Study of $\pi^- p \to \Sigma^- K^+$ at 1170 MeV/c *

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We have measured the differential and total cross sections of the reaction $\pi^- p \to \Sigma^- K^+$ with 1170-MeV/c pions incident on the Alvarez 72-in. hydrogen bubble chamber. Using 1338 events, we find the coefficients in the Legendre expansion of the differential cross section $d\sigma/d\Omega = A_0P_0 + A_1P_1 + A_2P_2$ to be $A_0 = 18.38 \pm 0.50$ μb/sr, $A_1 = 7.54 \pm 0.96$ μb/sr, and $A_2 = 11.10 \pm 1.21$ μb/sr, corresponding to a total cross section $\sigma = 231 \pm 6$ μb. No polynomials higher than P_2 are needed. The decay asymmetry $\alpha_- P_-$ of the decay $\Sigma^- \to n\pi^-$ is consistent with zero. Using previously reported results for the differential cross sections of the reactions $\pi^- p \to \Sigma^0 K^0$ and $\pi^+ p \to \Sigma^+ K^+$ at 1170-MeV/c incident pion momentum, we have compared the results with the predictions of the hypothesis of charge independence. We have found no significant violations of that hypothesis. We have compared the measured polarization of the Σ^0 with the prediction from the charge-independence hypothesis, using the measured differential cross sections of the three production reactions and the polarization of the Σ^+, as determined in the decay $\Sigma^+ \to p\pi^0$. The results agree with the prediction.

I. INTRODUCTION

DIFFERENTIAL and total cross sections for the process

$$\pi^- p \to \Sigma^- K^+, \quad (1a)$$

$$\Sigma^- \to n\pi^-, \quad (1b)$$

have been measured using 1170-MeV/c pions incident on the Alvarez 72-in. hydrogen bubble chamber. The amplitudes for reaction (1a) and for the reactions

$$\pi^- p \to \Sigma^0 K^0 \quad (2)$$

and

$$\pi^+ p \to \Sigma^+ K^+ \quad (3)$$

are related by the hypothesis of charge independence to only two independent isotopic spin amplitudes.[1] Thus the amplitudes for reactions (1)–(3) are subject to a constraint, producing the well-known "triangle" inequalities on the cross sections σ^-, σ^0, and σ^+. We compare the hypothesis of charge independence with our results for reaction (1), combined with previously reported results for reactions (2) and (3),[2,3] in Sec. IV below.

II. EXPERIMENTAL PROCEDURE

The π^- beam-transport system has been reported previously.[4] Events corresponding to Σ^- production are topologically distinguishable from other reactions present at this momentum. Since the production reaction is kinematically overdetermined, we use the fitting program PACKAGE and select events on the basis of χ^2. This procedure yields 1500 events. This number is reduced by fiducial criteria to 1338 events. (The largest fiducial loss results from our requirement that the Σ^- travel at least 0.3 cm before decaying.)

In calculations of physically interesting quantities, each event (i) carries a weighting factor $b_i \geqslant 1$ that includes all fiducial corrections as well as correction for the attenuation of the pion beam in the chamber. We choose the decay fiducial volume larger than the production fiducial volume to avoid fluctuations from accidentally large values of b_i. The weighted or "true" number of events N is given in terms of the observed number N_{obs} by

$$N \equiv \sum_i b_i \pm (\sum_i b_i^2)^{1/2} = N_{\text{obs}}\langle b\rangle \pm (N_{\text{obs}}\langle b^2\rangle)^{1/2}, \quad (4)$$

where the sums extend over the observed events from $i=1$ to N_{obs}. The brackets $\langle\ \rangle$ mean an average over the data. [We use weighted counts as in Eq. (4), not only for the entire sample to find the total cross section, but also for subsamples to find the angular distribution.] For the entire sample we find the values $\langle b\rangle = 1.1977$ and $\langle b^2\rangle^{1/2} = 1.1997$.

