frac{(dp)}{(2\pi)^4}\frac{1}{p^2}\left(\frac{m_\rho{}^2}{p^2+m_\rho{}^2}\right)^2.$$

In Euclidean spherical coordinates,

$$(1/i)(dp)\rightarrow\pi^2p^2dp^2$$

and

$$\delta m_\pi{}^{2\prime}=(3\alpha/4\pi)m_\rho{}^2.$$

This contribution to δm_π is

$$\delta m_\pi{}'=3.6\text{ MeV},$$

which is a substantial fraction (~80%) of the observed 4.6 MeV.

There has been much attention recently to the role of the π meson in chiral transformations and the related significance of A_1(1080 MeV) as the axial partner of ρ. Chiral invariance is violated by the π mass term. An independent approach to the pion electromagnetic mass is based on computing the additional violation of chiral invariance that is produced by

$$w_e=\tfrac{1}{2}(m_\rho{}^2e/g)^2\int(dx)(dx')\rho^\mu{}_3(x)D_+(x-x')\rho_{\mu3}(x').$$

The response to infinitesimal homogeneous chiral transformations is given by

$$\delta\rho_\mu=-(2g/m_A)\delta\varphi\times(A_\mu),$$
$$\delta(A_\mu)=-(2g/m_A)\delta\varphi\times\rho_\mu,$$

where

$$m_A=2^{1/2}m_\rho$$

and[6]

$$(A_\mu)=A_{1\mu}+(1/m_A)D_\mu\pi$$

combine the field of the 1^+ A_1 particle with that of the 0^- pion. Accordingly,

$$\delta_\varphi w_e=(m_\rho{}^2e/g)^2(2g/m_A)\int(dx)(dx')\times[(A^\mu)\times\delta\varphi]_3(x)D_+(x-x')\rho_{\mu3}(x')$$

and

$$\delta_\varphi{}^2w_e=2e^2m_\rho{}^2\int(dx)(dx')D_+(x-x')\{[(A^\mu)\times\delta\varphi]_3(x)\times[(A_\mu)\times\delta\varphi]_3(x')+[(\rho^\mu\times\delta\varphi)\times\delta\varphi]_3(x)\rho_{\mu3}(x')\}.$$

[6] The notation differs slightly from the Physics Letter of Ref. 1(b), where $-P_\mu$ is used for (A_μ) and $A_{1\mu}$.

We emphasize again that we are now considering processes that are not contemplated in the strong interaction phenomenology. The consideration of a virtual A_1, π, or ρ meson gives

$$\delta_\varphi{}^2w_e=2e^2m_\rho{}^2(\delta\varphi)^2i\int(dx)(dx')D_+(x-x')\times[G^\mu{}_\mu(x-x')_\rho-G^\mu{}_\mu(x-x')_{(A)}],$$

where $\delta\varphi$ here refers only to charged components, and meson Green's functions for any one isotopic component are used. This structure is identified as the chiral response of the additional charged pion term

$$\delta w_\pi=-e^2m_\rho{}^2\int(dx)(\phi(x))^2(1/i)\times\int\frac{(dp)}{(2\pi)^4}D_+(p)[G^\mu{}_\mu(p)_\rho-G^\mu{}_\mu(p)_{(A)}]$$

and gives

$$\delta m_\pi{}^2=2e^2m_\rho{}^2(1/i)\int\frac{(dp)}{(2\pi)^4}\frac{1}{p^2}[G^\mu{}_\mu(p)_\rho-G^\mu{}_\mu(p)_{(A)}].$$

Here

$$G_{\mu\nu}(p)_{(A)}=\frac{g_{\mu\nu}+(1/m_A{}^2)p_\mu p_\nu}{p^2+m_A{}^2}+\frac{(1/m_A{}^2)p_\mu p_\nu}{p^2+m_\pi{}^2},$$

and $(m_\pi{}^2\ll m_\rho{}^2)$

$$G^\mu{}_\mu(p)_{(A)}=3/(p^2+m_A{}^2)+1/m_A{}^2.$$

That combines with

$$G^\mu{}_\mu(p)_\rho=3/(p^2+m_\rho{}^2)+1/m_\rho{}^2$$

to give $(m_A{}^2=2m_\rho{}^2)$

$$\delta m_\pi{}^2=6e^2m_\rho{}^2(1/i)\int\frac{(dp)}{(2\pi)^4}\frac{1}{p^2}\left[\frac{1}{p^2+m_\rho{}^2}-\frac{1}{p^2+m_A{}^2}\right]$$
$$=(3\alpha/2\pi)(\ln 2)m_\rho{}^2.$$

