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Multilevel Analysis of the ^{239}Pu Fission Cross Section from 14 to 90 eV*

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The ^{239}Pu fission cross section has been fitted with a multilevel R-matrix formula from a neutron energy of 14 to 90 eV. Spin assignments have been made from the interference between resonances. The average fission widths are 1033 meV with at least two open fission channels for the 0^+ spin state, and 66 meV and one channel for the 1^+ state. The number of undetected levels is deduced from the statistical distributions of resonance parameters.

I. INTRODUCTION

PREVIOUS measurements of the neutron resonance parameters of ^{239}Pu have been based on the single-level Breit-Wigner formula except for the few lowest-energy resonances.[1–12] The ^{239}Pu fission cross section was measured by Shunk, Brown, and LaBauve by using the nuclear explosion Petrel as a neutron source.[13] The data were taken using a flight path of 185 m with a time resolution of about 3 μsec in the 20- to 200-eV region. The experimental technique has been described in detail elsewhere.[14] The nuclear explosion source has the advantage of a much lower background than conventional neutron sources. As a result, interference effects in the wings of the resonances are more clearly seen. This interference is, in many cases, sufficient to introduce considerable error into any method of resonance analysis that does not take it into account. For this reason, it seemed desirable to make a multilevel analysis of the data to as high an energy as possible.

Since the spin assignments of the most prominent ^{239}Pu resonances are known in most cases,[15,16] it was hoped that spin assignments could be made on the weaker resonances by observing their interference with neighboring levels. This has been the case for most of the resonances but some ambiguities remain, mostly for narrow resonances that are isolated from their neighbors.

II. METHOD OF ANALYSIS

The multilevel approach used was a trial and error shape fit to the data using the Reich-Moore multilevel formula[17] for two fission channels. The fit was generated by a computer code that was a modification of a program developed at Phillips Petroleum Co. The computed cross section was Doppler- and resolution-broadened and superimposed on a plot of the experimental data. The parameters were then adjusted and a new cross section generated. In the two-fission-channel Reich-Moore formalism, each resonance is described by five parameters: the resonance energy, the reduced neutron width, the capture width, and a width and associated sign in each of the two fission channels. In order to reduce the number of parameters, two assumptions were made: The fission width of each resonance was put entirely in one channel or the other and, because little capture information was available, the capture widths were held constant at 40 meV. The first assumption seemed justified since it is unlikely, from the Porter-Thomas distribution of widths, that a resonance will have a large width in both channels so that those resonances that have partial widths of similar magnitude in both channels will usually be narrow and interference effects will be small. The assumption of a constant capture width has little effect on the other parameters as long as the fission width is considerably larger than the capture width. This is true for many of the resonances considered but some error may be introduced in the parameters obtained for those resonances with small fission width.

Figures 1 and 2 show the multilevel fit obtained. The cross section in the low valleys at 20, 24, 30, and 38 eV is quite uncertain because of low signal input to the amplifiers; consequently no attempt was made to reproduce the exact level. The Petrel data have not been published below 20 eV because of rapidly diminishing neutron flux but since there are several strong reso-

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[1] J. F. Raffle and B. T. Price, in *Proceedings of the International Conference on the Peaceful Uses of Atomic Energy, Geneva, 1955* (United Nations, New York, 1956), Vol. 4, p. 187.

[2] L. M. Bollinger and R. E. Coté, in *Proceedings of the Second International Conference on the Peaceful Uses of Atomic Energy, Geneva, 1958* (United Nations, Geneva, 1958), Vol. 15, p. 127.

[3] P. A. Eglestaff and D. J. Hughes, Progr. Nucl. Energy **1**, 55 (1956).

[4] V. V. Sokolovski *et al.*, J. Nucl. Energy **5**, 389 (1957).

[5] I. V. Kurpichnikov *et al.*, At. Energ. **2**, 299 (1957).

[6] V. V. Vladimirsky *et al.*, in *Proceedings of the Second International Conference on the Peaceful Uses of Atomic Energy, Gevena, 1958* (United Nations, Geneva, 1958), Vol. 15, p. 309.

