Phenomenological Phase-Parameter Fits to N-N Data up to 350 MeV*

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Results of phenomenological phase-parameter fits intended primarily for energies between somewhat below 10 and 350 MeV are presented in the form of tables suitable for interpolation. Comparisons of the pion-nucleon coupling constant as derived from p-p and n-p scattering data are made. Indications of a difference in the value of this constant are obtained. Though relatively small, this difference is outside the apparent statistical uncertainty of the determination. Some possible reasons for the difference are briefly discussed. A total of 1988 experimental data has been used. Corrections for the effect of nucleon magnetic moments, for Coulomb effects, and for short-range charge dependence are included.

I. INTRODUCTION

TWO new phenomenological phase-parameter fits to N-N scattering data in the energy region up to 350 MeV were briefly described in a report[1] at the Gainesville Conference. The information available at the time regarding values obtained for the pion-nucleon coupling constant from p-p and n-p data was stated in reasonable detail and values of some of the more important phase parameters were presented in graphical form. Since then, the fit to n-p data has been supplemented by another one employing a wider selection of p-d data used with the spectator proton interpretation. The first of the two n-p fits has been combined with the p-p data fit and used as a starting point for a "p-p and n-p combined" phase-parameter search. Since this search used the individual p-p and n-p fits as starting points, it gave a somewhat better over-all fit. In view of requests for phase parameters and values of experimentally measurable quantities obtained with the new fits, as well as the large delay that would be caused by preparing a paper describing several years' work with reasonable completeness, the present brief paper has been prepared largely from the point of view of the phase-shift user. The values of the phase parameters for the p-p and n-p combined fit are provided at intervals convenient for interpolation, with supplementary tables intended to take care of special requirements in the case of the ${}^1S_0$ phase shift K_0 and the ${}^3S_1$ phase parameter ${}^3\theta^S{}_1$. Since the search was made using different values of the $T=1$ phases for p-p and n-p data, the values of both types of quantities are provided. The differences in these quantities were precalculated making use of the Yale potential. This way of making an allowance for the Coulomb interaction and for the apparent violation of short-range charge independence, the latter being strongly indicated in the case of the ${}^1S_0$ state, is admittedly subject to question. Since these effects are small through most of the energy region under consideration and since an exact calculation of the effects is difficult, a more conscientious evaluation of the corrections was not attempted. The number of data used in the searches was 947 for p-p and 1023 for n-p scattering. This number is appreciably greater than that used by the Livermore group on account of greater caution regarding data rejection. Reasons for such caution have been alluded to in Ref. 1 and will be discussed more fully in a forthcoming publication which will describe the criteria used. The data fits described below and in Ref. 1 are intended primarily for incident energies (E) above 10 MeV. An account of the data input at lower energies and of its treatment may be found in Ref. 1. Although it is not intended to provide as precise values of K_0 and ${}^3\theta^S{}_1$ as possible, it is believed that the Y-IV values of these phases below 10 MeV are considerably more accurate than those of most phases at $E>10$ MeV. The searches reported on here and in Ref. 1 are the only multiple-energy searches which include properly made corrections for the effect of nuclear magnetic moments (MAG) as well as Coulomb and apparent violation of short-range charge-independence corrections. The results were considered acceptable only if they satisfied causality tests.

II. MODIFIED $(\text{Y-IV})_{n-p}$ FIT

The $(\text{Y-IV})_{n-p}$ fit of Ref. 1 included only a small part of data from the bombardment of deuterons by protons, henceforth referred to as p-d data. This was done in order to minimize the uncertainties arising in the evaluation of corrections for the effect of the spectator particle. The modified n-p fit, $(\text{Y-IV})_{n-p}\text{M}$, included more of the p-d data and a somewhat more conscientious treatment of the spectator corrections. The number of n-p data deck entries in the analysis has been increased from 921 to 939 as a result. The number of data involved in the analysis increased from 1023 to 1041. The additions consist of three measurements of $A_t'(\theta)$ at 128 MeV by Wilson and Strax,[2] three values of $D_t(\theta)$ at 197 MeV by Thorndike *et al.*,[3]

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[1] G. Breit, Rev. Mod. Phys. 39, 560 (1967). This report was based primarily on work done in collaboration with K. A. Friedman, R. D. Haracz, John M. Holt, A. Prakash, and R. E. Seamon.

[2] Private communication from Professor Richard Wilson in which a graph and numerical values for A_t' from the Ph.D. thesis of N. Strax were given.

[3] D. Spalding, A. Thomas, and E. H. Thorndike, Phys. Rev. 158, 1338 (1967).

TABLE I. Values of D and of g_0^2 for fits of the Y-IV series.

Fit[a] Data[a]	$(p-p)$ $(p-p)$	$(n-p)$ $(n-p)$	$(n-p)\alpha''$ $(n-p)\alpha''$	$(n-p)$M $(n-p)$M	$pp+np$ $pp+np$	$pp+np$ $(p-p)$	$pp+np$ $(n-p)$	$pp+np$ $pp+np$M	$pp+np$ $(n-p)$M
N	886	921	921	939	1807	886	921	1825	939
N_T	947	1023	1023	1041	1970	947	1023	1988	1041
D[b]	1.269[d]	1.253[d]	1.248	1.271	1.230	1.232	1.228	1.248	1.262
D[c]	1.281[d]	1.236[d]	1.232	1.253	1.230	1.247	1.214	1.246	1.245
Smoothed K_5 increase					0.0007		0.0014	0.0006	0.0011
g_0^2	15.99±0.62[d]	13.77±0.70	13.86±0.78	13.61±0.78	15.43±0.44	(+0.41)[e]	(+0.40)[e]		(+0.55)[e]

[a] In these rows the subscript of Y-IV is listed and is followed by type description such as M or α''.
[b] For data deck entries.
[c] For data.
[d] Same as in Ref. 1.
[e] Estimates of increases in value for data of second row that would result if $(\text{Y-IV})_{pp+np}$ fit were used. Explained more fully in text.

