Determination of Kaon-Nucleon Coupling Constants from New Meson-Nucleon Dispersion Relations*

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Two superconvergent dispersion relations are proposed for the sum of three charge-conjugated meson-nucleon forward scattering amplitudes. These two sum rules can be evaluated exactly without using an approximation of saturation. They are obtained under the assumption that asymptotically the ρ pole couples to the pion and kaon universally. We use them here for a determination of the kaon-nucleon coupling constants.

IT was first pointed out by deAlfaro, Fubini, Rossetti, and Furlan[1] that by combining Regge theory with fixed-momentum-transfer dispersion relations for certain amplitudes there exist superconvergent sum rules of the form

$$\int_{-\infty}^{+\infty} d\nu \,\mathrm{Im}F(\nu,t)=0. \tag{1}$$

This type of sum rule [Eq. (1)] was recently generalized by Logunov, Soloviev, and Tavkhelidze[2] and independently by Igi and Matsuda[3] to the case where the amplitude does not converge but whose high-energy behavior is given. As an example, they considered the pion-nucleon charge-exchange forward scattering amplitude and assumed that its high-energy behavior is given by the ρ Regge pole. They were then able to write down a superconvergent dispersion relation for the difference between the amplitude and the ρ pole term.

In this paper we propose to write down two new superconvergent sum rules for the sum of three different charge-conjugated meson-nucleon forward scattering amplitudes. These sum rules unlike the previous ones, can be evaluated exactly without using an approximation of saturation and they are also independent of any unknown parameter. We consider

$$F(\nu)=f_\pi^{(-)}(\nu)+f_{K^-}^{(-)}(\nu)+f_{K^0}^{(-)}(\nu), \tag{2}$$

where $f_\pi^{(-)}(\nu)$ is the pion-nucleon charge-exchange forward scattering amplitude. It satisfies the ordinary dispersion relation[4]:

$$\frac{f_\pi^{(-)}(\nu)}{\nu}=\frac{g_{NN\pi}^2}{4\pi}\frac{\nu_B}{M_N}\frac{1}{\nu_B^2-\nu^2}+\frac{1}{4\pi^2}\int_\mu^\infty d\nu'\frac{(\nu'^2-m_\pi^2)^{1/2}}{\nu'^2-\nu^2}\times[\sigma_{\pi^-p}(\nu')-\sigma_{\pi^+p}(\nu')],$$

where

$$\nu_B=-m_\pi^2/2M_N.$$

Similarly we have two more ordinary dispersion relations for $f_{K^-}^{(-)}(\nu)$ and $f_{K^0}^{(-)}(\nu)$, where $f_{K^-}^{(-)}(\nu)$ and $f_{K^0}^{(-)}(\nu)$ are the $(K^+p\text{-}K^-p)$ and $(\bar{K}^0p\text{-}K^0p)$ forward scattering amplitudes, respectively. The $f(\nu)$'s are normalized such that

$$\mathrm{Im}f_\pi^{(-)}(\nu)=\frac{(\nu^2-m_\pi^2)^{1/2}}{4\pi}\tfrac{1}{2}[\sigma_{\pi^-p}(\nu)-\sigma_{\pi^+p}(\nu)], \tag{3a}$$

$$\mathrm{Im}f_{K^-}^{(-)}(\nu)=\frac{(\nu^2-m_K^2)^{1/2}}{4\pi}\tfrac{1}{2}[\sigma_{K^+p}(\nu)-\sigma_{K^-p}(\nu)], \tag{3b}$$

and

$$\mathrm{Im}f_{K^0}^{(-)}(\nu)=\frac{(\nu^2-m_K^2)^{1/2}}{4\pi}\tfrac{1}{2}[\sigma_{\bar{K}^0p}(\nu)-\sigma_{K^0p}(\nu)]. \tag{3c}$$

The absorptive part of this amplitude satisfies the crossing property

$$\mathrm{Im}F(-\nu)=\mathrm{Im}F(\nu). \tag{4}$$

We now examine the asymptotic behavior of $F(\nu)$. First we note that at high energy, only Regge trajectories with the ρ quantum numbers can contribute to $F(\nu)$. We now make the following assumptions:

(1) The ρ trajectory belongs to an $SU(3)$ octet and its couplings to pion and kaon at infinite energy satisfy the $SU(3)$ symmetry. (An alternative assumption will be that at sufficiently high energy, the ρ pole couples to the pion and kaon universally.)

