The experimental results are displayed in Table I, with previous work on ^{98}Mo and the isobaric analog levels in ^{98}Tc included for comparison. In general, good agreement is noted. The presence of a state at 0.74 MeV, below the well-known 2^+ state at 0.78 MeV, is unexpected and very interesting. {*Note added in proof.* K. Hubenthal, J. Berthier, J.-C. Hocquenghem, and A. Moussa [Compt. Rend. **B265**, 162 (1967)] find that this state is involved in the β decay of the ground state of ^{98}Nb: $E_x=735$ keV, $J^\pi=0^+$, $\tau_{1/2}=22$ nsec.} The 0.74-MeV state was observed by Moore *et al.*[5] and apparently has an isobaric analog at 0.76 MeV in ^{98}Tc.

ACKNOWLEDGMENTS

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Neutron Total Cross Sections of Even Isotopes of Zr in the Energy Range 2–60 keV*

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The zirconium nuclei are located at the beginning of the region of masses, viz., about 90–130, in which the measured strength functions are anomalously small, as far as the original optical model is concerned. We have examined the neutron resonance structure of ^{90,92,94,96}Zr in the energy region 2 keV $<E_n<$60keV, and we find that the s-wave strength function, in the region in which the s waves can be identified, has a mean value of $(1.2\pm0.4)\times10^{-4}$. At energies roughly above 25 keV, the ratio of the mean resonance width at 1 eV to the mean resonance spacing rises to a value of $(2.5\pm0.6)\times10^{-4}$; the latter value can not be uniquely interpreted because s-wave identification is not positive. Level densities for nuclides with neutron numbers in the vicinity of the magic number 50 are considered.

I. INTRODUCTION

THE optical model for nuclear reactions supplemented with appropriate terms to take account of various deformations has had considerable success in describing and predicting average nuclear properties as they pertain to reaction cross sections. An exception has been the behavior of the neutron s-wave strength function in the mass region $90<A<130$, where optical models usually predict values in the range 0.6×10^{-4} to 1×10^{-4} expressed in the convention of the ratio of the average resonance width at 1 eV to the average resonance spacing. The experimental situation is that in the range $90<A<130$ the s-wave strength function has an average value of about 0.3×10^{-4}. To remove the discrepancy between theory and experiment, a number of revisions to the theory have been suggested.[1] While these have had some degree of success, the fact is that more experimental data are needed also.

Useful information for understanding this mass region of low observed s-wave strength functions could conceivably be obtained from those nuclides whose number of neutrons is just deficient or just in excess of filling the 50-particle major shell. Total cross-section measurements of the Zr isotopes of masses 90, 92, and 94 were attempted by Newson *et al.*[2] Resonance structure was observed, but the data were difficult to interpret and the conclusion was that the relatively rapid rise in average cross section was in large measure due to p waves with a strength function $\approx4.5\times10^{-4}$ and that the s-wave strength function was probably very small.

At the Gattlinburg Conference on Compound States[3] we reported measurements on ^{90,92,94,96}Zr in the energy range 2.5–40 keV. These measurements by their very appearance indicated that the principal resonances in

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[2] H. W. Newson, R. C. Block, P. F. Nichols, A. Taylor, A. K. Furr, and E. Merzbacher, Ann. Phys. (N. Y.) **8**, 218 (1959).

[3] J. A. Biggerstaff, W. M. Good, and H. J. Kim, Bull. Am. Phys. Soc. **9**, 167 (1964).

the energy region below about 25 keV were s wave in character and that the strength function was about 1×10^{-4}. This value was at least twice larger than expected on the basis of other masses in the neighborhood of 90, although an earlier indication that the strength functions for Zr might be larger than expected had been obtained by Block.[4] who found a value of $(0.7\pm0.2)\times10^{-4}$ in the energy region up to 15 keV. About the same time that we reported our results, Moskalev *et al.*[5] reported measurements on the isotopes of Zr; there was generally good agreement between the results quoted at the compound states[3] conference and those of Moskalev.

This paper reports the results of the measurements already referred to on ^{90,92,94,96}Zr, as well as on subsequent measurements which were made to improve statistics and also to extend the energy range to 60 keV.

