interest that a recent value[16] for the reduced transition probability of the 3.90-MeV $2^+ \rightarrow 0^+$ transition in Ca^{40} is also smaller, by a factor of about 2, than the corresponding $B(E2)$ value reported by Ref. 3.

On the theoretical side, Gerace and Green[17] considered the lowest even-parity states in Ca^{40} as mixtures of the doubly closed $2s$-$1d$ shell-model state with intrinsic two-particle–two-hole and four-particle–four-hole states. For the 6.91-MeV $E2$ ground-state transition, they predicted a $B(E2)$ of $\simeq 7e^2$ F^4, i.e., just about half the experimental value $B(E2\downarrow)=(14.1\pm2.4)e^2$ F^4 determined in this paper. It might be mentioned, that the experimental $B(E2)$ value is 1.7 times the single-particle estimate.[14] This enhancement factor is the same as that observed for the 3.90-MeV $E2$ transition in Ca^{40} and for the 6.92-MeV $E2$ transition in O^{16}.

[16] J. R. MacDonald, D. F. H. Start, and R. Anderson, private communication, quoted by W. J. Gerace, Ph.D. thesis, Princeton University, Technical Report No. PUC-937-264, 1967 (unpublished).

[17] W. J. Gerace and A. M. Green, Nucl. Phys. **93**, 110 (1967).

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Inelastic Scattering of 42-MeV Alpha Particles by Mg^{24}†

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Angular distributions for elastic and inelastic scattering of 42-MeV α particles by Mg^{24} have been measured in the range $\theta(\text{lab})=10°$–$60°$. Eight inelastic α groups were identified with the following $-Q$ values: 1.37, 4.16 (doublet), 5.22, 6.005, 6.44, 7.35, 7.62, and 8.37 MeV. Two theoretical models, the distorted-wave Born-approximation version of the extended optical model and the smoothed Fraunhofer inelastic diffraction model, have comparable degrees of success in fitting the observed angular distributions. Further, extracted values of the deformation distances $\delta_\lambda \equiv \beta_\lambda R_0$, which parametrize the nuclear matrix elements, show good agreement between the two methods of analysis for low Q values and angular momentum transfer; as these quantities increase, however, the smoothed Fraunhofer model tends to underestimate δ_λ by as much as 30%. The present results on the nature and strength of the transitions to the positive-parity levels at 1.37, 4.16 (doublet), 5.22, 6.005, 6.44, and 7.35 MeV are in agreement with the broad features of the rotational-vibrational collective-model predictions for these levels, but not with its literal interpretation in terms of macroscopic quadrupole surface excitations of the nuclear intrinsic states. The present results indicate the presence of the two 3^- levels, strongly collective in nature, at 7.62 and 8.37 MeV. Comparisons are made with other experiments of similar nature on the Mg^{24} nucleus.

I. INTRODUCTION

THIS is the first of a set of papers devoted to the elastic and inelastic scattering of 42-MeV α particles from isotopes which lie in the middle of the s-d shell.[1–3] In this paper, the general experimental procedures (Sec. II) and methods of theoretical analysis (Sec. IV) are discussed, as well as scattering from the specific isotope Mg^{24} (Sec. V). Scattering from Mg^{25}, Mg^{26}, and Al^{27}, as well as the over-all systematics of the observations, will be dealt with in later publications.

The inelastic scattering of medium-energy α particles has been a valuable tool for studying discrete nuclear levels. The shapes of the angular distributions yield spin-parity assignments, and the magnitudes of the cross sections provide measures of the transition strengths from the ground state. The middle of the s-d shell is a particularly fertile ground for investigation. The level densities are low enough so that levels even at excitation energies of the order of 8 MeV may be resolved with the aid of solid-state detectors. Further, simple collective models have been successful in describing many features of the low-lying levels of these nuclei, and such models are aptly tested through inelastic scattering experiments.

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[1] Portions of this work are contained in the Ph.D. thesis of I. M. Naqib at the University of Washington (unpublished).

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[3] I. M. Naqib and G. W. Farwell, Bull. Am. Phys. Soc. **8**, 318 (1963).

II. EXPERIMENTAL PROCEDURES

1. General

The external 42-MeV α beam of the University of Washington cyclotron was used in conjunction with the 60-in. scattering chamber.[4] The beam was collimated upon entering the scattering chamber, so that the scattering source was less than $\frac{1}{16}$ in. in width and about $\frac{3}{8}$ in. high. Normalization of the number of incident α particles was made through measurement of the integrated charge collected on a Faraday cup. Relative normalization was also obtained at the same

[4] See, for example, R. E. Brown, J. S. Blair, D. Bodansky, N. Cue, and C. D. Kavaloski, Phys. Rev. **138**, B1394 (1965).

time by means of a fixed monitor counter mounted at a fixed angle relative to the beam.

The scattered particles were detected by a single silicon-diffused junction detector for which the depletion thickness was maintained slightly above the range of 42-MeV α particles in silicon. This choice of detector thickness (and the fact that α-induced reactions producing heavy ionizing particles like He^3, Li^6, or Li^7 have high negative Q values) provided natural bias against all particles other than α's in the energy range of interest. Thus no particle identification system was needed.

The over-all energy resolution of the detector was about 140 keV (FWHM) for scattering angles below 30° and increased up to about 300 keV for angles near 60°. This increase was due to the finite width of the counter acceptance angle, which was about 1°.

The Q values of the various excited nuclear levels investigated in this experiment were measured directly from the pulse-height spectra obtained with the solid-state detector; a typical accuracy of $\pm 1\%$ was achieved. Since, for each element investigated, the Q value of at least one strongly excited level is known[5] with high precision, it was possible to calibrate each pulse-height spectrum independently and with good accuracy. The final Q-value measurement for each level was obtained as an average of several measurements corresponding to different scattering angles.

The errors in the measurement of relative cross sections were caused mainly by the uncertainty in the number of background counts which had to be subtracted from each peak, and also by statistical uncertainties. Normally the statistical standard deviation was reduced below $\pm 8\%$ for the weakest investigated α group in each spectrum. The over-all uncertainties in the relative cross sections are indicated by error bars for typical points in each angular distribution presented in this work. The accuracy in the scales of absolute cross sections is generally better than $\pm 10\%$. The main difficulty in the determination of absolute cross section was associated with charge integration and not target-thickness measurements.

2. Beam Energy

The spread in beam energy was about 200 keV and was mainly due to target thickness; the inherent spread in the analyzed and collimated beam incident on the target was only about 50 keV.

A simple method was adopted by which the energy of the collimated beam could be measured during each run without alteration of the experimental arrangement used for the (α,α') experiments.[6] The accuracy achieved was better than ± 0.25 MeV, and only 1 to 2 h of operating time were required for the measurements. The method consists of finding a pair of scattering angles (θ_1,θ_2), such that the α particles elastically scattered by a thin C^{12} target at θ_1 have the same pulse height as the inelastic α's ($Q=-4.433$ MeV) scattered at θ_2. For each pair (θ_1,θ_2) we have the equality

$$E_3(Q=0,\,\theta=\theta_1,\,E_1)=E_3(Q=-4.433,\,\theta=\theta_2,\,E_1)\,, \quad (2.1)$$

where E_1 and E_3 are the energies of the incident and scattered particles, respectively. Since the explicit form of the function $E_3(Q,\theta,E_1)$ is known,[7] the solution of this equation can be obtained numerically or graphically and yields the value of the incident beam energy E_1.

In order to maintain a constant energy of the collimated α beam throughout the experiment, the duct slit opening was narrowed to less than 1 mm in width and was kept at a fixed lateral position. The analyzing magnet field was also kept at a fixed value within $\pm 0.1\%$. Further, the same beam-collimating system was used in all the runs. It was found that the beam energy under these conditions was always very close to an average value of 42.0 MeV; the deviations from this value were well within the experimental error of the measurement. The present result was also supported by measurements obtained with a broad-range magnetic spectrometer.

3. Targets

A self-supporting natural magnesium (78.7% Mg^{24}, 10.13% Mg^{25}, and 11.17% Mg^{25}) foil (1.1 mg/cm^2) was used during most of our runs. Over-all nonuniformity in target thickness was about 4%; this was measured by recording the changes in the energy of α particles from our α source when different parts of the targets were used as a degrader. Partial use was also made of an enriched Mg^{24} (99.98% Mg^{24}) target[8] which became available at the later stages of this work.

III. PULSE-HEIGHT SPECTRA OF Mg^{24}

Two typical pulse-height spectra of α particles scattered by a natural magnesium target are shown in Fig. 1. In addition to the elastic peak A, seven inelastic peaks (B, C, E, F, G, H, and I) have been investigated in the range $\theta(\text{lab})=10°$–$60°$. With the exception of the two components of the doublet C (4.16 MeV), all the peaks appear to be fully resolved. A list of the corresponding measured Q values is given in Table I. A diagram of the known low-lying excited levels in Mg^{24}, together with the known parity and spin assignments,[5] are shown in Fig. 2; the levels which correspond (or most probably correspond) to the investigated α groups A–I are indicated by the same letters. Definitive spin-parity assignments now exist[9] for many of the closely

[5] P. M. Endt and C. van der Leun, Nucl. Phys. 34, 1 (1962).

[6] We wish to thank D. Bodansky for suggesting this method to us.