After correcting for a scanning efficiency of 0.921, we obtain the total weighted number of Σ^--production events corresponding to 1338 observed events:

$$N = 1775.4 \pm 48.0. \quad (5)$$

III. RESULTS

Using the result (5) and our total pion track length of 2.1967×10^8 cm, we obtain the total cross section for reaction (1a) at 1170 MeV/c:

$$\sigma(\pi^- p \to \Sigma^- K^+) = 231.0 \pm 6.3\ \mu\text{b}.$$

We write the differential cross section for reaction (1a) in the Legendre polynomial expansion

$$d\sigma/d\Omega = \sum_n A_n P_n(\cos\theta_\Sigma), \quad (6)$$

where $\cos\theta_\Sigma$ is the cosine of the angle between the

* Work done under the auspices of the U. S. Atomic Energy Commission.
[1] J. J. Sakurai, Phys. Rev. **107**, 908 (1957).
[2] F. S. Crawford, Jr., F. Grard, and G. A. Smith, Phys. Rev. **128**, 368 (1962). The Σ^+ data were reanalyzed by the procedure with which the Σ^0 and Σ^- data were analyzed. The results quoted in this paper are the results of the reanalysis, which are consistent with those reported previously.
[3] J. Anderson, F. S. Crawford, Jr., and J. C. Doyle, Phys. Rev. **152**, 1139 (1967).
[4] S. Wolf, N. Schmitz, L. Lloyd, W. Laskar, F. Crawford, Jr., J. Button, J. Anderson, and G. Alexander, Rev. Mod. Phys. **33**, 439 (1961).

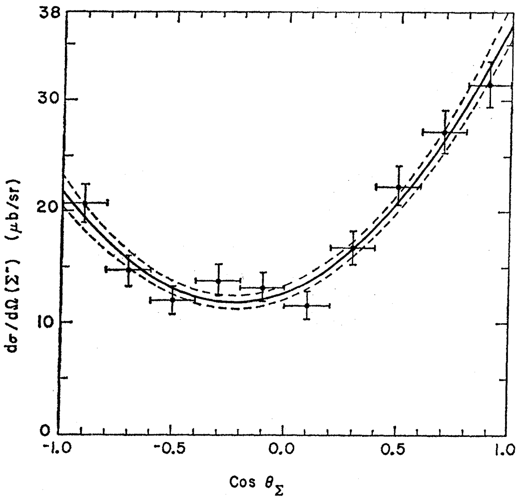

FIG. 1. Corrected absolute differential cross section in μb/sr for the reaction $\pi^- p \rightarrow \Sigma^- K^+$ and least-squares-fitted function with error corridor (corresponding to one standard deviation fluctuation), plotted as functions of $\cos\theta_\Sigma$, the angle between the incident pion and produced hyperon directions in the center-of-mass system.

incident pion direction and the produced Σ direction in the center-of-mass system. By the method of least squares, using weighted counts as in Eq. (4), we find the coefficients to be

$$A_0 = 18.38 \pm 0.50\ \mu\text{b/sr}, \quad (7a)$$

$$A_1 = 7.54 \pm 0.96\ \mu\text{b/sr}, \quad (7b)$$

$$A_2 = 11.10 \pm 1.21\ \mu\text{b/sr}, \quad (7c)$$

with off-diagonal error terms

$$\delta A_0 \delta A_1 = 0.0957\ (\mu\text{b/sr})^2, \quad (7d)$$

$$\delta A_0 \delta A_2 = 0.1565\ (\mu\text{b/sr})^2, \quad (7e)$$

$$\delta A_1 \delta A_2 = 0.1529\ (\mu\text{b/sr})^2, \quad (7f)$$

corresponding to χ^2 probability 31.43%. No polynomials higher than $P_2(\cos\theta_\Sigma)$ are needed. The differential cross section is plotted in Fig. 1. The decay asymmetry $\alpha_- P_-$ is found to be -0.028 ± 0.048, which is consistent with zero.