five values of $P_t(\theta)$ at 211 MeV by Spalding *et al.*,[4] and measurements of $P_t(\theta)$ at 310 MeV by Chamberlain *et al.*[5] The 939 deck entries used in $(\text{Y-IV})_{n-p}$M searches contain a total of 53 based on p-d data. For proton detection, published spectator corrections were used when available, the values being usually the same as in $(\text{Y-IV})_{n-p}$. For $P(\theta)$ and $D(\theta)$ at 310 MeV, extrapolation of the spectator corrections from 142 and 210 MeV was employed. The associated estimated uncertainties of ±0.032 and ±0.05, respectively, were added in quadrature to the other uncertainties. For $R(\theta)$ at 310 MeV, the spectator correction was taken to be the same as at 137.5 MeV in the $(\text{Y-IV})_{n-p}$ searches. In those for $(\text{Y-IV})_{n-p}$M it was halved since the correction is expected to decrease with E. The effect of the change was included in Cromer's validity uncertainty and the estimated uncertainty of the extrapolation of ±0.05 was added in quadrature to the other uncertainties.

Neutron-detection data from p-d measurements were used in $(\text{Y-IV})_{n-p}$ only in the case of $R_t(\theta)$ at 203 MeV, but in $(\text{Y-IV})_{n-p}$M all available neutron-detection data were used. Calculations of spectator corrections by Thorndike and collaborators around 200 MeV were reproduced employing the same method. For $R_t(\theta)$ at 203 MeV the spectator corrections in $(\text{Y-IV})_{n-p}$ were read off a graph kindly supplied by Professor Thorndike in correspondence concerning the value of $R_t(\theta)$ as expected from older Yale data fits. The value of the correction in $(\text{Y-IV})_{n-p}$M is the mean of those calculated for YLAN3M and $(\text{Y-IV})_{n-p}$. Fit YLAN3M was preferred over YLAN4M in this case because it agrees better with $P_t(\theta)$ at 211 MeV. A "statistical uncertainty" determined by comparison of calculations with YLAN3M, YLAN4M, and $(\text{Y-IV})_{n-p}$ and a "validity uncertainty" of 25% were added in quadrature. For $D_t(\theta)$ at 197 MeV and for $P_t(\theta)$ at 211 MeV, Thorndike's values of the corrections and statistical uncertainties were compounded in quadrature with 25% of the spectator correction. For high-angle $P_t(\theta)$ values at 310 MeV which were added in the $(\text{Y-IV})_{n-p}$M searches, the mean of corrections for fits YLAN4M and $(\text{Y-IV})_{n-p}$ was used. Statistical and validity uncertainties were added to the experimental uncertainty in quadrature as previously.

The D value[6] for fit $(\text{Y-IV})_{n-p}$ is given in three ways in Table V of Ref. 1. The last of these is for data with $E>24$ MeV and will not be considered here. For the $N=921$ deck entries, some of which represent lumped data, the recorded value of D was 1.253 and for the $N_T=1023$ data it was 1.236. The latter value includes an allowance for $\delta\chi^2$, the increase in χ^2 corresponding to the dispersion of deviations within each lumped group from the fit, and also for the increase of N to N_T. These values were obtained using Coulomb and short-range charge-dependence corrections for $T=1$ phases, as obtained from fit $(\text{Y-IV})_{p-p}$ for $g_0^2=15$. Such searches are referred to as of the α type. The $(\text{Y-IV})_{p-p}$ phases at $g_0^2=15$ were obtained by interpolation from searches at $g_0^2=12$, 14, 16, and 18 which were available from $(g_0^2)_{p-p}$ determinations. In $(g_0^2)_{n-p}$ determinations of the α type, the $T=1$ phases for each preset g_0^2 obtained in p-p searches for the same preset g_0^2 were used. Since then, the procedure was improved by applying the Coulomb and short-range charge-dependence corrections to $T=1$ phases from p-p data corresponding to $g_0^2=16$, very nearly the value 15.99 corresponding to $(\text{Y-IV})_{p-p}$ according to Table VI of Ref. 1, and further searching of $T=0$ phases at $g_0^2=15$ was carried out when the $T=1$ phases derived from those obtained in p-p searches for $g_0^2=15$ were replaced by values from $(\text{Y-IV})_{p-p}$ corrected for Coulomb and short-range charge dependence. This procedure is referred to as of the α'' type. For the 921 deck entries of $(\text{Y-IV})_{n-p}$ it gave $D=1.248$ and for the data $D=1.232$. The α'' procedure, when applied to the 939 deck entries of $(\text{Y-IV})_{n-p}$M, gave $D=1.271$ and for the 1041 data 1.253. The $(\text{Y-IV})_{n-p}$M fit to $(\text{Y-IV})_{n-p}$M data is seen to be slightly worse statistically than that of the result of the earlier $(\text{Y-IV})_{n-p}$ fit to $(\text{Y-IV})_{n-p}$ data.

[4] D. Spalding, A. R. Thomas, N. W. Reay, and E. H. Thorndike, Phys. Rev. **150**, 806 (1966).
[5] O. Chamberlain, E. Segrè, R. D. Tripp, C. Wiegand, and T. Ypsilantis, Phys. Rev. **105**, 288 (1957).
[6] G. Breit and R. D. Haracz, in *High Energy Physics*, edited by E. H. S. Burhop (Academic Press Inc., New York, 1967), Vol. I, p. 21.

TABLE II. Values of $T=1$ phase parameters for p-p scattering for fit $(\text{Y-IV})_{pp+np}$.