[7] See, for example, J. B. Marion, T. I. Arnette, and H. C. Owens, Oak Ridge National Laboratory Report No. ORNL-2574, 1959 (unpublished).

[8] Obtained from Isotopes Sales Department, Oak Ridge National Laboratory, Oak Ridge, Tenn.

[9] R. W. Ollerhead, J. A. Kuehner, R. J. A. Levesque, and E. W. Blackmore, Bull. Am. Phys. Soc. 11, 64 (1966).

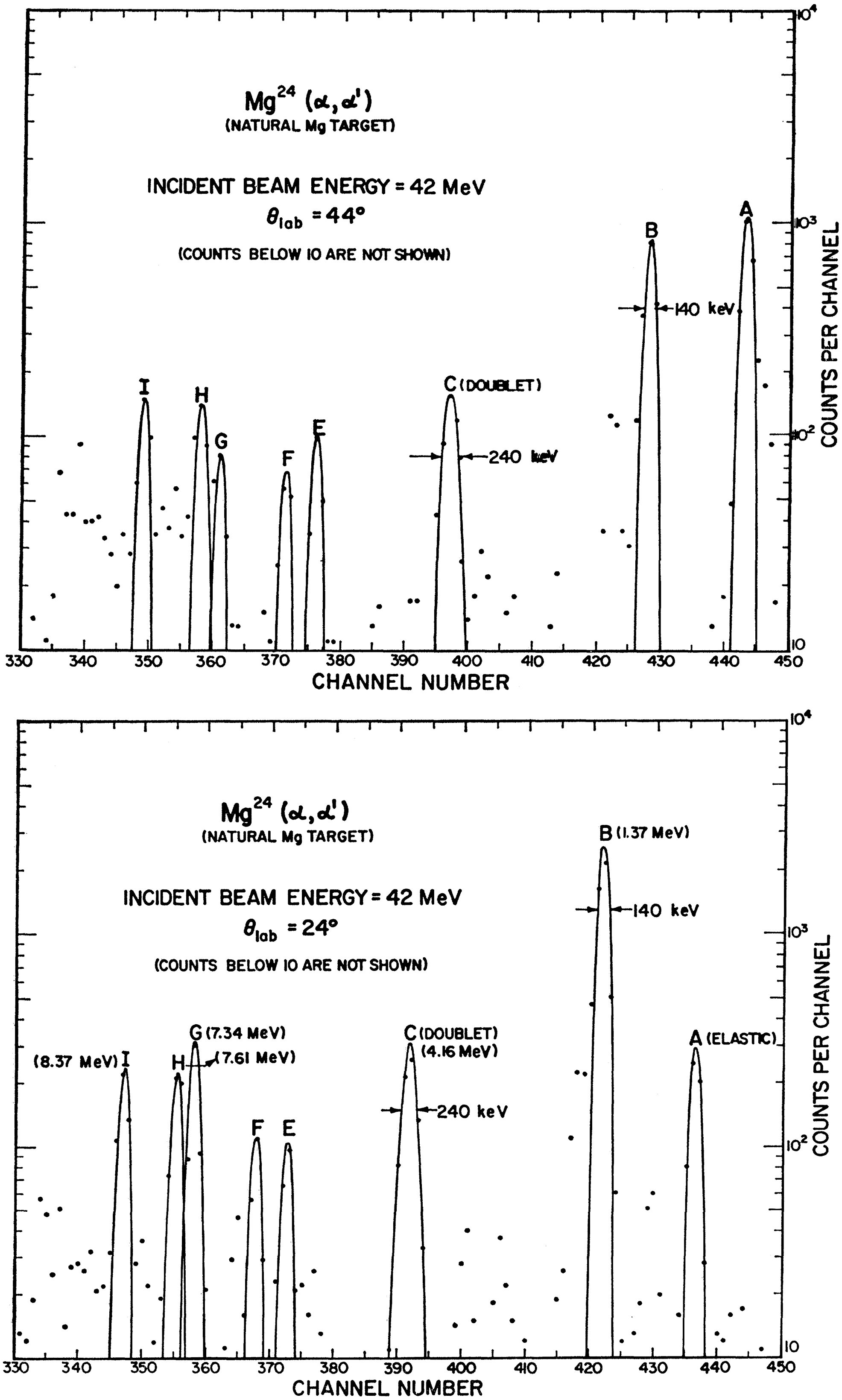

FIG. 1. Two typical pulse-height spectra of α particles scattered by Mg^{24}. A foil (1.1 mg/cm^2) of natural magnesium was used. The labeled peaks were all identified as belonging to Mg^{24}. The energies indicated for individual peaks are the corresponding values of $(-Q)$ as measured in the present experiment (Table I).

spaced levels shown in Fig. 2; these assignments are discussed further in Sec. V 6.

The measured angular distributions ($A, B, C, E, F, G,$ and I) are presented and analyzed in Sec. V. Angular distributions have also been obtained for the so-called "anomalous parity" level D which occurs at 5.22 MeV in Mg^{24}, with an enriched (99.98% Mg^{24}) target.

IV. THEORETICAL ANALYSIS

1. General

Two different methods of theoretical analysis, with contrasting levels of sophistication, have been emphasized in this study: (1) The distorted-wave Born-approximation (DWBA) version of the optical model, extended to include excitation of collective surface modes, has proved to be very successful in describing quantitatively a wide range of inelastic scattering experiments. (2) The Fraunhofer diffraction theory,[10–12] with the inclusion of the smooth-edge modification of Inopin and Berezhnoy,[13] has the virtue that it also is simple to apply, is physically *anschaulich*, and yet corresponds surprisingly well to experimental results. Brief discussions of both theories are given below.

(1) We adopt a particular version of the extended optical model,[14] in which the optical potential is assumed to have the form $V(R+\alpha(\hat{r}),r)$, where R is the radius parameter of a spherical optical potential and $\alpha(\hat{r})$ is the displacement of the nuclear surface in the direction $\hat{r}$. This displacement may be related to the canonical collective coordinates $\xi_{\lambda,\mu}$ through the multipole expansion

$$\alpha(\hat{r})=\sum_{\lambda,\mu}\xi_{\lambda,\mu}Y_\lambda^{\mu*}(\hat{r}). \tag{4.1}$$

When the potential is expanded through first order in $\alpha(\hat{r})$, the transition amplitude for scattering from an

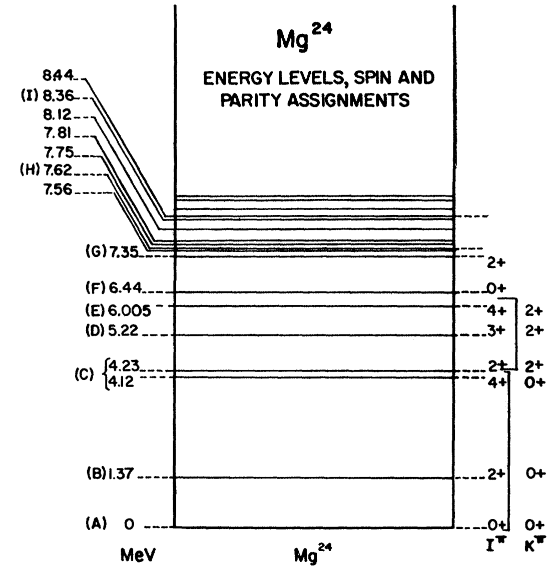

FIG. 2. Nuclear-level diagram of Mg^{24}. The excitation energies, spin, parity, and K assignments are those given in Ref. 5. K is defined as the projection of the total angular momentum on the nuclear symmetry axis. The K assignments are those suggested by the rotational-vibrational interpretation of the low-lying Mg^{24} levels. The levels labeled A, B, $\cdots$, I are the ones apparently excited in the present work.

initial nuclear state a to a different final state b is

$$T_{ba}=\int d\mathbf{r}\,\chi^{(-)*}(\mathbf{k}_b,\mathbf{r})\langle b|\alpha(\hat{r})|a\rangle\frac{\partial V(R,r)}{\partial R}\chi^{(+)}(\mathbf{k}_a,\mathbf{r}). \tag{4.2}$$

The functions $\chi^{(+)}$ and $\chi^{(-)}$ are exact solutions for elastic scattering in the presence of $V(R,r)$. This amplitude is termed the DWBA transition amplitude for single excitation of a collective surface mode.

In this DWBA approximation, the inelastic cross sections between states with angular momenta I and I' may always be written

$$\frac{d\sigma}{d\Omega}(I\rightarrow I')=\sum_\lambda S_\lambda(I\rightarrow I')\frac{d\bar{\sigma}}{d\Omega}(\lambda,Q), \tag{4.3}$$

where $(d\bar{\sigma}/d\Omega)$ (λ,Q) is a reduced cross section independent of nuclear matrix elements, which can be evaluated with existing computer codes, and the transition strength $S_\lambda(I\rightarrow I')$ is defined by

$$S_\lambda(I\rightarrow I')\equiv\frac{1}{2I+1}\sum_{\mu,M,M'}|\langle I',M'|\xi_{\lambda,\mu}|I,M\rangle|^2. \tag{4.4}$$

For the particular case of a permanently deformed axially symmetric nucleus, the surface deformation is most easily described in terms of body-fixed coordinates:

$$\alpha(\hat{r})=\sum_\lambda\delta_\lambda Y_\lambda^0(\theta',0)\equiv\sum_\lambda(\beta_\lambda R)Y_\lambda^0(\theta',0), \tag{4.5}$$

TABLE I. Observed inelastic groups and corresponding measured Q values.

