Some Consequences of the Saturation of Covariant Sum Rules Derived from Quark Charge-Commutation Relations*

GEORGE PATSAKOS
Institute of Theoretical Physics, Department of Physics, Stanford University, Stanford, California
(Received 11 September 1967)

Some features of a model introduced recently by Fubini, Segrè, and Walecka and by Segrè and Walecka are examined. A high-energy contribution to a sum rule appearing in the model is calculated, producing better agreement with experiment. The results of the model are then applied to electromagnetic meson decays, the $N^* \to N\pi$ decay, and low-momentum-transfer N^* production by neutrinos.

1. INTRODUCTION

In two recent papers Fubini, Segrè, and Walecka[1] and Segrè and Walecka[2] have obtained a consistent solution of the sum rules derived by considering the algebra of the generators of $U(12)$ in the framework of a quark model. They first obtained the largest subset of "good" charges contained among the 144 charges appearing in this model. These good charges are distinguished by the fact that they lead to sum rules which are not likely to require subtractions since they can be expressed in terms of differences of cross sections which, by the Pomeranchuk or other high-energy theorems, vanish at high energy. The authors then showed that, by assuming these good charge sum rules to be saturated by a few low-lying particles and resonances and interpreting the couplings appearing by "polology," many of the results of higher-symmetry groups, such as $SU(6)_W$, can be derived in an unambiguous, Lorentz-covariant manner without any assumption of higher symmetry. The solution obtained in I and II has some other interesting consequences which will be discussed in the present paper.

Segrè and Walecka found that, as first pointed out by Adler,[3] the sum rules obtained by taking the matrix elements of the quark-model axial-vector charge commutator between pseudoscalar-meson states are not saturated by vector-meson intermediate states. In particular, the vector-meson–pseudoscalar-meson–pseudoscalar-meson coupling constant ($G_{V\Pi\Pi}$) obtained by assuming such saturation is too large by about a factor of 3. On the other hand, the ratios of the couplings are given correctly by the consistent solution obtained in I and II. Adler's suggestion[3] that a large s-wave π-π scattering is needed to saturate the sum rule is incompatible with the vector-meson-dominance solution obtained in II, and so would ruin the correct coupling ratios given by that solution. This is the case due to the fact that, in the framework of exact $SU(3)$ and vector-meson dominance, the structure of the equations is such that simply adding an $SU(3)$ singlet s-wave resonance to the vector-meson-dominance solution no longer gives a solution consistent with the known G-parity assignments for the mesons. This point will be made clearer when the solutions are discussed in Sec. 2. In Sec. 2 it is also shown that the dominant high-energy contributions to the sum rule, which arise from the vector-meson Regge pole, can be included in a way which changes the scale of the V-Π-Π couplings obtained in II, bringing them into better agreement with experiment, while at the same time preserving the results for the ratios. Some comments on the relation of this result to the recent work of Meiere and Sugawara[4] on the same problem are also presented.

A by-product of the consistency solution obtained in I and II was the direct determination of the vector-meson–photon coupling constant $g_{V\gamma}$. The derivation is given in Appendix A. Using the value of $g_{V\gamma}$ obtained, together with the Gell-Mann, Sharp, and Wagner model[5] which is built into the approach of I and II, results for electromagnetic decays of vector and pseudoscalar mesons are presented in Sec. 3. While many of these decays have been considered previously by various authors,[6] it should be noted that the value of $g_{V\gamma}$ used most often is obtained by assuming vector-meson dominance of the pion electromagnetic form factor, an assumption which is equivalent to assuming an unsubtracted dispersion relation for the electric charge. The value of $g_{V\gamma}$ used in the present calculations is obtained by assuming that only the magnetic form factors of the nucleon have an unsubtracted representation. The value of $g_{V\gamma}$ used here is thus based on much more plausible assumptions.

In Sec. 4 we illustrate the higher-symmetry results obtainable in this model with the example of the $N^* \to N+\pi$ decay. Some of the other symmetry features of the solution are also mentioned.

Finally, in Sec. 5 a calculation of the low-momentum-transfer N^* production by neutrinos on protons is presented. A new result for the N^*N weak axial-vector vertex obtained in I is used in the calculation and the result is compared with the existing experimental data.

* Research sponsored by the Air Force Office of Scientific Research, Office of Aerospace Research, U. S. Air Force, under AFOSR Contract No. AF49(638)-1389.

[1] S. Fubini, G. Segrè, and J. D. Walecka, Ann. Phys. (N. Y.) **39**, 381 (1966), hereafter referred to as I.
[2] G. Segrè and J. D. Walecka, Ann. Phys. (N. Y.) **40**, 337 (1966), hereafter referred to as II.
[3] S. L. Adler, Phys. Rev. **140**, B736 (1965).
[4] F. T. Meiere and M. Sugawara, Phys. Rev. **153**, 1702 (1967).
[5] M. Gell-Mann, D. Sharp, and W. Wagner, Phys. Rev. Letters **8**, 261 (1962).
[6] References to previous work are given in Sec. 3.

2. HIGH-ENERGY CONTRIBUTION TO THE π-π SUM RULE

To fix the notation we review briefly the derivation of the sum rule. For further details, see II. The commutator of interest is that of the axial charges

$$Q_A{}^\alpha=\int \psi^\dagger(x)\gamma_5\lambda^\alpha\psi(x)d^3x$$

which satisfy the commutation relations

$$[Q_A{}^\alpha,Q_A{}^\beta]=-2\sqrt{3}\begin{pmatrix}8 & 8 & 8_2\\ \alpha & \beta & \gamma\end{pmatrix}Q_V{}^\gamma. \qquad (2.1)$$

Here

$$Q_V{}^\gamma=\int \psi^\dagger(x)\lambda^\gamma\psi(x)d^3x,$$

$\psi(x)$ is a 12-component quark field and the λ^α are the usual $SU(3)$ matrices in a spherical tensor basis. For convenience we work in the limit of exact $SU(3)$ symmetry.

Taking the matrix element of (2.1) between pseudoscalar-meson states of momentum P and applying the method of Fubini, Furlan, and Rossetti,[7] we arrive at the sum rule

$$m_\Pi\frac{\partial}{\partial\nu}\Pi^{\alpha\beta}(\nu)\bigg|_{\nu=0}=12\begin{pmatrix}8 & 8 & 8_2\\ \alpha & \beta & \gamma\end{pmatrix}\begin{pmatrix}8 & 8 & 8_2\\ \sigma & \gamma & \rho\end{pmatrix}, \qquad (2.2)$$

where m_Π is the pseudoscalar octet mass,

$$\Pi^{\alpha\beta}(\nu)=-\tfrac{1}{2}i[F^{\alpha\beta}(\nu)-F^{\beta\alpha}(\nu)],$$

$$F^{\alpha\beta}(\nu)=\left(\frac{E^2\Omega^2}{m_\Pi{}^2}\right)^{1/2}\int e^{-iq\cdot x}\theta(x_0)$$
$$\times\langle P8(\rho)|[D_A{}^\alpha(x),D_A{}^\beta(0)]|P8(\sigma)\rangle d^4x. \qquad (2.3)$$

Here E is the meson energy, Ω is the quantization volume, $\nu=-q\cdot P$, and $D_A{}^\alpha(x)$ is the divergence of the axial-vector current $J_\mu{}^{A\alpha}(x)=i\bar{\psi}(x)\gamma_\mu\gamma_5\lambda^\alpha\psi(x)$. The 4-vector q is, as usual, taken to be null, $q^2=0$.

Owing to the coupling coefficients appearing on the right side of (2.2), it is convenient to decompose $\Pi^{\alpha\beta}$ into t-channel amplitudes:

$$\Pi^{\alpha\beta}(\nu)=\sum_{\mu\epsilon\gamma\delta}\begin{pmatrix}8 & 8 & \mu_\gamma\\ \alpha & \beta & \epsilon\end{pmatrix}\begin{pmatrix}8 & \mu & 8_\delta\\ \sigma & \epsilon & \rho\end{pmatrix}\Pi_{\mu_\gamma\delta}(\nu),$$

so that finally we obtain the sum rules

$$0=\frac{2m_\Pi}{\pi}\int_{\nu_0}^\infty \mathrm{Im}\Pi_{10}(\nu)\frac{d\nu}{\nu^2}, \qquad 0=\frac{2m_\Pi}{\pi}\int_{\nu_0}^\infty \mathrm{Im}\Pi_{\overline{10}}(\nu)\frac{d\nu}{\nu^2},$$

$$0=\frac{2m_\Pi}{\pi}\int_{\nu_0}^\infty \mathrm{Im}\Pi_{8_{21}}(\nu)\frac{d\nu}{\nu^2}, \qquad 12=\frac{2m_\Pi}{\pi}\int_{\nu_0}^\infty \mathrm{Im}\Pi_{8_{22}}(\nu)\frac{d\nu}{\nu^2},$$

where $\nu_0=\frac{3}{2}m_\Pi{}^2$ and unsubtracted dispersion relations for the $\Pi_{\mu_\gamma\delta}$ and their odd crossing symmetry have been used.

Assuming saturation by a nonet of vector mesons of mass m_V, we have the equations

$$\begin{aligned}
0&=(\tfrac{1}{8}\sqrt{10})a_1+(\sqrt{\tfrac{1}{5}})a_{8_{11}}+\tfrac{1}{2}a_{8_{12}}-\tfrac{1}{2}a_{8_{21}},\\
0&=(-\tfrac{1}{8}\sqrt{10})a_1-(\sqrt{\tfrac{1}{5}})a_{8_{11}}+\tfrac{1}{2}a_{8_{12}}-\tfrac{1}{2}a_{8_{21}},\\
0&=-\tfrac{1}{2}a_{8_{12}}-\tfrac{1}{2}a_{8_{21}},\\
12&=(\sqrt{\tfrac{1}{8}})a_1-\tfrac{1}{2}a_{8_{11}}-\tfrac{1}{2}a_{8_{22}}, \qquad (2.4)
\end{aligned}$$

where the $a_{\mu_\gamma\delta}$ are the amplitudes coupled in the direct channel

$$\Pi^{\alpha\beta}(\nu)=\sum_{\mu\epsilon\gamma\delta}\begin{pmatrix}8 & 8 & \mu_\gamma\\ \sigma & \beta & \epsilon\end{pmatrix}\begin{pmatrix}\mu & 8 & 8_\delta\\ \epsilon & \alpha & \rho\end{pmatrix}a_{\mu_\gamma\delta}.$$

Equations (2.4) admit the solution

$$\begin{aligned}
a_{8_{21}}&=a_{8_{12}}=0,\\
a_{8_{11}}&=-\tfrac{5}{8}\sqrt{2}a_1, \qquad (2.5)\\
12&=-\tfrac{1}{2}a_{8_{22}}-\tfrac{9}{10}a_{8_{11}}.
\end{aligned}$$

If we represent pseudoscalar mesons by a dashed line, vector mesons by a solid line, and axial-vector vertices by crosses, then vector-meson dominance in the sum rule (2.2) and pion pole dominance of the axial-vector current imply that the right side of (2.3) can be represented schematically as in Fig. 1. If $\bar{f}$ is the pseudoscalar-meson decay amplitude defined by

$$\langle P8(\gamma)|J_\mu{}^{A\delta}(0)|0\rangle=i\bar{f}\frac{P_\mu}{(2E\Omega)^{1/2}}\begin{pmatrix}1 & 8 & 8\\ 000 & \delta & \gamma\end{pmatrix},$$

and

$$\left(\frac{E_1E_2\Omega^2}{m_\Pi m_V}\right)^{1/2}\langle P+qV_\mu8(\eta)|[j_\Pi{}^\epsilon(0)]^\dagger|P8(\sigma)\rangle$$
$$=(V\cdot q)\left\{G_{V\Pi\Pi}{}^D\begin{pmatrix}8 & 8 & 8_1\\ \sigma & \epsilon & \eta\end{pmatrix}+G_{V\Pi\Pi}{}^F\begin{pmatrix}8 & 8 & 8_2\\ \sigma & \epsilon & \eta\end{pmatrix}\right\}$$

defines the V-Π-Π coupling constants, where V_μ is the

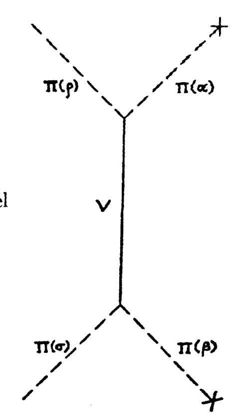

FIG. 1. Meson-pole-dominance model for the right side of (2.3).