E (MeV)	K_0	${}^3\delta^{P}{}_0$	${}^3\delta^{P}{}_1$	${}^3\theta^{P}{}_2$	ρ_2	K_2	${}^3\theta^{F}{}_2$
10	0.9604	0.0599	−0.0405	0.0133	−0.0085	0.0033	0.0003
20	0.8929	0.1121	−0.0695	0.0314	−0.0292	0.0110	0.0013
30	0.8067	0.1615	−0.0951	0.0541	−0.0500	0.0198	0.0028
40	0.7318	0.1912	−0.1176	0.0796	−0.0647	0.0279	0.0048
50	0.6730	0.2038	−0.1394	0.1020	−0.0736	0.0345	0.0068
60	0.6189	0.2071	−0.1598	0.1234	−0.0790	0.0410	0.0084
70	0.5712	0.2040	−0.1799	0.1441	−0.0841	0.0475	0.0094
80	0.5251	0.1949	−0.1982	0.1625	−0.0882	0.0540	0.0098
90	0.4835	0.1829	−0.2161	0.1791	−0.0918	0.0604	0.0102
100	0.4460	0.1677	−0.2332	0.1948	−0.0946	0.0664	0.0105
110	0.4096	0.1530	−0.2488	0.2072	−0.0968	0.0725	0.0107
120	0.3751	0.1375	−0.2643	0.2193	−0.0984	0.0781	0.0110
130	0.3411	0.1216	−0.2796	0.2295	−0.0995	0.0838	0.0111
150	0.2750	0.0902	−0.3068	0.2463	−0.1007	0.0950	0.0114
170	0.2121	0.0581	−0.3332	0.2592	−0.1009	0.1055	0.0114
190	0.1559	0.0240	−0.3579	0.2677	−0.1006	0.1161	0.0111
210	0.1000	−0.0093	−0.3811	0.2729	−0.0999	0.1264	0.0106
230	0.0447	−0.0420	−0.4026	0.2756	−0.0986	0.1363	0.0099
250	−0.0015	−0.0774	−0.4216	0.2768	−0.0968	0.1463	0.0090
270	−0.0479	−0.1154	−0.4391	0.2770	−0.0947	0.1559	0.0080
290	−0.0881	−0.1536	−0.4558	0.2771	−0.0921	0.1649	0.0069
310	−0.1278	−0.1938	−0.4715	0.2771	−0.0895	0.1741	0.0059
330	−0.1646	−0.2327	−0.4853	0.2762	−0.0870	0.1831	0.0050
350	−0.2004	−0.2704	−0.4999	0.2751	−0.0845	0.1920	0.0041

E (MeV)	${}^3\delta^{F}{}_3$	${}^3\theta^{F}{}_4$	ρ_4	K_4	${}^3\theta^{H}{}_4$	${}^3\delta^{H}{}_5$	${}^3\theta^{H}{}_6$
10	−0.0007	0.0000	−0.0002	0.0001	0.0000	0.0000	0.0000
20	−0.0031	0.0002	−0.0011	0.0005	0.0000	−0.0002	0.0000
30	−0.0065	0.0005	−0.0029	0.0012	0.0001	−0.0005	0.0000
40	−0.0103	0.0009	−0.0051	0.0020	0.0003	−0.0010	0.0001
50	−0.0143	0.0014	−0.0076	0.0029	0.0005	−0.0017	0.0001
60	−0.0181	0.0019	−0.0102	0.0038	0.0008	−0.0025	0.0002
70	−0.0216	0.0027	−0.0129	0.0047	0.0011	−0.0033	0.0003
80	−0.0244	0.0035	−0.0156	0.0056	0.0014	−0.0042	0.0004
90	−0.0266	0.0045	−0.0182	0.0064	0.0017	−0.0052	0.0006
100	−0.0285	0.0056	−0.0208	0.0072	0.0021	−0.0061	0.0007
110	−0.0304	0.0068	−0.0234	0.0080	0.0025	−0.0071	0.0009
120	−0.0319	0.0080	−0.0258	0.0087	0.0029	−0.0081	0.0010
130	−0.0333	0.0095	−0.0280	0.0096	0.0033	−0.0090	0.0012
150	−0.0363	0.0129	−0.0313	0.0117	0.0041	−0.0110	0.0015
170	−0.0389	0.0170	−0.0334	0.0141	0.0049	−0.0129	0.0019
190	−0.0417	0.0213	−0.0347	0.0164	0.0058	−0.0148	0.0023
210	−0.0445	0.0258	−0.0357	0.0182	0.0067	−0.0172	0.0028
230	−0.0475	0.0303	−0.0366	0.0195	0.0076	−0.0200	0.0035
250	−0.0506	0.0348	−0.0375	0.0203	0.0087	−0.0226	0.0045
270	−0.0541	0.0396	−0.0388	0.0206	0.0099	−0.0248	0.0058
290	−0.0576	0.0445	−0.0404	0.0206	0.0115	−0.0265	0.0073
310	−0.0615	0.0497	−0.0421	0.0203	0.0133	−0.0280	0.0091
330	−0.0653	0.0549	−0.0441	0.0198	0.0152	−0.0291	0.0112
350	−0.0692	0.0604	−0.0463	0.0192	0.0172	−0.0301	0.0134

III. FIT TO p-p AND n-p DATA COMBINED

The data decks prepared for the $(\text{Y-IV})_{p-p}$ and $(\text{Y-IV})_{n-p}$ searches were combined into one, giving a deck with $886+921=1807$ deck entries. The calculation of p-p and n-p observables from p-p and n-p phases was made in each case as though p-p and n-p data were treated separately, but for $T=1$ phases the difference between n-p and p-p phases was kept equal to the precalculated corrections for Coulomb and short-range charge dependence. The starting values of the phases were those obtained in the $(\text{Y-IV})_{p-p}$ and $(\text{Y-IV})_{n-p}$ fits for $g_0^2=15$. On account of the Coulomb and charge-dependence corrections and the difference in pion mass effects on the one-pion-exchange (OPE) phases, this procedure gives discontinuities in the $T=1$ phases at the transition energies from the OPE to the searched regions. These discontinuities were removed by applying correction functions of the form $(E_c'-E)^3[A+B(E-E_c)+C(E-E_c)^2]$, with E_c equal to the transition energy, E_c' determined by judgment, and A, B, and C arranged to give an osculating fit to the n-p OPE phase in question. Correction functions to the resultant phases with a smooth energy dependence were then searched for by the usual procedure. The resultant fit is called $(\text{Y-IV})_{pp+np}$. This fit was also used for a g_0^2 determination, intended to give the best g_0^2 for fitting as a whole the p-p and n-p data previously mentioned. The $(\text{Y-IV})_{pp+np}$ fit gives somewhat smaller values of D when used with the $(\text{Y-IV})_{p-p}$, $(\text{Y-IV})_{n-p}$, and $(\text{Y-IV})_{n-p}\text{M}$ data decks than fits $(\text{Y-IV})_{p-p}$, $(\text{Y-IV})_{n-p}$, and $(\text{Y-IV})_{n-p}\text{M}$, respectively, perhaps be-

TABLE III. Values of $T=1$ phase parameters for n-p scattering for fit $(\text{Y-IV})_{pp+np}$.