α group	$-Q$ (MeV)
B	1.37 ±0.05
C (doublet)	4.16 ±0.05
E	6.005±0.005[a]
F	6.43 ±0.05
G	7.34 ±0.05
H	7.61 ±0.05
I	8.37 ±0.05

[a] Given by Ref. 5 and used here for absolute energy calibration (see Sec. II 1).

[10] S. I. Drozdov, Zh. Eksperim. i Teor. Fiz. **28**, 734 (1955); **28**, 736 (1955) [English transls.: Soviet Phys.—JETP **1**, 591 (1955); **1**, 588 (1955)].
[11] E. V. Inopin, Zh. Eksperim. i Teor. Fiz. **31**, 901 (1956) [English transl.: Soviet Phys.—JETP **4**, 784 (1957)].
[12] J. S. Blair, Phys. Rev. **115**, 928 (1959).
[13] E. V. Inopin and Yu. A. Berezhnoy, Nucl. Phys. **63**, 689 (1965).
[14] R. H. Bassel, G. R. Satchler, R. M. Drisko, and E. Rost, Phys. Rev. **128**, 2693 (1962).

where θ' is the polar angle between the field point and the body axis of the nucleus, and the transition strength for excitation of an even-mass nucleus then becomes

$$S_\lambda(0\rightarrow\lambda)=\delta_\lambda{}^2=(\beta_\lambda R)^2. \tag{4.6}$$

It has become customary to characterize the strength of single excitation matrix elements through Eq. (4.6) even when such a description in terms of a permanent deformation is clearly inappropriate.

Since this paper is concerned only with scattering from an even-even target, we postpone discussion of the relation between the values of S for even- and odd-mass nuclei until a later paper.

Because there are only a few instances in which our data betray any presence of double excitation, we have not used the more sophisticated method of coupled channels[15–17] in our analyses. Further, although it is obvious that a collective model is not an appropriate description for many of our weaker transitions, we have not attempted to analyze our results in terms of microscopic DWBA models.[18]

(2) In the simplest version of Fraunhofer diffraction theory, the sharp-edge Fraunhofer model[10–12] with neglect of Coulomb corrections, the nucleus is assumed to be opaque within a deformed surface $R_{\mathrm{def}}=R_0+\alpha(\hat{r})$, where R_0 is the strong-absorption radius and $\alpha(\hat{r})$ is the nuclear deformation defined as in Eq. (4.1). When the transition amplitude is expanded only through first order in $\alpha(\hat{r})$, the elastic cross section on an even-mass target is given by the familiar black-disk formula

$$\frac{d\sigma}{d\Omega}(0\rightarrow 0)_{\mathrm{el}}=(kR_0{}^2)^2\left[\frac{J_1(x)}{x}\right]^2, \tag{4.7}$$

while the inelastic cross section for single excitation of a collective surface mode is[12]

$$\frac{d\sigma}{d\Omega}(I\rightarrow I')=\sum_\lambda S_\lambda(I\rightarrow I')\frac{d\ddot{\sigma}}{d\Omega}(\lambda), \tag{4.8}$$

where $S_\lambda(I\rightarrow I')$ is defined in Eq. (4.4), and $(d\ddot{\sigma}/d\Omega)(\lambda)$ is a reduced cross section independent of nuclear matrix elements:

$$\frac{d\ddot{\sigma}}{d\Omega}(\lambda)=\frac{(kR_0)^2}{4\pi}\sum_{\substack{\mu\\(\lambda+\mu\,\mathrm{even})}}\frac{(\lambda-\mu)!(\lambda+\mu)!}{[(\lambda+\mu)!!(\lambda-\mu)!!]^2}\times J_{|\mu|}{}^2(x). \tag{4.9}$$

Here $J_{|\mu|}$ is the Bessel function of order $|\mu|$ and $(2n)!!\equiv 2\times4\times6\times\cdots 2n$ and $(2n+1)!!\equiv1\times3\times5\times\cdots(2n+1)$. When $x>(\lambda\pi/2)$, the reduced cross sections obey the familiar phase rule[12] and, at smaller values of x, are sufficiently different to allow one to make an assignment of angular momentum transfer.

In such Fraunhofer formulas there is some ambiguity concerning the most appropriate form of the argument x when these formulas are extrapolated to large angles. Derivations of these formulas as limiting cases of parametrized phase-shift theories[19–22] indicate that the proper choice is $x=kR_0\theta$. However, the observed angular distributions of medium-energy α particles scattered from light nuclei tend to be in better agreement with the choice $x=2kR_0\sin\frac{1}{2}\theta$; accordingly, this latter identification will be made throughout the present study.

As long as $n(\equiv ZZ'e^2/\hbar v)$ is not too large, inclusion of Coulomb distortion does not alter significantly the magnitude or shape of the Fraunhofer results for inelastic scattering of natural projectiles. However, the radius R_0 occurring in these expressions is more properly considered to be[20]

$$R_0=R_{0C}\left(1-\frac{ZZ'e^2}{ER_{0C}}\right)^{1/2}, \tag{4.10}$$

where R_{0C} is the Coulomb-corrected strong-absorption radius.

A major defect of the sharp-edge Fraunhofer model is that the envelopes to the cross sections do not decrease fast enough with increasing scattering angle at the larger angles. However, Inopin and Berezhnoy[13] have shown that, when the shadow function is assumed to be a convolution of the sharp-edge shadow function with a smoothing function, the resulting cross sections for both elastic and inelastic scattering are products of the sharp-edge Fraunhofer formulas and the common damping factor $[f(k\theta)]^2$. There are several choices for the smoothing function which lead to simple expressions for the form factor $f(k\theta)$. The Inopin-Berezhnoy modification of the sharp-edge Fraunhofer theory will be used throughout this study in obtaining values of $S_\lambda(I\rightarrow I')$.

There are a variety of models for both elastic and inelastic scattering, intermediate in sophistication between the Fraunhofer model and the extended optical model, in which the scattering amplitudes are expressed in terms of parametrized partial-wave amplitudes. Only limited use will be made of these models in the present work, but it is pertinent to make the following comments concerning them:

(a) When the partial-wave amplitudes for elastic scattering are parametrized by smoothed step functions or derivatives thereof, the resulting "smooth-cutoff"

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[16] J. G. Wills, Ph.D. thesis, University of Washington, 1963 (unpublished).
[17] T. Tamura, Rev. Mod. Phys. **37**, 679 (1965).
[18] For references to such models see, for example, G. R. Satchler, Nucl. Phys. **77**, 481 (1966).
[19] J. S. Blair, D. Sharp, and L. Wilets, Phys. Rev. **125**, 1625 (1962).
[20] W. E. Frahn and R. H. Venter, Ann. Phys. (N. Y.) **24**, 243 (1963).
[21] N. Austern and J. S. Blair, Ann. Phys. (N. Y.) **33**, 15 (1965).
[22] J. M. Potgieter and W. E. Frahn, Nucl. Phys. **80**, 434 (1966).

elastic cross sections can either be evaluated by numerical means[19,23–25] or be approximated by analytic expressions.[20] For η_l real, it can be shown[20] that the cross section for elastic scattering of neutral projectiles is approximated, for $L \gg 1$, by a formula essentially equivalent to the smooth-edge Fraunhofer formula. With the simple parametrization

$$\eta_l(L,\Delta) = 1/\{1+\exp[(l-L)/\Delta]\}, \quad (4.11)$$

the form factor is

$$f(k\theta) = (\pi k d\theta)/\sinh(\pi k d\theta), \quad (4.12)$$

where $\Delta \equiv kd$ and $(L+\frac{1}{2}) \equiv kR_0$.

(b) It is also possible to derive,[19,21] as approximations to the extended optical model, expressions for inelastic scattering involving derivatives of the parametrized partial-wave amplitudes with respect to l. Again, these expressions have been evaluated numerically[19,25,26] or approximated by analytic expressions.[21,22,27,28] For the parametrization given in Eq. (4.11) Potgieter and Frahn[22] show that the cross sections for inelastic scattering of neutral projectiles with single excitation of the target nucleus are well approximated by expressions which are nearly equivalent to the Inopin-Berezhnoy modification of the Fraunhofer formulas with the form factor given by Eq. (4.12). In other words, the smooth-edge Fraunhofer theory has sounder theoretical underpinnings than might be surmised initially. Indeed, comparison with direct numerical evaluations[19] shows that modified Fraunhofer formulas are even better justified, at values of L and Δ appropriate to our experiments, for inelastic scattering than for elastic scattering.

2. Application

We have made no attempt to search for optical-model parameters which give a best fit to the observed elastic scattering. Rather, we have adopted the parameters used by Rost,[29] which provide a reasonable but common garden-variety fit to our data. Here both the real and imaginary potentials are described by the same Woods-Saxon radial dependence; the parameters for Mg^{24} are $V = -47.6$ MeV, $W = -13.8$ MeV, $R = 4.76$ F, and $a = 0.549$ F. The charge distribution is assumed to be uniform within the same radius 4.76 F. Coulomb excitation is neglected. The code ABACUS-2 [30] has been used to compute the elastic cross section, while the code T-SALLY [31] and one constructed by Wills[16] have been used to compute the DWBA inelastic cross sections. Complex coupling has been assumed in computing the inelastic cross sections; this assumption has no bearing on the computed angular distributions, since both the real and imaginary potentials have the same form factor, but does reduce the values of δ from what would result if only real coupling were adopted.