II. DESCRIPTION OF THE EXPERIMENT

The Oak Ridge National Laboratory pulsed Van de Graaff, which has been used for these experiments, has a precision in energy of 1–2% in the energy range 2.5–60 keV, achieved by increasing the distance in steps as the energy increases so that the flight time remains approximately constant. The resolution figure (time resolution/flight path) decreases typically from about 8 nsec/m to about 2 nsec/m in the course of a set of measurements. This resolution is adequate for the study of some of the properties, notably the strength function, associated with resonances in nuclei whose resonant level spacings are of the order 1–2 keV or more.

Samples for this experiment were available only in the form of oxides because the technique for conversion of Zr to metal has not proved successful on small quantities of material. Each nuclide was fabricated into right-circular cylinders. For the nuclides of masses 90, 92, 94, each cylinder was 32 mm in diam and consisted of two pieces whose thickness ratios were about 3:2, to permit measurements at thicknesses corresponding to reciprocal atom/area of about 30 and 50 b/atom. The isotope of mass 96 was available only in limited quantity with the result that in choosing a suitable thickness corresponding to about 48 b/atom, it was necessary to accept a sample diameter of 12 mm.

In an average time of about 10 h, 2000 counts per 2-nsec time interval can be acquired. The accumulated number of flight-time intervals are stored in a 2048-channel or a 4096-channel analyzer. The data are then converted to energy and cross sections by means of a computer which simultaneously prepared a tape by means of which a Calcomp plotter can plot either the cross section, or the transmission-versus-neutron energy. From the various flight paths, a single σ_T-versus-E_n plot is assembled which is composed of energy segments each containing the highest resolution results for a given energy interval. Such a plot is a valuable check on the correctness of the background subtractions. The data corresponding to the σ_T-versus-E_n plot are finally replotted in the form of transmission-versus-neutron energy. From the transmission-versus-E_n plots, areas are obtained and resonance widths extracted by means of area analysis.[6]

III. RESULTS

Figures 1(a)–1(d) show the total cross sections of the even isotopes of zirconium. Table I is a summary of the observed levels and their widths. Above about 25 keV the ability to distinguish s-wave character is lost and hence all are tentatively assumed to be s wave. Unless otherwise stated, the level positions can be assumed to have an accuracy of about 2% or better. The measurements of the two sample thicknesses provided two independent measurements of level width for a number of resonances. These exhibited a standard error of about 20% which we take to be a more or less representative figure. Also given in Table I for comparison are the corresponding results obtained by Moskalev *et al.* With few exceptions the agreement is good.

The quantities of greatest physical significance in these measurements are the strength function and mean level spacing. These are summarized in Table II, where the strength functions are quoted on energy intervals roughly the same as those used by Moskalev *et al.*; the errors in the strength function are appropriate to Porter-Thomas distribution of widths and Wigner distribution of spacings. Within these error limits our values of strength functions agree with those of Moskalev *et al.*, and the mean s-wave strength function for all four isotopes is $(1.2\pm0.4)\times10^{-4}$. Beyond about 25 keV there is an apparent rise in the s-wave strength function to a mean value of about 2.5×10^{-4}, which, since the s and p waves are not clearly distinguished, can not be uniquely interpreted.

The 5th column in Table II gives the mean level spacing as determined in this experiment from plots of the accumulated number of resonances versus energy. Since the transmission factor for p-wave neutrons rises approximately linearly with energy, the p-wave cross section can be expected to rise from a few percent of that of the s wave at 2 keV to perhaps 65% at 80 keV. This fact implies that the mean level spacing should decrease with energy as the p-wave resonances gradually grow into prominence. The level spacings observed in this experiment show no such definite trend, which is likely a consequence of the fact that the small s-wave levels missed due to resolution loss are compensated by the emergence of the larger p-wave resonances. Thus, the mean level spacings deduced on the energy intervals shown in Table II may seem arbitrary, but in point of