This prediction is[7]

$$\delta m_\pi=5.0\text{ MeV}.$$

There is another chirality-based derivation of this result that brings dynamics more into evidence. It uses $\delta_\varphi w_e$ rather than $\delta_\varphi{}^2w_e$. We write

$$\delta_\varphi w_e=(m_\rho{}^2e^2/g)\int(dx)(dx')[(\partial^\mu\pi+g\rho^\mu\times\phi)\times\delta\varphi]_3(x)\times D_+(x-x')\rho_{\mu3}(x')+(m_\rho{}^2e/g)^2(2g/m_A)\times\int(dx)(dx')[A_1{}^\mu\times\delta\varphi]_3(x)D_+(x-x')\rho_{\mu3}(x'),$$

[7] We have reproduced the result of a current-algebra derivation by T. Das, G. Guralnick, V. Mathur, F. Low, and J. Young, Phys. Rev. Letters 18, 759 (1967).

which exhibits several new processes induced by the electromagnetic interaction. Only one of these is self-contained in its contribution to a structure that can be recognized as the chiral response of a charged pion mass term. It is

$$-e^2 m_\rho^2(1/i)\int (dx)(dx')\phi(x)\cdot\delta\varphi D_+(x-x')G^\mu{}_\mu(x-x')_\rho.$$

For the others, we use relevant strong-interaction couplings.[1b] They are

$$\mathcal{L}_{\rm int}=g\rho^\mu\cdot\partial_\mu\pi\times\phi+(g/m_A)\tfrac{1}{2}A_1{}^{\mu\nu}\times\phi\cdot\rho_{\mu\nu}.$$

The ρ-π coupling term gives the contribution

$$e^2 m_\rho^2(1/i)\int (dx)\cdots(dx'')\phi(x)\cdot\delta\varphi D_+(x'-x'')\times G^{\mu\nu}(x-x'')_\rho\partial_\mu\partial_\nu{}'\Delta_+(x-x')_\pi,$$

which combines with the simple ρ-exchange contribution to produce the following partial evaluation of the (mass)^2 displacement:

$$e^2 m_\rho^2(1/i)\int\frac{(dp)}{(2\pi)^4}\frac{1}{p^2}G^{\mu\nu}(p)_\rho\left[g_{\mu\nu}-\frac{p_\mu p_\nu}{p^2}\right]=3e^2m_\rho^2(1/i)\int\frac{(dp)}{(2\pi)^4}\frac{1}{p^2}\frac{1}{p^2+m_\rho^2}.$$

The complete result obtained by adding the effect of $A_1+\rho$ exchange is

$$\delta m_\pi{}^2=3e^2m_\rho^2(1/i)\int\frac{(dp)}{(2\pi)^4}\frac{1}{p^2+m_\rho^2}\left[\frac{1}{p^2}-\frac{1}{p^2+m_A{}^2}\right]=6e^2(m_\rho^2)^2(1/i)\int\frac{(dp)}{(2\pi)^4}\frac{1}{p^2}\frac{1}{p^2+m_\rho^2}\frac{1}{p^2+m_A{}^2},$$

which is equivalent to the first chiral evaluation. This is indirect evidence for the consistency of the dynamical scheme with chirality requirements.

The chirality calculations seem to suggest that the particle A_1 is essential to a pion electromagnetic mass calculation. And yet, a not unsatisfactory result was obtained from the $\pi+\rho$ system alone. We shall implement the idea that the consideration of A_1 is significant, but not fundamental, by exhibiting the physical processes that relate the two distinct calculational schemes.