The parameters occurring in the smooth-edge Fraunhofer formulas have been chosen in the following fashion. The locations of the minima in the elastic scattering cross sections are used to determine a value for the strong-absorption radius R_0 for each isotope studied. The value for R_0 is readily translated via Eq. (4.10) into the Coulomb-corrected strong-absorption radius R_{0C}.

The form factor has been chosen so that the smooth-edge Fraunhofer theory and one version of a parameterized partial-wave amplitude theory give consistent results for inelastic scattering. Specifically, a good fit to the angular distribution following excitation of the 1.37-MeV 2^+ level in Mg^{24} was obtained with the parametrized model of Blair, Sharp, and Wilets[19]; here η_l is parametrized according to Eq. (4.11), and the ratio Δ/L was found to be 0.06. It was also observed that the reduced cross section of the BSW calculation could be duplicated as the product of the Fraunhofer sharp-edge cross section and a damping factor; the damping factor, so determined by dividing the BSW cross section by the sharp-edge Fraunhofer cross section, is then used throughout this work. The resulting form factor is, for all practical purposes, equivalent to the expression given in Eq. (4.12) with $(\Delta/L) = 0.06$; for $x < 10$, the two form factors differ by less than 1%.

V. ANGULAR DISTRIBUTIONS AND DISCUSSION—Mg^{24}

1. A (Elastic) and B (1.37 MeV)

The measured angular distributions A (elastic) and B (1.37 MeV) are shown in Fig. 3. We note that both angular distributions are in basic agreement with previous measurements in this laboratory.[32,33] However, the present results display deeper minima and sharper diffractionlike structure, which is to be expected as a result of the enhanced energy and angular resolution in the present experiment. Further, the sharp forward rise of the inelastic cross section at $\theta_{c.m.} = 12°$, reported in Ref. 33, appears to have been in error.

An analysis of the present measurements in terms of diffraction theory differs only slightly from similar analyses[33] of earlier experiments.[32–36] A plot of the

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[33] J. S. Blair, G. W. Farwell, and D. K. McDaniels, Nucl. Phys. **17**, 641 (1960).
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[36] C. Hu, S. Kato, Y. Oda, and M. Takeda, J. Phys. Soc., Japan **14**, 539 (1959).

angular distribution A (elastic) is shown in Fig. 4 on a "universal" scale, i.e., the cross section divided by $(kR_0^2)^2$ has been plotted versus $x \equiv 2kR_0 \sin\frac{1}{2}\theta$; the value of the strong-absorption radius $R_0 = 6.06$ F has been chosen to give the best average fit of the Fraunhofer black-disk cross section, also shown in this figure, to the locations of the observed minima. This value is slightly larger than the value derived in earlier analyses, $R_0 = 5.97$ F. The value of the Coulomb-corrected strong-absorption radius corresponding to $R_0 = 6.06$ F is $R_{0C} = 6.56$ F. Also indicated are the values of the maxima of a "smooth-cutoff" evaluation of the elastic cross section; here η_l has been represented by the parametrization of Eq. (4.11), and the partial-wave sums have been evaluated numerically for the choice $(\Delta/L = 0.06)$ and projectile charge set equal to zero. It is worth mentioning that the maxima predicted by the smooth-edge Fraunhofer formula with the form factor of Eq. (4.12) differ by an appreciable amount from those evaluated by numerical summation of the partial-wave expansion.

A similar "universal" plot[33] of angular distribution B is shown in Fig. 5. Here the deformation distance δ_2 is determined to be 1.66 F, in order that there be a good fit over the first three maxima between the smooth-edge Fraunhofer angular distribution and experiment. The detailed manner in which this smooth-edge curve was constructed has been described in the last paragraph of Sec. IV.

The experimental results for elastic scattering are compared to an optical-model calculation in Fig. 6. The choice of parameters is discussed in Sec. IV 2. Again the maxima of the smooth-cutoff calculation are shown for comparison. There appears to be a shift of about a degree between the experimental and optical-model curves, particularly at the smaller angles. We believe that this probably reflects a systematic experimental error.

In Fig. 7 angular distribution B is compared to the DWBA calculation as well as to the smooth-edge Fraunhofer curve. The deformation parameter resulting from this optical-model fit, $\delta_2 = 1.68$ F, is exceedingly close to that found from the smooth-edge Fraunhofer analysis. The value obtained differs from that found by Rost,[29] $\delta_2 = (0.38)(4.76\text{ F}) = 1.81$ F, because (a) the data have changed slightly from the preliminary values available to Rost, and (b) we have assumed the coupling potential to be complex.

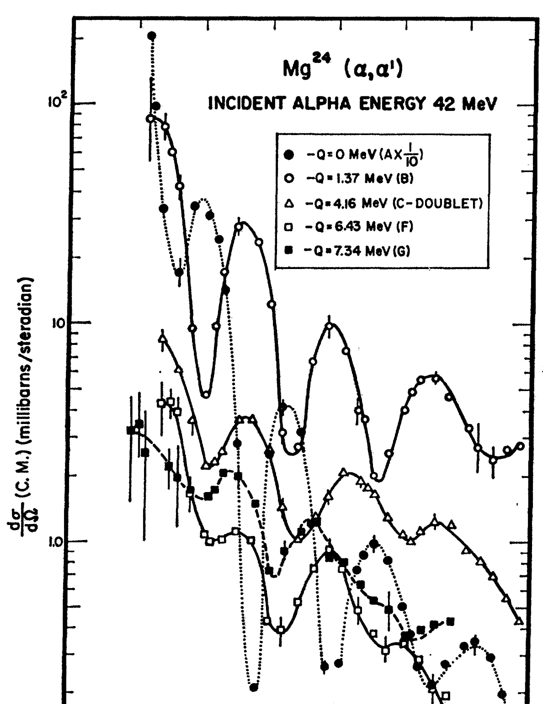

FIG. 3. Angular distributions of α particles scattered by Mg^{24} for peaks A (0 MeV), B (1.37 MeV), the doublet C (4.16 MeV), F (6.43 MeV), and G (7.34 MeV).

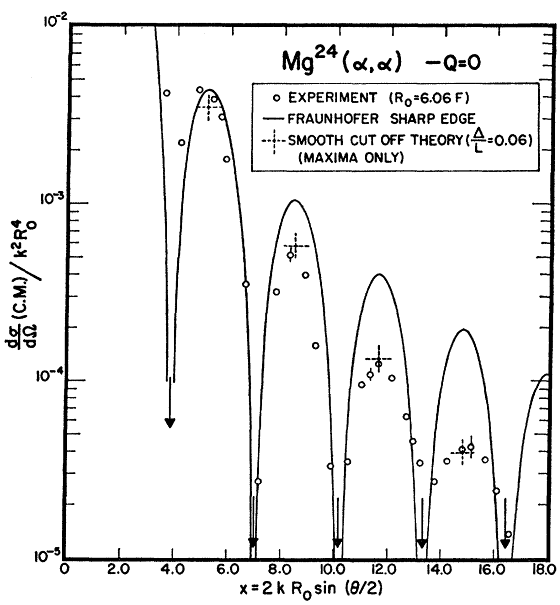

FIG. 4. "Universal" plot of the differential cross section for elastic scattering of 42-MeV α particles by Mg^{24}, i.e., the abscissa is $[2kR_0 \sin(\frac{1}{2}\theta)]$, where R_0, the strong-absorption radius, is chosen so that the Fraunhofer formula gives the best average fit to the locations of the observed minima, and the ordinate is the cross section divided by the factor $(k^2R_0^4)$. The solid curve is the sharp-edge Fraunhofer prediction $[J_1(x)/x]^2$. The arrows indicate the zeros of this expression. Also shown are the values of maxima when the partial-wave amplitudes are parametrized by the smoothed step function of Eq. (4.11) and the ratio of the diffuseness parameter to the cutoff angular momentum (Δ/L) is taken to be 0.06.

2. The Doublet C (4.16 MeV)

The angular distribution corresponding to the excitation of the unresolved doublet C (4.16 MeV) is also shown in Fig. 3. Near $\theta_{c.m.}=26°$ and 40° it is in phase with that of B (1.37 MeV) and out of phase with A (elastic). Beyond the maximum near 42°, the oscillations of C start to fall behind those of B and the diffraction structure becomes distinctly shallow.

Since the 4.12-MeV 4^+ and 4.23-MeV 2^+ levels are not resolved, our discussion of this angular distribution must be rather speculative. The 4.23-MeV member of the C doublet is, according to the rotational-vibrational interpretation of the first excited states in Mg^{24}, the first member ($I=2^+$) of a rotational band built on a $K=2^+$ γ vibration; the second and third members of this band are presumably the 3^+ state at 5.22 MeV and the 4^+ state at 6.005 MeV, respectively. The angular distribution for inelastic scattering to the 4.23-MeV level should therefore have the familiar shape associated with a 2^+ single-phonon excitation. The similarity of the observed angular distribution of B and C at angles less than 45° is consistent with this interpretation. Under the extreme and undoubtedly erroneous assumption that the first maximum is due completely to the 2^+ level, the reduced collective matrix element for this transition would then be 0.33 times for the 1.37-MeV transition when analyzed with the smooth-edge Fraunhofer model; the corresponding parameter δ_2 becomes 0.55 F, a reasonable value for a vibrational excitation.