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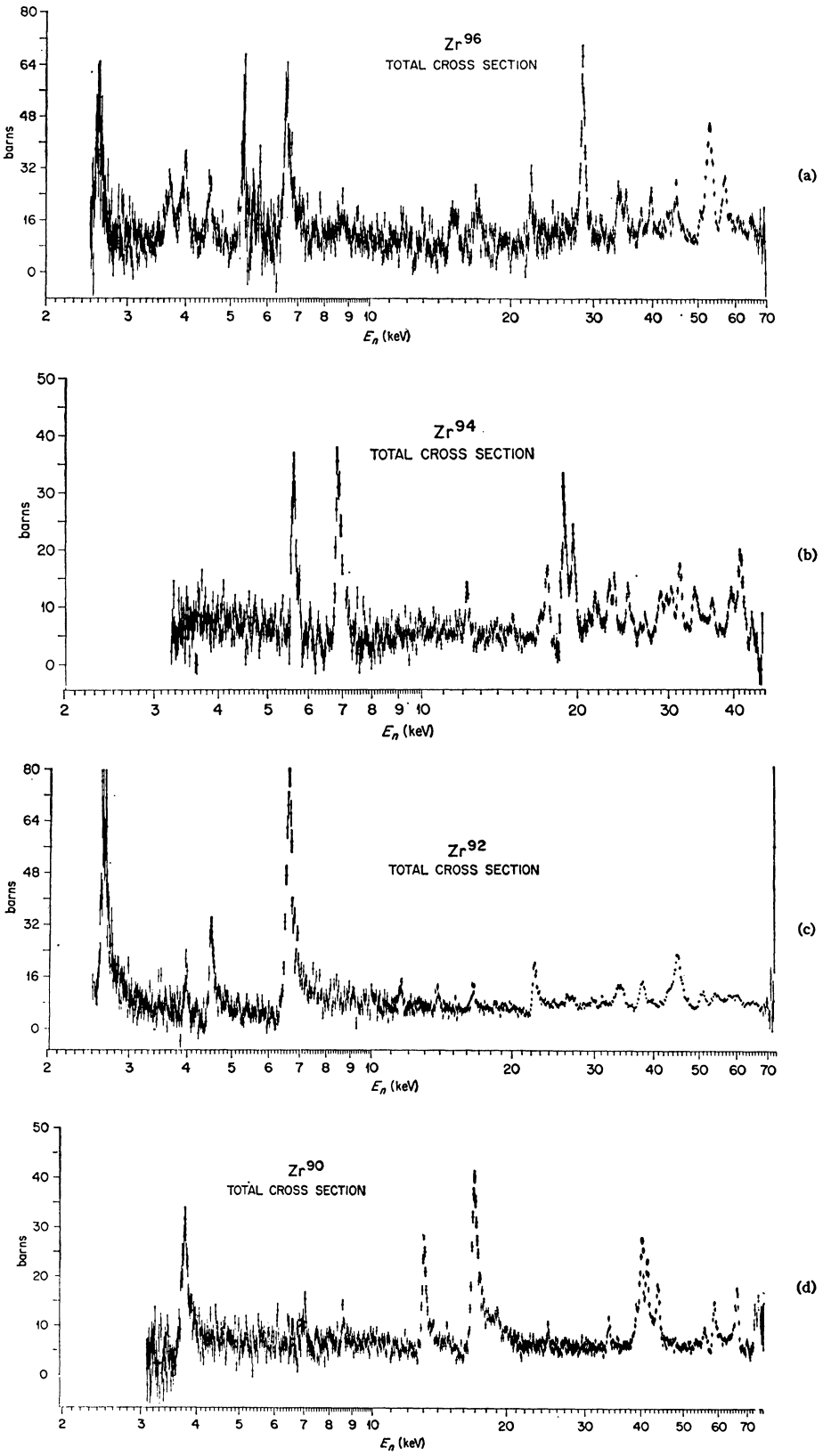

FIG. 1. Total neutron cross sections of the even isotopes of zirconium.

TABLE I. Summary of zirconium resonance energies, neutron widths, and reduced widths.