[7] K. G. Ignat'ev and I. V. Kurpichnikov, EuroNuclear **2**, 77 (1965).

[8] G. D. James, Atomic Energy Research Establishment Report No. AERE-NP/PR-8, 5, 1965 (unpublished).

[9] C. A. Uttley, European Nuclear Energy Agency Report EANDC(UK)-40 "L", 1964 (unpublished).

[10] H. Derrien *et al.*, in *Proceedings of the Paris Conference on Nuclear Data for Reactors, 1966* (International Atomic Energy Agency, Vienna, 1967), Vol. 2, p. 195.

[11] J. J. Schmidt, European Nuclear Energy Agency Report EANDC-E-35U, 1966 (unpublished).

[12] E. Vogt, Phys. Rev. **118**, 724 (1960).

[13] E. R. Shunk, W. K. Brown, and R. LaBauve, in Proceedings of the Conference on Neutron Cross Section Technology, Washington, D. C., 1966 (unpublished).

[14] P. A. Seeger, A. Hemmendinger, and B. C. Diven, Nucl. Phys. **A96**, 605 (1967).

[15] G. D. Sauter and C. D. Bowman, Phys. Rev. Letters **15**, 761 (1965).

[16] G. A. Cowan *et al.*, Phys. Rev. **144**, 979 (1966).

[17] C. W. Reich and M. S. Moore, Phys. Rev. **111**, 929 (1958).

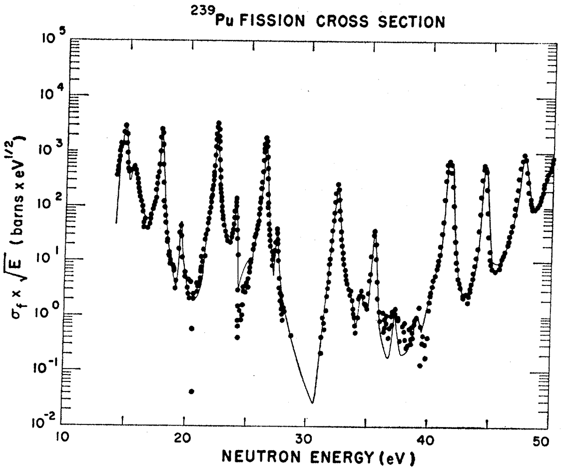

FIG. 1. The fission cross section of ^{239}Pu times $\sqrt{E}$ from 14 to 50 eV measured by Shunk *et al.* The solid curve is the multilevel fit.

nances below this energy, it was necessary to consider this region. Since the peak heights above 20 eV match those observed by deSaussure,[18] it was decided to match the lower-energy peak heights to his data. Resonance parameters for the resonances below 14 eV were taken from Vogt[12] with the relative signs of the fission widths adjusted to give the observed shape.

III. RESULTS

^{239}Pu has a ground-state spin of $\frac{1}{2}^+$ so that the possible spin states for s-wave neutrons are 0^+ and 1^+. The first attempt to fit the observed cross section was made under the assumption of only one fission channel in each spin state, which proved unsuccessful. It soon became apparent that there are at least three groups of resonances which interfere with each other but not with members of other groups. For example, the resonance at 47 eV does not appear to interfere with either the resonance at 44 eV or the one at 50 eV and the two latter resonances do not interfere with each other. Two of these groups have an average fission width of about 1 eV and have been assigned to the 0^+ spin state. The other group, which contains most of the resonances, has an average width of less than 0.07 eV and has been assigned to the 1^+ spin state. The fit is quite good except at 45.5 eV where the existence of one or more hidden levels is indicated.

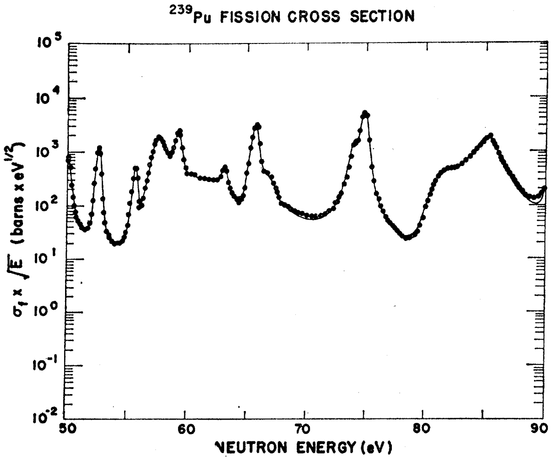

FIG. 2. The fission cross section of ^{239}Pu times $\sqrt{E}$ from 50 to 90 eV.