[7] S. Fubini, G. Furlan, and C. Rossetti, Nuovo Cimento **40**, 1171 (1965).

vector-meson polarization and $j_\Pi{}^\epsilon$ is the source current of the pseudoscalar octet, then we find

$$a_1=\frac{2}{\sqrt{8}}\left(\frac{m_\Pi}{m_V}\right)\bar{f}^2(G_{V\Pi\Pi}{}^0)^2,$$

$$a_{8_{12}}=a_{8_{21}}=-2\left(\frac{m_\Pi}{m_V}\right)\bar{f}^2G_{V\Pi\Pi}{}^FG_{V\Pi\Pi}{}^D,$$

$$a_{8_{11}}=-2\left(\frac{m_\Pi}{m_V}\right)\bar{f}^2(G_{V\Pi\Pi}{}^D)^2,$$

$$a_{8_{22}}=-2\left(\frac{m_\Pi}{m_V}\right)\bar{f}^2(G_{V\Pi\Pi}{}^F)^2.$$

Inserting these values into the solution (2.5) yields two solutions for the couplings:

$$G_{V\Pi\Pi}{}^D=G_{V\Pi\Pi}{}^0=0,\quad 4m_\Pi m_V(G_{V\Pi\Pi}{}^F)^2=48m_V{}^2/\bar{f}^2,$$

and

$$G_{V\Pi\Pi}{}^F=0,$$
$$4m_\Pi m_V(G_{V\Pi\Pi}{}^D)^2=(80/3)m_V{}^2/\bar{f}^2,$$
$$(G_{V\Pi\Pi}{}^0)^2=(16/5)(G_{V\Pi\Pi}{}^D)^2.$$

Charge-conjugation invariance requires that we choose the first solution, so that

$$G_{V\Pi\Pi}{}^D=G_{V\Pi\Pi}{}^0=0,\quad 4m_\Pi m_V(G_{V\Pi\Pi}{}^F)^2=48m_V{}^2/\bar{f}^2.$$

As mentioned earlier, this result predicts vector-meson widths which are too large by a factor of 3; the ratios of the widths, which, once the F or D nature of the coupling is fixed follow from $SU(3)$ and phase space, are given correctly. The inclusion of an s-wave $SU(3)$ singlet resonance contributes to a_1 and thus would require $G_{V\Pi\Pi}{}^D\neq0$, which is not consistent with the desired G-parity assignments. An examination of Eqs. (2.4) and (2.5) immediately indicates that the amplitude $a_{8_{22}}$ (or alternatively $\Pi_{8_{22}}$) controls the scale of the couplings. Unfortunately, an s-wave octet cannot be coupled antisymmetrically to two pseudoscalar mesons because of Bose statistics, so that $a_{8_{22}}$ cannot be altered by considering s-wave resonances alone. The conclusion is that in the model considered here we cannot utilize s waves as the mechanism for altering the scale of the couplings.

A process which contributes only to $\Pi_{8_{22}}$, on the other hand, would, in effect, change only the value of $a_{8_{22}}$ and so would alter the scale of the couplings while preserving the other desirable features of the solution. The dominant high-energy contribution to $\Pi^{\alpha\beta}$, which involves the vector-meson Regge trajectory, is precisely of this type.

We first relate the $\Pi^{\alpha\beta}$ to the pseudoscalar-meson scattering process

$$p_18(\beta)+p_28(\sigma)\rightarrow p_38(\alpha)+p_48(\rho).$$

Defining the invariant amplitude $T_{\beta\sigma}{}^{\alpha\rho}$ by

$$\left(\frac{d\sigma_{\beta\sigma}{}^{\alpha\rho}}{d\Omega}\right)_{\text{c.m.}}=\left|\frac{-m_\Pi T_{\beta\sigma}{}^{\alpha\rho}}{4\pi E_T{}^{\text{c.m.}}}\right|^2,$$

we find in the usual way that

$$\text{Im}T_{\beta\sigma}{}^{\alpha\rho}=-\frac{1}{2}\left(\frac{E_2E_3\Omega^2}{m_\Pi{}^2}\right)^{1/2}(2\pi)^4\sum_N\big[\langle p_38(\rho)|j_\Pi{}^\alpha(0)|N\rangle$$
$$\times\langle N|[j_\Pi{}^\beta(0)]^\dagger|p_28(\sigma)\rangle\delta^4(p_3+p_4-p_N)$$
$$-\langle p_38(\rho)|[j_\Pi{}^\beta(0)]^\dagger|N\rangle$$
$$\times\langle N|j_\Pi{}^\alpha(0)|p_28(\sigma)\rangle\delta^4(p_4+p_N-p_2)\big].$$

Again making use of pion pole dominance of the axial-vector current divergence, which in this notation is equivalent to the replacement

$$D_A{}^\alpha(x)=-m_\Pi{}^2\bar{f}[\phi_\Pi{}^\alpha(x)]^\dagger,$$

the properties of spherical tensor operators $[O^\alpha]^\dagger=(-1)^{\bar{\nu}_\alpha}O^{-\alpha}$ ($\bar{\nu}_\alpha=\frac{1}{2}Y+I_3$ is the charge of the field), and extrapolating to $p_3=p_2=P$, $p_1=p_4=q$, $q^2=0$, we find

$$\text{Im}T_{\beta\sigma}{}^{\alpha\rho}=-\frac{1}{2}\left(\frac{E^2\Omega^2}{m_\Pi{}^2}\right)^{1/2}\frac{(-1)^{\bar{\nu}_\alpha}}{\bar{f}^2}\int d^4x\,e^{-iq\cdot x}$$
$$\times\langle P8(\rho)|[D^{-\alpha}(x),D^\beta(0)]|P8(\sigma)\rangle.$$

Comparing this with (2.3) implies

$$\text{Im}\Pi^{\alpha\beta}=\tfrac{1}{2}\bar{f}^2[(-1)^{\bar{\nu}_\alpha}\,\text{Im}T_{\beta\sigma}{}^{-\alpha\rho}-(-1)^{\bar{\nu}_\beta}\,\text{Im}T_{\alpha\sigma}{}^{-\beta\rho}].$$

Expanding the scattering amplitude into its t-channel components

$$\text{Im}T_{\beta\sigma}{}^{\alpha\rho}=(-1)^{\bar{\nu}_\alpha}\sum_{\mu\epsilon\gamma\delta}\begin{pmatrix}8&8&\mu_\gamma\\-\alpha&\beta&\epsilon\end{pmatrix}\begin{pmatrix}8&\mu&8_\delta\\\sigma&\epsilon&\rho\end{pmatrix}\text{Im}T_{\mu_{\gamma\delta}},$$

and using (2.3), we find

$$\text{Im}\Pi_{\mu_{\gamma\delta}}=\bar{f}^2\,\text{Im}T_{\mu_{\gamma\delta}}\quad[\text{odd } SU(3) \text{ amplitudes only}]$$

so that, in particular,

$$\text{Im}\Pi_{8_{22}}=\bar{f}^2\,\text{Im}T_{8_{22}}.\qquad(2.6)$$

The exchange of a vector-meson Regge pole in the t channel contributes a high-energy term to $T_{8_{22}}$ which can be written

$$T_{8_{22}}\underset{s\to\infty}{\longrightarrow}4m_\Pi{}^2\left[\frac{1-\exp(-i\pi\alpha_V(t))}{2\sin\pi\alpha_V(t)}\right]b_V(t)\left(\frac{s}{4m_\Pi{}^2}\right)^{\alpha_V(t)},$$

where $\alpha_V(t)$ is the vector-meson Regge trajectory and s is the usual Mandelstam energy variable. Comparison with the exchange of an elementary V in the t channel yields the result

$$b_V(m_V{}^2)=\pi m_V\,\text{Re}\alpha_V{}'(m_V{}^2)(G_{V\Pi\Pi}{}^F)^2,$$

so that setting $b_V(t)=b_V(m_V{}^2)\gamma_V(t)$ and substituting into (2.6) gives the high-energy behavior (note that $s \to 2\nu$ as $\nu \to \infty$),

$$\mathrm{Im}\Pi_{8_{22}} \underset{\nu\to\infty}{\longrightarrow} 2f^2 m_\Pi{}^2 m_V \pi \times \mathrm{Re}\alpha_V'(m_V{}^2)(G_{V\Pi\Pi}{}^F)^2 \gamma_V(0)\left(\frac{\nu}{2m_\Pi{}^2}\right)^{\alpha_V(0)}.$$

Assuming that the Regge asymptotic behavior sets in at $\nu=\bar{\nu}$, we then find that the high-energy contribution to the 8_{22} sum rule is

$$\frac{2m_\Pi}{\pi}\int_{\bar{\nu}}^{\infty} \mathrm{Im}\Pi_{8_{22}}(\nu)\frac{d\nu}{\nu^2} = \frac{4\bar{f}^2 m_\Pi{}^3 m_V\,\mathrm{Re}\alpha_V'(m_V{}^2)(G_{V\Pi\Pi}{}^F)^2\gamma_V(0)}{(2m_\Pi{}^2)^{\alpha_V(0)}}\int_{\bar{\nu}}^{\infty}\nu^{\alpha_V(0)-2}d\nu = (m_\Pi/m_V)\bar{f}^2(G_{V\Pi\Pi}{}^F)^2 X\,,$$

where

$$X=\frac{4m_\Pi{}^2 m_V{}^2\,\mathrm{Re}\alpha_V'(m_V{}^2)\gamma_V(0)(\bar{\nu})^{\alpha_V(0)-1}}{(2m_\Pi{}^2)^{\alpha_V(0)}[1-\alpha_V(0)]}.$$

The 8_{22} equation then reads

$$12=(m_\Pi/m_V)\bar{f}^2(G_{V\Pi\Pi}{}^F)^2+(m_\Pi/m_V)\bar{f}^2(G_{V\Pi\Pi}{}^F)^2X\,,$$

so that the new value of $(G_{V\Pi\Pi}{}^F)^2$ is given by

$$(G_{V\Pi\Pi}{}^F)^2=\frac{12}{(m_\Pi/m_V)\bar{f}^2}\left(\frac{1}{1+X}\right),$$

while the rest of the solution is as given in II.

The best fits to πN charge-exchange scattering[8] give for the ρ trajectory

$$\alpha_\rho(0)=0.57\,,\quad \alpha_\rho'(0)\approx\alpha_\rho{}^1(m_\rho{}^2)=0.96\ \mathrm{GeV}^{-2}.$$

Using these values together with $m_\pi=0.14$ GeV, $m_V=0.76$ GeV we find

$$X=0.64\gamma_V(0)/(\bar{\nu})^{0.43}.$$

If we take $\gamma_V(0)\approx\gamma_V(m_V{}^2)=1$ and assume $\bar{\nu}=1$ GeV^2, then $X=0.64$ and we find

$$(G_{V\Pi\Pi}{}^F)^2=[12/(m_\Pi/m_V)\bar{f}^2](0.61).$$

Vector-meson exchange in the t channel certainly contributes not only in the asymptotic region considered here but also down to threshold. We have found, however, no clear way to include this additional contribution. The difficulty is that the vector-meson contributions in the s and u channels which are included trivially in (2.3) arise at least in part from the background vector-meson exchange in the t channel. Simply extending the asymptotic result down to threshold would therefore count some contributions twice. There is also no unambiguous method of joining a low-energy approximation to an asymptotic Regge contribution. Note that if we do continue the Regge contribution down to threshold, i.e., take $\bar{\nu}=0.02$ GeV^2, we find $X\approx3.4$ so that the vector-meson widths are reduced by a factor of 4. This may indicate that proper inclusion of the background, if one could do it, might produce substantially better agreement with experiment than is obtained here.

At any rate, the reduction in the strength of the coupling produced by including only the Regge pole asymptotically brings the predicted widths of the vector mesons to within better than a factor of 2 agreement with experiment, which, in the light of the approximations involved and the approach of the work in I and II, is not unreasonable. It is not surprising that we are able to do this well, neglecting s waves, since Meiere and Sugawara[4] have recently shown that the Adler sum rule can be saturated by an s-wave scattering only $\frac{1}{4}$ that originally suggested by Adler as a lower limit.

3. ELECTROMAGNETIC MESON DECAYS

The consistency solution obtained in I and II resulted in a new determination of the vector-meson–photon and the vector-meson–photon–pseudoscalar-meson coupling constants (see Appendix A). In this section[9] we make use of our approach of pole dominance, which is the Gell-Mann, Sharp, and Wagner model,[5] and these new values for the couplings to reexamine electromagnetic meson decays. The new determination of $g_{V\gamma}$ and $G_{\gamma V\Pi}$ used here requires as experimental input only the meson masses and $SU(3)$ assignments (including the ω-ϕ and η-X mixing angles)[10] and the properties of the nucleon (neutron magnetic moment, $\mu_n=-1.91$; pion-nucleon coupling constant, $g_{\Pi NN}{}^2/4\pi=14.6$; weak axial-vector renormalization ratio, $G_A=-1.18$). Given this input, the absolute partial widths for each decay mode are determined and, of course, the ratios of these to the dominant modes, which are more amenable to experiment, follow.

A. Pseudoscalar-Meson Decays

(i) $\Pi\to2\gamma$

The model used here treats this decay as proceeding through a two-vector-meson intermediate state as in Fig. 2(a). Because we have already developed an expression for the $\Pi V\gamma$ vertex in Appendix A, we can treat the decay chain in the simpler fashion indicated in Fig. 2(b). The decay width is given by

$$\Gamma(\Pi\to2\gamma)=(1/32\pi m_\Pi)|T|^2, \tag{3.1}$$

[8] G. Höhler, J. Baacke, and G. Eisenbeiss, Phys. Letters **22**, 203 (1966).