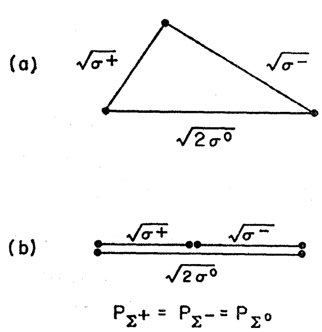

FIG. 2. Charge-independence triangle for the reactions $\pi^+ p \rightarrow \Sigma^+ K^+$, $\pi^- p \rightarrow \Sigma^0 K^0$, and $\pi^- p \rightarrow \Sigma^- K^+$. (a) General configuration of the constraint on the corresponding cross sections σ^+, σ^0, and σ^- implied by the charge-independence hypothesis. (b) "Flat" triangle, a limiting case of Fig. 2(a), for which $P_{\Sigma^+} = P_{\Sigma^-} = P_{\Sigma^0}$.

IV. CHARGE INDEPENDENCE

The hypothesis of charge independence implies that the complex amplitudes a^+, a^0, and a^- for the three reactions

$$\pi^+ p \rightarrow \Sigma^+ K^+, \quad (8a)$$

$$\pi^- p \rightarrow \Sigma^0 K^0, \quad (8b)$$

$$\pi^- p \rightarrow \Sigma^- K^+, \quad (8c)$$

are determined by only two independent isotopic spin amplitudes a_3 and a_1, corresponding to $T = \frac{3}{2}$ and $T = \frac{1}{2}$.[1] Therefore, a constraint is implied on the amplitudes a^+, a^0, and a^-, namely

$$\sqrt{2} a^0 = a^+ - a^-. \quad (9)$$

The three amplitudes form a triangle in the complex plane. This implies three "triangle inequalities" constraining the differential (and total) cross sections σ^+, σ^0, and σ^-, namely

$$(2\sigma^0)^{1/2} \leqslant (\sigma^+)^{1/2} + (\sigma^-)^{1/2}, \quad (10a)$$

$$(\sigma^+)^{1/2} \leqslant (2\sigma^0)^{1/2} + (\sigma^-)^{1/2}, \quad (10b)$$

$$(\sigma^-)^{1/2} \leqslant (2\sigma^0)^{1/2} + (\sigma^+)^{1/2}. \quad (10c)$$

The cross sections σ^+, σ^0, and σ^- can refer to completely specified configurations or to spin-summed or integrated cross sections.[5] (See Fig. 2.)

The results (7) for the differential cross section for reaction (8c) can be combined with previously reported results for reactions (8a) and (8b) at the same pion momentum[2,3] to test the charge-independence hypothesis. The coefficients A_n in Legendre-polynomial expansions of the form of Eq. (6) for the σ^0 and σ^+ differential cross sections are given in Table I. The differential cross-section coefficients and complete error matrices were used to calculate the quantities $(2\sigma^0)^{1/2}$, $(\sigma^+)^{1/2}$, and $(\sigma^-)^{1/2}$ and the error corridors (corresponding to one standard deviation fluctuation) plotted in Fig. 3. In Fig. 4 are plotted $(2\sigma^0)^{1/2}$ and $(\sigma^+)^{1/2} + (\sigma^-)^{1/2}$, corresponding to the inequality of Eq. (10a). It can

TABLE I. Coefficients in the Legendre-polynomial expansion of the differential cross section.