We have already referred to the $A_1\rho\pi$ coupling, which arises from the following Lagrangian term ($\partial_\mu\phi$ contributions are omitted):

$$-\tfrac{1}{4}[A_{1\mu\nu}+(g/m_A)\rho_{\mu\nu}\times\phi]^2.$$

Thus, a more complete account of this interaction is

$$\mathcal{L}_{A_1\rho\pi}=(g'/m_A)\tfrac{1}{2}A_1{}^{\mu\nu}\times\phi\cdot\rho_{\mu\nu}-(g'/m_A)^2\tfrac{1}{4}(\rho_{\mu\nu}\times\phi)^2,$$

where we have written g' rather than g in order to incorporate form-factor effects that are not considered in the simple Lagrange function. Their presence is indicated by the observed A_1 width, which requires[1b]

$$g'\simeq g/1.2.$$

We now consider electromagnetic modifications in the exchange of $\rho+A_1$ and of ρ alone, corresponding to the two terms of $\mathcal{L}_{A_1\rho\pi}$. This gives

$$\begin{aligned}\delta w_\pi{}''&=(g'/2m_A{}^2)(1/2i)\int(dx)(dx')\phi(x)\cdot\phi(x')\\&\quad\times\delta G_{\mu\nu,\lambda\kappa}(x-x')_\rho G_{\mu\nu,\lambda\kappa}(x-x')_{A_1}-(g'/2m_A)^2(1/i)\\&\quad\times\int(dx)(\phi(x))^2\delta G_{\mu\nu,\mu\nu}(0)_\rho\\&\cong(g'/2m_A)^2\int(dx)(\phi(x))^2(1/i)\int\frac{(dp)}{(2\pi)^4}\\&\quad\times[\tfrac{1}{2}\delta G_{\mu\nu,\lambda\kappa}(p)_\rho G_{\mu\nu,\lambda\kappa}(p)_{A_1}-\delta G_{\mu\nu,\mu\nu}(p)_\rho].\end{aligned}$$

Here again $\phi(x)$ refers to charged π mesons, in the approximation $m_\pi{}^2\ll m_\rho{}^2$. Also

$$G_{\mu\nu,\lambda\kappa}(p)_{A_1}=f_{\mu\nu,\lambda\kappa}[1/(p^2+m_A{}^2)]$$

and

$$\delta G_{\mu\nu,\lambda\kappa}(p)_\rho=f_{\mu\nu,\lambda\kappa}(e/g)^2(1/p^2)[m_\rho{}^2/(p^2+m_\rho{}^2)]^2,$$

where

$$f_{\mu\nu,\lambda\kappa}=p_\mu p_\lambda g_{\nu\kappa}-p_\nu p_\lambda g_{\mu\kappa}+p_\nu p_\kappa g_{\mu\lambda}-p_\mu p_\kappa g_{\nu\lambda}.$$

We note that

$$\tfrac{1}{4}(f_{\mu\nu,\lambda\kappa})^2=3(p^2)^2$$

and

$$\tfrac{1}{2}f_{\mu\nu,\mu\nu}=3p^2.$$

Hence, this contribution to $\delta m_\pi{}^2$ is

$$\begin{aligned}\delta m_\pi{}^{2\prime\prime}&=3(g'/m_A)^2(e/g)^2(1/i)\\&\quad\times\int\frac{(dp)}{(2\pi)^4}\left(\frac{m_\rho{}^2}{p^2+m_\rho{}^2}\right)^2\left[1-\frac{p^2}{p^2+m_A{}^2}\right]\\&=3(g'/g)^2e^2(1/i)\int\frac{(dp)}{(2\pi)^4}\left(\frac{m_\rho{}^2}{p^2+m_\rho{}^2}\right)^2\frac{1}{p^2+m_A{}^2}.\end{aligned}$$