TABLE I. Individual resonance parameters for ^{239}Pu from 14 to 90 eV. Γ_{f1} and Γ_{f2} are the partial fission widths in the two channels assumed open. $\Gamma_n{}^0$ is the reduced neutron width $\Gamma_n{}^0=\Gamma_n/[E_0/(1\text{ eV})]^{1/2}$. The capture width was assumed constant at 40 meV.

E_0 (eV)	$\Gamma_n{}^0$ (meV)	Γ_{f1} (meV)	Γ_{f2} (meV)	Γ_γ (meV)	J
14.29	0.183	−67.00		40	1
14.68	0.429	68.00		40	1
15.38	0.537	608.00		40	0
17.65	0.390	−68.36		40	1
19.32[a]	0.017	−9.93		40	1
21.45[b]	0.018	250.00		40	0
21.70[b]	0.0087	−110.00		40	1
22.26	0.585	47.24		40	1
23.91	0.038	−14.8		40	1
25.15[b]	0.000	45.00		40	1
26.25	0.360	−44.0		40	1
27.25	0.080	−1.58		40	1
30.60[c]	0.000	2000.00		40	0
32.34	0.147	99.10		40	0
33.50[c]	0.0001	−50.0		40	1
34.30	0.0013	50.00		40	0
35.47	0.010	−25.0		40	1
37.25[b]	0.0002	80.00		40	1
39.25[b]	0.0004	50.00		40	0
40.95[c]	0.015	760.00		40	0
41.43	0.101	39.35		40	1
41.72	0.112	−79.2		40	1
44.51	0.220	23.75		40	1
46.00[c]	0.0002	30.00		40	1
47.64	0.718		230.00	40	0
49.60	0.536	900.00		40	0
50.10	0.231	−23.70		40	1
51.60[b]	0.0021	−30.0		40	1
52.59	0.405	32.43		40	1
55.66	0.143	−43.7		40	1
57.30	2.607	1040.00		40	0
59.22	0.563	−141.00		40	1
60.65[c]	0.011	185.0		40	1
62.70	1.689	−4250.00		40	0
63.16	0.087	−54.4		40	1
65.40[c]	0.099	−39.2		40	1
65.75	0.934	−127.00		40	1
66.75	0.515		1355.00	40	0
68.05[c]	0.0017	−250.00		40	1
74.19	0.385	91.90		40	1
74.97	1.893	148.00		40	1
77.80[c]	0.040		2000.00	40	0
78.60	0.0009	50.00		40	1
81.10	0.948	1950.00		40	0
83.62	0.024	−74.30		40	1
85.40	0.160	48.30		40	1
85.60	4.702	−1916.00		40	0

[a] Parameters very uncertain due to close proximity to a resonance in the target backing.

[b] Resonance included to improve the fit, primarily in the valleys. Their existence is probable but positions and widths are uncertain.

[c] Resonance previously unreported but whose presence seems well established by the present fit.

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[18] G. DeSaussure *et al.*, in *Proceedings of the IAEA Symposium on the Physics and Chemistry of Fission, Salzburg, 1965* (International Atomic Energy Agency, Vienna, 1965), Vol. I, p. 205.

The most striking feature of the data is the existence of several valleys between resonances that are very much lower than has previously been observed. At four of these minima the cross section is low enough to permit the counting of individual fission events on the photographic recording of the data. The amplitude of the pulses was small and there was considerable overlap so that the errors are large. The values obtained for the fission cross section in these minima are

E (eV)	σ_f (b)
20.4	0.5 ±0.3
24.2	0.2 ±0.1
30.0	0.01±0.01
38.0	0.2 ±0.1

These low valleys are a direct result of interference between resonances. In order to reproduce the minimum at 30 eV, it was necessary to assume the existence of a resonance in the 0^+ channel with a very small neutron width, and a large but somewhat uncertain fission width. A number of other weak resonances that were previously unknown have been revealed by the fit. Some of these, such as at 41 eV, are quite certain; others were included to improve the fit locally but their exact widths and positions are uncertain.