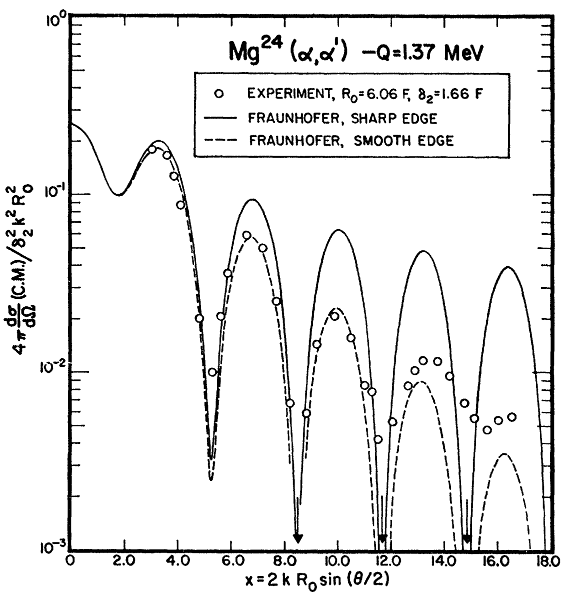

FIG. 5. "Universal" plot of the differential cross section for excitation of level B (2^+, $-Q=1.37$ MeV) in Mg^{24}. The deformation distance $\delta_2=1.66$ F has been chosen so that there is agreement between the smooth-edge Fraunhofer curve and the experimental points over the first three maxima. The arrows indicate minima in the Fraunhofer angular distribution.

In the rotational-vibrational interpretation, the 4.12-MeV level is the third member ($I=4^+$, $K=0^+$) of the ground-state rotational band. The present experiment appears to be consistent with this description but does not force one to such a conclusion. Both the adiabatic Fraunhofer theory, when carried to higher

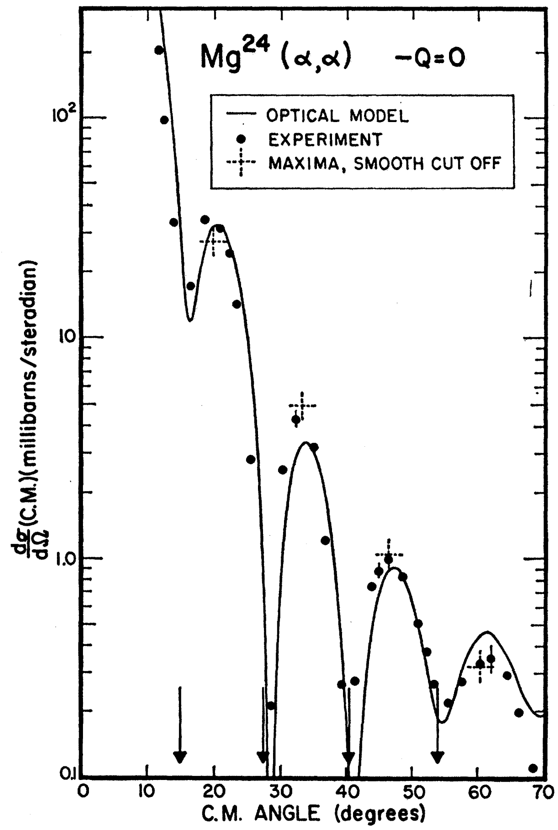

FIG. 6. Comparison between the observed differential cross section for elastic scattering from Mg^{24} and the cross section computed with an optical model. The optical parameters used are those of Rost (Ref. 29). Also shown are the maxima of the smooth cutoff calculation and arrows indicating the zeros of the Fraunhofer angular distribution.

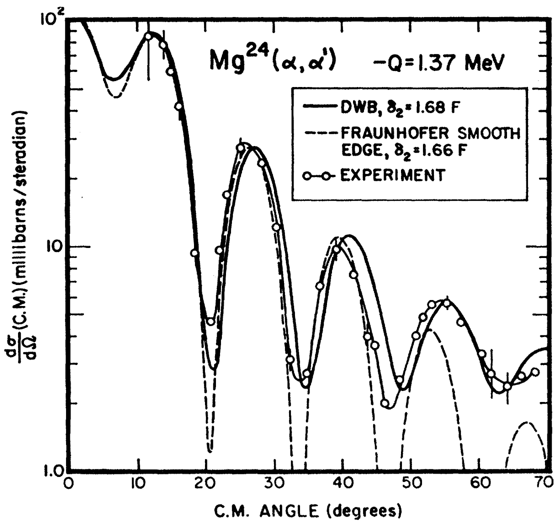

FIG. 7. Comparison between the observed differential cross section for excitation of level B (2^+, $-Q=1.37$ MeV) in Mg^{24}, the DWBA calculation, and the smooth-edge Fraunhofer calculation

order,[21,37] and the method of coupled channels[15–17] predict that the double-excitation cross section, produced by a quadrupole deformation, oscillates in phase with the elastic cross section at the larger angles and has a flatter envelope than is characteristic of single-excitation cross sections. The fact that angular distributions C and B deviate at the larger angles suggests sizeable double-excitation contributions. The magnitudes of the maxima in the double-excitation cross section predicted by the adiabatic Fraunhofer smooth-edge model, with δ_2 chosen to fit curve B, are slightly larger than 1 mb/sr; when computed by the method of coupled channels,[16] these magnitudes are predicted to be somewhat less than 1 mb/sr. Such values are not inconsistent with the assumption that the measured cross sections for the doublet C at angles larger than 30° are a superposition of the double-excitation angular distribution to the 4^+ state plus a single-excitation angular distribution to the 2^+ state.

However, it does not appear to be true that the 4^+ cross section is weak relative to the 2^+ cross section at angles smaller than 30°. Inspection of the pulse-height spectra in Fig. 1 shows that the width of the doublet C is larger than that for known single levels even when $\theta=24°$, suggesting that the previously estimated value for the collective matrix element to the 2^+ level should be reduced. This supposition is borne out by recent experiments with α particles of 50 MeV at the Berkeley 88-in. cyclotron, in which the two members of the doublet have been resolved[38]; it is there found that the cross section to the 4^+ state exceeds that to the 2^+ state at the larger angles and is roughly 60% of the 2^+ cross section at the maximum corresponding to our peak at 24°.

3. *D* (5.22 MeV)

Inelastic scattering to the level at 5.22 MeV, which has the "anomalous" parity assignment 3^+, is strongly inhibited over the angular range of the present experiments. In fact, it was not possible to identify positively an inelastic group belonging to this level in the spectra obtained with the natural Mg target (as typified by Fig. 1). However, the spectrum with a target enriched to 99.98% Mg^{24}, shown in Fig. 8, indicates the presence of a weak but measurable inelastic α group corresponding to the 5.22-MeV level. The resulting angular distribution is shown in Fig. 9, the large experimental errors in this measurement being due mainly to poor statistics

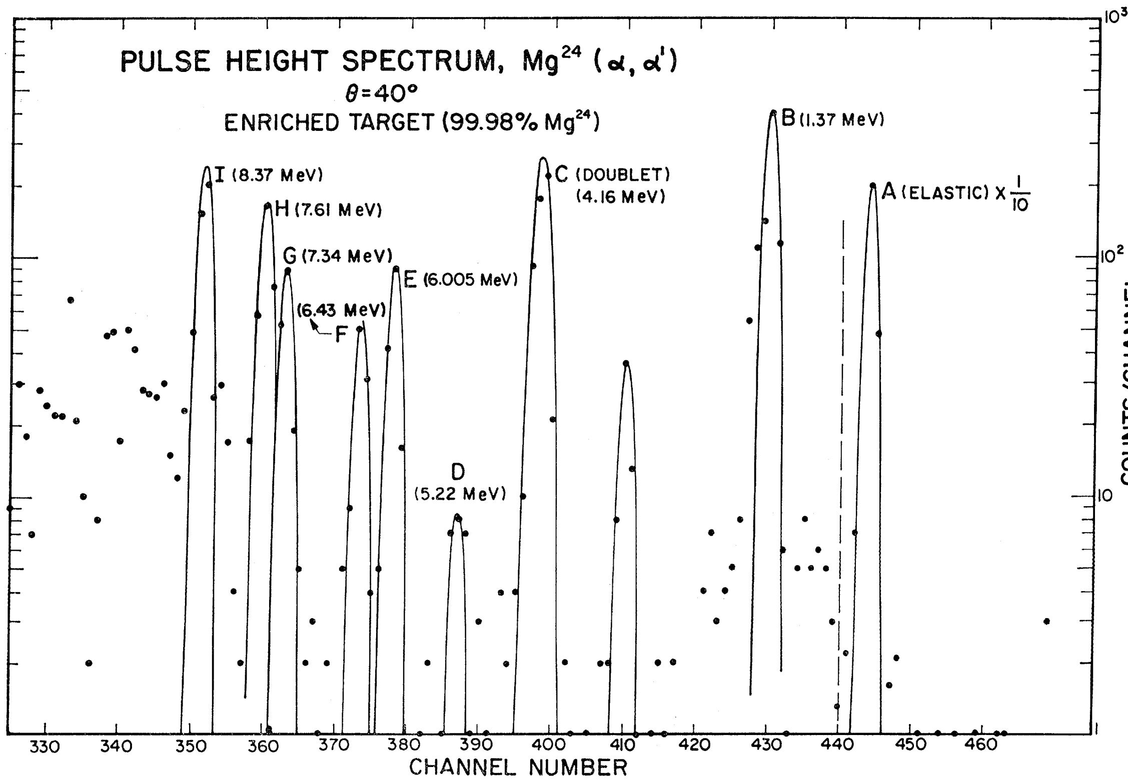

FIG. 8. Pulse-height spectrum of α particles scattered by an an enriched magnesium foil (99.98% Mg^{24}), taken at $\theta(\text{lab})=40°$. The energies indicated for individual peaks are the corresponding $(-Q)$ values as measured in the present experiment (Table I). The spectrum is substantially cleaner than the corresponding one at $\theta(\text{lab})=44°$ for natural Mg shown in Fig. 1.