Indeed, if $g'=g$, the two physical contributions to δm_π^2 combine, according to

$$\frac{1}{(p^2+m_\rho^2)^2}\left[\frac{1}{p^2}+\frac{1}{p^2+m_A^2}\right]=2\frac{1}{p^2}\frac{1}{p^2+m_\rho^2}\frac{1}{p^2+m_A^2},$$

to reproduce the result of the chirality calculations. But if we retain the empirical distinction between g' and g, the second contribution is

$$\delta m_\pi^{2\prime\prime}=(3\alpha/4\pi)(g'/g)^2(2\ln 2-1)m_\rho^2,$$

or

$$\delta m_\pi''=1.4/(1.2)^2=1.0 \text{ MeV}.$$

Now the predicted value is

$$\delta m_\pi=3.6+1.0=4.6 \text{ MeV},$$

which is embarrassingly accurate.

The deceptiveness of this agreement is emphasized by examining the heretofore ignored $(m_\pi/m_\rho)^2$ corrections. While that is outside the framework of the chirality methods, it is a straightforward calculational question in our dynamical approach. We consider the principal mechanism of $\rho+\pi$ exchange. It is easily seen that $\delta m_\pi^{2\prime}$ is modified into

$$\delta m_\pi^{2\prime}=(1/i)\int\frac{(dp)}{(2\pi)^4}g^2\delta G^{\mu\nu}(p)_\rho \times\left[g_{\mu\nu}-\frac{(p-2P)_\mu(p-2P)_\nu}{(p-P)^2+m_\pi^2}\right],$$

where

$$P^2+m_\pi^2=0.$$

The longitudinal terms in $\delta G^{\mu\nu}(p)_\rho$ still give no contribution, since

$$p^\mu p^\nu[g_{\mu\nu}-(p-2P)_\mu(p-2P)_\nu/(p^2-2pP)]=2p^\mu P_\mu$$

vanishes on integration. Accordingly,

$$\delta m_\pi^{2\prime}=e^2(1/i)\int\frac{(dp)}{(2\pi)^4}\frac{1}{p^2}\frac{1}{(p^2+m_\rho^2)^2} \times\left[3+\frac{2pP+4m_\pi^2}{p^2-2pP}\right],$$

where the factor in brackets can also be written as

$$3+(p^2+4m_\pi^2)[1/(p^2-2pP)]-1.$$

The result of averaging over the four-dimensional Euclidean angle between p and P, with the aid of

$$\frac{2}{\pi}\int_0^\pi \sin^2\vartheta d\vartheta\frac{1}{1+t^2-2t\cos\vartheta}=1,$$

is given by

$$\left\langle\frac{1}{p^2-2pP}\right\rangle=\frac{1}{p^2}\frac{2}{(1+4m_\pi^2/p^2)^{1/2}+1}.$$

Therefore,

$$\delta m_\pi^{2\prime}=\frac{\alpha}{4\pi}\int_0\frac{dx}{(x+1)^2}\left[3+2\frac{1+\lambda x}{(1+\lambda x)^{1/2}+1}-1\right],$$

where

$$x=m_\rho^2/p^2,\quad \lambda=(2m_\pi/m_\rho)^2.$$

An asymptotic evaluation based on the smallness of λ gives

$$\delta m_\pi^{2\prime}\simeq(3\alpha/4\pi)m_\rho^2[1+(m_\pi^2/m_\rho^2)(\ln(m_\rho^2/m_\pi^2)+\tfrac{1}{2})].$$

Owing to the logarithmic factor, this is a 13% correction. It raises the computed value of δm_π by somewhat less than $\frac{1}{2}$ MeV.

There are processes that contribute only to the $(m_\pi/m_\rho)^2$ corrections. The best established of these is the $\omega\rho\pi$ coupling[8]

$$(g/m_\rho)({}^*\omega^{\mu\nu}\rho_\nu+{}^*\rho^{\mu\nu}\omega_\nu)\cdot\partial_\mu\pi.$$

The electromagnetic modification of ρ^0 exchange gives the following action term for neutral pions:

$$2(g/m_\rho)^2\int(dx)(dx')\partial_\mu\phi_3(x){}^*\omega^{\mu\lambda}(x)\delta G_{\lambda\kappa}(x-x')_\rho \times\partial_\nu\phi_3(x'){}^*\omega^{\nu\kappa}(x').$$