(2) There are no other singularities in the complex J plane for $\alpha\geq-1$.[5]

From assumption (1), the leading term of the ρ pole does not contribute to $F(\nu)$. Consequently, the contribution from the ρ pole behaves like $\nu^{\alpha_\rho(0)-2}$ at high energy, where $\alpha_\rho(0)$ is the intercept of the ρ trajectory at $t=0$. Combining this result with assumption 2, we are led to the superconvergent sum rule

$$\int_{-\infty}^{\infty} d\nu\,\mathrm{Im}F(\nu)=0.$$

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[1] V. deAlfaro, S. Fubini, G. Rossetti, and G. Furlan, Phys. Letters **21**, 576 (1966).

[2] A. A. Logunov, L. D. Soloviev, and A. N. Tavkhelidze, Phys. Letters **24B**, 181 (1967).

[3] K. Igi and S. Matsuda, Phys. Rev. Letters **18**, 625 (1967).

[4] M. L. Goldberger, Phys. Rev. **99**, 979 (1955); M. L. Goldberger, H. Miyazawa, and R. Oehme, *ibid.* **99**, 986 (1955).

[5] There can be other singularities, but they have all to belong to an $SU(3)$ octet.

Using Eqs. (3) and (4), and charge symmetry, we finally obtain

$$R=I+\int_{m_\pi}^{\infty} d\nu(\nu^2-m_\pi)^{1/2}[\sigma_{\pi^-p}(\nu)-\sigma_{\pi^+p}(\nu)]$$
$$+\int_{m_K}^{\infty} d\nu(\nu^2-m_K^2)^{1/2}[\sigma_{K^+p}(\nu)-\sigma_{K^-p}(\nu)+\sigma_{K^-n}(\nu)-\sigma_{K^+n}(\nu)], \quad (5)$$

where

$$R=\pi^2\left[\frac{2m_\pi^2}{M_N^2}\frac{g_{NN\pi}^2}{4\pi}+\frac{m_K^2-(M_\Sigma-M_N)^2}{M_N^2}\frac{g_{\Sigma NK}^2}{4\pi}-\frac{m_K^2-(M_\Lambda-M_N^2)}{M_N^2}\frac{g_{\Lambda NK}^2}{4\pi}\right]$$

corresponds to the pole terms, and

$$I=8\pi\int_{\nu_0}^{m_K} d\nu\ \mathrm{Im}[f_{K^-}{}^{(-)}(\nu)+f_{K^0}{}^{(-)}(\nu)]$$

is the contribution due to absorption below threshold. The energy $\nu_0=(M_N^2+M_K^2-M_\Lambda^2)/2M_N$ corresponds to the lowest many-particle state threshold which has the same quantum numbers as the K^-p system.

Sum rule (5) can be used as an extra means of determining the K-N coupling constants in addition to the ordinary K-N forward scattering dispersion relations. To obtain some numerical results, the following data were used. With $g_{NN\pi}^2/4\pi=14.6$,

$$R=6.31+2.03\frac{g_{\Sigma KN}^2}{4\pi}-2.38\frac{g_{\Lambda KN}^2}{4\pi}. \quad (6)$$

For the evaluation of the two integrals in Eq. (5), we have used the $\pi^\pm p$ data[6] up to 6 BeV from the papers listed in Ref. 6 and the KN data[7] from the papers listed

TABLE I. A numerical evaluation of the integrals

$$\int_{m_\pi}^{\nu_c} (\nu^2-m_\pi^2)^{1/2}[\sigma_{\pi^-p}(\nu)-\sigma_{\pi^+p}(\nu)]d\nu$$

and

$$\int_{m_K}^{\nu_c} (\nu^2-m_K^2)^{1/2}[\sigma_{K^+p}(\nu)-\sigma_{K^-p}(\nu)+\sigma_{K^-n}(\nu)-\sigma_{K^+n}(\nu)]d\nu.$$