	E_0 (keV)		Γ_n (eV)		Γ_n^0 (eV)	
	Present expt.	Ref. 5	Present expt.	Ref. 5	Present expt.	Ref. 5
^{90}Zr	3.8 ±0.06	3.91±0.025	12±2	13.6±0.5	0.20±0.07	0.22 ±0.08
	6.9 ±0.03		8±1		0.10±0.02	
	7.9 ±0.05		9±1		0.10±0.02	
	13.3 ±0.1	13.67±0.16	69±9	65 ±15	0.60±0.08	0.56 ±0.13
	17.2 ±0.1	17.85±0.25	205±14	200 ±70	1.6 ±0.1	1.50 ±0.52
	34.7 ±0.1		89		0.84	
	40.1 ±0.1		94		0.47	
	41.6		362		1.79	
	42.4 ±0.2		342		1.68	
	44.1 ±1		320		1.03	
	56		126		0.54	
	59		310		1.29	
	68		563		2.20	
^{92}Zr	2.7 ±0.06	2.72±0.02	23±4	14 ±2	0.48±0.1	0.28 ±0.04
	4.10±0.06	4.15±0.03	5±2	3.5±0.5	0.07±0.02	0.055±0.008
	4.71±0.05	4.66±0.03	16±3	10 ±4	0.25±0.04	0.15 ±0.06
	6.88±0.02	6.88±0.06	78±9	80 ±10	0.94±0.1	0.96 ±0.11
	12.1		25		0.23	
	14.5		38		0.32	
	17.2		38		0.30	
	23.1		121		0.81	
	27.0					
	31.2					
	34.3		217		1.19	
	39.3		208		1.07	
	44.7		186		0.90	
	46.8		607		2.70	
	52.0					
^{94}Zr	5.7 ±0.1	5.87±0.06	22	27 ±7	0.29	0.35 ±0.09
	6.98±0.2	7.22±0.06	30	72 ±8	0.37	0.85 ±0.10
	12.7	12.80±0.16	20	20 ±7	0.19	0.175±0.06
	15.5		10		0.08	
	17.7		16		0.12	
	17.95±0.2		54±8		0.41	
	19.4 ±0.5	19.86±0.27	107±9	125 ±40	0.78±0.09	0.9 ±0.3
	20.2 ±0.2		85		0.61	
	21.7					
	23.1		50		0.32	
	23.7		43.5		0.28	
	25.1		57		0.36	
	27.0					
	29.0		57		0.34	
	29.8		57		0.34	
	30.2		78		0.45	
	31.8		134		0.75	
	33.8		130		0.71	
	36.3		69		0.36	
	40.0		145		0.73	
	41.6		267		1.29	
	43.7					
^{96}Zr		2.38±0.015		1.5±0.3		0.03 ±0.006
	2.68		4		0.08±3	
	3.85±0.06	3.84±0.04	7±1	4.5±1	0.12±0.01	0.07 ±0.016
	4.11±0.02	4.15±0.03	13±1	12.5±2	0.19±0.03	0.195±0.032
	4.61±0.01		6±2		0.08±0.04	
	5.48±0.1	5.51±0.05	15.6±3	16.0±8	0.215±0.05	0.215±0.110
	5.84±0.07		4		0.05	
	5.97		4		0.05	
	6.80		0		0	
	8.96					
	15.45					
	17.51					
	22.76		46		0.3	
	29.4		402		2.39	
	34.5		70		0.4	
	36.1		93		0.50	
	37.2		43		0.23	
	38.9		96		0.49	
	40.7		180		0.91	
	45.8		96		0.5	
	51.6		60		0.25	
	54.2		1170		5.11	
	58.3		558		2.35	

fact in interpreting the mean spacing from the plots of accumulated resonances versus energy, the energy regions known to be s wave were weighted most heavily. The mean level spacings in Table II show general agreement between the present experiment and that of Moskalev *et al.* The biggest discrepancy is that of ^{92}Zr, in which case it will be seen from Fig. 1, in an energy interval which includes an s-wave resonance at 23 keV, that the spacing is more nearly of the order of 2.5 keV, even if three resonances that may be p wave are included. A bigger uncertainty actually exists in the case of ^{94}Zr, where the mean spacing quoted for this experiment is calculated on the interval 3–40 keV, and in this interval some p-wave resonances are no doubt included.

The strength function, since it is the ratio of an integrated width divided by an energy interval, is not critically dependent upon small resonances that can be readily missed. However, meaningful distribution functions on widths and spacings, and indeed the mean spacing itself, depend upon not missing a significant number of levels. The energy region common to the two experiments listed in Table I certainly contains only s-wave resonances. The distributions on reduced width amplitudes and nearest-neighbor spacings for the resonances on these energy intervals are shown in Figs. 2 and 3, respectively.

IV. DISCUSSION

The S-Wave Strength and Distribution Functions

The results of these measurements give for the s-wave strength function of Zr an average value of $(1.2\pm0.4)\times10^{-4}$ near neutron dissociation energy. If one considers all values of the strength function which the various optical models predict for the mass-90 region, one will find that they range from 0.05×10^{-4} to 1×10^{-4}. Moldauer,[1] for example, employs a surface absorption outside the nuclear surface and obtains 0.15×10^{-4}; Krueger and Margolis,[1] and also Fredeley and Frahn,[1] concentrate the absorption on the surface and obtain

TABLE II. Summary of zirconium s-wave strength functions and mean level spacings.