There is no contribution from the longitudinal part of $\delta G_{\lambda\kappa}$, and the latter is effectively proportional to $g_{\lambda\kappa}$. The consideration of a virtual ω meson ($m_\omega\simeq m_\rho$) then implies, approximately, the neutral pion term

$$-3(em_\rho)^2\int(dx)(\partial_\mu\phi_3)^2(1/i)\int\frac{(dp)}{(2\pi)^4}\frac{1}{(p^2+m_\rho^2)^3} =-\frac{3\alpha}{4\pi}\frac{1}{2}\int(dx)(\partial_\mu\phi_3)^2.$$

A redefinition of the π^0 field, or the equivalence $(\partial_\mu\phi)^2\rightarrow -m_\pi^2\phi^2$, identifies this as producing a π^0 mass decrease. Thus δm_π^2 is further increased by $(3\alpha/4\pi)m_\pi^2$, which raises the discrepancy to 16%.

Another relevant mechanism is the $A_2\rho\pi$ coupling. But we shall terminate this discussion here, with the general remark that to achieve better than $\sim$10% accuracy in computing the pion electromagnetic mass splitting seems to require detailed reference to fairly

[8] J. Schwinger, Phys. Rev. Letters 18, 923 (1967).

high-energy phenomena. There may include multiple exchange modifications of the processes that give the bulk of the electromagnetic mass effect. A particularly interesting subset of the latter can be described as electromagnetic modification of $\rho^{\pm}$ exchange, with these modifications produced within the coupled ρ system by the primary mechanism of electromagnetically modified ρ^0 exchange.

GAUGE INVARIANCE II

The possibility of realizing electromagnetic gauge invariance by compensation between the electromagnetic potential A_μ and a neutral meson gauge field has been illustrated by the $T=1$ field ρ_μ. We now include the $T=0$ fields ω_μ and ϕ_μ. The only problem is the relative coupling strengths of the various mesons. For simplicity, we place ω entirely in the U_2 subspace of the three-dimensional unitary space. Then, following the suggestion[8] that mass is a significant factor, we consider the 3×3 matrix

$$\tfrac{1}{2}(t_{11}-t_{22})m_\rho\rho_{\mu3}+\tfrac{1}{2}(t_{11}+t_{22})m_\omega\omega_\mu+2^{-1/2}t_{33}m_\phi\phi_\mu\,,$$

where t_{aa} has unit entry in the ath row and column and is zero elsewhere. The above matrix is such that the trace of its square equals

$$\tfrac{1}{2}[m_\rho^2(\rho_{\mu3})^2+m_\omega^2(\omega_\mu)^2+m_\phi^2(\phi_\mu)^2].$$

We assume this matrix to be the significant structure in coupling the vector fields to a given system, with the U_3 matrices T_{aa} of that system replacing the elementary matrices t_{aa}. It is then required that the linear interaction of ρ_μ, ω_μ, and ϕ_μ with A_μ reproduce the coupling of A_μ to the electric charge, as represented by the SU_3 matrix

$$T_{11}-\tfrac{1}{3}(T_{11}+T_{22}+T_{33})=\tfrac{1}{2}(T_{11}-T_{22})+\tfrac{1}{6}(T_{11}+T_{22})-\tfrac{1}{3}T_{33}.$$

Using the normalization already established in the ρ discussion, we infer the following gauge-invariant structure of the neutral gauge-field mass terms:

$$-\tfrac{1}{2}m_\rho^2[\rho_{\mu3}-(e_0/g)A_\mu]^2-\tfrac{1}{2}m_\omega^2[\omega_\mu-\tfrac{1}{3}(e_0/g)(m_\rho/m_\omega)A_\mu]^2 -\tfrac{1}{2}m_\phi^2[\phi_\mu+\tfrac{1}{3}2^{1/2}(e_0/g)(m_\rho/m_\phi)A_\mu]^2.$$