[9] A preliminary version of some of this work has been reported previously in G. Patsakos, G. Segrè, and J. D. Walecka, Phys. Letters **23**, 141 (1966); G. Patsakos, Bull. Am. Phys. Soc. **11**, 901 (1966).

[10] A. H. Rosenfeld, A. Barbaro-Galtieri, W. J. Podolsky, L. R. Price, P. Soding, C. G. Wohl, M. Roos, and W. J. Willis, Rev. Mod. Phys. **39**, 1 (1967). All empirical data are taken from this reference unless otherwise noted.

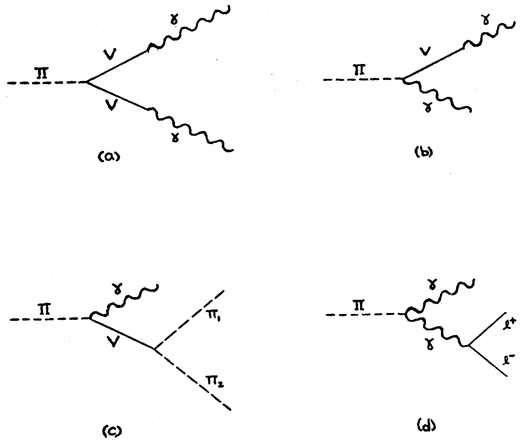

FIG. 2. Vector-meson-dominance models for pseudoscalar-meson decays.

where T is the invariant amplitude.[11] Assuming the decay chain indicated above and the couplings defined in Appendix A, we find

$$T(\Pi\to 2\gamma)=4\pi\alpha(g_{V\gamma}/m_V{}^2)(4m_\Pi m_V)^{1/2}G_{\gamma V\Pi}{}^D \times\varepsilon_{\mu\eta\gamma\delta}(p_1)_\mu\epsilon_\eta(p_2)\epsilon_\gamma(p_1)(p_2)_\delta\Lambda_1\,, \quad (3.2)$$

where $\alpha=1/137$, p_1 and p_2 are the photon momenta, $\epsilon(p_1)$ and $\epsilon(p_2)$ are their polarization vectors, and Λ_1 is an $SU(3)$ factor:

$$\Lambda_1=\sum_{\mu,\beta}\left[\begin{pmatrix}1 & 8 & \mu\\ 000 & 010 & \beta\end{pmatrix}+\frac{1}{\sqrt{3}}\begin{pmatrix}1 & 8 & \mu\\ 000 & 000 & \beta\end{pmatrix}\right] \times\left[\begin{pmatrix}\xi & 8 & \mu_1\\ \nu & 010 & \beta\end{pmatrix}+\frac{1}{\sqrt{3}}\begin{pmatrix}\xi & 8 & \mu_1\\ \nu & 000 & \beta\end{pmatrix}\right].$$

Here $\mu(\beta)$ refers to the intermediate vector meson and $\xi(\nu)$ to the decaying pseudoscalar meson. Squaring (3.2), summing over photon polarizations, inserting into (3.1), and using the values for the coupling constants given in Appendix A, we find

$$\Gamma(\Pi\to 2\gamma)=\frac{5\alpha^2}{12}\left(\frac{m_\Pi}{M}\right)^2\left(\frac{3\mu_n}{4}\right)^4\frac{G_A{}^2}{g_{\Pi NN}{}^2/4\pi}m_\Pi\Lambda_1{}^2\,, \quad (3.3)$$

so that the width is independent of the vector-meson mass. The results for the π^0 $[\xi(\nu)=8(010)]$ and η^0 $[\xi(\nu)=8(000)]$ are given in Table I together with the implied total width of the η. In the case of the η we have taken account of the η-X mixing through a correction factor[12]

$$C(\eta\to 2\gamma)=[\cos\lambda_1+2\sqrt{2}\sin|\lambda_1|]^2\,,$$

where λ_1 is the η-X mixing angle, $\tan\lambda_1=0.19$.[13]

TABLE I. Pseudoscalar-meson decays.

	Theory	Experiment	Ref.
$\tau(\pi^0\to 2\gamma)$	1.01×10^{-16} sec	$(0.73\pm0.10)\times10^{-16}$ sec	a
		$(0.89\pm0.18)\times10^{-16}$ sec	10
$\Gamma(\pi^0\to\gamma+e^++e^-)$	7.82×10^{-2} eV		
$\Gamma(\eta^0\to 2\gamma)$	0.34 keV	1.21 ± 0.26 keV[c]	
$\Gamma(\eta^0\to\pi^++\pi^-+\gamma)$	0.082 keV		
$\Gamma(\eta^0\to\gamma+e^++e^-)$	5.5 eV		
$\Gamma(\eta^0\to\gamma+\mu^++\mu^-)$	0.19 eV		
$\Gamma(\eta^0)_{\text{total}}$	1.08 keV	1 keV$<\Gamma<$10 keV	10,b

[a] G. Bellettini, C. Bemporad, P. L. Braccini, and L. Foa, in *Proceedings of the International Symposium on Electron and Photon Interactions at High Energy*, edited by G. Höhler *et al.* (Deutsche Physikalische Gesellschaft, Hamburg, 1965).
[b] M. Feldman, W. Frati, R. Gleeson, J. Halpern, M. Nussbaum, and S. Richert, Phys. Rev. Letters **18**, 868 (1967).
[c] *Note added in proof.* The measurement, made by a Pisa-Bonn collaboration using the Primakoff effect, was reported by Haim Harari at the 1967 International Symposium on Electron and Photon Interactions at High Energy held at SLAC, September, 1967 (unpublished).

The good agreement with experiment in the case of the π^0 decay is particularly heartening, since this decay has previously eluded theoretical calculations. The static quark model,[14] in addition to its other shortcomings, predicts a width about five times larger than that obtained here for the $\pi^0\to 2\gamma$. As far as the vector-meson-dominance model, in addition to the original work[5] and its extension by Dashen and Sharp to include the ω-ϕ mixing,[15] we can mention here the work of Faier,[16] who, using the $\pi^0\to 2\gamma$ width as input obtains an $\eta\to 2\gamma$ width about two times smaller than ours. Yellin[17] has obtained a $\pi^0\to 2\gamma$ width, essentially identical to that first calculated by Dashen and Sharp,[15] which is also about five times larger than ours. Using the experimental π^0 width as input, Yellin also obtains an $\eta\to 2\gamma$ width about 2.5 times smaller than that indicated in Table I. Recently, Pagels[18] has derived an expression for the π^0 width remarkably similar to that given here but based on an analysis of nucleon Compton scattering sum rules. He obtains a lifetime about twice as large as ours for the π^0 but is unable to give a reliable figure for the $\eta\to 2\gamma$ due to the uncertainty in the η-nucleon coupling. Maiani and Preparata,[19] using

---

[11] In all cases we define the invariant amplitude T in the usual way,

$$S_{fi}=\delta_{fi}-i(2\pi)^4\delta^4(P_f-P_i)N_iN_fT_{fi},$$

where

$$N_\alpha=\prod_k\left(\frac{M_k}{E_k\Omega}\right)^{1/2}\prod_l\left(\frac{1}{2E_l\Omega}\right)^{1/2}$$

and the $k(l)$ product is over all fermions (bosons) contained in the state α.

[12] In obtaining this factor one must know the ratio of the $\gamma V\Pi$ coupling for singlet pseudoscalar mesons $G_{\gamma V\Pi}{}^0$ to that for octet pseudoscalar mesons $G_{\gamma V\Pi}{}^D$. The solution in II gave

$$(4m_Vm_{\Pi^0})^{1/2}G_{\gamma V\Pi^0}{}^0/(4m_Vm_\Pi)^{1/2}G_{\gamma V\Pi}{}^D=\sqrt{\tfrac{2}{5}}.$$

See Appendix B for further discussion of mixing corrections.

[13] R. H. Dalitz and D. G. Sutherland, Nuovo Cimento **37**, 1777 (1965).

[14] H. Pietschmann and W. Thirring, Phys. Letters **21**, 713 (1966).

[15] R. F. Dashen and D. H. Sharp, Phys. Rev. **133**, B1585 (1964).

[16] H. Faier, Nuovo Cimento **41**, 127 (1966).

[17] J. Yellin, Phys. Rev. **147**, 1080 (1966).

[18] H. Pagels, Phys. Rev. **158**, 1566 (1967).

[19] L. Maiani and G. Preparata, Nuovo Cimento **48**, 550 (1967).

arguments similar to those here, have also obtained π^0 and η widths slightly larger than those given in the present work.

(*ii*) $\Pi \rightarrow \gamma + \Pi_1 + \Pi_2$

In the model under consideration this decay proceeds through a vector-meson intermediate state as pictured in Fig. 2(c). The coupling at the first vertex is given by (A18) while that at the second was discussed in Sec. 2. We find for the amplitude

$$T(\Pi \rightarrow \gamma + \Pi_1 + \Pi_2) = (4\pi\alpha)^{1/2} 4 m_\Pi m_V \times G_{V\Pi\Pi}{}^F G_{\gamma V\Pi}{}^D \frac{E_{\alpha\beta\gamma\delta}(p_1)_\alpha \epsilon_\beta(p_3)(p_2)_\gamma P_\delta}{(p_1+p_2)^2 + m_V{}^2} \Lambda_2 ,$$

where p_1, p_2, p_3, and P are the momenta of the Π_1, Π_2, γ, and Π; ϵ is the photon polarization vector, and

$$\Lambda_2 = \sum_\beta \begin{pmatrix} 8 & 8 & 8_2 \\ \lambda & \sigma & \beta \end{pmatrix} \left[\begin{pmatrix} 8 & 8 & \pi \\ \beta & 010 & \kappa \end{pmatrix} + \frac{1}{\sqrt{3}} \begin{pmatrix} 8 & 8 & \pi \\ \beta & 000 & \kappa \end{pmatrix} \right],$$

where λ,σ refer to Π_1, Π_2, β to the intermediate vector meson, and $\pi(\kappa)$ to the decaying pseudoscalar meson. Note that here, as well as in Sec. 3B(*iii*), we are making additional assumptions since we neglect other diagrams which can contribute. For the other decays considered the diagrams indicated in Fig. 2 and Fig. 3 are unique. Using the couplings from Appendix A we find

$$\Gamma(\Pi \rightarrow \Pi_1\Pi_2\gamma) = \frac{5\alpha}{6\pi}\left(\frac{m_V}{M}\right)^2 \left(\frac{m_\Pi}{M}\right)^2 \times \frac{g_{\Pi NN}{}^2/4\pi}{G_A{}^2}\left(\frac{3\mu_n}{4}\right)^2 m_\Pi \Lambda_2{}^2 I_1 \times (0.61) , \quad (3.4)$$

where m_Π is the mass of the Π, 0.61 is the correction to $(G_{V\Pi\Pi}{}^F)^2$ obtained in Sec. 2, and

$$I_1 = \int_0^{\alpha/2} \frac{x^3(\alpha - 2x)^{3/2}}{(1-2x)^{1/2}(\beta - 2x)^2} dx ,$$

with

$$\alpha = 1 - 4\mu^2/m_\Pi{}^2, \quad \beta = 1 - m_V{}^2/m_\Pi{}^2 .$$

μ is the mass of Π_1 and Π_2.