Isotope	ΔE (keV)	$S_n^0=\sum\Gamma_n^0/\Delta E$ (10^{-4}) Present expt.	Ref. 5	$\bar{D}$ (keV) Present expt.	Ref. 5
^{90}Zr	0 –28	0.93±0.6	0.85±0.65		4.5±1.6
	28 –70	2.46±1.2			
	0 –70			5.0 ±1.0	
^{92}Zr	0 –12	1.5 ±0.9	1.2 ±0.8		1.2±0.4
	12 –52	1.82±0.9		3.3 ±0.7	
	0 –52				
^{94}Zr	0 –21	1.1 ±0.8	1.1 ±0.8		2.4±0.9
	21 –44	2.84±1			
	0 –44			1.7 ±0.3	
^{96}Zr	0 –5.8	1.0 ±0.6	0.9 ±0.6	0.750±0.2	1.0±0.3
	5.8–58	2.62±1			

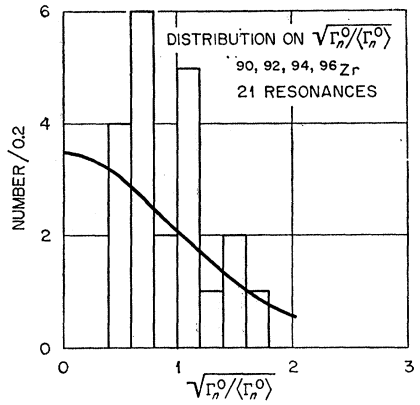

FIG. 2. Distribution of the reduced width amplitudes for the even isotopes of zirconium over the energy region of known s-wave resonances.

0.2×10^{-4}. Elagin, however, finds that the low values that these authors obtain are only partly attributable to the surface concentration of the absorptive part of the potential, the remainder being due to differences in the actual form of the potential. Lane *et al.*[1] suggested that the absorptive part of the potential could be expected to fluctuate in magnitude and were actually first to propose that it might actually, under certain circumstances, concentrate on the surface, thus causing the strength function to fluctuate with mass. In this connection, Moldauer observed that near mass 100, the strength function for the case of the even target nuclides might be expected to have rather larger values than neighboring odd nuclides. In another context, Feshbach and Block[1] extended the optical model of the strength function to include shell effects, and by means of semiempirical approximations arrived at strength-function values of about 0.3×10^{-4} for the mass-100 region. The original spherical model with Saxon potential gave values ranging from 0.6×10^{-4} to 1×10^{-4}, depending upon choice of parameters.

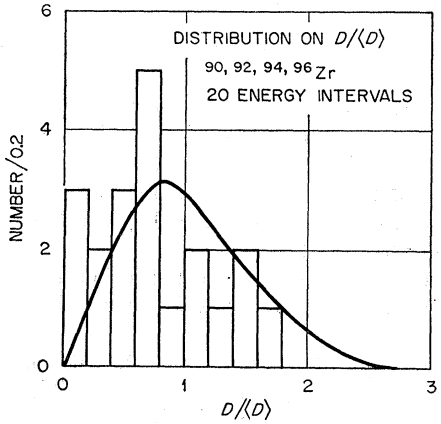

FIG. 3. Distribution of nearest-neighbor spacings for the even isotopes of zirconium over the energy region of known s-wave resonances.

TABLE III. Table comparing the experimentally observed mean level spacing in selenium and zirconium with various theories of level density.

1	2	3	4	5	6	7	8	9	10	11
Nuclide	N	$\bar{D}(U)$ (keV)	Binding energy (MeV)	U (MeV)	$\bar{D}(6)$ (keV)	$350/n_z n_N$ (keV) Ref. 17, 18	a (MeV^{-1}) expt.	a (MeV^{-1}) Refs. 12, 13	a (MeV^{-1}) Ref. 14	a (MeV^{-1}) Ref. 15
^{77}Se	43	1.3 ±0.35	7.4	6.2	1.6	1.3	11.8	10.5	12.4	
^{79}Se	45	4.2 ±1	6.9	5.7	3.0	1.9	11.0	10.7	12.3	12
^{81}Se	47	4.2 ±1	6.8	5.6	2.7	3.6	11.1	10.7	11.4	11.4
^{83}Se	49	7.1 ±2.5	6.7	5.5	4.2	9.2	10.8	10.3	9.9	9.4
^{91}Zr	51	5.0 ±1	7.2	6.0	5.0	5.1	10.4	9.0	9.9	11.3
^{93}Zr	53	3.3 ±0.7	6.7	5.5	1.8	1.6	11.8	7.9	11.3	12.3
^{95}Zr	55	1.7 ±0.3	6.4	5.2	0.7	0.7	13.5	8.7	12.9	12.3
^{97}Zr	57	0.75±0.2	5.6	4.4	0.01		17.0	9.8	14.2	

^{93}Nb has been carefully studied[7,8,9] and its s-wave strength function is $(0.36\pm0.06)\times10^{-4}$. Unfortunately, ^{89}Y has not been so carefully investigated and an s-wave strength-function value of $(0.15\pm0.75)\times10^{-4}$ for this nuclide leaves the experimental situation at mass 90, and especially below, still in doubt.