E (MeV)	K_0	$^3\delta_{P_0}$	$^3\delta_{P_1}$	$^3\theta_{P_2}$	ρ_2	K_2	$^3\theta_{F_2}$
10	1.0476	0.0665	−0.0441	0.0147	−0.0091	0.0035	0.0002
20	0.9310	0.1185	−0.0738	0.0341	−0.0303	0.0115	0.0010
30	0.8322	0.1660	−0.0993	0.0577	−0.0511	0.0205	0.0024
40	0.7509	0.1940	−0.1221	0.0836	−0.0660	0.0286	0.0040
50	0.6875	0.2051	−0.1436	0.1065	−0.0746	0.0354	0.0059
60	0.6312	0.2071	−0.1640	0.1279	−0.0797	0.0421	0.0074
70	0.5813	0.2032	−0.1840	0.1487	−0.0845	0.0486	0.0087
80	0.5339	0.1934	−0.2023	0.1669	−0.0884	0.0552	0.0096
90	0.4912	0.1808	−0.2201	0.1834	−0.0917	0.0616	0.0102
100	0.4528	0.1651	−0.2372	0.1989	−0.0943	0.0677	0.0106
110	0.4157	0.1500	−0.2527	0.2111	−0.0963	0.0739	0.0108
120	0.3805	0.1342	−0.2682	0.2230	−0.0977	0.0795	0.0110
130	0.3460	0.1181	−0.2835	0.2330	−0.0987	0.0852	0.0112
150	0.2792	0.0862	−0.3107	0.2492	−0.0997	0.0964	0.0114
170	0.2157	0.0538	−0.3372	0.2616	−0.0997	0.1069	0.0113
190	0.1588	0.0195	−0.3619	0.2695	−0.0993	0.1175	0.0110
210	0.1027	−0.0138	−0.3851	0.2742	−0.0986	0.1277	0.0105
230	0.0473	−0.0464	−0.4066	0.2765	−0.0973	0.1374	0.0097
250	−0.0009	−0.0820	−0.4256	0.2771	−0.0956	0.1473	0.0088
270	−0.0459	−0.1202	−0.4431	0.2770	−0.0933	0.1569	0.0078
290	−0.0863	−0.1584	−0.4598	0.2767	−0.0908	0.1659	0.0066
310	−0.1262	−0.1987	−0.4756	0.2764	−0.0883	0.1749	0.0056
330	−0.1641	−0.2381	−0.4902	0.2754	−0.0858	0.1840	0.0047
350	−0.1990	−0.2753	−0.5040	0.2737	−0.0834	0.1927	0.0037

E (MeV)	$^3\delta_{F_3}$	$^3\theta_{F_4}$	ρ_4	K_4	$^3\theta_{H_4}$	$^3\delta_{H_5}$	$^3\theta_{H_6}$
10	−0.0005	0.0000	−0.0001	0.0000	0.0000	0.0000	0.0000
20	−0.0025	0.0001	−0.0009	0.0004	0.0000	−0.0001	0.0000
30	−0.0055	0.0004	−0.0023	0.0009	0.0001	−0.0004	0.0000
40	−0.0089	0.0007	−0.0042	0.0017	0.0002	−0.0008	0.0000
50	−0.0126	0.0011	−0.0064	0.0025	0.0004	−0.0013	0.0001
60	−0.0165	0.0017	−0.0088	0.0033	0.0006	−0.0020	0.0002
70	−0.0205	0.0025	−0.0112	0.0042	0.0008	−0.0027	0.0002
80	−0.0241	0.0035	−0.0137	0.0050	0.0011	−0.0035	0.0003
90	−0.0269	0.0046	−0.0162	0.0058	0.0014	−0.0044	0.0004
100	−0.0289	0.0057	−0.0187	0.0066	0.0017	−0.0052	0.0006
110	−0.0308	0.0069	−0.0212	0.0074	0.0021	−0.0061	0.0007
120	−0.0323	0.0082	−0.0235	0.0081	0.0024	−0.0070	0.0008
130	−0.0337	0.0096	−0.0257	0.0090	0.0028	−0.0079	0.0010
150	−0.0366	0.0130	−0.0293	0.0113	0.0035	−0.0097	0.0013
170	−0.0393	0.0171	−0.0321	0.0139	0.0043	−0.0115	0.0016
190	−0.0421	0.0215	−0.0341	0.0163	0.0051	−0.0134	0.0019
210	−0.0448	0.0260	−0.0356	0.0183	0.0061	−0.0160	0.0025
230	−0.0477	0.0305	−0.0367	0.0196	0.0072	−0.0192	0.0034
250	−0.0509	0.0350	−0.0377	0.0204	0.0085	−0.0224	0.0045
270	−0.0543	0.0397	−0.0389	0.0207	0.0100	−0.0249	0.0058
290	−0.0578	0.0446	−0.0405	0.0208	0.0116	−0.0267	0.0074
310	−0.0617	0.0498	−0.0422	0.0205	0.0134	−0.0281	0.0092
330	−0.0656	0.0552	−0.0441	0.0200	0.0153	−0.0293	0.0112
350	−0.0693	0.0606	−0.0463	0.0194	0.0173	−0.0302	0.0135

cause of the additional searching which was done after the smoothing close to $E=E_c$. The main tabular material concerning phase parameters is presented in terms of the combined fit. A slight change was made regarding the high-energy end of the $T=0\ {}^1H_5$ phase shift K_5, the plot of which against energy showed an improbable bump. Smoothing it affected D by quite small amounts. However, the g_0^2 determination for $(\text{Y-IV})_{pp+np}$ was carried out without smoothing K_5. The smallness of the effect on D just mentioned and past experience indicate that the effect on g_0^2 should be insignificant.