	Reaction $\pi^- p \rightarrow \Sigma^0 K^0$	Reaction $\pi^+ p \rightarrow \Sigma^+ K^+$
Coefficients (μb/sr)	$A_0 = 19.68 \pm 0.59$	$A_0 = 16.31 \pm 1.06$
	$A_1 = -0.04 \pm 1.19$	$A_1 = 12.11 \pm 1.87$
	$A_2 = 14.39 \pm 1.58$	$A_2 = 1.95 \pm 2.50$
		$A_3 = -9.63 \pm 2.85$
Error correlations $(\mu\text{b/sr})^2$	$\delta A_0 \delta A_1 = +0.056$	$\delta A_0 \delta A_1 = +0.749$
	$\delta A_0 \delta A_2 = +0.298$	$\delta A_0 \delta A_2 = +0.087$
	$\delta A_1 \delta A_2 = +0.017$	$\delta A_0 \delta A_3 = -0.613$
		$\delta A_1 \delta A_2 = +0.865$
		$\delta A_1 \delta A_3 = +0.561$
		$\delta A_2 \delta A_3 = +2.550$

[5] L. Michel, Nuovo Cimento **22**, 203 (1961).

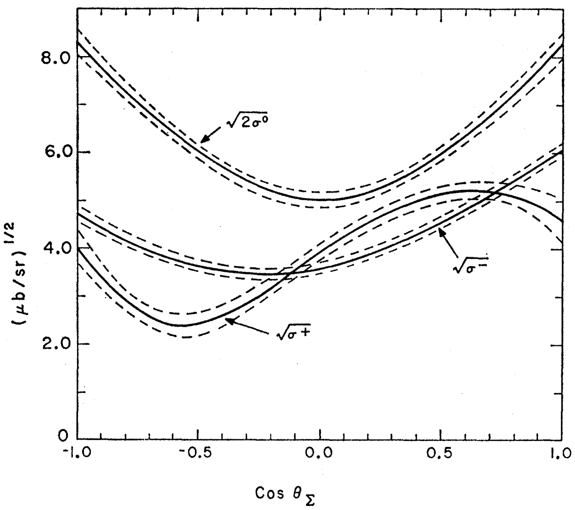

FIG. 3. The quantities $(\sigma^+)^{1/2}$, $(\sigma^-)^{1/2}$, and $(2\sigma^0)^{1/2}$ plotted versus $\cos\theta_\Sigma$ at 1170-MeV/c incident pion momentum, as calculated from the results of Refs. 2 and 3 and this experiment. The error corridors correspond to one standard deviation fluctuation and were calculated by propagating the complete error matrices.

be seen that the inequality is not significantly violated. The closest that the data come to violating the inequality is for values of $\cos\theta_\Sigma$ between about -0.5 and -0.9. The inequalities of Eqs. (10b) and (10c) are satisfied by the experimental results. Thus reactions (8) at 1170-MeV/c incident pion momentum satisfy the hypothesis of charge independence.

The constraint on the polarizations of the produced Σ's resulting from Eq. (9) has been analyzed by Michel.[5,6] In the limit in which the equality holds in one of Eqs. (10), corresponding to a "flat" triangle such as that in Fig. 2(b), this constraint reduces to

$$P_{\Sigma^+} = P_{\Sigma^-} = P_{\Sigma^0}.$$

As the triangle in Fig. 2 departs from flatness, this

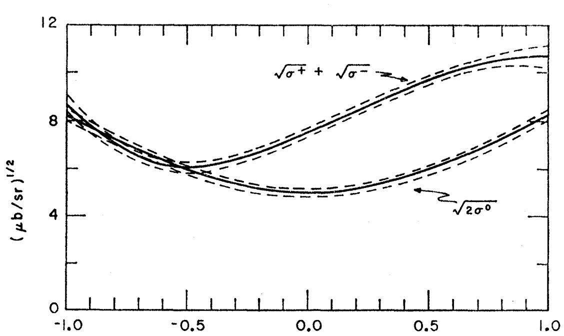

FIG. 4. Comparison of the quantities $(2\sigma^0)^{1/2}$ and $(\sigma^+)^{1/2} + (\sigma^-)^{1/2}$ as functions of $\cos\theta_\Sigma$, corresponding to the triangle inequality of Eq. (10a).