At neutron energies above about 20 keV, s-wave resonances can not in general be positively identified. Thus, the rise in the apparent s-wave strength function from about 1.2×10^{-4} to 2.5×10^{-4} can not be rigorously interpreted. There are two limiting situations for which the variation in apparent strength function can be interpreted with results of some interest: viz., under the assumption that the s-wave strength function of 1.2×10^{-4} is constant, and that the p-wave strength function has the value 4.5×10^{-4}. In either case, the widths derived assuming s waves are used to estimate the p-wave widths. In the latter case the area analysis graphs give approximately $g\Gamma_n$ rather than Γ_n for a given value of $\sigma_0=4\pi\lambda\!\!\!^{-2}$. Furthermore, the p-wave strength function $S_n{}^1$ is given by

$$S_n{}^1=[\Delta E(2\cdot1+1)]^{-1}\sum_r(g\Gamma_n{}^{(1)})_r,$$

in which

$$\Gamma_n{}^{(1)}=\Gamma_r E_r{}^{-1/2}(k_rR)^{-2}[(k_rR)^2+1].$$

If the s-wave strength function is taken to have a value of about 1×10^{-4}, independent of energy, then on the 30-keV energy interval centered at about 35 keV the p-wave strength function has a value $(10\pm4)\times10^{-4}$. In the other situation, if the p-wave strength function is assumed to have the reasonable value of 4.5×10^{-4}, then $S_n{}^0$ rises to a value $(1.8\pm0.7)\times10^{-4}$ in an energy range of about 29 keV.

As mentioned earlier, the distributions on widths and spacings depend more critically upon the number of missed levels of a given spin and parity than does the strength function. While Porter-Thomas and Wigner distributions have been strongly substantiated in general, such behavior has not been extensively studied in cases of nuclides so special as the isotopes of Zr.

The number of levels observed in these measurements of known spin and parity are too few, and indeed it may be said that the data are not sufficiently good to open the question of what distribution functions are obeyed best. However, if the separate distributions for the individual isotopes are combined, as in Figs. 2 and 3, the resulting distributions are consistent with the Porter-Thomas on reduced widths and, with some reservations, with the Wigner distribution on spacings. A reasonable estimate is that levels might start to be lost when $\Gamma_n{}^0<0.25\langle\Gamma_n{}^0\rangle$; Fig. 2 is consistent with this. Figure 3 can be used to argue that the pairs of levels; 4.1 and 4.7 keV in ^{92}Zr, 17.7 and 17.95 keV in ^{94}Zr, and 5.84 and 5.95 keV in ^{96}Zr which define the small intervals that are clearly (according to the Wigner distribution) in excess number, each contain at least one level that is mistakenly assigned—probably as regards parity.

The Average Level Spacing

The isotopes of Zr combined with the isotopes of Se[10] afford an account of the level spacings for neutron numbers 43, 45, 47, 49, 51, 53, 55, and 57 based upon proton numbers 34 and 40. Since the target spins are all zero, these nuclei present a favorable situation to study level spacings in the vicinity of the magnetic number 50. Table III is a summary of the data and some comparisons with various theories which purport to give an account of level spacings in the neighborhood of a closed shell.

The various level-spacing theories are reviewed by Ericson.[11] In the region between magic numbers, a rather satisfactory account of level spacing is given in terms of a degenerate gas model. For this model, the level *density* for both parities of level is given by an

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[10] H. Munzer, S. Nishimura, and W. M. Good (unpublished).
[11] T. Ericson, Phil. Mag. Suppl. 9, 425 (1960).

expression of the form

$$\rho(UJ)=\tfrac{1}{12}\sqrt{\pi}\frac{e^{2\sqrt{(aU)}}}{a^{1/4}U^{5/4}}\frac{(2J+1)}{2\sigma^3\sqrt{(2\pi)}}e^{-(2J+1)^2/8\sigma^2},$$

where U is the binding energy, less the pairing energy; a=density of single particle states; $\sigma^2\approx 0.09$ at $A^{2/3}$; $t\approx(U/a)^{1/2}$. Hence

$$\rho(UJ)=1.1(2J+1)(a/A)[e^{2\sqrt{(aU)}}/(aU)^2],$$

in the present case where $J=\frac{1}{2}$. For s-wave neutrons, half this value is to be used in comparing with experiment since only one parity is observed and both parities are assumed equally probable in the formula for $\rho(UJ)$.