We shall not discuss the new diagonalization problem or the altered relation between e_0 and e. Let us ignore that distinction and write the linear coupling between A_μ and the neutral mesons as

$$(m_\rho^2e/g)A^\mu V_\mu^{(0)}\,,$$

with

$$V_\mu^{(0)}=\rho_{\mu3}+\tfrac{1}{3}(m_\omega/m_\rho)\omega_\mu-\tfrac{1}{3}2^{1/2}(m_\phi/m_\rho)\phi_\mu.$$

Viewed as a photon source, the self-couping of the latter is described by

$$w_e=\tfrac{1}{2}(m_\rho^2e/g)^2\int(dx)(dx')V^{(0)\mu}(x)D_+(x-x')V^{(0)}{}_\mu(x').$$

The following application of this electromagnetic term falls quite short of the quantitative ambition of the preceding section.

BARYON ELECTROMAGNETIC MASSES

We give a brief and highly empirical discussion of baryon electromagnetic mass splittings. This is patterned after a recent observation[9] concerning the gross mass spectrum of the baryons. It was noted that

$$M=M_0(1+\vartheta H_3)\,,$$

where M_0 varies from octuplet to decuplet but is fixed within each multiplet, while ϑ is a universal constant,[10]

$$\vartheta=0.119.$$

The quantum number H_3 is defined additively on the unitary indices in ψ_{abc}, with the basic values: $+1$, for a single 3 index; -1, for an antisymmetrical 12 pair; 0 otherwise. From the viewpoint of the three isotopic spins that characterize SU_3 symmetry, T(12 plane), U(23 plane), V(31 plane), the symmetry-breaking effect of H_3 is described by $\Delta T=0$, $\Delta U=1$, $\Delta V=1$, which is known[11] to produce Gell-Mann–Okubo mass relations. The H_3 mass formula also supplies specific connections among the arbitrary constants of such relations. If Λ is omitted, one has the following correspondences with hypercharge:

$$8(N,\Sigma,\Xi):\quad H_3=1-\tfrac{3}{2}Y-\tfrac{1}{2}Y^2,$$
$$10:\quad H_3=1-Y.$$

The electromagnetic coupling term contains $\Delta T=2$ ($\rho\rho$) and $\Delta T=1$ ($\rho\omega,\rho\phi$) contributions. The square of the electric charge

$$Q=T_3+\tfrac{1}{2}Y=T_{11}-\tfrac{1}{3}\sum_{a=1}^{3}T_{aa}$$

also has that character. We propose to represent the $\Delta T=2$ electromagnetic term as a multiple of Q^2, and treat the residual $\Delta T=1$ component by analogy with H_3. Thus, we define H_1 additively in its action on ψ_{abc}, with the basic values: $+1$ for a single 1 index; -1 for an antisymmetrical 23 pair; 0, otherwise. For the dec-

[9] J. Schwinger, Phys. Rev. Letters **18**, 797 (1967).

[10] As a curiousity, we remark that $m=m_\rho(1+\frac{3}{2}\vartheta H_3)$, $m_\rho=755$ MeV, where H_3 is defined on ψ_{ab^*}, gives quite a good account of the masses of $K^*(H_3=1)$ and $\phi(H_3=2$, if it is assumed that φ is very closely represented by ψ_{33^*}).

[11] J. Schwinger, Phys. Rev. **136B**, 1821 (1964). There is a brief discussion here of the use of meson fields to represent electromagnetic and weak interactions.

uplet we have, simply,

$$10:\quad H_1=1+Q\,,$$

while

$$N:\quad H_1=1+Q\,,$$
$$\Sigma:\quad H_1=\tfrac{1}{2}+\tfrac{3}{2}Q\,,$$
$$\Xi:\quad H_1=1+2Q\,.$$

These are represented collectively by

$$8(N,\Sigma,\Xi):\quad H_1=\tfrac{1}{2}+\tfrac{1}{2}Y^2+(\tfrac{3}{2}-\tfrac{1}{2}Y)Q\,.$$