The numerical result for the $\eta^0 \rightarrow \pi^+\pi^-\gamma$ (for which $I_1 \approx 2.6\times10^{-4}$), again corrected for η-X mixing in this case through a factor[12]

$$C(\eta \rightarrow \pi^+\pi^-\gamma) = [\cos\lambda_1 + \sqrt{2}\sin|\lambda_1|]^2 ,$$

is given in Table I. Note that combining (3.4) and (3.3) yields the result

$$R \equiv \frac{\Gamma(\eta \rightarrow \pi^+\pi^-\gamma)}{\Gamma(\eta \rightarrow 2\gamma)} = \frac{3}{2\pi\alpha}\left(\frac{m_V}{M}\right)^2 \left(\frac{g_{\Pi NN}{}^2/4\pi}{G_A{}^2}\right)^2 \times \frac{1}{(\frac{3}{4}\mu_n)^2}\left[\frac{1+\sqrt{2}\tan|\lambda_1|}{1+2\sqrt{2}\tan|\lambda_1|}\right]^2 I_1 \times (0.61) ,$$

which gives the numerical result $R = 0.24$ to be compared with the experimental value[20] of $R_{\text{expt}} = 0.19 \pm 0.06$

The static quark model[14] combined with the experimental $\rho\pi\pi$ width yields $\Gamma = 120$ eV and a value of $R = 0.2$. Faier,[16] again using the π^0 width as input, finds $\Gamma = 152$ eV and $R = 0.23$, while Yellin[17] obtains $\Gamma = 175$ eV and the rather poor result $R = 1.3$. Despite the lack of direct experimental measurement of the $\eta \rightarrow \pi^+\pi^-\gamma$ partial width we believe that the value presented in the present work is less open to suspicion than those mentioned above since the same model, without appreciable experimental input, predicts reasonable values for R, the $\eta \rightarrow 2\gamma$ *and* the $\pi^0 \rightarrow 2\gamma$. We should also mention here the interesting work of Ademollo and Gatto,[21] who, making use of pole dominance of the axial-vector divergence and an assumed commutation relation between an axial charge and the axial-vector current components, have found a relation between $\eta \rightarrow 2\gamma$ and $\eta \rightarrow \pi^+\pi^-\gamma$ which yields $R = 0.14$. The $\eta \rightarrow \pi^+\pi^-\gamma$ has also been considered, in a different model, by Okubo and Sakita.[22]

(*iii*) $\Pi \rightarrow \gamma + l^+ + l^-$

This decay[23] is just $\Pi \rightarrow 2\gamma$ followed by conversion of one of the photons via the usual electromagnetic interaction as pictured in Fig. 2(d). We discuss this simple decay here because a new experimental determination of the $\eta \rightarrow \gamma + e^+ + e^-$ is now in progress by a group at Berkeley.[24] Using the results of Sec. 3A(*i*), in an obvious notation,

$$T(\Pi \rightarrow \gamma + l^+ + l^-) = 4\pi\alpha \frac{g_{V\gamma}}{m_V{}^2}(4m_\Pi m_V)^{1/2} \varepsilon_{\mu\eta\gamma\delta}(p_\gamma)_\mu \epsilon_\gamma(p_\gamma)(p_+ + p_-)_\delta \times \left[\frac{-\delta_{\eta\nu}}{(p_+ + p_-)^2}\right](4\pi\alpha)^{1/2} \bar{v}(p_+)\gamma_\nu u(p_-)\Lambda_1(G_{\gamma V\Pi}{}^D) .$$

The width is then given by

$$\Gamma(\Pi \rightarrow \gamma l^+ l^-) = \tfrac{1}{6}\alpha^3 (g_{V\gamma}/m_V{}^2)^2 4 m_\Pi m_V (G_{\gamma V\Pi}{}^D)^2 m_\Pi{}^3 (1-\Delta)^{9/2} I_2 \Lambda_1{}^2 ,$$

---

[20] We have obtained this result by combining the data of F. S. Crawford and L. R. Price, Phys. Rev. Letters **16**, 333 (1966) with those of Ref. 10.

[21] M. Ademollo and R. Gatto, Nuovo Cimento **44**, 282 (1966); J. Pasupathy and R. E. Marshak, Phys. Rev. Letters **17**, 888 (1966).

[22] S. Okubo and B. Sakita, Phys. Rev. Letters **11**, 50 (1963).

[23] This decay was first considered by R. H. Dalitz, Proc. Phys. Soc. (London) **A64**, 667 (1951); and later by N. Kroll and M. Wada, Phys. Rev. **98**, 1355 (1955); S. M. Berman and D. A. Geffen, Nuovo Cimento **18**, 1192 (1960); E. Celeghini and R. Gatto, Nuovo Cimento **28**, 1497 (1963).

[24] H. H. Bingham (private communication).

where $\Delta=4m_l^2/m_\Pi^2$ and

$$I_2=\int_0^1 \frac{x^3(1-x)^{1/2}[1+\frac{1}{2}\Delta-(1-\Delta)x]}{[1-(1-\Delta)x]^{5/2}}dx$$

$$=\frac{1}{(1-\Delta)^{9/2}}\left[\left(1-\frac{9}{8}\Delta^2+\frac{\Delta^3}{4}\right)\ln\left\{\frac{1+(1-\Delta)^{1/2}}{1-(1-\Delta)^{1/2}}\right\}\right.$$
$$\left.-(1-\Delta)^{1/2}\left(\frac{7}{2}-\frac{11}{4}\Delta-\frac{1}{2}\Delta^2\right)\right].$$

Again using the couplings from Appendix A, we get

$$\Gamma(\Pi\to\gamma l^+l^-)$$
$$=\frac{5\alpha^3}{18\pi}\left(\frac{3\mu_n}{4}\right)^4\frac{G_A{}^2}{g_{\Pi NN}{}^2/4\pi}\left(\frac{m_\Pi}{M}\right)^2 m_\Pi(1-\Delta)^{9/2}I_2\Lambda_1{}^2. \quad (3.5)$$

In Table I we give the results obtained from (3.5) for η^0 and π^0 decays into e^+e^- and η^0 decay into $\mu^+\mu^-$. The correction factor for η-X mixing is the same here as for the 2γ mode.

Combining (3.5) and (3.3) we obtain the coupling-independent result

$$\frac{\Gamma(\Pi\to\gamma+l^++l^-)}{\Gamma(\Pi\to 2\gamma)}=\frac{2\alpha}{3\pi}(1-\Delta)^{9/2}I_2.$$

For the sake of completeness we also present these well-known ratios in Table III, although they do not reflect our new values for the coupling constants.

B. Vector-Meson Decays

(i) $V\to\Pi+\gamma$

We treat this decay as proceeding through a ΠV intermediate state as in Fig. 3(a). This mechanism was included in the calculation of the $\Pi V\gamma$ coupling constant presented in Appendix A. The decay width is

$$\Gamma(V\to\Pi+\gamma)=\frac{1}{8\pi m_V{}^2}\left(\frac{m_V{}^2-m_\Pi{}^2}{2m_V}\right)|T|^2. \quad (3.6)$$

Using the coupling defined in (A17), we find

$$T(V\to\Pi\gamma)=(4\pi\alpha)^{1/2}(4m_Vm_\Pi)^{1/2}G_{\gamma V\Pi}{}^D$$
$$\times\varepsilon_{\alpha\beta\gamma\delta}(p_1)_\alpha\epsilon_\beta(p_1)E_\gamma(p_2)_\delta\Lambda_3, \quad (3.7)$$

where p_1 and p_2 are the momenta of the photon and pseudoscalar meson, ϵ is the photon polarization, E is the vector-meson polarization, and

$$\Lambda_3=\begin{pmatrix}8 & 8 & 8_1\\ \sigma & 010 & \tau\end{pmatrix}+\frac{1}{\sqrt{3}}\begin{pmatrix}8 & 8 & 8_1\\ \sigma & 000 & \tau\end{pmatrix};$$

here σ refers to the pseudoscalar meson and τ to the vector meson.

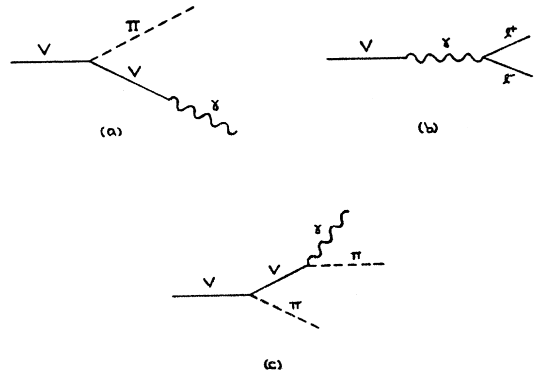

FIG. 3. Pole-dominance models for vector-meson decays.

Using (3.7) in (3.6) we find, after taking the appropriate sums and averages over polarizations,

$$\Gamma(V_8\to\Pi+\gamma)=(\alpha/24)[4m_\Pi m_V(G_{\gamma V\Pi}{}^D)^2]$$
$$\times(1-m_\Pi{}^2/m_V{}^2)^3m_V{}^3\Lambda_3{}^2.$$

Similarly, for singlet vector-meson decay, we get

$$\Gamma(V_0\to\Pi+\gamma)=(\alpha/24)[4m_\Pi m_V{}^0(G_{\gamma V^0\Pi}{}^0)^2]$$
$$\times[1-m_\Pi{}^2/(m_V{}^0)^2]^3(m_V{}^0)^3(\Lambda_3{}')^2,$$

where

$$\Lambda_3{}^1=\begin{pmatrix}8 & 8 & 1\\ \sigma & 010 & 000\end{pmatrix}+\frac{1}{\sqrt{3}}\begin{pmatrix}8 & 8 & 1\\ \sigma & 000 & 000\end{pmatrix}$$

and $G_{\gamma V^0\Pi}{}^0$ is the coupling constant for singlet vector mesons.[25]

Making use of the results given in Appendix A yields

$$\Gamma(V_8\to\Pi\gamma)=\frac{5\alpha}{18}\left(\frac{3\mu_n}{4}\right)^2\left(\frac{m_V}{M}\right)^2\left(1-\frac{m_\Pi{}^2}{m_V{}^2}\right)^3m_V\Lambda_3{}^2, \quad (3.8)$$

$$\Gamma(V_0\to\Pi\gamma)=\frac{8\alpha}{9}\left(\frac{3\mu_n}{4}\right)^2\left(\frac{m_V{}^0}{M}\right)^2\left(1-\frac{m_\Pi{}^2}{(m_V{}^0)^2}\right)^3$$
$$\times m_V{}^0(\Lambda_3{}')^2. \quad (3.9)$$

We give results for the ρ, ω, and ϕ decays to $\pi\gamma$ and $\eta\gamma$ in Table II. Where necessary we have applied corrections to take account of the ω-ϕ and η-X mixing. The procedure used to fix these corrections is discussed in Appendix B.

Yellin,[17] using the experimental $\omega\to\pi\gamma$ width as input, has obtained results similar to those of Table II with the notable exception of his $\omega\to\eta\gamma$ width, which is more than 10 times smaller than that presented here, and his $\phi\to\eta\gamma$, which is somewhat larger than ours.

[25] The solution in II gave

$$(4m_V{}^0m_\Pi)^{1/2}G_{\gamma V^0\Pi}{}^0/(4m_Vm_\Pi)^{1/2}G_{\gamma V\Pi}{}^D=4/\sqrt{5}.$$

We use this ratio and that given in Ref. 12 in calculating the corrections due to ω-ϕ and η-X mixing in the $V\to\Pi\gamma$ decays. See Appendix B for further discussion of mixing corrections.

(ii) $V \to l^+ + l^-$

Here the decay is pictured as proceeding through a photon intermediate state. The vector meson, after converting to a photon with coupling $g_{V\gamma}$, proceeds through the usual electromagnetic interaction to a lepton pair [see Fig. 3(b)]. The invariant amplitude is

$$T(V \to l^+l^-) = 4\pi\alpha(g_{V\gamma}/m_V^2)\bar{v}(p_+)Eu(p_-)\Lambda_4\,,$$

where $u(p_-)$ and $v(p_+)$ are the usual lepton and antilepton spinors, E is the vector-meson polarization, and

$$\Lambda_4 = \begin{pmatrix} 1 & 8 & \mu \\ 000 & 010 & \beta \end{pmatrix} + \frac{1}{\sqrt{3}}\begin{pmatrix} 1 & 8 & \mu \\ 000 & 000 & \beta \end{pmatrix}, \quad (3.10)$$

with $\mu(\beta)$ refering to the decaying vector meson. For the width we find the well-known result[23]

$$\Gamma(V \to l^+l^-) = \tfrac{4}{3}\pi\alpha^2\left(\frac{g_{V\gamma}}{m_V^2}\right)^2 \times\left(1-\frac{4m_l^2}{m_V^2}\right)^{1/2}\left(1+\frac{2m_l^2}{m_V^2}\right)m_V\Lambda_4^2\,,$$

or using (A16),

$$\Gamma(V \to l^+l^-) = \frac{\alpha^2}{3}\left(\frac{3\mu_n}{4}\right)^2 \frac{G_A^2}{g_{\Pi NN}^2/4\pi} \times\left(1-\frac{4m_l^2}{m_V^2}\right)^{1/2}\left(1+\frac{2m_l^2}{m_V^2}\right)m_V\Lambda_4^2\,,$$

which is very insensitive to the lepton mass. We present the results for ρ, ω, and ϕ in Table II.

Yellin,[17] combining the work of Nambu and Sakurai[26] with that of Dashen and Sharp,[15] obtains values for these decays two to three times larger than those calculated here.

(iii) $V \to \Pi_1 + \Pi_2 + \gamma$

For this decay we use the same model as Singer,[27] that is, we view the decay as proceeding $V \to V\Pi \to \Pi\gamma\Pi$ as in Fig. 3(c). We consider here only the $\omega^0 \to \pi^+\pi^-\gamma$, for which we find

$$T(\omega \to \pi^+\pi^-\gamma) = G\varepsilon_{\alpha\beta\gamma\delta}\varepsilon_{\kappa\lambda\gamma\epsilon}(p_\gamma)_\alpha\epsilon_\beta(p_\gamma) \times V_\lambda\left[\frac{(p_2)_\delta(p_1)_\kappa(p_2+p_\gamma)_\epsilon}{(p_2+p_\gamma)^2+m_V^2}+\frac{(p_1)_\delta(p_2)_\kappa(p_1+p_\gamma)_\epsilon}{(p_1+p_\gamma)^2+m_V^2}\right],$$

where p_1, p_2, and p_γ are the pion and photon momenta, ϵ is the photon polarization, V is the ω polarization, and

[26] Y. Nambu and J. J. Sakurai, Phys. Rev. Letters 8, 49 (1962).