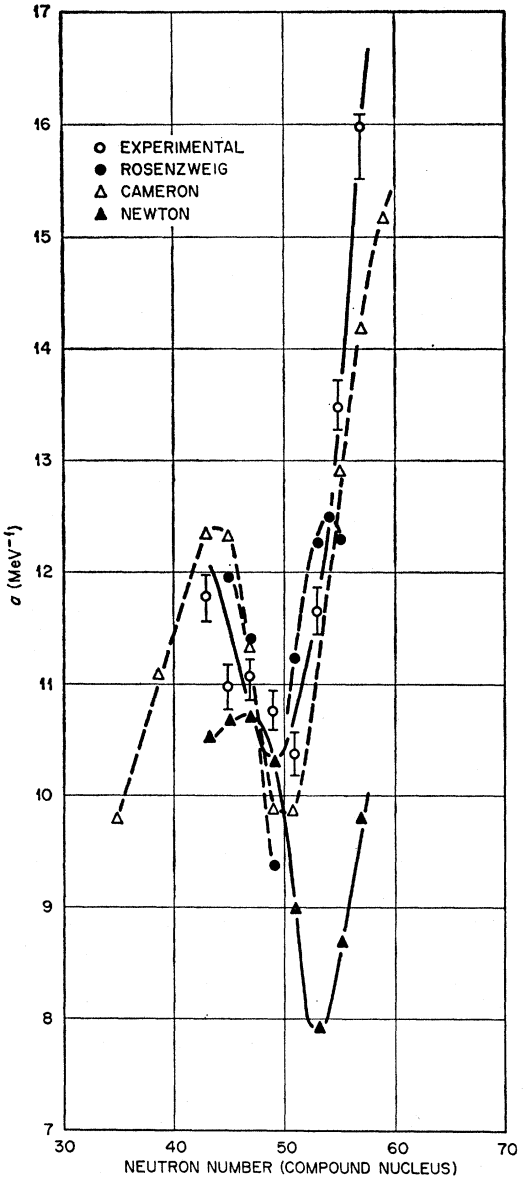
The quantity a, the density of individual particle states, is expected to decrease in the vicinity of closed shell where shell-model effects are most pronounced. There are basically three approaches that have been taken to predict such behavior. These give for a, respectively,

(a) $a\approx 0.0748\,(\vec{j}_n+\vec{j},+1)A^{2/3}$, according to Newton[12] and Lang[13];

(b) $a=A[0.01S+0.14]$, in the treatment by Gilbert and Cameron,[14] whereby the shell correction to the mass formula, like the pairing correction, is included in the excitation energy to give for the argument of the radical in the exponent an expression of the form aU;

(c) $a\approx(\pi^2/6)(g/U)[U/g+\frac{1}{12}g-(1/2g)(n-\frac{1}{2}g)^2]$, if the argument in the exponential of the expression derived by Rosenzweig[15] for level density is rearranged in the form aU.

The values of $\vec{j}$ for use in the first expression are given by Klinkenburg.[16] Use of the second expression necessitates knowing S, and tables for calculating this quantity are given by Gilbert and Cameron.[14] Rosenzweig's calculation shows that the degree of filling of the ground-state shell of degeneracy $g=2j+1$ can significantly influence the spacing of energy levels, even at high excitation. In order to apply his result we have assumed that the effect is significant only in the final filling of the $g_{9/2}$ shell and the initial filling of the $g_{7/2}$ shell which lie just inside and just outside the major 50-particle shell. In the region of mass where Rosenzweig's[15] expression may be expected to be valid, a has its maximum value when the shell in question is half-filled, i.e., $(n-\frac{1}{2}g)=0$. If at such masses we assume that a takes its conventional gas model value,



Level Density Parameter a for ^{77,79,81,83}Se and ^{91,93,95,97}Zr.