[6] Michel's paper contains an algebraic error. His equations (16) and (17) are correct, but his Eq. (18) is incorrect. As a result, the allowed regions of polarization in his Fig. 1 include not only the interior of the ellipse, as stated by Michel, but also the "symmetric corners" outside the ellipse, in the regions where η_α and η_β have the same sign.

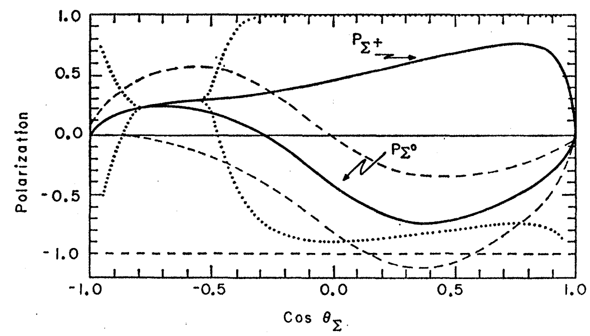

FIG. 5. Charge-independence constraints on the polarization of the produced Σ^0 (indicated by dotted lines), as calculated by using the measured differential cross sections of the three reactions (8) and the measured polarization of the Σ^+ from reaction (8a). Also plotted are the measured Σ^+ polarization (solid line) and the measured Σ^0 polarization (solid line) with error corridor (dashed lines) as reported in Ref. 3. Between $\cos\theta_\Sigma = -0.5$ and $\cos\theta_\Sigma = -0.9$, in the region of apparent violation of the inequality of Eq. (10a), the triangle is assumed to be flat.

constraint is rapidly relaxed. In Fig. 5 are shown the constraints on the polarization of the Σ^0 at 1170-MeV/c beam momentum as a function of $\cos\theta_\Sigma$, calculated from the measured differential cross sections for the three reactions (8) and the least-squares-fitted values for the polarization of the Σ^+ as measured from the asymmetry of the decay $\Sigma^+ \rightarrow p\pi^0$.[2] The least-squares coefficients for the Σ^+ polarization times differential cross section are given in Table II. Also plotted in Fig. 5 are the fitted Σ^+ polarization used and the Σ^0 polarization and error corridor reported in Ref. 3. (In the region between $\cos\theta_\Sigma = -0.5$ and $\cos\theta_\Sigma = -0.9$, the triangle is assumed to be flat.) It can be seen that the experimental results agree with the charge-independence constraints.

Binford, Good, and Kofler have pointed out[7] that the Σ charge-independence triangle is flat or nearly flat in the backward-hyperon direction for beam momenta from threshold up to 1275 MeV/c, as well as at higher momenta, where backward Σ^- production is nearly zero, whereas backward Σ^+ and Σ^0 production is large (presumably because of the exchange of strange mesons such as K^*). When the charge-independence triangle is flat, the two isotopic spin amplitudes a_1 and a_3 (as well as the amplitudes a^+, a^-, and a^0) are constrained to be relatively real. Then the measured

TABLE II. Least-squares coefficients in an expansion of the form $P_{\Sigma^+} d\sigma/d\Omega = \sin\theta_\Sigma \sum_n B_n \cos^n\theta_\Sigma$ for the Σ^+ polarization times differential cross section.

	Reaction $\pi^+ p \rightarrow \Sigma^+ K^+$
Coefficients (μb/sr)	$B_0 = 7.03 \pm 3.19$
	$B_1 = 18.80 \pm 5.95$
	$B_2 = 17.40 \pm 12.60$
Error correlations (μb/sr)^2	$\delta B_0 \delta B_1 = +3.42$
	$\delta B_0 \delta B_2 = -27.90$
	$\delta B_1 \delta B_2 = +23.70$

[7] T. O. Binford, M. L. Good, and R. R. Kofler, University of Wisconsin (unpublished).