[37] S. I. Drozdov, Zh. Eksperim. i Teor. Fiz. **38**, 499 (1960) [English transl.: Soviet Phys.—JETP **11**, 362 (1960)].
[38] D. L. Hendrie, B. G. Harvey, J. Mahoney, and J. R. Meriwether, Bull. Am. Phys. Soc. **12**, 555 (1967).

Marked inhibition of the cross section to this level at moderate bombarding energies is to be expected on theoretical grounds. The excitation of states in even-even nuclei with "anomalous" parity [i.e., $\Pi=(-)^{J+1}$] by spinless projectiles is forbidden in a single-step process which proceeds via a local interaction between the projectile and the nuclear coordinates.[39]

Further, there is an approximate selection rule which may inhibit even a two-step direct interaction for this particular transition. According to the rotational-vibrational model, the 3^+ state is a member of the rotational band built on a $K=2$ quadrupole vibration. This level could then be excited by two-step processes which proceed via two different routes: (1) First, there is an interaction with the fixed axially-symmetric quadrupole field of the ground-state rotational band, and thus the first 2^+ level of Mg^{24} becomes the intermediate state. This step is followed by interaction with a non-axially-symmetric quadrupole field which excites a γ vibration. (2) In the second route, the projectile interacts initially with the non-axially-symmetric field so that the second 2^+ level at 4.23 MeV is the intermediate state. Subsequent interaction with the axially-symmetric quadrupole field of the $K=2^+$ band leads to the 3^+ level. With the two approximations that (a) the energies of the intermediate states, as they occur in the intermediate Green's function, are degenerate, and (b) the radial shapes of all three quadrupole fields are the same, it is an exercise in the manipulation of Clebsch-Gordan coefficients to show that the amplitudes for these two routes are equal in magnitude but opposite in sign; consequently, the total double scattering amplitude vanishes.

For incident α particles of 42 MeV, approximation (a) should be rather reasonable; it amounts to making the adiabatic approximation for the two-step amplitude. Approximation (b) is typical in many phenomenological treatments of nuclear deformations, the usual radial dependence being the derivative of the spherical potential $\partial V/\partial r$. When these assumptions are dropped, nonzero cross sections can be produced, as has been shown in the coupled-channel calculations of Tamura.[40]

Our small cross section and observations made at lower bombarding energies are consistent with the preceding theoretical remarks. Moderate cross sections for excitation of the 3^+ level which fluctuate with energy have been measured at bombarding energies below 24 MeV.[39,41] These decrease as the bombarding energy is raised to 28.5 MeV,[42] although they are about five times what we have observed. This trend is consistent with our discussion in that the adiabatic approximation,

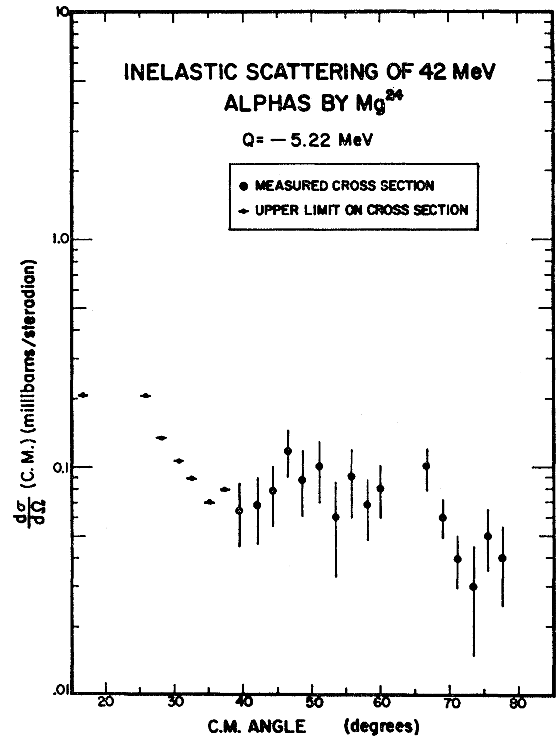

FIG. 9. Observed differential cross section for excitation of level D (3^+, $-Q=5.22$ MeV) in Mg^{24}. For small scattering angles only an upper limit (indicated in the figure by horizontal segments) on the cross sections was obtained.

necessary for the cancellation in the two-step amplitude, is expected to improve with increasing bombarding energy.

Two recent experiments indicate, however, that the situation is much more complex than that envisaged here. The observation has been made by Vincent, Boschitz, and Priest[43] that the cross section at a bombarding energy of 42 MeV increases considerably at the back angles. More surprisingly, it has been found by Wood, Harvey, and Hendrie[44] that these maxima maintain about the same magnitude but move forward in angle as the energy is further increased.

4. E (6.005 MeV) and F (6.43 MeV)

The spin and parity assignments of the levels E (6.005 MeV) and F (6.43 MeV) are known to be 4^+ and 0^+ respectively.[5] According to the rotational-vibrational collective interpretation of the low-lying levels in Mg^{24}, the E (6.005 MeV) level is the third member of the $K=2^+$ rotational band built on a γ-quadrupole vibration, while the F (6.43 MeV) level is the first member of the $K=0^+$ rotational band built on a β-quadrupole

[39] W. W. Eidson and J. G. Cramer, Jr., Phys. Rev. Letters 9, 497 (1962).
[40] T. Tamura, Nucl. Phys. 73, 241 (1965).
[41] W. J. Braithwaite, J. G. Cramer, and R. A. Hinrichs, Nuclear Physics Laboratory Annual Report, 1966, University of Washington (unpublished).
[42] K. Kokame, K. Fukunaga, N. Inoue, and H. Nakamura, Phys. Letters 8, 342 (1964).
[43] J. S. Vincent, E. T. Boschitz, and J. R. Priest, Bull. Am. Phys. Soc. 11, 333 (1966).
[44] M. Wood, B. G. Harvey, and D. L. Hendrie, University of California Radiation Laboratory Report No. 17299, 1967 (unpublished).

vibration. We therefore expected the inelastic scattering to these two levels to proceed solely through the double-excitation mechanism and the corresponding angular distributions to display the characteristic double-excitation features.

We see in Figs. 3, 10, 11, and 12, however, that the observed angular distributions E and F resemble the single-excitation $\lambda=4$ and $\lambda=0$ curves, respectively. The observed positions of most of the maxima and minima, their relatively large cross sections, and the fall of their envelopes with angle, all indicate the dominance of single- rather than double-excitation processes. However, detailed comparison with the single-excitation curves does show some discrepancies; in particular, the shift in the deep minimum of E to 30° in Fig. 11 argues for some double-excitation contribution, although the oscillations at larger angles do not show the reversal of phase so characteristic of double excitation. The extracted values of δ_4 and δ_0 are not unreasonable for vibrational excitations.

Although the single-excitation character of the angular distribution E is inconsistent with the most literal interpretation of the collective model, in which the 4^+ state is a rotational level built on a $K=2^+$ quadrupole vibration, it is consistent with a less phenomenological description of the intrinsic state. When analyzed in terms of individual nucleon motions, the intrinsic "vibrational" states are linear combinations of particle-hole configurations excited from the intrinsic ground state; it is easy to show that configurations prominent in the $K=2^+$ intrinsic state lead to sizeable components in the nuclear transition density with $\lambda=4$ as well as $\lambda=2$. Such $\lambda=4$ components will give rise to single-excitation amplitudes characteristic of that angular momentum transfer.

An analogous discussion may be given for the 0^+ state. A preliminary estimate indicates that the nuclear transition density for the $K=0^+$ excitation contains a considerable contribution characterized by $\lambda=0$.

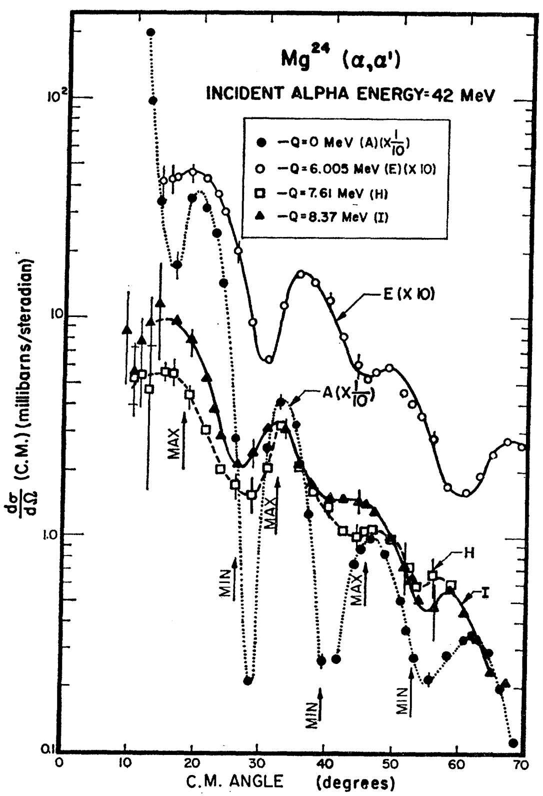

FIG. 10. Angular distributions of α particles scattered by Mg^{24} for the peaks A (0 MeV), E (6.005 MeV), H (7.34 MeV), and I (8.37 MeV). The veritical arrows indicate the locations of the maxima and minima as predicted by the Fraunhofer expression for single excitation of an octupole mode.