Our proposed formula for the electromagnetic splitting of an isotopic multiplet with central mass M_0 is

$$M=M_0[1-\epsilon(H_1-\langle H_1\rangle)]+\lambda(Q^2-\langle Q^2\rangle)\,,$$

where $\langle\quad\rangle$ refers to a multiplet average. We note that

$$8:\quad \langle H_1\rangle=1-\tfrac{1}{2}H_8\,,$$
$$10:\quad \langle H_1\rangle=\tfrac{3}{2}-\tfrac{1}{2}H_8\,.$$

The constants ϵ and λ, assumed to be universal, can be determined from the properties of the Σ multiplet,[12]

$$\Sigma^- - \Sigma^0=4.88\pm0.06\ \text{MeV}\,,$$
$$\Sigma^- - \Sigma^+=7.97\pm0.11\ \text{MeV}\,.$$

An acceptable fit is given by

$$3\epsilon M(\Sigma)=8.0\ \text{MeV}\,,$$
$$\lambda+4.0\ \text{MeV}=4.8\ \text{MeV}\,,$$

or

$$\lambda=0.8\ \text{MeV}\,,\quad \epsilon=2.2(4)\times10^{-3}\,.$$

The nucleon splitting is now predicted:

$$N^0-N^+=-0.8+(8/3)(939/1192)=1.3\ \text{MeV}\,,$$

in agreement with the observed 1.29 MeV. For the cascade particle we anticipate

$$\Xi^- - \Xi^0=0.8+2(8/3)(1318/1192)=6.7\ \text{MeV}\,,$$

which is compatible with the measured 6.5 ± 0.2 MeV. It is interesting that the Coleman-Glashow electromagnetic mass formula[13]

$$\Sigma^- - \Sigma^+=\Xi^- - \Xi^0+N^0-N^+$$

now appears as a consequence of the gross structure mass relation

$$3\Sigma=2\Xi+N$$

or

$$\Sigma-N=2(\Xi-\Sigma)\,.$$

This connection is built into the H_8 mass formula, and is accurately obeyed.

The comparison with the present crude data on decuplet electromagnetic mass splittings is as follows:

$$\Xi^{*-}-\Xi^{*0}=0.8+(8/3)(1532/1192)=4.2\ \text{MeV}$$
$$\text{Exp: }4.9\pm2.2\ \text{MeV}\,,$$
$$Y_1^{*-}-Y_1^{*+}=2(8/3)(1385/1192)=6.2\ \text{MeV}\,,$$
$$\text{Exp: }5.8\pm3.0\ \text{MeV}\,,$$
$$N^{*-}-N^{*++}=-3(0.8)+3(8/3)(1238/1192)=5.9\ \text{MeV}$$
$$\text{Exp: }7.9\pm6.8\ \text{MeV}\,,$$
$$N^{*0}-N^{*++}=-4(0.8)+2(8/3)(1238/1192)=2.3\ \text{MeV}$$
$$\text{Exp: }0.5\pm0.9\ \text{MeV}\,.$$

The agreement is satisfactory, with the possible exception of the last entry.

The $\Delta T=1$ component of electromagnetic mass splitting is compared in strength with the SU_3 symmetry-breaking mechanism by the ratio

$$\epsilon/\vartheta=1.9\times10^{-2}\,.$$

When one contrasts mass intervals, rather than the universal parameters, the ratio varies within a unitary multiplet. In the uniformly spaced decuplet, for example, the ratio of the H_1 coefficient to the H_8 coefficient changes from 1.9×10^{-2} to 2.4×10^{-2}. An analogous comparison is produced within the 0^- meson spectrum by using the (mass)^2 difference $\pi^+-\pi^0$ to remove the $\Delta T=2$ part of K^0-K^+, giving $(K-\pi)^0-(K-\pi)^+$, and dividing this by the SU_3 (mass)^2 splitting, $K-\pi$,

$$[(K-\pi)^0-(K-\pi)^+]/(K-\pi)=2.3\times10^{-2}\,.$$

It would seem that the same mechanisms are at work to shape the meson mass spectrum.

[12] All experimental mass values are taken from A. Rosenfeld *et al.*, Rev. Mod. Phys. **39**, 1 (1967).

[13] S. Coleman and S. Glashow, Phys. Rev. Letters **6**, 423 (1961).