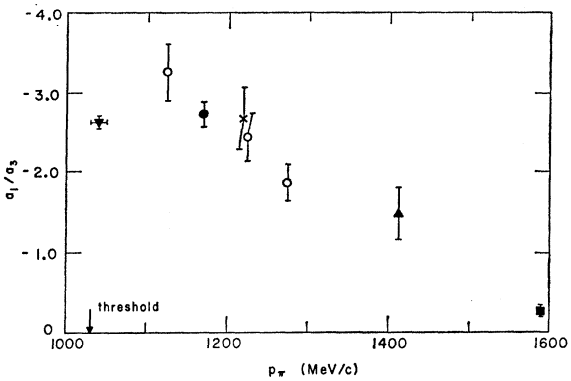

FIG. 6. The ratio a_1/a_3 of isotopic spin amplitudes corresponding to $T=\frac{1}{2}$ and $T=\frac{3}{2}$ in reactions (8), calculated for values of $\cos\theta_\Sigma$ at which the charge-independence triangle is flat, plotted as a function of beam momentum. ▼, Ref. 8; ○, Ref. 9; ×, Ref. 10; ▲, Ref. 11; ■, Ref. 12; ●, this experiment.

values of the differential cross sections for any two of the three reactions (8) can be used to calculate values of a_1 and a_3 (up to a common phase). From the differential cross sections of all three reactions, these amplitudes can be calculated by least-squares analysis with one constraint. From the measured differential cross sections of this experiment and of Refs. 2 and 3, evaluated at $\cos\theta_\Sigma=-0.7$ (which corresponds to the center of the region in which the triangle is assumed to be flat), the amplitudes are calculated to be $a_1=-7.37\pm0.13$ $(\mu\text{b/sr})^{1/2}$ and $a_3=2.70\pm0.14$ $(\mu\text{b/sr})^{1/2}$ at 1170-MeV/c pion momentum. Their ratio is $a_1/a_3=-2.73\pm0.15$. We have also calculated this ratio in a similar fashion from previously reported data for the three reactions (8) in angular regions where the triangle is flat at a number of other beam momenta from threshold up to 1.59 GeV/c.[8–12] The results are plotted in Fig. 6.

Tripp *et al.*[13] have arranged most of the known baryon resonances into $SU(3)$ multiplets by means of mass formulas, checking the consistency of their assignments by comparison of various decay rates within each multiplet. Using the values of coupling coefficients thus determined, one can predict rates for the decay of various resonances into previously unobserved channels. We have calculated the predicted rates at the center-of-mass energy of this experiment for decay into Σ^-+K^+ of four baryon resonances: $N_{1/2}^*(1570)$, $J^P=\frac{1}{2}^-$; $N_{1/2}^*(1518)$, $J^P=\frac{3}{2}^-$; $N_{1/2}^*(1670)$, $J^P=\frac{5}{2}^-$; and $N_{1/2}^*(1688)$, $J^P=\frac{5}{2}^+$. The combined effects of these predictions should contribute to the coefficients A_n of the Legendre-polynomial expansion of the differential cross section, Eq. (6), as follows:

$$A_0=12.39\ \mu\text{b/sr},$$
$$A_1=0.15\ \mu\text{b/sr},$$
$$A_2=25.67\ \mu\text{b/sr},$$
$$A_3=-2.89\ \mu\text{b/sr},$$
$$A_4=3.43\ \mu\text{b/sr},$$
$$A_5=-4.48\ \mu\text{b/sr}.$$