Table I gives the parameters which produced the fit shown in Figs. 1 and 2. The reduced neutron width given is $\Gamma_n^0=\Gamma_n/[E/(1\text{ eV})]^{1/2}$. The fit was terminated at 86 eV because of increasingly poor energy resolution although resonances above the region fitted had to be included to improve the fit at the ends. Parameters for these were taken from Derrien.[10] The widths Γ_{f1} and Γ_{f2} are for the two fission channels, assumed open. Only one channel was necessary for the 1^+ resonances.

IV. INTERPRETATION

Thirty-two of the 47 resonances included in the fit have been assigned to the 1^+ spin state, and the remaining 15 to the 0^+ spin state. The resonance at 47 eV has been assigned to the 0^+ state, which is in agreement with the assignment by Sauter and Bowman[15] rather than that by Cowan.[16] However, the fission width of this resonance is predominantly in a different channel from the other prominent 0^+ resonances, so that the mass distribution of fission fragments may be different. On the other hand, the resonance at 66.8 eV has been placed in the same channel as the 47-eV resonance, but was assigned 0^+ by Cowan. Table II gives the average parameters for each spin state. Since the level spacing should be proportional to $2J+1$, there should be three times as many $J=1$ levels as there are with $J=0$. The observed ratio is 2.1. Since the average width of the $J=1$ levels is less than 1/10 that of the $J=0$ levels, it is likely that many more of the 1^+ resonances were missed because of widths too narrow to be observed. If we assume that none of the 0^+ levels were missed then at least 30% of the 1^+ levels have not been found even though more have been observed than by previous workers.

TABLE II. Average resonance parameters of ^{239}Pu. D_{deduced} is the level spacing corrected for missed levels.

	$J^\pi=0^+$	$J^\pi=1^+$
No. of resonances	15	32
No. of fission channels	≥ 2	≥ 1
$\bar{\Gamma}_f$	1033±375 meV	66±16 meV
$\bar{\Gamma}_n^0$	0.83±0.3 meV	0.23±0.06 meV
D_{observed}	5.0±1.3 eV	2.3±0.4 eV
D_{deduced}	5.0±1.3 eV	1.7±0.5 eV
$10^4\bar{\Gamma}_n^0/D$	1.7±0.6	1.0±0.3

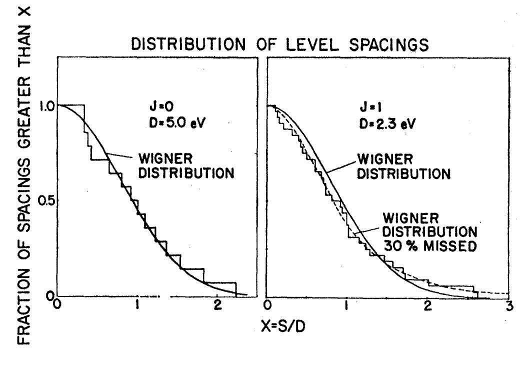

FIG. 3. The integral distributions of local level spacings for the two spin states. The solid curve on each side is the Wigner distribution. The dashed curve on the right is a Wigner distribution with 30% of the levels missed at random.

The errors of the average parameters for the 0^+ levels are large because of the small number of levels in the region considered. The number of fission channels necessary to fit the data and the average fission widths obtained are in agreement with current fission theory which predicts at least two open channels for the 0^+ spin state and one partially open channel for the 1^+ state.[19] The errors of the strength functions for the two neutron channels overlap so that it is not possible to determine if the strength function is spin-dependent.