IV. DISCUSSION AND TABLES

The values of D and of g_0^2 derived from the new fits are summarized in Table I. Some of the values that have appeared in Ref. 1 are listed again, but only in cases concerned with the whole energy region used in the analysis. For the effect of excluding low-energy parts of the data collection in a few representative cases and for comparison with some older fits, Tables I, V, and VII of the reference just mentioned may be helpful. In Table I of the present paper, the first two rows give, respectively, the fits and the data decks used to calculate D and g_0^2. The subscript of (Y-IV) such as n-p appears in parentheses, and an additional specification such as M appears immediately afterwards. The entry (n-p)M, for example, means $(\text{Y-IV})_{n-p}$M. The numbers N and N_T appearing in the third and fourth rows are, respectively, the number of data deck entries used in the calculation of D or of g_0^2 and the number of elementary pieces of data from which the data deck

TABLE IV. Values of $T=0$ phase parameters for fit $(\text{Y-IV})_{pp+np}$.

E (MeV)	$^3\theta_{S_1}$	ρ_1	K_1	$^3\theta_{D_1}$	$^3\delta_{D_2}$	$^3\theta_{D_3}$	ρ_3
10	1.7984	0.0404	−0.0450	−0.0122	0.0150	0.0004	0.0031
20	1.4957	0.0586	−0.0753	−0.0384	0.0481	0.0016	0.0141
30	1.3034	0.0682	−0.0956	−0.0662	0.0911	0.0033	0.0292
40	1.1579	0.0744	−0.1101	−0.0931	0.1392	0.0052	0.0458
50	1.0458	0.0792	−0.1224	−0.1189	0.1858	0.0072	0.0625
60	0.9535	0.0831	−0.1365	−0.1431	0.2234	0.0093	0.0778
70	0.8750	0.0862	−0.1515	−0.1651	0.2552	0.0116	0.0899
80	0.8091	0.0886	−0.1683	−0.1846	0.2823	0.0140	0.0997
90	0.7499	0.0904	−0.1860	−0.2016	0.3053	0.0168	0.1080
100	0.7003	0.0917	−0.2048	−0.2163	0.3257	0.0198	0.1160
110	0.6496	0.0927	−0.2254	−0.2285	0.3443	0.0230	0.1237
120	0.6038	0.0937	−0.2463	−0.2392	0.3618	0.0265	0.1322
130	0.5632	0.0947	−0.2689	−0.2478	0.3776	0.0301	0.1408
150	0.4859	0.0975	−0.3145	−0.2638	0.4056	0.0379	0.1584
170	0.4203	0.1021	−0.3599	−0.2802	0.4274	0.0460	0.1755
190	0.3620	0.1088	−0.4050	−0.2983	0.4436	0.0541	0.1915
210	0.3109	0.1179	−0.4505	−0.3174	0.4553	0.0620	0.2057
230	0.2662	0.1289	−0.4950	−0.3397	0.4627	0.0696	0.2182
250	0.2259	0.1418	−0.5408	−0.3629	0.4661	0.0768	0.2287
270	0.1883	0.1559	−0.5866	−0.3881	0.4665	0.0834	0.2376
290	0.1525	0.1707	−0.6318	−0.4152	0.4644	0.0895	0.2449
310	0.1180	0.1859	−0.6798	−0.4432	0.4602	0.0951	0.2506
330	0.0859	0.2009	−0.7278	−0.4720	0.4546	0.1002	0.2553
350	0.0561	0.2155	−0.7745	−0.5015	0.4478	0.1048	0.2591

E (MeV)	K_3	$^3\theta_{G_3}$	$^3\delta_{G_4}$	$^3\theta_{G_5}$	ρ_5	K_5
10	−0.0013	−0.0001	0.0003	0.0000	0.0001	0.0000
20	−0.0055	−0.0006	0.0019	−0.0001	0.0007	−0.0003
30	−0.0110	−0.0016	0.0049	−0.0003	0.0023	−0.0010
40	−0.0167	−0.0031	0.0089	−0.0007	0.0048	−0.0019
50	−0.0222	−0.0049	0.0135	−0.0011	0.0078	−0.0031
60	−0.0268	−0.0069	0.0185	−0.0017	0.0114	−0.0045
70	−0.0301	−0.0091	0.0237	−0.0024	0.0152	−0.0059
80	−0.0328	−0.0115	0.0290	−0.0032	0.0192	−0.0073
90	−0.0352	−0.0138	0.0344	−0.0040	0.0234	−0.0088
100	−0.0373	−0.0163	0.0397	−0.0048	0.0276	−0.0103
110	−0.0389	−0.0188	0.0450	−0.0057	0.0318	−0.0117
120	−0.0400	−0.0213	0.0501	−0.0066	0.0361	−0.0131
130	−0.0406	−0.0236	0.0553	−0.0074	0.0403	−0.0144
150	−0.0404	−0.0275	0.0651	−0.0081	0.0485	−0.0170
170	−0.0390	−0.0298	0.0728	−0.0075	0.0565	−0.0194
190	−0.0368	−0.0306	0.0780	−0.0058	0.0645	−0.0217
210	−0.0337	−0.0300	0.0812	−0.0032	0.0722	−0.0240
230	−0.0300	−0.0285	0.0824	0.0001	0.0799	−0.0265
250	−0.0261	−0.0264	0.0819	0.0039	0.0868	−0.0291
270	−0.0223	−0.0238	0.0795	0.0081	0.0933	−0.0317
290	−0.0183	−0.0212	0.0755	0.0126	0.0996	−0.0345
310	−0.0144	−0.0187	0.0704	0.0173	0.1058	−0.0374
330	−0.0105	−0.0164	0.0644	0.0222	0.1121	−0.0404
350	−0.0067	−0.0142	0.0573	0.0271	0.1185	−0.0435