[27] P. Singer, Phys. Rev. 128, 2789 (1962). The $\omega \to \pi^+\pi^-\gamma$ decay has also been considered earlier by L. M. Brown and P. Singer, Phys. Rev. Letters 8, 155, 353 (1962); and Gell-Mann, Sharp, and Wagner (Ref. 5); and, more recently, with emphasis on possible C violation in electromagnetic interactions, by J. Bernstein, G. Feinberg, and T. D. Lee, Phys. Rev. 139, B1650 (1965); J. Yellin, Ref. 17.

TABLE II. Vector-meson decays.

	Theory	Experiment	Ref.
$\Gamma(\rho^0 \to \pi^0\gamma)$	0.13 MeV	<0.56 MeV	10
$\Gamma(\rho^0 \to \eta^0\gamma)$	0.055 MeV		
$\Gamma(\omega \to \pi^0\gamma)$	1.25 MeV	1.15 ± 0.15 MeV	10
$\Gamma(\omega \to \eta^0\gamma)$	0.094 MeV	<0.18 MeV	10
$\Gamma(\phi \to \pi^0\gamma)$	0.019 MeV		
$\Gamma(\phi \to \eta^0\gamma)$	0.26 MeV	<0.32 MeV	10
$\Gamma(\rho^0 \to e^+e^-)$	2.65 keV	$9.1_{-7.0}^{+15.4}$ keV	a,10
$\Gamma(\rho^0 \to \mu^+\mu^-)$	2.65 keV	7.14 ± 1.68 keV	b,10
$\Gamma(\omega \to e^+e^-)$	0.345 keV	$1.19_{-0.89}^{+2.02}$ keV	a,10
$\Gamma(\omega \to \mu^+\mu^-)$	0.345 keV		
$\Gamma(\phi \to l^+l^-)$	0.735 keV		
$\Gamma(\omega \to \pi^+\pi^-\gamma)$	3.34×10^{-4} MeV	<0.6 MeV	10

[a] S. S. Hertzbach, R. W. Kraemer, L. Madansky, R. A. Zdanis, and R. Strand, Phys. Rev. 155, 1461 (1967).

[b] A. Wehmann, E. Engles, Jr., C. M. Hoffman, P. G. Innocenti, R. Wilson, W. A. Blanpied, D. J. Drickey, L. N. Hand, and D. G. Stairs, Phys. Rev. Letters 18, 929 (1967).

$$G = \frac{4m_\Pi m_V(G_{\gamma V\Pi}{}^D)^2(4\pi\alpha)^{1/2}}{\sqrt{15}}\left\{\frac{\cos\lambda_2}{\sqrt{8}}\left(\frac{G_{V^0\gamma\Pi^0}}{G_{\gamma V\Pi}{}^D}\right)-\frac{\sin|\lambda_2|}{\sqrt{5}}\right\},$$

with λ_2 the ω-ϕ mixing angle. Using the values of the coupling constants from Appendix A, we get

$$\Gamma(\omega \to \pi^+\pi^-\gamma) = \frac{\alpha}{72}\frac{g_{\Pi NN}^2/4\pi}{G_A^2}(\mu_n)^2\left(\frac{M_\omega}{M}\right)^4 \times M_\omega[\sqrt{2}\cos\lambda_2-\sin\lambda_2]^2(1.92\times10^{-3})\,,$$

where the last factor is the result of a numerical phase-space integration given in Ref. 27.

We present the numerical results for the partial width in Table II and for the ratio of $\Gamma(\omega \to \pi^+\pi^-\gamma)$ to $\Gamma(\omega \to \pi\gamma)$ in Table III.

4. $N^* \to N + \pi$

As an illustration of the higher-symmetry results obtainable with the model introduced in I and II, we present here a calculation of the N^* width.

The $N^*N\pi$ coupling constant is defined by

$$\left(\frac{EE^*\Omega^2}{MM^*}\right)^{1/2}\langle 10(\eta)p+q|[j_\pi{}^\epsilon(0)]^\dagger|p8(\xi)\rangle = G_{N^*N\pi}\bar{\omega}_\rho(p+q)q_\rho u(p)\begin{pmatrix} 8 & 8 & 10 \\ \xi & \epsilon & \eta \end{pmatrix},$$

where ω_ρ is a Rarita-Schwinger spin-$\frac{3}{2}$ wave function.

The calculations in I, which dealt directly with the matrix elements of divergences of currents, gave a value for the coupling α_{31}, defined by

$$\left(\frac{EE^*\Omega^2}{MM^*}\right)^{1/2}\langle 10(\eta)p+q|D_A{}^\epsilon(x)|p8(\xi)\rangle = e^{-iq\cdot x}\bar{\omega}_\rho(p+q)q_\rho u(p)\alpha_{31}\begin{pmatrix} 8 & 8 & 10 \\ \xi & \epsilon & \eta \end{pmatrix}.$$

By making use of pion pole dominance of the axial-vector divergence we find, at $q^2=0$,

$$G_{N^*N\pi}=\alpha_{31}/\bar{f}\,,$$

where $\bar{f}$ is the pseudoscalar-meson decay amplitude defined in Sec. 2. We then find for the N^* width the result

$$\Gamma(N^*\to N\Pi)=\frac{1}{12\pi}\left\{\left(\frac{p\cdot p^*}{M^*}\right)^2-M^2\right\}^{3/2}\times\left\{1-\frac{p\cdot p^*}{MM^*}\right\}\frac{M}{M^*}\frac{\alpha_{31}{}^2}{\bar{f}^2}\begin{pmatrix}8 & 8 & 10\\ \xi & \epsilon & \eta\end{pmatrix}^2.$$

Using the Goldberger-Treiman (GT) value for $\bar{f}$: $\bar{f}=-2MG_A/g_{\Pi NN}$ and the value of α_{31} obtained from the consistent solution in I, $\alpha_{31}=4[2M^*/(M+M^*)]$, we find

$$\Gamma(N^*\to N\Pi)=\frac{4}{3}\frac{(g_{\Pi NN}{}^2/4\pi)}{G_A{}^2}\left\{\frac{(M+M^*)^2-m_\Pi{}^2}{2MM^*}\right\}^{5/2}\times\left\{\frac{(M^*-M)^2-m_\Pi{}^2}{2MM^*}\right\}^{3/2}\frac{M^2}{M^*}\begin{pmatrix}8 & 8 & 10\\ \xi & \epsilon & \eta\end{pmatrix}^2\left(\frac{2M^*}{M+M^*}\right)^2.$$

It was found in I that a consistent solution could be obtained for the baryon couplings only if the mass of the octet and decuplet were taken as degenerate, i.e., only if $M=M^*$. With this provision $\alpha_{31}=4$, and we find for $N^*\to p+\pi^+$ the result

$$\Gamma(N^*\to p+\pi^+)=63.2\text{ MeV},$$

a well-known $SU(6)$ result. Note that a first symmetry-breaking correction (note that corrections of all orders can be obtained simply by not truncating the sum rules in I) might be included by allowing $M^*\neq M$ in the coupling. This results in $\Gamma(N^*\to p+\pi^+)=83.5$ MeV. Both of these numbers are to be compared with the experimental value $\Gamma=120\pm1.5$ MeV.[10]

TABLE III. Decay ratios.

	Theory	Experiment	Ref.
$\frac{\Gamma(\pi^0\to\gamma+e^+e^-)}{\Gamma(\pi^0\to2\gamma)}$	1.185×10^{-2}	$(1.166\pm0.047)\times10^{-2}$	a
$\frac{\Gamma(\eta^0\to\gamma+e^+e^-)}{\Gamma(\eta^0\to2\gamma)}$	1.62×10^{-2}		
$\frac{\Gamma(\eta^0\to\gamma+\mu^++\mu^-)}{\Gamma(\eta^0\to2\gamma)}$	5.58×10^{-4}		
$\frac{\Gamma(\eta^0\to\pi^++\pi^-+\gamma)}{\Gamma(\eta^0\to2\gamma)}$	0.24	0.19 ± 0.06	20
$\frac{\Gamma(\omega^0\to\pi^++\pi^-+\gamma)}{\Gamma(\omega^0\to\pi^0+\gamma)}$	2.7×10^{-4}	<0.5	10

[a] H. Samios, Phys. Rev. 121, 275 (1961)

It is not surprising that octet-decuplet mass degeneracy must be assumed for the baryons to obtain a consistent approximate solution to the sum rules in I which reproduces higher-symmetry results since, for example, $SU(6)$ places these particles in the same multiplet. A very interesting feature of the approach in I and II, on the other hand, is that a more thorough investigation of all the meson relations (some of which were not mentioned in II) has shown that the corresponding assumption of degeneracy for the pseudoscalar and vector mesons [which are also placed in the same multiplet by $SU(6)$] is not needed to obtain consistency.

5. LOW-MOMENTUM-TRANSFER N^* PRODUCTION BY NEUTRINOS

We consider here the process[28]

$$\nu_\mu+p\to N^{++}+\mu^-.$$

The weak Hamiltonian is taken to be

$$H_W=(G/\sqrt{2})(J_\alpha j_\alpha{}^\dagger+j_\alpha J_\alpha{}^\dagger)\,,$$

with

$$G=1.023\times10^{-5}m_p{}^2\,,\quad J_\alpha=J_\alpha{}^V+J_\alpha{}^A\,,$$

and j_α the conventional $V-A$ lepton current. The amplitude for the reaction is then

$$T=\left(\frac{EE^*\Omega^2}{MM^*}\right)^{1/2}\frac{ig}{\sqrt{2}}\times\langle N^*10(1\tfrac{3}{2}\tfrac{3}{2})|(J_\alpha{}^{V(+)}+J_\alpha{}^{A(+)})|N8(1\tfrac{1}{2}\tfrac{1}{2})\rangle\times\bar{u}(p_\mu)\gamma_\alpha(1+\gamma_5)u(p_\nu)\,,$$

where $g=G\cos\theta_c$ with θ_c the Cabibbo angle. We then find that

$$\frac{d\sigma}{dt}=-\frac{1}{2\pi}\frac{MM^*m_\mu m_\nu}{(s-M^2)^2}|T|^2,$$

where $s=-(p_\nu+P)^2$ and $t=-(p_\nu-p_\mu)^2$.

Adler[29] has given an exhaustive analysis of high-energy neutrino reactions from the point of view of testing proposed local current commutation relations and the conserved vector current (CVC) and partially conserved axial-vector current (PCAC) hypotheses. Here we will give a simple analysis of N^* production at very low momentum transfer, retaining only the most important pieces of the invariant amplitude in this region. The reason for doing this is that the calculations performed in I resulted in a direct determination of the leading term in the N^*N axial-vector vertex. We can thus give a theoretical result independent of PCAC or other GT-like arguments.

[28] This reaction has also been analyzed by S. Berman and M. Veltman, Nuovo Cimento 38, 993 (1965); C. H. Albright and L. S. Lin, Phys. Rev. 140, B748 (1965); Ph. Salin, Nuovo Cimento 48, 506 (1967).

[29] S. L. Adler, Phys. Rev. 135, B963 (1964); 143, 1144 (1966).

In the axial-vector matrix element ($q=P^*-P$):

$$\langle 10(\eta)P^*|J_\mu{}^{A\epsilon}(0)|8(\xi)P\rangle=(MM^*/EE^*\Omega^2)^{1/2}i\bar{\omega}_\nu(P^*)$$
$$\times\left[G_1\delta_{\mu\nu}+\frac{iG_2}{m_\Pi}q_\nu\gamma_\mu+\frac{G_3}{m_\Pi{}^2}q_\nu(P+P^*)_\mu+\frac{G_4}{m_\Pi{}^2}q_\nu q_\mu\right]$$
$$\times u(P)\begin{pmatrix}8 & 8 & 10\\ \xi & \epsilon & \eta\end{pmatrix},$$

we will retain only $G_1(q^2=0)$ for which the results of I give a value of $G_1=+4[2M^*/(M+M^*)]$. We stress again that this value of G_1 is the main new input in this calculation. By way of comparison, an argument of the GT variety combined with the known residue of the pion pole in G_4 yields $G_1(0)=\bar{f}G_{N^*N\pi}$, which, using the experimental N^* width, has a value of 5.6. The value we use here, determined entirely from the consistency solution in I, is $G_1(0)=4.6$.