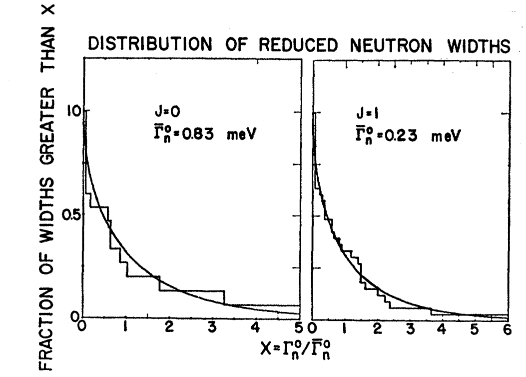

FIG. 4. The integral distributions of reduced neutron widths for the two spin states. The solid curves are Porter-Thomas distributions for one channel.

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[19] J. E. Lynn, in *Proceedings of the International Conference on the Study of Nuclear Structure with Neutrons, Antwerp, 1965* (North-Holland Publishing Co., Amsterdam, 1966), p. 205.

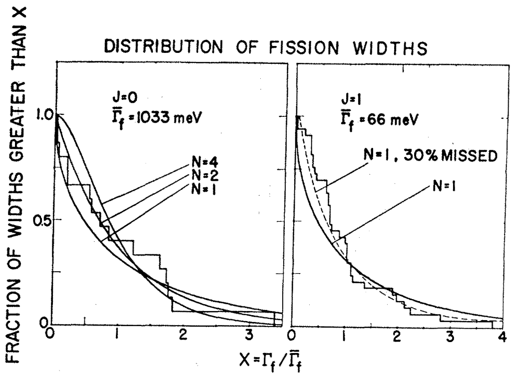

Fig. 5. The integral distributions of fission widths for the two spin states. Porter-Thomas distributions for 1, 2, and 4 open channels are plotted for the 0^+ state. The solid curve for the 1^+ state is the Porter-Thomas distribution for one channel. The dashed curve is truncated Porter-Thomas distribution with the 30% smallest widths left out.

Figure 3 shows the integral distributions of local level spacings for each spin state. The solid curve is the Wigner distribution.[20] The agreement is good for the 0^+ levels but there appear to be too few narrow spacings for the 1^+ group. However, if we correct the Wigner distribution for 30% of the levels missed at random, there is excellent agreement as shown by the dashed curve in Fig. 3. From this, we conclude that few, if any, 0^+ levels have been missed and that about 30% of the 1^+ levels remain unobserved.

Figure 4 shows the integral distributions of reduced neutron widths for the two spin states fitted with the Porter-Thomas distribution[21] for one channel. The agreement in both cases is very good which is somewhat surprising for the 1^+ levels since a sizeable number of them have been missed. Apparently the majority of these have not been missed because of small neutron width.

The integral distributions of total fission widths have been plotted in Fig. 5. For the $J=0$ levels, Porter-Thomas distributions for 1, 2, and 4 open channels have been plotted. There are too few resonances to make an accurate determination of the number of fission channels but the two-channel distribution appears to fit best. For the 1^+ levels, the solid curve is the Porter-Thomas distribution for one fission channel which fits poorly. The dashed curve is a truncated Porter-Thomas distribution assuming that the 30% narrowest levels have not been seen. The agreement in this case is much better but there is still a small excess of levels with widths less than the average which may indicate the presence of a second slightly open fission channel for the 1^+ state.

V. CONCLUSIONS

There are at least two open fission channels in the 0^+ spin state and one, possibly two, partially open channels in the 1^+ state. The presence of more than one open 1^+ channel would be difficult to determine as the fission widths are small and interference is less important than for the 0^+ spin state. Most of the 0^+ resonances have been observed but approximately 30% of the 1^+ resonances have been missed so that the true 1^+ level spacing is approximately 1.7 eV rather than the 2.3 eV observed. From the observed distributions of widths, it appears that many of these resonances have been missed because of small fission widths rather than small neutron widths as would be expected. This may be due to the fact that resonance with a very small neutron width can be detected from the interference of its fission width with neighboring resonances whereas those resonances with a small fission width would generally have a total width less than the experimental resolution and would probably not be seen.

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[20] E. P. Wigner, Columbia University Report No. TID-7547, 1957, p. 49 (unpublished).

[21] C. E. Porter and R. G. Thomas, Phys. Rev. **104**, 483 (1956).