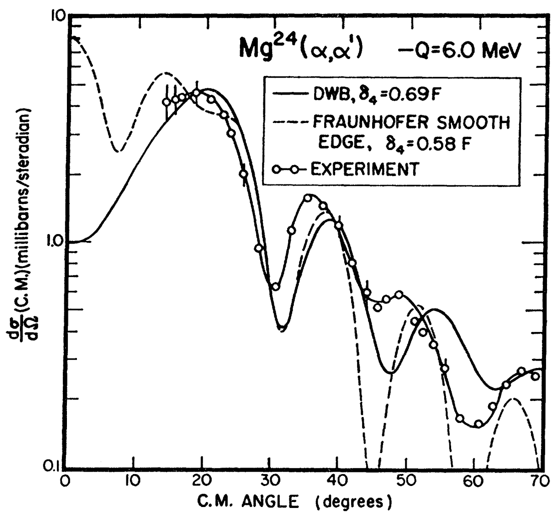

FIG. 11. Comparison between the observed differential cross section for excitation of level E (4^+, $-Q=6.005$ MeV) in Mg^{24}, the DWBA calculation, and the smooth-edge Fraunhofer calculation.

5. G (7.34 MeV)

The angular distribution for inelastic scattering to this level is shown in Fig. 3. The distribution displays an oscillatory structure for angles below 45°, and the two oscillations in that region are nearly out of phase with those of A (elastic). Comparison with the Fraunhofer prediction therefore suggests an even-parity assignment, with $\lambda=0$ or 2, in agreement with the known assignment of 2^+ for this level[5] (Fig. 2). The exact positions of the maxima and minima are definitely shifted from those of curve B, however, and in fact do not correspond in detail to either Fraunhofer or DWBA patterns. The apparent single-excitation character of this transition is consistent with the assumption that this state is the second member of the β-vibrational band which commenced with the $I=0^+$ state at 6.43 MeV.

6. H (7.61 MeV) and I (8.37 MeV)

The observed angular distributions for excitation of levels H and I are shown in Fig. 10. Also displayed for comparison are the elastic angular distribution and the locations of the maxima and minima predicted by the adiabatic Fraunhofer model for an $\lambda=3$ transition. The two observed angular distributions are quite similar, correspond qualitatively to the phase rule for odd-parity transitions, and, more precisely, match the predicted locations of the maxima and minima of an $\lambda=3$ transition. The presence of a pronounced minimum near 25° followed by a maximum near 32° rules out the possibility of an $\lambda=5$ transition. On the other hand, the angular

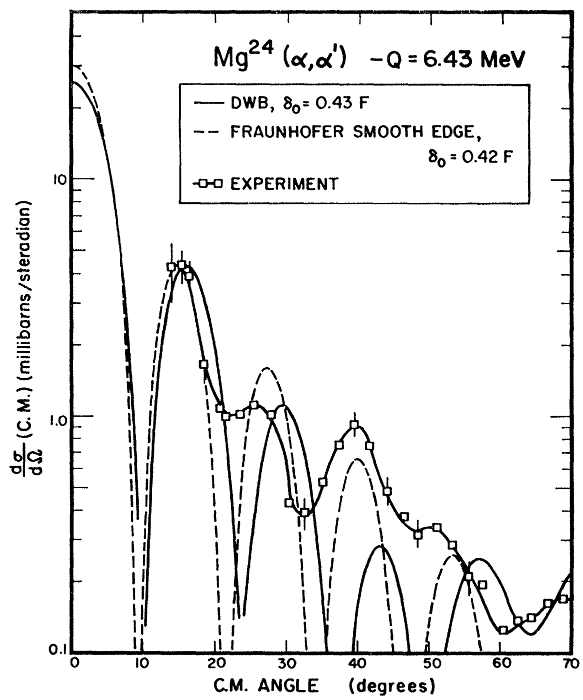

FIG. 12. Comparison between the observed differential cross section for excitation of level F (0^+, $-Q=6.43$ MeV) in Mg^{24}, the DWBA calculation, and the smooth-edge Fraunhofer calculation.

distribution for an $\lambda=1$ transition should be proportional to $[J_1(x)]^2$ and thus should have a deep minimum near 15°. We note, to the contrary, that both H and I have a maximum around 15° (although the errors associated with data in this region are large). The observed angular distributions H (7.61 MeV) and I (8.37 MeV) thus favor strongly an $I=3$ assignment for the two levels at 7.61 and 8.37 MeV. The observed angular distribution to the level at 8.37 MeV is shown in Fig. 13, together with calculated smooth-edge Fraunhofer and DWBA curves.

Definitive spin-parity assignments for many of the levels in Mg^{24} with excitation energies between 7 and 9 have emerged from recent α–γ correlation experiments.[9] The assignments are: 1^-, 7.56 MeV; 3^-, 7.62 MeV;

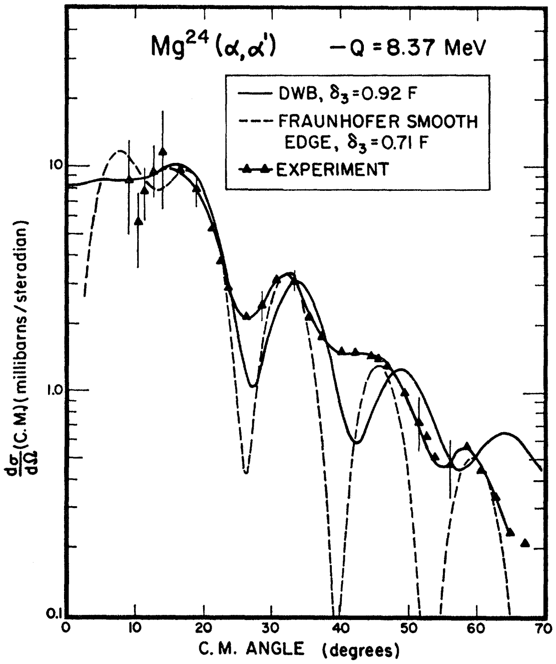

FIG. 13. Comparison between the observed differential cross section for excitation of level I [(3^-), $-Q=8.37$ MeV] in Mg^{24}, the DWBA calculation, and the smooth-edge Fraunhofer calculation.

6^+, 8.12 MeV; 1^- and (4^+), doublet at 8.44 MeV; 2^+, 8.65 MeV. We feel rather confident in assigning group I to the state at 8.36 MeV, although we cannot exclude the possibility that there is contamination of this group by levels at 8.44 MeV; further, some contamination of the angular distribution to the 3^- level at 7.62 MeV due to the 1^- level at 7.65 MeV cannot be excluded. All of the groups observed in the correlation studies mentioned above plus the level at 8.36 MeV were excited in the inelastic proton experiments of Crawley and Garvey.[45]

7. Transition Strengths

The values extracted for δ_λ are summarized in Table II. The indicated errors reflect both the relative errors in the data and the uncertainties in matching the data to the computed theoretical curves. They do not include an uncertainty, which is probably less than $\pm10\%$, in the absolute cross sections.

These values are also translated into electromagnetic transition strengths via the following prescription: It is assumed that (a) the electromagnetic transition is collective, (b) the deformation distance δ_λ is the same for both the nuclear and electromagnetic transitions, (c) the spherical-charge density is uniform within a radius, $R_{EM}=1.2A^{1/3}$ F, and is zero beyond, and (d) the matrix elements may be considered only to first order in the collective coordinates. The reduced transi-

[45] G. M. Crawley and G. T. Garvey, Phys. Rev. 160, 981 (1967).

TABLE II. Values obtained for deformation distances δ_λ, and G_λ, the ratio of reduced electromagnetic transition probability $B(E\lambda)_{ex}$, as estimated by Eq. (5.1), to the corresponding single-particle estimate $B(E\lambda)_{ex,sp}$.