was constructed. The two are not the same because some data-deck entries represent the result of data lumping. Corresponding to these two numbers there are two values of D in the table. The first corresponds to χ^2 and N, the second to $\chi^2+\delta\chi^2$ and N_T as mentioned in Sec. II. In the last four columns the data used are different from those employed in arriving at the whole fit. In columns 7 and 8 the data in the D computation occurred as a unit in the searches that gave the fit. In columns 9 and 10 the changes in going from $(n-p)$ to $(n-p)$M are also involved. The seventh row gives the increase in D caused by smoothing K_5, the procedure for which is described in Sec. III. The increase is nearly the same for D^b as for D^c. Only the arithmetic mean of the two values is therefore listed. All D values reported here are slightly under the true value because N and N_T were not corrected for the number of adjustable parameters. There is a slight difference between the $(g_0^2)_{n-p}$ value for $(\text{Y-IV})_{n-p}$ which appears as 13.77 ± 0.70 in Table I and the corresponding number 13.76 ± 0.74 in Table VII of Ref. 1. This difference has been caused by the uniform use of cubic curves through four points for the g_0^2 values reported here, in contrast to quadratic parabolas through three points nearest to the minimum used for the value mentioned in Table VII in Ref. 1. The last three entries in the g_0^2 row are the increases in g_0^2 obtained through the employment of an admittedly questionable procedure, the object of which was to ascertain without too much additional work the values of $(g_0^2)_{p-p}$ and $(g_0^2)_{n-p}$ that would be obtained had the p-p and n-p parts of fit $(\text{Y-IV})_{pp+np}$ been used. This procedure used $(\chi^2)_{p-p}$ and

TABLE V. Values of the f functions, $k\cot(K_0)_{p-p}$, and $k\cot(K_0)_{n-p}$, and $k\cot{}^3\theta^{S_1}$ for fit $(\text{Y-IV})_{pp+np}$.

E (MeV)	f function	$k\cot(K_0)_{pp}$ (F^{-1})	$k\cot(K_0)_{np}$ (F^{-1})	$k\cot({}^3\theta^{S_1})$ (F^{-1})
2	9.63835		0.07446	−0.16353
4	11.51745		0.10633	−0.14236
6	13.38336		0.13756	−0.12153
8	15.23067		0.16871	−0.10141
10	17.06566	0.24729	0.20023	−0.08040
12	18.86881	0.27194	0.23236	−0.05924
14	20.61687	0.30061	0.26475	−0.03629
16	22.43156	0.33106	0.29814	−0.01185
18	24.28862	0.36263	0.33175	0.01197
20	26.18932	0.39526	0.36536	0.03694

$(\chi^2)_{n-p}$, the contributions to χ^2 in the determination that gave $g_0^2=15.43\pm0.44$ for $(\text{Y-IV})_{pp+np}$ in column 6. The questionable feature is seen by considering the determination of $(g_0^2)_{p-p}$. As the preset g_0^2 is varied, all of the phases are readjusted in the method used and n-p as well as p-p data influence the resultant D. Were fit $(\text{Y-IV})_{pp+np}$ used with p-p data alone, the n-p data would not influence the result. Their entry in the procedure used introduces a questionable element. The differences between the tabulated increases may be more significant than their values. The comparison of $(g_0^2)_{p-p}$ with $(g_0^2)_{n-p}$ on the basis of $(\text{Y-IV})_{n-p}$ is then practically unaffected because the difference $0.40-0.41=-0.01$ is barely perceptible. The same comparison on the basis of $(\text{Y-IV})_{n-p}\text{M}$ would be in effect as though 13.61 were increased by $0.55-0.41=0.14$, i.e., up to 13.75 which is very close to the 13.77 in the $(\text{Y-IV})_{n-p}$ column. Were the -0.01 correction included, the comparison would be between 13.76 and 13.75. Taking the value in the $(\text{Y-IV})_{n-p}$ column and adding the uncertainty, $(g_0^2)_{n-p}$ is seen to be probably less than 14.47 and similarly $(g_0^2)_{p-p}$ to be probably greater than 15.37. The calculation of the uncertainties includes the $D^{1/2}$ safety factor and the error increase factor of Eq. (VII-37a) discussed in a review[6] of N-N scattering analysis. The chance that $(g_0^2)_{p-p}$ is exactly equal to $(g_0^2)_{n-p}$ appears to be small. However, there are still some improvements to be made, especially in the $(g_0^2)_{n-p}$ case. Furthermore, as mentioned more

TABLE VI. Phase-parameter uncertainties in radians for fit $(\text{Y-IV})_{pp+np}$ obtained by the parallel-shift method.

Phase parameters \ Energy region:	0–69 (MeV)	69–155 (MeV)	155–275 (MeV)	275–350 (MeV)
K_0	0.0002	0.0059	0.0080	0.0229
${}^3\delta^{P_0}$	0.0030	0.0067	0.0134	0.0246
${}^3\delta^{P_1}$	0.0010	0.0021	0.0067	0.0150
${}^3\theta^{P_2}$	0.0009	0.0016	0.0040	0.0098
ρ_2	0.0014	0.0022	0.0047	0.0141
K_2	0.0003	0.0019	0.0031	0.0101
${}^3\theta^{F_2}$	0.0017	0.0027	0.0061	0.0099
${}^3\delta^{F_3}$	0.0011	0.0024	0.0037	0.0082
${}^3\theta^{F_4}$	0.0006	0.0014	0.0030	0.0052
ρ_4		0.0011	0.0031	0.0098
K_4		0.0009	0.0018	0.0047
${}^3\theta^{H_4}$			0.0040	0.0062
${}^3\delta^{H_5}$			0.0027	0.0074
${}^3\theta^{H_6}$			0.0027	0.0041
${}^3\theta^{S_1}$	0.0024	0.0064	0.0710	0.0872
ρ_1	0.0171	0.0220	0.1151	0.1321
K_1	0.0085	0.0194	0.1045	0.0910
${}^3\theta^{D_1}$	0.0074	0.0044	0.0970	0.0584
${}^3\delta^{D_2}$	0.0073	0.0078	0.0784	0.0860
${}^3\theta^{D_3}$	0.0035	0.0037	0.0405	0.0248
ρ_3	0.0266	0.0085	0.0826	0.0530
K_3	0.0102	0.0063	0.0647	0.0326
${}^3\theta^{G_3}$		0.0047	0.0457	0.0473
${}^3\delta^{G_4}$		0.0058	0.0413	0.0481
${}^3\theta^{G_5}$		0.0035	0.0493	0.0173
ρ_5			0.0241	0.0223
K_5			0.0262	0.0288

TABLE VII. Values of phase-parameter differences $\delta[(\text{Y-IV})_{p-p}]-\delta[(\text{Y-IV})_{pp+np}]$.