The coefficients in the fifth-order least-squares fit to the experimental data are

$$A_0=18.38\pm0.50\ \mu\text{b/sr},$$
$$A_1=7.33\pm0.98\ \mu\text{b/sr},$$
$$A_2=10.89\pm1.25\ \mu\text{b/sr},$$
$$A_3=-2.15\pm1.60\ \mu\text{b/sr},$$
$$A_4=-0.90\pm1.80\ \mu\text{b/sr},$$
$$A_5=-4.59\pm2.25\ \mu\text{b/sr},$$

with a χ^2 probability of 46.58%. The experimental total cross section, which is proportional to A_0, is half again as large as that predicted from the baryon resonances. Thus there must be some other contributions to the total cross section. This makes elaborate comparisons of the higher coefficients quite speculative. However, since the other contributions are likely to be of low angular momentum, it may be significant that the predicted coefficient A_5 arising from the interference between the $D_{5/2}$ and $F_{5/2}$ resonances agrees within the experimental error with the measured value of A_5. We conclude that the experimental data are consistent with the predictions, even though an adequate fit required Legendre polynomials only through the second order.

[8] F. S. Crawford, Jr., F. Grard, and G. A. Smith, in *Proceedings of the 1962 International Conference on High-Energy Physics at CERN*, edited by J. Prentki (CERN, Geneva, 1962), p. 270; beam momentum 1030 MeV/c (threshold). The angular distributions were found to be consistent with pure S wave, and the triangle for the total cross sections was found to be flat. The data used to calculate a_1/a_3 are coefficients in the expressions manifesting the threshold energy dependence of the total cross sections.

[9] Reference 7; beam momenta 1125, 1225, and 1275 MeV/c. The data used to calculate a_1/a_3 are obtained by evaluating the least-squares fits to the angular distributions at $\cos\theta_\Sigma=-0.7$, which is within the region in which the triangle can be assumed flat at each momentum.

[10] F. S. Crawford, Jr., R. L. Douglass, M. L. Good, G. R. Kalbfleisch, M. L. Stevenson, and H. K. Ticho, Phys. Rev. Letters **3**, 394 (1959); beam momentum 1220 MeV/c. Differential cross sections were found to be consistent with a flat triangle at all angles, and values of a_1 and a_3 were calculated by the authors.

[11] For Σ^- and Σ^0 data: F. Eisler, R. Plano, A. Prodell, N. Samios, M. Schwartz, J. Steinberger, P. Bassi, V. Borelli, G. Puppi, H. Tanaka, P. Waloschek, V. Zoboli, M. Conversi, P. Franzini, I. Mannelli, R. Santangelo, and V. Silvestrini, Nuovo Cimento **10**, 468 (1958); beam momentum 1430 MeV/c. For Σ^+ data: C. Baltay, H. Courant, W. J. Fickinger, E. C. Fowler, H. L. Kraybill, J. Sandweiss, J. R. Sanford, D. L. Stonehill, and H. D. Taft, Rev. Mod. Phys. **33**, 374 (1961); beam momentum 1390 MeV/c. For both references, differential cross-section data for which $-1.0\leq\cos\theta_\Sigma\leq-0.8$ were used to calculate a_1/a_3. The result was plotted in Fig. 6 at 1410 MeV/c.

[12] For Σ^- and Σ^0 data: O. Goussu, M. Sené, B. Ghidini, S. Mongelli, A. Romano, P. Waloschek, and V. Alles-Borelli, Nuovo Cimento **42A**, 606 (1966); beam momentum 1590 MeV/c. For Σ^+ data: P. Daronian, A. Daudin, M. A. Jabiol, C. Lewin, C. Kochowski, B. Ghidini, S. Mongelli, and V. Picciarelli, Nuovo Cimento **41A**, 503 (1966); beam momentum 1590 MeV/c. For both references, differential cross-section data for which $-1.0\leq\cos\theta_\Sigma\leq-0.8$ were used to calculate a_1/a_3.

[13] R. D. Tripp, D. W. G. Leith, A. Minten, R. Armenteros, M. Ferro-Luzzi, R. Levi-Setti, H. Filthuth, V. Hepp, E. Kluge, H. Schneider, R. Barloutaud, P. Granet, J. Meyer, and J. P. Porte, Lawrence Radiation Laboratory Report UCRL-17385, revised, 1967 (unpublished).