FIG. 4. Observed level-density parameter a compared with theories of Cameron, Rosenzweig, and Newton.

we get

$$a\approx A/12-(0.21/U)(2n-g)^2,$$
for $N<50$; $g=10$, $n=5, 3, 1$.
$$a\approx A/13-(0.21/U)(2n-g)^2,$$
for $N>50$; $g=8$, $n=1, 3, 5$.

The values of a for the isotopes of Se and Zr according to the three expressions just given are listed in columns 9, 10, and 11, respectively, of Table III. The experimental values of a that are to be compared with theory

[12] T. D. Newton, Can. J. Phys. **34**, 804 (1956).
[13] D. W. Lang, Nucl. Phys. **26**, 434 (1961).
[14] A. Gilbert and A. G. W. Cameron, Institute for Space Studies, Goddard Space Flight Center, NASA Report (unpublished).
[15] N. Rosenzweig, Phys. Rev. **108**, 817 (1957); *Conference on the Study of Nuclear Structure with Neutrons, Antwerp, 1965* (North-Holland Publishing Company, Amsterdam, 1966).
[16] P. F. A. Klinkenberg, Rev. Mod. Phys. **24**, 63 (1952).

are given in column 8 of Table III. The extraction of a from the expression for s waves

$$\rho(UJ)=1.1\frac{a}{A}\frac{e^{2\sqrt{(aU)}}}{(aU)^2}=\frac{1}{D_{\text{obs}}}$$

was accomplished by trial and error assisted by a graphical plot of $1/D_{\text{obs}}$ versus a.

Finally an entirely different model for level density is proposed by Newson and Duncan[17] in which unbroken pairs of like particles, excited together, are presumed to be principally responsible for the states seen in resonance. In the neighborhood of shell closure, these states can be counted to give a spacing

$$D_0\approx S'/n_Nn_Z,$$
$$D_0=2(2I+1)D_{\text{obs}},$$

in which S' is the pair spacing and n_Nn_Z are the number of arrangements of the indistinguishable neutron and proton pairs. Although the model has been applied by these authors most extensively to the odd-odd and even-even compound nuclides, it is interesting to inquire what the model predicts in the case of even-even target nuclides for the relative dependence of level spacing on mass as compared with measurement. Column 7 of Table III lists 350 keV/n_Zn_N. In this expression, the significant numbers are n_Nn_Z which are given by Newson,[18] and the parameter 350 keV is empirically chosen so that theory and experiment agree at ^{91}Zr. The mass dependence for the spacing predicted by the model is compared with observation. For this comparison tabulated in Table III, it is necessary to correct all observations to a common excitation energy arbitrarily chosen to be 6 MeV. Correction to 6-MeV excitation was accomplished by means of the formula

$$D(6)=D_{\text{obs}}e^{2(\sqrt{a})(\sqrt{U}-\sqrt{6})}/(U/6)^2$$

with $a=\frac{1}{8}A$; spacings at 6 MeV, $D(6)$, are given in column 6 of Table III. In Fig. 4 the experimental and theoretical values of a are shown as a function of neutron number. The data given in this paper were used by Gilbert and Cameron in arriving at their semiempirical formula. The discrepancy that appears in Fig. 4 between their theory and our experiment is a consequence of their "smoothing" to average out fluctuations.

[17] H. W. Newson and M. M. Duncan, Phys. Rev. Letters 3, 45 (1959).
[18] H. W. Newson (private communication).

V. CONCLUSIONS

We believe that our observed s-wave strength-function value of $(1.2\pm0.4)\times10^{-4}$ confirms earlier findings that the zirconium nuclei do not have the small s-wave strength functions which nuclides in the mass-90 region seem generally to possess. Inadequate resolution at energies above about 20 keV prohibited positive identification of the corresponding s-wave resonances; therefore, the apparent rise in the s-wave strength function could not be uniquely interpreted. Extreme alternatives in interpretation are that the p-wave strength function is 50–100% larger than the 4.5×10^{-4} usually assumed, or the s-wave strength function increases with energy. The densities of $\frac{1}{2}^+$ levels near neutron dissociation energy in the even-odd nuclides that span the 50-neutron shell are reasonably well described by recent level-density formulas; further study is needed to be sure however, especially in the case of ^{96}Zr. Because the Zr nuclides have neutrons which progressively fill the 50-shell, because these nuclides have large s-wave strength functions with respect to neighboring nuclides, and because mass 90 is in the region of resonating p waves whose resonances emerge strongly, the inadequacies of the present experiment emphasize the need for further measurements at significantly better resolution.

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