E (MeV)	K_0	${}^3\delta^{P_0}$	${}^3\delta^{P_1}$	${}^3\theta^{P_2}$	ρ_2	K_2
9.69	−0.0006	0.0006	−0.0005	0.0003	0.0000	0.0000
30.0	−0.0036	0.0002	−0.0026	0.0010	0.0000	0.0000
50.0	−0.0045	−0.0030	−0.0037	0.0013	−0.0003	0.0001
78.0	−0.0009	−0.0005	−0.0002	0.0013	−0.0003	0.0000
108.0	−0.0006	0.0025	0.0017	0.0014	0.0001	−0.0002
138.0	−0.0021	0.0035	0.0018	0.0016	0.0006	−0.0003
170.0	−0.0025	0.0038	0.0020	0.0016	0.0012	−0.0002
210.0	−0.0019	0.0036	0.0024	0.0014	0.0022	0.0001
276.0	0.0008	0.0035	0.0036	0.0011	0.0044	0.0016
310.0	0.0025	0.0036	0.0042	0.0010	0.0057	0.0026
345.0	0.0042	0.0040	0.0050	0.0009	0.0070	0.0038

E (MeV)	${}^3\theta^{F_2}$	${}^3\delta^{F_3}$	${}^3\theta^{F_4}$	ρ_4	K_4	${}^3\theta^{H_4}$	${}^3\delta^{H_5}$	${}^3\theta^{H_6}$
50.0	0.0004	−0.0007	0.0001					
78.0	0.0002	−0.0001	0.0001					
108.0	0.0003	0.0000	−0.0002			OPE regions		
138.0	0.0003	0.0000	−0.0004	−0.0004	0.0006			
170.0	0.0001	0.0000	−0.0006	0.0003	−0.0006			
210.0	−0.0005	−0.0003	−0.0005	0.0006	−0.0024	0.0002	−0.0005	0.0000
276.0	−0.0021	−0.0009	0.0001	0.0006	−0.0043	−0.0013	0.0012	−0.0016
310.0	−0.0030	−0.0013	0.0005	0.0003	−0.0050	−0.0033	0.0019	−0.0033
345.0	−0.0039	−0.0016	0.0008	−0.0002	−0.0056	−0.0058	0.0026	−0.0054

TABLE VIII. Values of the $T=0$ phase-parameter differences $\delta[(\text{Y-IV})_{n-p}\text{M}]-\delta[(\text{Y-IV})_{n-p}\alpha'']$, $\delta[(\text{Y-IV})_{n-p}\alpha'']-\delta[(\text{Y-IV})_{pp+np}]$, and $\delta[(\text{Y-IV})_{n-p}\alpha'']-\delta[(\text{Y-IV})_{n-p}]$ in radians listed in above order for each entry and reading from the top down in each triple entry.

E (MeV)	${}^3\theta_{S_1}$	ρ_1	K_1	${}^3\theta_{D_1}$	${}^3\delta_{D_2}$	${}^3\theta_{D_3}$	ρ_3	K_3	${}^3\theta_{G_3}$	${}^3\delta_{G_4}$	${}^3\theta_{G_5}$	ρ_5	K_5
10.42	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000							
	−0.0056	0.0000	0.0000	0.0000	0.0000	0.0000							
	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000							
30.0	0.0000	0.0000	−0.0001	0.0001	0.0000	0.0000							
	−0.0123	0.0001	−0.0003	−0.0002	0.0000	0.0001							
	−0.0002	0.0000	0.0002	−0.0003	0.0000	0.0001							
50.0	−0.0002	0.0000	−0.0008	0.0003	−0.0002	−0.0001	0.0000	0.0000					
	−0.0058	0.0006	−0.0010	−0.0010	0.0004	0.0002	0.0000	0.0000					
	−0.0010	0.0000	0.0008	−0.0011	0.0004	0.0002	0.0000	0.0000					
80.0	−0.0004	0.0009	−0.0016	0.0004	−0.0007	0.0000	0.0000	−0.0001		OPE regions			
	−0.0020	0.0027	−0.0005	−0.0006	0.0020	−0.0002	−0.0007	0.0000					
	−0.0035	0.0001	0.0001	−0.0005	0.0015	−0.0006	0.0000	0.0000					
110.0	−0.0010	0.0039	−0.0015	−0.0001	−0.0008	0.0004	−0.0002	−0.0005					
	−0.0027	0.0064	0.0002	0.0016	0.0031	−0.0009	−0.0021	−0.0002					
	−0.0062	0.0006	−0.0016	0.0025	0.0030	−0.0017	0.0000	0.0002					
140.0	−0.0023	0.0100	−0.0012	−0.0006	−0.0004	0.0005	−0.0005	−0.0014	−0.0003	0.0000	−0.0001		
	−0.0042	0.0106	−0.0008	0.0044	0.0034	−0.0010	−0.0036	−0.0003	−0.0003	−0.0002	0.0000		
	−0.0080	0.0022	−0.0020	0.0061	0.0047	−0.0021	−0.0001	0.0006	0.0000	0.0001	0.0000		
172.0	−0.0042	0.0200	−0.0010	−0.0009	0.0002	0.0004	−0.0011	−0.0027	−0.0034	−0.0002	−0.0012		
	−0.0056	0.0148	−0.0042	0.0074	0.0029	−0.0007	−0.0051	−0.0002	−0.0016	−0.0008	−0.0002		
	−0.0089	0.0055	−0.0006	0.0095	0.0066	−0.0017	−0.0004	0.0012	0.0003	0.0009	0.0001		
217.0	−0.0067	0.0380	−0.0012	−0.0009	0.0010	0.0001	−0.0023	−0.0049	−0.0139	−0.0009	−0.0050	0.0001	−0.0002
	−0.0073	0.0196	−0.0129	0.0105	0.0013	−0.0001	−0.0072	0.0005	−0.0048	−0.0016	−0.0011	0.0001	−0.0003
	−0.0089	0.0140	0.0024	0.0124	0.0085	−0.0006	−0.0009	0.0025	0.0011	0.0042	0.0003	0.0000	0.0000
272.0	−0.0082	0.0624	−0.0014	−0.0006	0.0015	−0.0002	−0.0043	−0.0077	−0.0321	−0.0020	−0.0115	0.0013	−0.0033
	−0.0087	0.0260	−0.0277	0.0126	−0.0017	0.0003	−0.0101	0.0025	−0.0093	−0.0013	−0.0028	0.0010	−0.0024
	−0.0080	0.0306	0.0051	0.0132	0.0093	0.0005	−0.0019	0.0045	0.0016	0.0112	0.0007	−0.0003	0.0004
310.0	−0.0083	0.0787	−0.0014	−0.0004	0.0016	−0.0003	−0.0059	−0.0096	−0.0455	−0.0028	−0.0163	0.0027	−0.0074
	−0.0092	0.0331	−0.0389	0.0132	−0.0040	0.0003	−0.0124	0.0043	−0.0125	0.0000	−0.0043	0.0019	−0.0048
	−0.0071	0.0457	0.0059	0.0126	0.0090	0.0009	−0.0027	0.0060	0.0014	0.0171	0.0008	−0.0006	0.0006
350.0	−0.0078	0.0943	−0.0014	−0.0002	0.0016	−0.0004	−0.0076	−0.0114	−0.0589	−0.0038	−0.0210	0.0044	−0.0127
	−0.0094	0.0441	−0.0501	0.0133	−0.0065	0.0000	−0.0148	0.0064	−0.0158	0.0021	−0.0060	0.0027	−0.0073
	−0.0061	0.0639	0.0060	0.0114	0.0083	0.0011	−0.0037	0.0076	0.0007	0.0238	0.0009	−0.0008	0.0007