For the vector vertex we use the CVC theory and the results of I to write

$$\langle 10(\eta)P^*|J_\mu{}^{V\epsilon}(0)|8(\xi)P\rangle$$
$$=\left(\frac{MM^*}{EE^*\Omega^2}\right)^{1/2}i\bar{\omega}_\nu(P^*)\bar{C}_3\gamma_5[q_\nu\gamma_\mu-q\delta_{\mu\nu}]$$
$$\times u(P)\begin{pmatrix}8 & 8 & 10\\ \xi & \epsilon & \eta\end{pmatrix},$$

where $\bar{C}_3=-\sqrt{3}C_3/m_\Pi$ and C_3 is defined in I. The total hadronic current used in these calculations is then

$$\left(\frac{EE^*\Omega^2}{MM^*}\right)^{1/2}\langle 10(\eta)P^*|J_\mu{}^\epsilon(0)|8(\xi)P\rangle$$
$$=i\bar{\omega}_\nu(P^*)[G_1\delta_{\mu\nu}+i\bar{C}_3\delta_{\mu\nu}\gamma_5(M+M^*)+\bar{C}_3q_\nu\gamma_5\gamma_\mu]$$
$$\times u(P)\begin{pmatrix}8 & 8 & 10\\ \xi & \epsilon & \eta\end{pmatrix},$$

which leads to

$$T=-\bar{g}\bar{\omega}_\nu(P^*)[G_1\delta_{\mu\nu}+i\bar{C}_3\delta_{\mu\nu}(M+M^*)\gamma_5+\bar{C}_3q_\nu\gamma_5\gamma_\mu]$$
$$\times u(P)\bar{u}(p_\mu)\gamma_\mu(1+\gamma_5)u(p_\nu),$$

where we have set

$$\bar{g}=\frac{g}{\sqrt{2}}\begin{pmatrix}8 & 8 & 10\\ \xi & \epsilon & \eta\end{pmatrix}.$$

We then find, after squaring and performing the appropriate polarization sums,

$$\sum_{\text{polarizations}}|T|^2=\frac{-\bar{g}^2}{2m_\mu m_\nu}\left\{\frac{G_1{}^2}{3M^2}(4M^2-t)(t-m_\mu{}^2)\right.$$
$$+\bar{C}_3{}^2\frac{t[(4M^2-t)t-16M^4](t-m_\mu{}^2)}{12M^4}$$
$$+\left[\frac{G_1{}^2(4M^2-t)}{3M^2}+\frac{\bar{C}_3{}^2(t^2-12M^4)}{6M^2}\right]$$
$$\left.\times\frac{(M^2-s)(M^2-s-t)}{2M^2}\right\},$$

where for simplicity we have set $M^*=M$.

Using the data for the CERN bubble-chamber experiments,[30] we find that the number of events (taken to be single pion production proceeding entirely through N^* production) per unit momentum transfer is

$$\frac{\Delta N(t)}{\Delta t}=(2.97\times10^{12})\int_{s_0}^{s_m}\left(\frac{d\sigma}{dt}\right)e^{-0.38s}ds,$$

where s_0 and s_m are the threshold and maximum values of s [for the CERN neutrino spectrum $s_m\lesssim19.7$ GeV^2], and the exponential arises from a fit to the neutrino spectrum. Dropping all terms of higher than the second power in t and neglecting $m_\mu{}^2$ compared to s and M^2, we get

$$\Delta N/\Delta t=-(2.97\times10^{12})(\bar{g}^2/12\pi)$$
$$\times\{[4M^2G_1{}^2(t-m_\mu{}^2)-4M^2\bar{C}_3{}^2t(t-m_\mu{}^2)]A$$
$$+[2G_1{}^2-2M^2\bar{C}_3{}^2]B(t)\},$$

where

$$A=\int_{s_0}^{s_m}\frac{\exp[-0.38s]}{(s-M^2)^2}ds$$

and

$$B=\int_{s_0}^{s_m}\frac{(M^2-s)(M^2-s-t)}{(s-M^2)^2}\exp[-0.38s]ds.$$

Gathering results, we find

$$\Delta N/\Delta t=-36.4-297.5t+206.4t^2.$$

In Fig. 4, we plot

$$N(Q^2)=\int_0^{Q^2}\frac{\Delta N}{\Delta q^2}d(q^2),$$

[30] C. Franzinetti, CERN Report 66-13 (unpublished).

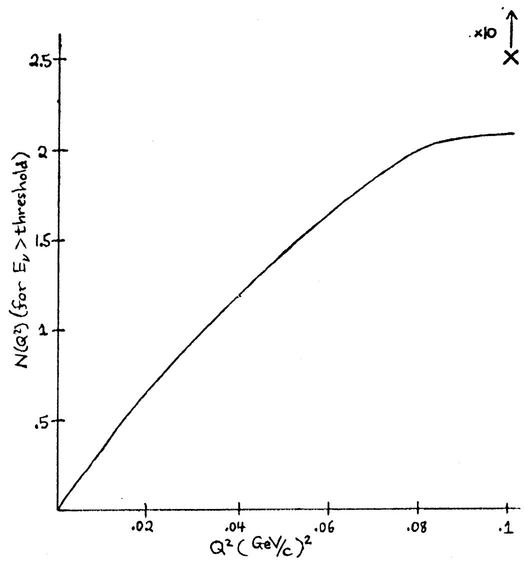

FIG. 4. Total number N^* events for $q^2<Q^2$, $N(Q^2)$, versus Q^2. The single experimental point (actually off scale) is taken from the low end of a histogram of single pion events given in Ref. 29.

which is the total number of events observed with $0<q^2<Q^2$ for $E_\nu>E_\nu^{\text{thresh}}$. The CERN data are not fine enough as yet to allow a detailed comparison with experiment, although the data indicate a higher number of events for $Q^2<0.1$ GeV^2 than predicted here.

6. CONCLUSION

In this paper we have examined some further consequences of the model introduced in I and II. The aim of this model was not so much to produce any exact results but rather to extend the Adler-Weisberger[31] relations to cover as wide a field as possible with the view of correlating diverse phenomena. At the same time it was desired that the model incorporate certain features of the quark model, but in a Lorentz-covariant way and without the need for any detailed assumptions on the composite nature of the hadrons. In addition, the model was an attempt to understand higher-symmetry results from a dispersion-theory point of view. The assumptions of the model are:

(i) Integrated quark charge commutation relations. This is a much weaker assumption than any bound-state quark model or even local quark commutation relations since possible Schwinger terms are integrated out.

(ii) Unsubtracted dispersion relations for "good" charge sum rules. This assumption is supported by Pomeranchuk and other high-energy theorems.

(iii) Approximate saturation of these sum rules by low-lying $SU(3)$ multiplets, an assumption inherent in almost any group-theory or dispersion-theory approach.

(iv) When necessary, the interpretation of couplings through the use of meson pole dominance just as in the Adler-Weisberger case for the pion dominance of the axial-vector divergence. In the case of vector-meson dominance of the tensor current divergence, the fact that a Compton scattering sum rule, derived in I, which relies on extrapolating from the vector-meson pole twice, agrees with experiment to within 40% indicates that this assumption cannot be far in error.

As demonstrated in I, II, and the present work, these few reasonable assumptions seem to be sufficient to allow one to calculate and correlate many diverse physical processes to at worst within a factor of 2 agreement with experiment.

ACKNOWLEDGMENT

The author would like to thank Professor J. D. Walecka for his continued guidance and encouragement during the course of this work. The author would also like to thank the National Science Foundation for a predoctoral fellowship which he held while this work was in progress.

APPENDIX A: DETERMINATION OF $g_{v\gamma}$

We define the coupling of a vector meson to the divergence of the tensor current, $J_{\mu\nu}{}^{\alpha}(x)=\frac{1}{2}i\bar{\psi}(x)[\gamma_\mu,\gamma_\nu]\lambda^\alpha\psi(x)$, by

$$\epsilon_\nu\langle pV_\sigma 8(\eta)|\partial_\mu J_{\mu\nu}{}^{\alpha}(0)|0\rangle=\frac{1}{(2p_0\Omega)^{1/2}}\frac{a_{SV}}{m_V{}^2}(p_\nu p_\mu-p^2\delta_{\mu\nu})\epsilon_\nu V_\mu\begin{pmatrix}1 & 8 & 8\\ 000 & \alpha & \eta\end{pmatrix}. \tag{A1}$$

The vector-meson–pseudoscalar-meson matrix element of this same divergence is given by

$$\left(\frac{E_1E_2\Omega^2}{m_{\Pi}m_V}\right)^{1/2}\langle p+qV_\sigma 8(\eta)|\partial_\mu J_{\mu\nu}{}^{\alpha}(0)|p8(\rho)\rangle\epsilon_\nu=i\varepsilon_{\beta\gamma\delta\kappa}q_\beta\epsilon_\gamma V_\delta p_\kappa\left\{G_T{}^D\begin{pmatrix}8 & 8 & 8_1\\ \rho & \alpha & \eta\end{pmatrix}+G_T{}^F\begin{pmatrix}8 & 8 & 8_2\\ \rho & \alpha & \eta\end{pmatrix}\right\}. \tag{A2}$$

In the following we take ϵ such that $\epsilon\cdot P=\epsilon\cdot q=0$.

Assuming that the matrix elements of the divergence of the tensor current are dominated by their vector-meson poles and writing the vector-meson–pseudoscalar-meson–vector-meson coupling as

$$\left(\frac{E_1E_2\Omega^2}{m_{\Pi}m_V}\right)^{1/2}\epsilon_\nu\langle p+qV_\kappa 8(\eta)|[J_\nu{}^{V\epsilon}(0)]^\dagger|p8(\sigma)\rangle=i\varepsilon_{\alpha\beta\gamma\delta}q_\alpha\epsilon_\beta V_\gamma p_\delta\left\{G_{VV\Pi}{}^D\begin{pmatrix}8 & 8 & 8_1\\ \sigma & \epsilon & \eta\end{pmatrix}+G_{VV\Pi}{}^F\begin{pmatrix}8 & 8 & 8_2\\ \sigma & \epsilon & \eta\end{pmatrix}\right\}, \tag{A3}$$

where $J_\mu{}^{V\epsilon}(x)$ is the vector-meson octet source current, we find

$$\left(\frac{E_1E_2\Omega^2}{m_{\Pi}m_V}\right)^{1/2}\epsilon_\nu\langle p+qV_\sigma 8(\eta)|\partial_\mu J_{\mu\nu}{}^{\epsilon}(0)|p8(\rho)\rangle=\left(\frac{E_1E_2\Omega^2}{m_{\Pi}m_V}\right)^{1/2}\frac{a_{SV}}{q^2+m_{\Pi}{}^2}\epsilon_\nu\langle p+qV_\sigma 8(\eta)|[J_\nu{}^{V\epsilon}(0)]^\dagger|p8(\rho)\rangle$$

$$=\frac{a_{SV}}{q^2+m_V{}^2}\left\{G_{VV\Pi}{}^D\begin{pmatrix}8 & 8 & 8_1\\ \rho & \epsilon & \eta\end{pmatrix}+G_{VV\Pi}{}^F\begin{pmatrix}8 & 8 & 8_2\\ \rho & \epsilon & \eta\end{pmatrix}\right\}i\varepsilon_{\beta\gamma\delta\kappa}q_\beta\epsilon_\gamma V_\delta p_\kappa.$$

[31] S. Adler, Phys. Rev. Letters **14**, 1051 (1965); W. Weisberger, Phys. Rev. Letters **14**, 1047 (1965).