$-Q$ [a] (MeV)	Spin parity of excited state	δ_λ in F Fraunhofer	G_λ	δ_λ in F DWBA	G_λ
1.37	2^+	1.66±0.04	13.2	1.68±0.04	13.5
4.24[b]	(2^+)	(0.55±0.05)	(1.45)	(0.59±0.05)	(1.66)
6.005[c]	4^+	0.58±0.05	1.75	0.69±0.05	2.48
6.44[d]	0^+	0.42±0.04		0.43±0.04	
7.36[e]	2^+	0.31±0.08	0.46	0.39±0.08	0.73
7.61[f]	3^-	0.54±0.05	1.44	0.69±0.05	2.34
8.36	$[3^-]$[g]	0.71±0.04	2.48	0.92±0.04	4.17

[a] The excitation energies are taken to be those of Endt and van der Leun (Ref. 5) rather than the values measured in the present experiment in Table I.
[b] Upper limit obtained by assuming that cross section at 14° is due solely to the 2^+ member of the doublet.
[c] Mediocre fits to theory.
[d] Poor fits. Data and theory normalized at 15°.
[e] Poor fits. Data and theory normalized at 12°. Normalization at second maximum would increase deformation parameter by 50%.
[f] Data and theory normalized at 15°. Both theories too low at back angles.
[g] Spin-parity assignment obtained from inelastic α-particle experiment.

tion probability for excitation of the ground state is then[46]

$$B(E\lambda)_{ex}=\left[\frac{3eZ\delta_\lambda R_{EM}{}^{(\lambda-1)}}{4\pi}\right]^2. \tag{5.1}$$

Rather than quote these in the table, we tabulate instead the ratio $G_\lambda\equiv[B(E\lambda)_{ex}/B(E\lambda)_{ex,sp}]$, which is a measure of collective enhancement. The quantity $B(E\lambda)_{ex,sp}$ is the usual estimate of single-particle strength

$$B(E\lambda)_{ex,sp}=\frac{e^2}{4\pi}\left[(2\lambda+1)\left(\frac{3}{3+\lambda}\right)^2\right]R_{EM}{}^{2\lambda}. \tag{5.2}$$

It is obvious that the above procedure for obtaining electromagnetic transition strengths is rather crude, even when it is a good assumption to use the collective description and to equate nuclear and electromagnetic deformation distances, primarily because this method underestimates the contribution of the charge density in the nuclear-surface region.[47,48]

We do believe that it is definitely preferable[21] to estimate an electromagnetic strength by equating the nuclear and electric deformation distances δ_l rather than the dimensionless parameters β_l. The dimensionless parameters have no meaning apart from the radii to which they refer. It is clear that the nuclear radii which enter the description of the scattering of complex projectiles, whether they be the strong-absorption radii of the Fraunhofer model or the (nonuniquely determined) radii of the optical potentials, are much larger than the half-density or mean-square radii of the nuclear-matter distribution; in contrast, the electromagnetic radii presumably parametrize only the charge density of the target nucleus. The electromagnetic rates would be decreased by a factor between 2 and 3.5 were we to replace δ_λ in Eq. (5.2) by $\beta_\lambda R_{EM}$ where β_λ is the dimensionless parameter corresponding either to optical-model or strong-absorption radii.

The actual electromagnetic transition probabilities have been measured for some of these transitions. From consideration of several experiments, Stelson and Grodzins[49] arrive at the value $G_2=(25.0\pm3.9)$ for the transition to the 2^+ level at 1.37 MeV. It has been found recently[50] that $G_2=0.9$ for the ground-state transition to the 2^+ level at 4.23 MeV, while $G_3=4.3$ for the transition to the 3^- level at 7.62 MeV.

Inspection of the table suggests that most of the transitions we observed have a collective character. It is also apparent that the deformation distances tend to be increasingly underestimated by the adiabatic smooth-edge Fraunhofer model as the excitation energy is increased.

8. Comparison to Other Experiments

The element Mg^{24} has been a time-honored target material for inelastic scattering experiments. We here limit our discussion to three studies of α-particle scattering which closely parallel the present investigation, plus some results from nucleon scattering:

(1) Cross sections to all of the levels below 7 MeV as well as to a level at 8.4 MeV have been measured by a group at the University of Kyoto,[42,51,52] using a beam of 28.5-MeV α particles. The qualitative aspect of their cross sections is much the same as in the present experiment, except that excitation of the 3^+ level at 5.22 MeV is roughly 5 times stronger at 28.5 MeV than at 42 MeV. A DWBA analysis of their cross sections to the 2^+ level at 1.37 MeV and to the presumed 3^- level at 8.4 MeV gave the values $\delta_2=\beta_2R=(0.35)$ 4.90$=1.72$ F and $\delta_3=\beta_3R=(0.22)$ 4.90$=1.08$ F, respectively; these may be compared with our DWBA values, $\delta_2=1.68$ F and $\delta_3=0.92$ F, respectively. Apparently the values given by the Kyoto group for electromagnetic enhancement factors G_λ are computed by equating the nuclear and electromagnetic dimensionless parameters β_λ rather than the corresponding deformation distances δ_λ, and thus are somewhat reduced from our estimates.

(2) Cross sections have been measured over the same range of excitation energy by a group at Saclay,[53]

[46] K. Alder, A. Bohr, T. Huus, B. Mottelson, and A. Winther, Rev. Mod. Phys. **28**, 432 (1956).
[47] A. M. Lane and E. D. Pendlebury, Nucl. Phys. **15**, 39 (1960).
[48] L. W. Owen and G. R. Satchler, Nucl. Phys. **51**, 155 (1964).
[49] P. H. Stelson and L. Grodzins, Nucl. Data **1**, 21 (1965).
[50] T. K. Alexander, C. Broude, A. J. Ferguson, J. A. Kuehner, A. E. Litherland, R. W. Ollerhead, and P. J. M. Smulders, in Proceedings of the International Conference on Nuclear Physics, Gatlinburg, Tennessee, September, 1966 (to be published).
[51] J. Kokame, K. Fukunaga, H. Nakamura, and N. Inoue, J. Phys. Soc. Japan **20**, 475 (1965).
[52] J. Kokame, K. Fukunaga, and H. Nakamura, Phys. Letters **20**, 672 (1966).
[53] G. Bruge, J. C. Faivre, G. Vallois, A. Bussiere, and P. Roussel, J. Phys. (Paris) **27**, C1-44 (1966); and M. Barloutaud, R. Chaminade, H. Faraggi, D. Garetta, and B. G. Harvey (private communication).

using a beam of 44-MeV α particles. In general, the data are in accord with ours, although the maxima in their cross sections to the levels at 6.44 and 8.4 MeV are as much as 20 or 30% less than ours.

Values for deformation distances have been extracted by the Saclay group, using the parametrized phase-shift theory of Austern and Blair[21]; the values obtained tend to be some 25% less than those resulting from the smooth-edge Fraunhofer analyses and are thus even further distant from the results of our DWBA analysis. This discrepancy seems rather puzzling at first sight, since the parametrized phase-shift theory should be an improvement over the less sophisticated smooth-edge Fraunhofer theory. This discrepancy has also appeared in similar analyses[54–56] of other inelastic scattering experiments.

Our belief is that the discrepancy between values extracted with a careful evaluation of the Austern-Blair theory and the DWBA model even at low values for $-Q$ is a real and generally present effect. Further, we believe that the closer agreement between the values obtained with the smooth-edge Fraunhofer and DWBA models is somewhat fortuitous and results from two compensating approximations: (a) For a given value of δ_λ and Q set equal to zero, the most accurate evaluations of the Austern-Blair theory will tend to overestimate the cross section since that theory overestimates those radial matrix elements of the DWBA theory for which the incoming and outgoing angular momenta are different. (b) On the other hand, our smooth-edge Fraunhofer model underestimates the cross section of the Austern-Blair theory; it emerges as an approximation to the latter theory when only the real, and not the imaginary, part of the partial-wave amplitude η_l is considered.

(3) Inelastic scattering of 50-MeV α particles has been studied recently by a group at Berkeley,[38] with energy resolution superior to that of any of the other experiments, including the present one. The values of the deformation parameters obtained from a DWBA analysis of these data are generally quite close to those of the present paper.

A DWBA analysis of the scattering of 17.5-MeV protons[45] leads to the following values for deformation parameters: 2^+, 1.37 MeV, $\delta_2=1.80$ F; 2^+, 7.35 MeV, $\delta_2=0.52$ F; 3^-, 7.62 MeV, $\delta_3=1.00$ F; (3^-), 8.36 MeV, $\delta_3=0.73$ F. Similar analyses of the scattering of 14-MeV neutrons[57,58] give $\delta_2=(0.62)\ (3.605)=2.24$ F for the first 2^+ level. The scattering of 185-MeV protons[59] yields spin-parity assignments which are consistent with our results.

[54] R. J. Peterson, Ph.D. thesis, University of Washington, 1966 (unpublished).

[55] D. L. Hendrie (private communication).

[56] B. Fernandez (private communication).

9. Summary for Mg^{24}

The observed angular distributions are consistent with the broad features of the rotational-vibrational model for Mg^{24}. The large value of δ_2 corresponding to excitation of the first excited state indicates that this nucleus has a permanent quadrupole deformation. The excitation of positive parity states at 4.23 MeV and beyond are not in contradiction with their classification as members of either a γ- or β-vibrational band. The single-excitation character of the 6.005-MeV (4^+) and 6.43-MeV (0^+) angular distributions rules out the most naive model, in which these bands are based on pure quadrupole vibrations, but is consistent with a particle-hole description of the intrinsic states. The apparent presence of two strong octupole excitations is also appropriate to the rotational-vibrational model, where the octupole strength will be distributed among 3^- levels from differing K bands.

The smooth-edge Fraunhofer model appears as a fairly reliable and simple substitute for DWBA calculations for α-particle scattering in the s-d shell, although the discrepancy between the values of deformation distances extracted with these two theories becomes substantial as angular momentum transfer and excitation energy increases. The exceedingly close agreement obtained for the strong excitation of the first 2^+ state is probably a bit deceptive.

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