Upper limit ν_c in BeV	Numerical evaluation of the first integral in Eq. (5) up to ν_c	Numerical evaluation of the second integral in Eq. (5) up to ν_c	Sum of these two integrals
3	42	−55	−13
4	71	−92	−21
5	103	−128	−25
6	140	−166	−26

in Ref. 7. Above 6 BeV, we assume that we have already reached the asymptotic region, hence from our assumptions (1) and (2), the contribution is negligible. As a check to our assumptions, we evaluated our integrals up to 3, 4, 5, and 6 BeV separately, and the contributions are listed in Table I. We note from Table I, that the sums of these two integrals evaluated up to 5 BeV and 6 BeV are almost the same. Above 6 BeV, the values of the integrands are consistent with zero within the experimental error,[8] ensuring the validity of our assumptions. For convenience, the integrands of these two integrals are plotted in Fig. 1.[9] One similarly

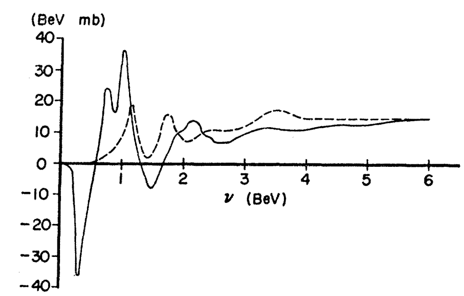

FIG. 1. A plot of the two integrands in Eq. (5), where the solid curve represents the integrand $(\nu^2-m_\pi^2)^{1/2}[\sigma_{\pi^-p}(\nu)-\sigma_{\pi^+p}(\nu)]$ and the dashed curve represents the integrand $(\nu^2-m_K^2)^{1/2}[\sigma_{K^-p}(\nu)-\sigma_{K^+p}(\nu)+\sigma_{K^+n}(\nu)-\sigma_{K^-n}(\nu)]$.

observes that these two terms approach each other quickly at high energy.

In order to evaluate the integral I in Eq. (5) over the K-N unphysical region, we separate it into two parts, I_1 and I_2. The first part I_1 is from the S-wave absorption below threshold and can be evaluated via the method of Dalitz and Tuan[10] using $\bar{K}N$ complex scattering

[6] A. Citron, W. Galbraith, T. F. Kycia, B. A. Leontic, R. H. Phillips, A. Roussett, and R. H. Sharp, Phys. Rev. **144**, 1101 (1966); T. J. Devlin, J. Solomon, and G. Bertsch, Phys. Rev. Letters **14**, 1031 (1965); A. N. Diddens, E. W. Jenkins, T. F. Kycia, and K. F. Riley, *ibid.* **10**, 262 (1963); V. S. Barashenkov and V. M. Maltsev, Fortsch. Physics **9**, 549 (1961); T. J. Devlin, B. J. Moyer, and V. Perez-Mendez, Phys. Rev. **125**, 690 (1962); M. J. Longo and B. J. Moyer, *ibid.* **125**, 701 (1962).

[7] R. L. Cool, G. Giacomelli, T. F. Kycia, B. A. Leontić, K. K. Li, A. Lundby, and J. Teiger, Phys. Rev. Letters, **16**, 1228 (1966); **17**, 102 (1966); R. Good and Nguyen-huu Xuong, *ibid.* **14**, 191 (1965); W. F. Baker, R. L. Cool, E. W. Jenkins, T. F. Kycia, R. H. Phillips, and A. L. Read, Phys. Rev. **129**, 2285 (1963); A. N. Diddens, E. W. Jenkins, T. F. Kycia, and K. F. Riley, *ibid.* **132**, 2721 (1963); P. Bastien and P. Berge, Phys. Rev. Letters **10**, 188 (1963); O. Chamberlain, K. M. Crowe, D. Keefe, L. T. Kerth, A. Lemonick, T. Maung, and T. F. Zipf, Phys. Rev. **125**, 1696 (1962); V. Cook, B. Cook, T. F. Hoang, D. Keefe, L. T. Kerth, W. A. Wenzel, and T. F. Zipf, *ibid.* **123**, 320 (1961); V. Cook, D. Keefe, L. T. Kerth, P. G. Murphy, W. A. Wenzel, and T. F. Zipf, Phys. Rev. Letters **7**, 182 (1961); W. Slater, D. H. Stork, H. K. Ticho, W. Lee, W. Chinowsky, G. Goldhaber, S. Goldhaber, and T. O'Halloran, *ibid.* **7**, 378 (1961); R. J. Abrams, R. L. Cool, G. Giacomelli, T. F. Kycia, B. A. Leontić, K. K. Li, and D. N. Michael *ibid.* **19**, 259 (1967); **19**, 678 (1967).