Comparing this with (A2) we find, at $q^2=0$,

$$G_T{}^D=\frac{a_{SV}}{m_V{}^2}G_{VV\Pi}{}^D\,,\quad G_T{}^F=\frac{a_{SV}}{m_V{}^2}G_{VV\Pi}{}^F\,. \tag{A4}$$

If we now define the vector-meson–vector-meson matrix element of the divergence of the axial-vector current by

$$\left(\frac{E_1E_2\Omega^2}{m_V{}^2}\right)^{1/2}\langle P+qV^{(1)}8(\tau)\,|\,\partial_\nu J_\nu{}^{A\gamma}(0)\,|\,PV^{(2)}8(\sigma)\rangle=i\varepsilon_{\nu\alpha\epsilon\delta}q_\nu V_\alpha{}^{(1)}V_\epsilon{}^{(2)}\frac{P_\delta}{m_V}\left\{g_S{}^F\begin{pmatrix}8 & 8 & 8_2\\ \sigma & \gamma & \tau\end{pmatrix}-g_S{}^D\begin{pmatrix}8 & 8 & 8_1\\ \sigma & \gamma & \tau\end{pmatrix}\right\}, \tag{A5}$$

and assume pion pole dominance, we get

$$\left(\frac{E_1E_2\Omega^2}{m_V{}^2}\right)^{1/2}\langle P+qV^{(1)}8(\tau)\,|\,\partial_\nu J_\nu{}^{A\gamma}(0)\,|\,PV^{(2)}8(\sigma)\rangle$$
$$=\frac{m_\Pi{}^2\bar{f}}{q^2+m_\Pi{}^2}i(m_\Pi m_V)^{1/2}\varepsilon_{\nu\alpha\epsilon\delta}q_\nu V_\alpha{}^{(1)}V_\epsilon{}^{(2)}\frac{P_\delta}{m_V}\left\{G_{VV\Pi}{}^D\begin{pmatrix}8 & 8 & 8_1\\ \sigma & \gamma & \tau\end{pmatrix}-G_{VV\Pi}{}^F\begin{pmatrix}8 & 8 & 8_2\\ \sigma & \gamma & \tau\end{pmatrix}\right\}, \tag{A6}$$

where we have used the fact that the V-V-π coupling defined in (A3) can also be written

$$\left(\frac{E_1E_2\Omega^2}{m_V{}^2}\right)^{1/2}\langle P+qV^{(1)}8(\tau)\,|\,[j_\Pi{}^\gamma(0)]^\dagger\,|\,PV^{(2)}8(\sigma)\rangle$$
$$=i(m_\Pi m_V)^{1/2}\varepsilon_{\nu\alpha\epsilon\delta}q_\nu V_\alpha{}^{(1)}V_\epsilon{}^{(2)}\frac{P_\delta}{m_V}\left\{G_{VV\Pi}{}^F\begin{pmatrix}8 & 8 & 8_2\\ \sigma & \gamma & \tau\end{pmatrix}-G_{VV\Pi}{}^D\begin{pmatrix}8 & 8 & 8_1\\ \sigma & \gamma & \tau\end{pmatrix}\right\}.$$

Comparing (A6) and (A5) we get, again at $q^2=0$,

$$g_S{}^D=-(m_\Pi m_V)^{1/2}\bar{f}G_{VV\Pi}{}^D\,,\quad g_S{}^F=-(m_\Pi m_V)^{1/2}\bar{f}G_{VV\Pi}{}^F\,. \tag{A7}$$

Charge conjugation invariance (or G parity) forces us to take $G_{VV\Pi}{}^F=0$. Combining the remaining elements of (A7) and (A4) then yields

$$a_{SV}/m_V{}^2=-(m_\Pi m_V)^{1/2}\bar{f}(G_T{}^D/g_S{}^D)\,.$$

The consistency solution for the couplings obtained in II gave

$$g_S{}^D=2\sqrt{(5/3)}\,,\quad G_T{}^D=2\sqrt{(5/3m_\Pi m_V)}\,,$$

so that we finally arrive at

$$a_{SV}/m_V{}^2=-\bar{f}. \tag{A8}$$

That this relation arises from the results of II is not surprising since the assumption of meson dominance is equivalent to the formal identifications

$$\partial_\mu J_\mu{}^{A\epsilon}(x)=-m_\Pi{}^2\bar{f}[\phi_\Pi{}^\epsilon(x)]^\dagger\,,\quad \partial_\mu J_{\mu\nu}{}^\epsilon(x)=a_{SV}[\phi_\nu{}^{V\epsilon}(x)]^\dagger\,.$$

Because the solution in I and II incorporates many $SU(6)$ results and this group places both the vector and pseudoscalar mesons in a single 35-dimensional representation, it is clear that some relation between a_{SV} and $\bar{f}$ must follow.

The vector meson-photon coupling constant is defined by

$$\langle pV_\sigma 8(\eta)\,|\,J_\mu{}^{\text{elect}}(0)\,|\,0\rangle=\frac{1}{(2p_0\Omega)^{1/2}}\frac{g_{V\gamma}}{m_V{}^2}(p_\mu p_\nu-p^2\delta_{\mu\nu})V_\nu\left\{\begin{pmatrix}1 & 8 & 8_1\\ 000 & 010 & \eta\end{pmatrix}+\frac{1}{\sqrt{3}}\begin{pmatrix}1 & 8 & 8\\ 000 & 000 & \eta\end{pmatrix}\right\}. \tag{A9}$$

We determine $g_{V\gamma}$ by relating it to a_{SV}. This is done by considering the nucleon-nucleon matrix elements of the divergence of the tensor current and the nucleon-nucleon matrix elements of the electromagnetic current, both in the limit of vanishing momentum transfer q^2. The first of these matrix elements is written as

$$\left(\frac{E_1E_2\Omega^2}{M^2}\right)^{1/2}\epsilon_\nu\langle p+q8(\sigma)\,|\,\partial_\mu J_{\mu\nu}{}^\epsilon(0)\,|\,p8(\tau)\rangle\xrightarrow[q^2\to 0]{}\bar{u}(p+q)\tfrac{1}{2}[\boldsymbol{q},\boldsymbol{\varepsilon}]u(p)\left\{\begin{pmatrix}8 & 8 & 8_1\\ \tau & \epsilon & \sigma\end{pmatrix}D+\begin{pmatrix}8 & 8 & 8_2\\ \tau & \epsilon & \sigma\end{pmatrix}F\right\}. \tag{A10}$$

Here M is the nucleon octet mass and $u(p)$ is the usual Dirac spinor. As was pointed out in I, only this "magnetic" coupling persists in the $q^2\to 0$ limit, so that we need only consider the magnetic coupling of the nucleon to the vector mesons, which we define by

$$\left(\frac{E_1E_2\Omega^2}{M^2}\right)^{1/2}\langle p+q8(\sigma)|[J_\mu{}^{V\epsilon}(0)]^\dagger|p8(\tau)\rangle_{\text{mag}}\epsilon_\mu=-i\bar{u}(p+q)\frac{\sigma_{\mu\nu}q_\nu}{2M}u(p)\times\left\{\begin{pmatrix}8&8&8_1\\ \tau&\epsilon&\sigma\end{pmatrix}\mu_{VN}{}^D+\begin{pmatrix}8&8&8_2\\ \tau&\epsilon&\sigma\end{pmatrix}\mu_{VN}{}^F\right\}\epsilon_\mu. \quad \text{(A11)}$$

Assuming the matrix element in (A10) to be dominated by the vector-meson pole and using (A11) and (A1), we find

$$\left(\frac{E_1E_2\Omega^2}{M^2}\right)^{1/2}\epsilon_\nu\langle p+q8(\sigma)|\partial_\mu J_{\mu\nu}{}^\epsilon(0)|p8(\tau)\rangle\xrightarrow[q^2\to 0]{}\frac{a_{SV}}{q^2+m_V{}^2}\bar{u}(p+q)\tfrac{1}{2}[\mathbf{q},\boldsymbol{\epsilon}]u(p)\left\{\begin{pmatrix}8&8&8_1\\ \tau&\epsilon&\sigma\end{pmatrix}\frac{\mu_{VN}{}^D}{2M}+\begin{pmatrix}8&8&8_2\\ \tau&\epsilon&\sigma\end{pmatrix}\frac{\mu_{VN}{}^F}{2M}\right\}.$$

Comparing this to (A10) then yields

$$D=\frac{a_{SV}}{m_V{}^2}\frac{\mu_{VN}{}^D}{2M},\quad F=\frac{a_{SV}}{m_V{}^2}\frac{\mu_{VN}{}^F}{2M}. \quad \text{(A12)}$$

For the magnetic coupling of the nucleon to the electromagnetic current, we write

$$\left(\frac{E_1E_2\Omega^2}{M^2}\right)^{1/2}\langle p+q8(\sigma)|J_\mu{}^{\text{elect}}(0)|p8(\tau)\rangle_{\text{mag}}\epsilon_\mu=-i\bar{u}(p+q)\frac{\epsilon_\mu\sigma_{\mu\nu}q_\nu}{2M}\times u(p)\left\{\left[\begin{pmatrix}8&8&8_1\\ \tau&010&\sigma\end{pmatrix}+\frac{1}{\sqrt{3}}\begin{pmatrix}8&8&8_1\\ \tau&000&\sigma\end{pmatrix}\right]\mu^D+\left[\begin{pmatrix}8&8&8_2\\ \tau&010&\sigma\end{pmatrix}+\frac{1}{\sqrt{3}}\begin{pmatrix}8&8&8_2\\ \tau&000&\sigma\end{pmatrix}\right]\mu^F\right\}. \quad \text{(A13)}$$

Now, assuming that only this magnetic coupling is dominated by vector-meson poles and using (A11) and (A9), we get

$$\left(\frac{E_1E_2\Omega^2}{M^2}\right)^{1/2}\langle p+q8(\sigma)|J_\mu{}^{\text{elect}}(0)|p8(\tau)\rangle_{\text{mag}}\epsilon_\mu=-\frac{ig_{V\gamma}}{q^2+m_V{}^2}\bar{u}(p+q)\frac{\epsilon_\mu\sigma_{\mu\nu}q_\nu}{2M}\times u(p)\left\{\left[\begin{pmatrix}8&8&8_1\\ \tau&010&\sigma\end{pmatrix}+\frac{1}{\sqrt{3}}\begin{pmatrix}8&8&8_1\\ \tau&000&\sigma\end{pmatrix}\right]\mu_{VN}{}^D+\left[\begin{pmatrix}8&8&8_2\\ \tau&010&\sigma\end{pmatrix}+\frac{1}{\sqrt{3}}\begin{pmatrix}8&8&8_2\\ \tau&000&\sigma\end{pmatrix}\right]\mu_{VN}{}^F\right\},$$

and comparison with (A13) then yields

$$\mu^D=(g_{V\gamma}/m_V{}^2)\mu_{VN}{}^D,\quad \mu^F=(g_{V\gamma}/m_V{}^2)\mu_{VN}{}^F. \quad \text{(A14)}$$

Eliminating $\mu_{VN}{}^{D,F}$ between Eqs. (A14) and (A12), we find

$$\frac{g_{V\gamma}}{a_{SV}}=\frac{\mu^D}{2MD},\quad \frac{g_{V\gamma}}{a_{SV}}=\frac{\mu^F}{2MF}.$$

The consistency solution obtained in I gave

$$D=2\surd(5/3),\quad F=4/\surd 3,$$

so noting that

$$\mu^D=-(\tfrac{1}{2}\surd 15)\mu_n,\quad \mu^F=\surd 3(\mu_p+\tfrac{1}{2}\mu_n),$$

where μ_n and μ_p are the magnetic moments of the neutron and proton, we find

$$\frac{g_{V\gamma}}{a_{SV}}=-\frac{3}{8M}\mu_n,\quad \frac{g_{V\gamma}}{a_{SV}}=\frac{3}{8M}(\mu_p+\tfrac{1}{2}\mu_n). \quad \text{(A15)}$$

As discussed in I, there is an ambiguity as to whether anomalous or total magnetic moments are to appear in (A14). This ambiguity is related to the choice of invariants made in writing (A13). For simplicity we have here presented the derivation for anomalous moments, although the entire argument carries through with the same resulting expressions (except for the replacement of anomalous by total moments) if we adopt a no-subtraction approach for the Sachs, rather than the Dirac, form factors. At the present time there is, to the author's knowledge, no compelling reason for choosing one moment over the other or even some linear combination of them. Therefore, we will let the internal consistency of the theory itself determine which moment, total or anomalous, we are assuming an unsubtracted dispersion relation for. An examination of (A15) indicates that if we are to preserve the internal consistency of our approach we must use total magnetic moments. Alternatively, one could just treat μ_p as a single parameter and make a best fit to all of the experimental quantities which we have calculated. In this case it is, of course, also the total moment which gives the best agreement with experiment.

Finally, combining (A15) and (A8), we obtain the value of $g_{V\gamma}$,

$$g_{V\gamma}/m_V^2=(3/8M)\bar{f}\mu_n\,.$$

Using the GT value for the pion decay amplitude $\bar{f}$:

$$\bar{f}=-2MG_A/g_{\Pi NN}\,,$$

where G_A is the nucleon axial-vector renormalization ratio, $G_A=-1.18$, and $G_{\Pi NN}$ is the renormalized pion-nucleon coupling constant, $g_{\Pi NN}^2/4\pi=14.6$, we relate $g_{V\gamma}$ to the properties of the nucleon alone:

$$g_{V\gamma}/m_V^2=-\tfrac{3}{4}\mu_n G_A/g_{\Pi NN}\,. \tag{A16}$$

For completeness we note that if the vector-meson–photon–pseudoscalar-meson coupling is defined by

$$(E_1E_2\Omega^2/m_\Pi m_V)^{1/2}\langle p+qV_\sigma 8(\eta)\,|\,J_\mu^{\text{elect}}(0)\,|\,p8(\tau)\rangle\epsilon_\mu=i\varepsilon_{\alpha\beta\gamma\delta}q_\alpha\epsilon_\beta V_\gamma p_\delta$$
$$\times\left\{G_{\gamma V\Pi}{}^D\left[\begin{pmatrix}8 & 8 & 8_1\\ \tau & 010 & \eta\end{pmatrix}+\frac{1}{\sqrt{3}}\begin{pmatrix}8 & 8 & 8_1\\ \tau & 000 & \eta\end{pmatrix}\right]+G_{\gamma V\Pi}{}^F\left[\begin{pmatrix}8 & 8 & 8_2\\ \tau & 010 & \eta\end{pmatrix}+\frac{1}{\sqrt{3}}\begin{pmatrix}8 & 8 & 8_2\\ \tau & 000 & \eta\end{pmatrix}\right]\right\}, \tag{A17}$$

then the same methods described above lead to the relations

$$G_{\gamma V\Pi}{}^D=\frac{g_{V\gamma}}{m_V^2}G_{VV\Pi}{}^D\,,\quad G_{\gamma V\Pi}{}^F=\frac{g_{V\gamma}}{m_V^2}G_{VV\Pi}{}^F\,.$$