fully elsewhere,[1,6,7] the effect of the pion-nucleon coupling, being partly of pseudovector type, of two-pion exchange, and of one-boson exchanges of other types, may be responsible for the apparent difference between $(g_0^2)_{p-p}$ and $(g_0^2)_{n-p}$. If long-range charge independence is assumed, then 15.43 ± 0.44 is indicated by the data, disregarding the uncertainties just mentioned. Approximate cancellation of one-boson exchange effects is also more probable[6] for it. The uncertainties in the spectator corrections for *p*-*d* data may be expected to affect this value less than $(g_0^2)_{n-p}$ as derived from $(\text{Y-IV})_{n-p}$ or $(\text{Y-IV})_{n-p}$M.

In Tables II–IV there are listed values of phase parameters obtained by interpolation from those at data-deck energies in the fit $(\text{Y-IV})_{pp+np}$. The energy spacings used are a compromise between economy of space and accuracy obtainable for interpolated values. With a six-point interpolation formula the worst disagreements noticed by comparison with $(\text{Y-IV})_{pp+np}$ values at data-deck energies were about 0.002 rad close to the maxima of phases of the $^3S_1$, K_0, and $^3P_0$ states. For any phase parameter the reproduction of the $(\text{Y-IV})_{pp+np}$ values may be claimed to be good to at least 0.4%, but more frequently to 0.2%, of the total range between the maximum and minimum of the phase parameter within the energy region analyzed. Tables II and III deal with $T=1$ phases for *p*-*p* and *n*-*p* scattering, respectively. Table IV is concerned with $T=0$ phases which occur only in the *n*-*p* case. Interpolation at the lower energies is more convenient, especially if simple interpolation formulas are preferred, employing the quantities in Table V, which lists values of the f function, $k\cot K_0$ for the *p*-*p* and *n*-*p* cases' and of $k\cot(^3\theta^S{}_1)$. The unit of length is 10^{-13} cm. The tabulation of $k\cot K_0$ is broken off at the low-energy end, where it ceases to be useful. The original f function is related to its modifications as follows:

$$\mathbf{K}=\tfrac{1}{2}f+1-2\gamma=\tfrac{1}{2}f-0.15443\cdots,$$
$$f_{\text{Bethe}}=f+2-2\gamma=f+0.84557.$$

Interpolation from Table V reproduces K_0 and $^3\theta^S{}_1$ of the $(\text{Y-IV})_{pp+np}$ fit with an accuracy sufficient to reproduce the corresponding value of $\sin^2K_0$ or $\sin^2(^3\theta^S{}_1)$ to within 0.1%. As mentioned in the Introduction, the determination of phase parameters in the energy region below about 8 MeV is not the main object of the present work. Table V is not intended to furnish and accurate reproduction of the scattering lengths and effective ranges but only to provide the approximate trend of the phases joining smoothly to those at higher energies. Table VI gives the values of phase-parameter uncertainties in radians obtained by parallel shifts of the phase-energy curves within specified energy intervals.[8] The energy regions in MeV are indicated in the first row. The uncertainties in the one-pion regions are not indicated. The standard errors in this table would be different had other energy intervals been used, and tend to increase with finer subdivisions of the energy regions, fewer data being then present in each energy interval. It is nevertheless a useful guide.

Tables VII and VIII are intended to give an idea of the differences in the phases for various fits described in this note. In the headings of these two tables the phase parameters are generically indicated by the letter δ. The first column in each of these tables gives the laboratory energy at which the comparison is made. The other columns are headed by the symbol for the phase parameter for which the differences are tabulated.

[7] G. Breit, M. H. Hull, Jr., F. A. McDonald, and H. M. Ruppel, in *Proceedings of the International Conference on High-Energy Physics at CERN, 1962*, edited by J. Prentki (CERN Scientific Information Service, Geneva, 1962), p. 134; G. Breit, in *Proceedings of the International Conference on Nucleon Structure, Stanford, 1963*, edited by R. Hofstadter and L. I. Schiff (Stanford University Press, Stanford, Calif., 1964), p. 188.

[8] G. Breit, M. H. Hull, Jr., K. E. Lassila, and K. D. Pyatt, Jr., Phys. Rev. **120**, 2227 (1960).