[8] W. Galbraith, E. W. Jenkins, T. F. Kycia, B. A. Leontić, R. H. Phillips, A. L. Read, and A. Rubenstein, Phys. Rev. **138**, B913 (1865).

[9] In Fig. 1, we plotted, instead of the second integrand, its negative for the sake of an easy comparison.

[10] R. H. Dalitz and S. F. Tuan, Ann. Phys. (N. Y.) **10**, 307 (1960).

lengths. The second part, I_2, is from a pole term at $Y_1^*(1385)$ which we assume is the only contribution other than the S wave. For the evaluation of I_1, we use the scattering-length parameters as obtained by Kim.[11] We obtain

$$I_1 = -6.3\,. \tag{7}$$

For I_2, we assume an $(Y_1^* N\bar{K})$ effective Hamiltonian

$$H = (h/M_{Y_1^*})(\bar{Y}_1^*)_\mu N(\partial_\mu \bar{K}),$$

where h is the coupling constant and can be related to the $N^*(1236)$ width via $SU(3)$. We obtain

$$I_2 = \frac{h^2}{4\pi}\frac{\pi^2}{6M_N^2 M_{Y_1^*}^4}[(M_{Y_1^*}+M_N)^2 - m_K^2]^2 \times [(M_{Y_1^*}-M_N)^2 - m_K^2] = -1.6\,. \tag{8}$$

These numbers cannot be taken too seriously because of the experimental uncertainties, but, fortunately, are small. Thus, their uncertainties do not affect our final result.

Substituting Eqs. (6), (7), (8) and the value of the integrations from Table I back into Eq. (5), we finally obtain

$$2.38\frac{g_{\Lambda KN}^2}{4\pi} - 2.03\frac{g_{\Sigma KN}^2}{4\pi} = 40.2\,. \tag{9}$$

This equation can be used to evaluate the K-N coupling constants. In order to determine $g_{\Lambda KN}$ and $g_{\Sigma KN}$ separately, it is possible to write down an additional superconvergent sum rule

$$\int_{-\infty}^{\infty} d\nu \frac{\operatorname{Re} F(\nu)}{(\nu^2 - m^2)^{1/2}} = 0 \tag{10}$$

involving Re $F(\nu)$ in analogy to the dispersion relation Gilbert[12] wrote down many years ago. However, because of the lack of experimental data, a numerical evaluation of Eq. (10) is difficult at the present time. We shall therefore assume $SU(3)$ symmetry which gives

$$\frac{g_{\Lambda NK}^2}{4\pi} = \frac{(1+2\alpha)^2}{3}\frac{g_{NN\pi}^2}{4\pi}, \qquad \frac{g_{\Sigma NK}^2}{4\pi} = (1-2\alpha)^2 \frac{g_{NN\pi}^2}{4\pi}, \tag{11}$$

where $\alpha = F/(F+D)$ and $g_{NN\pi}^2/4\pi = 14.6$. From Eqs. (9) and (11), we obtain $\alpha = 0.43$ and

$$g_{\Lambda NK}^2/4\pi = 16.9, \quad g_{\Sigma NK}^2/4\pi = 0.3.$$

The values we obtain here are very close to those determined recently by Kim[13] using a KN forward dispersion relation.

In conclusion, we have proposed here two superconvergent sum rules, Eqs. (5) and (10). These two sum rules can be easily compared with the experimental results without using an approximation of saturation. Alternatively, they can be used as an independent means of determining the K-N coupling constants. We hope to reevaluate them when more accurate experimental data at high energy become available.

[11] Jae Kwan Kim, Phys. Rev. Letters **14**, 29 (1965).

[12] W. Gilbert, Phys. Rev. **108**, 1078 (1957).

[13] J. K. Kim, Phys. Rev. Letters **19**, 1079 (1967).