Combining these with (A4) and (A15), we find

$$G_{\gamma V\Pi}{}^F=0\,,\quad G_{\gamma V\Pi}{}^D=-(\tfrac{1}{4}\sqrt{15})\mu_n/M(m_\Pi m_V)^{1/2}\,. \tag{A18}$$

APPENDIX B: MIXING CORRECTIONS

If we denote the η-X^0 mixing angle by λ_1, then[13]

$$|\eta\rangle=\cos\lambda_1|\eta_0\rangle-\sin\lambda_1|X_0\rangle\,,$$
$$|X\rangle=\sin\lambda_1|\eta_0\rangle+\cos\lambda_1|X_0\rangle\,,$$

where $|\eta_0\rangle$ is an $SU(3)$ octet state [8(000)] and $|X_0\rangle$ is an $SU(3)$ singlet [1(000)]. The correction factor which must be used with the $\eta\rightarrow 2\gamma$ width calculated from (3.3) assuming a pure octet η is then found to be

$$C(\eta\rightarrow 2\gamma)=\left[\cos\lambda_1-(2\sqrt{5})\frac{(4m_Vm_{\Pi^0})^{1/2}G_{\gamma V\Pi^0}{}^0}{(4m_Vm_\Pi)^{1/2}G_{\gamma V\Pi}{}^D}\sin\lambda_1\right]^2.$$

Likewise the correction to be applied to the $\eta\rightarrow\pi^+\pi^-\gamma$ width calculated with pure octet η from (3.4) is

$$C(\eta\rightarrow \pi^+\pi^-\gamma)=\left[\cos\lambda_1-(\sqrt{5})\frac{(4m_Vm_{\Pi^0})^{1/2}G_{\gamma V\Pi^0}{}^0}{(4m_Vm_\Pi)^{1/2}G_{\gamma V\Pi}{}^D}\sin\lambda_1\right]^2.$$

Now the solution for the couplings obtained in II does not determine their relative sign and, of course, the sign of λ_1 is also not determined. The results presented in Table I were obtained by making the choice

$$\frac{(4m_V m_{\Pi^0})^{1/2}G_{\gamma V\Pi^0}{}^0}{(4m_V m_\Pi)^{1/2}G_{\gamma V\Pi}{}^D}\sin\lambda_1<0\,, \tag{A19}$$

because this produces the best agreement with the experimentally determined ratio of $\Gamma(\eta\to\pi^+\pi^-\gamma)/\Gamma(\eta\to 2\gamma)$. With this choice the correction factors are[12]

$$C(\eta\to 2\gamma)=[\cos\lambda_1+2\sqrt{2}\,\sin|\lambda_1|]^2\,,$$
$$C(\eta\to\pi^+\pi^-\gamma)=[\cos\lambda_1+\sqrt{2}\,\sin|\lambda_1|]^2\,.$$

In the case of the $\rho^0\to\eta\gamma$ decay the correction factor to be applied to the results obtained from (3.8) assuming a pure octet η is

$$C(\rho^0\to\eta\gamma)=\left[\cos\lambda_1-\frac{1}{\sqrt{5}}\frac{(4m_V m_{\Pi^0})^{1/2}G_{\gamma V\Pi^0}{}^0}{(4m_V m_\Pi)^{1/2}G_{\gamma V\Pi}{}^D}\sin\lambda_1\right]^2.$$

The choice of sign is fixed by (A19), so that we have

$$C(\rho^0\to\eta\gamma)=[\cos\lambda_1+\tfrac{1}{5}\sqrt{2}\,\sin|\lambda_1|]^2\,.$$

For the ω-ϕ we write[32]

$$|\omega\rangle=\cos\lambda_2|\omega_0\rangle-\sin\lambda_2|\phi_0\rangle\,,$$
$$|\phi\rangle=\sin\lambda_2|\omega_0\rangle+\cos\lambda_2|\phi_0\rangle\,,$$

where $\lambda_2\approx\pm 38°$, $|\omega_0\rangle$ is an $SU(3)$ singlet $[1(000)]$ state, and $|\phi_0\rangle$ is an $SU(3)$ octet $[8(000)]$ state.

To the $\omega\to\Pi\gamma$ width calculated from (3.9), assuming a pure singlet ω, we then find the correction factor

$$C(\omega\to\Pi\gamma)=\left[\cos\lambda_2+\left(\frac{8}{5}\right)^{1/2}\frac{(4m_\Pi m_V)^{1/2}G_{\gamma V\Pi}{}^D}{(4m_\Pi m_{V^0})^{1/2}G_{\gamma V^0\Pi}{}^0}\sin\lambda_2\right]^2.$$

Again the relative signs are not fixed, so that we take

$$\frac{(4m_\Pi m_V)^{1/2}G_{\gamma V\Pi}{}^D}{(4m_\Pi m_{V^0})^{1/2}G_{\gamma V^0\Pi}{}^0}\sin\lambda_2>0\,, \tag{A20}$$

so as to produce the best agreement with the experimental suppression of $\phi\to\Pi\gamma$. The correction is then[25]

$$C(\omega\to\Pi\gamma)=[\cos\lambda_2+\tfrac{1}{2}\sqrt{2}\,\sin|\lambda_2|]^2\,.$$

For the $\phi\to\Pi\gamma$ calculated from (3.8) we find the correction

$$C(\phi\to\Pi\gamma)=\left[\cos\lambda_2-\left(\frac{5}{8}\right)^{1/2}\frac{(4m_\Pi m_{V^0})^{1/2}G_{\gamma V^0\Pi}{}^0}{(4m_\Pi m_V)^{1/2}G_{\gamma V\Pi}{}^D}\sin\lambda_2\right]^2.$$

The choice of sign made in (A20) then leads to[25]

$$C(\phi\to\Pi\gamma)=[\cos\lambda_2-\sqrt{2}\,\sin|\lambda_2|]^2\,.$$

The correction to be applied to the $\Gamma(\phi\to\eta\gamma)$ obtained from (3.8) is

$$C(\phi\to\eta\gamma)=\left[\cos\lambda_1\cos\lambda_2+(\sqrt{5})\frac{(4m_V m_{\Pi^0})^{1/2}G_{\gamma V\Pi^0}{}^0}{(4m_V m_\Pi)^{1/2}G_{\gamma V\Pi}{}^D}\cos\lambda_2\sin\lambda_1+\left(\frac{5}{8}\right)^{1/2}\frac{(4m_\Pi m_{V^0})^{1/2}G_{\gamma V^0\Pi}{}^0}{(4m_\Pi m_V)^{1/2}G_{\gamma V\Pi}{}^D}\sin\lambda_2\cos\lambda_1\right]^2.$$

Equations (A19) and (A20) fix the relative signs, so that we find[12,25]

$$C(\phi\to\eta\gamma)=[\cos\lambda_1\cos\lambda_2-\sqrt{2}\,\cos\lambda_2\sin|\lambda_1|+\sqrt{2}\,\sin|\lambda_2|\cos\lambda_1]^2\,.$$

---

[32] J. J. Sakurai, Phys. Rev. Letters **9**, 472 (1962); S. L. Glashow, *ibid.* **11**, 48 (1963). See also II.

The $\omega \to \eta\gamma$ width obtained from (3.8) requires the correction

$$C(\omega \to \eta\gamma)=\left[\cos\lambda_1 \cos\lambda_2-(\sqrt{8})\frac{(4m_V m_{\Pi^0})^{1/2}G_{\gamma V\Pi^0}{}^0}{(4m_\Pi m_{V^0})^{1/2}G_{\gamma V^0\Pi^0}}\sin\lambda_1 \sin\lambda_2-\left(\frac{8}{5}\right)^{1/2}\frac{(4m_\Pi m_V)^{1/2}G_{\gamma V\Pi}{}^D}{(4m_\Pi m_{V^0})^{1/2}G_{\gamma V^0\Pi^0}}\sin\lambda_2 \cos\lambda_1\right]^2.$$

Here again our choice of relative sign in (A19) and (A20) determines this correction uniquely as[12,25]

$$C(\omega \to \eta\gamma)=[\cos\lambda_1 \cos\lambda_2+\sin|\lambda_1| \sin|\lambda_2|-\tfrac{1}{2}\sqrt{2} \sin|\lambda_1| \cos\lambda_2]^2.$$

For the $V \to l^+l^-$ decays the ω-ϕ mixing presents no complications because the singlet component does not contribute. We find

$$\Lambda_4(\omega^0 \to l^+l^-)=-\tfrac{1}{3}\sqrt{3} \sin\lambda_2$$

and

$$\Lambda_4(\phi \to l^+l^-)=\tfrac{1}{3}\sqrt{3} \cos\lambda_2,$$

where Λ_4 is defined in (3.10).

PHYSICAL REVIEW VOLUME 165, NUMBER 5 25 JANUARY 1968

Algebra of Current Components and the Hypothesis of Partially Conserved Axial-Vector Current Applied at High Energies

P. Olesen*

The Niels Bohr Institute, University of Copenhagen, Copenhagen, Denmark

(Received 1 June 1967; revised manuscript received 22 September 1967)

Using the hypothesis of partially conserved axial-vector currents, the algebra of current components, and the assumption that the pion-hadron total cross section $\sigma(s)$ approaches its asymptotic value rapidly, a method is developed which allows a calculation of the elastic amplitude at high energies and small momentum transfers. This method uses the fact that asymptotically the dynamics is given by the commutator on the light cone. The results are $\sigma_{\pi p}(\infty)=25.7\pm4.2$ mb and $d\sigma/dt=(d\sigma/dt)_{t=0}[G_E{}^2-(t/4M^2)G_M{}^2](1-t/4M^2)^{-1}$ (for small values of the momentum transfer t), where $G_E(t)$ and $G_M(t)$ are the electric and magnetic form factors of the proton. It is shown that possible Schwinger terms in the equal-time commutators are without importance for our results. An important feature of our calculation is that the energy and the momentum are allowed to go to infinity simultaneously; our method therefore deviates essentially from the Bjorken limit, which in general involves a continuation of the amplitude infinitely off the mass shell.

1. INTRODUCTION

In the present paper we shall present a calculation of the high-energy total cross sections $\sigma(\infty)$ for pion-hadron scattering which gives good agreement with the value of $\sigma_{\pi p}(\infty)$ obtained by fitting forward dispersion relations. The main tools in our derivation are the following three assumptions.

(i) The partially conserved axial-vector current (PCAC) hypothesis: The divergence of the $\Delta S=0$ axial-vector current $j_\mu{}^\pm(x)$ is proportional to the pion field[1,2] in the $SU(3)$ limit,[3,4]

$$\langle\alpha|\partial^\mu j_\mu{}^\pm(0)|\beta\rangle_{SU(3)}=-if_\pi m^2\langle\alpha|\varphi^\pm(0)|\beta\rangle_{SU(3)}, \quad (1)$$

$$f_\pi=\sqrt{2}Mg_A/g, \quad (1')$$

where $\varphi^\pm(x)$ is the renormalized Heisenberg field of the charged pions, m is the pion mass, M is the nucleon mass, g is the renormalized pion-nucleon coupling constant, and g_A is the renormalization (by the strong interactions) of the axial-vector coupling constant in β-decay. The index "$SU(3)$" indicates that the matrix elements are evaluated in the mass-degenerate $SU(3)$ limit. As pointed out in Ref. 4, it is reasonable to expect that the $SU(3)$ limit is achieved at high energies, i.e., when the energy difference between the states $|\alpha\rangle$ and $|\beta\rangle$ becomes very large; at the same time the invariant momentum transfer between $|\alpha\rangle$ and $|\beta\rangle$ approaches zero.

(ii) The equal-time commutators ($x_0=0$):

$$[j_\mu{}^+(x), j_0{}^-(0)]=2j_\mu{}^{V3}(0)\delta(\mathbf{x})+\text{S.T.}, \quad (2)$$

$$[j_l{}^+(x), j_k{}^-(0)]=2\delta_{kl}j_0{}^{V3}(0)\delta(\mathbf{x})$$
$$+\text{tensor term antisymmetric in } k \text{ and } l+\text{S.T.} \quad (3)$$

are assumed. Here $j_\mu{}^{V3}(x)$ is the third component of the isovector current, and S.T. stands for possible Schwinger terms. The commutators (2) and (3) are obtained from a quark model for the currents. We assume that these commutators can be abstracted from the model and postulated as true for the physical

* Present address: Department of Physics and Astronomy, The University of Rochester, Rochester, N. Y.
[1] M. Gell-Mann and M. Lévy, Nuovo Cimento **16**, 705 (1960).
[2] Y. Nambu, Phys. Rev. Letters **4**, 380 (1960).
[3] M. Gell-Mann, Physics **1**, 63 (1964).
[4] P. Olesen, Phys. Rev. **157**, 1296 (1967).