Effects of T Violation in Nuclear Reactions*

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A violation of time-reversal invariance of the nuclear Hamiltonian results in a violation of the reciprocity relation connecting the magnitudes of nuclear reaction cross sections in which initial and final states are interchanged. The development of reaction theory in the absence of T invariance is outlined, and the connection between T violation and reciprocity violation is calculated for the cases of direct reactions, isolated resonances, average compound-nucleus cross sections, and fluctuating cross sections measured with good energy resolution. It is found that in a direct reaction the magnitude of the reciprocity violation is proportional to the matrix elements of the T-odd part of the Hamiltonian connecting different competing residual states, divided by the energy separations of these residual states. In isolated resonances and average cross sections the effect depends entirely on the presence of a competing direct reaction. In fluctuating cross sections the rms value of the reciprocity violation is proportional to the rms absolute value of the matrix elements of the T-odd part of H connecting different "compound states," divided by the geometric mean of the average spacing and the average width of these "compound states." Thus the effect is favored in fluctuating reactions over direction reactions by a factor of the order of the ratio of the mean spacing of residual levels to the geometric mean of the average widths and spacings of compound levels. An additional strong enhancement of the fluctuating effect appears in the presence of competing strongly absorbed channels. Various aspects of possible experimental tests of reciprocity violation are discussed.

INTRODUCTION

IT was shown by Wigner and Eisenbud,[1] and in greater generality by Coester[2] that invariance of the nuclear Hamiltonian under the time-reversal operation (T) results in a symmetric S matrix

$$S_{ab}=S_{ba}. \tag{1}$$

Since the cross section for the reaction proceeding from an initial asymptotic state a to a final state b is given by

$$\sigma_{ab}=\pi k_a^{-2}g_a|\delta_{ab}-S_{ab}|^2, \tag{2}$$

it is clear that the symmetry relation (1) implies the reciprocity relation for cross sections

$$k_a^2\sigma_{ab}/g_a=k_b^2\sigma_{ba}/g_b, \tag{3}$$

where k_a is the relative asymptotic momentum of the two reaction fragments and g_a is the statistical factor in state a.

The discovery of indications of T violation in the decay of K mesons[3] has caused interest in experimental tests of T invariance in nuclear physics by measurements of the validity of the reciprocity relation (3),[4,5] among other methods. Furthermore, one would like to be able to interpret any violation of Eq. (3) in terms of properties of the T-violating part of the nuclear Hamiltonian. For the purposes of this paper, we shall assume that all cross sections σ_{ab} are measured in units of $\pi g_a k_a^{-2}$ so that reciprocity becomes the symmetry relation

$$\sigma_{ab}=\sigma_{ba} \tag{3'}$$

and the magnitude of reciprocity violation is given by

$$\delta\sigma_{ab}=\sigma_{ab}-\sigma_{ba}. \tag{4}$$

EXPERIMENTS

Possible experiments to test the validity of (3) were discussed by Henley and Jacobsohn[6] who emphasized the fact that (3′) may hold for reasons other than T invariance. Thus, if only two independent competing reaction channels are open, Eq. (3) must be satisfied by flux conservation as expressed in the unitarity of the S matrix. Henley and Jacobsohn called this fact the "Two-State Theorem." Also, if σ_{ab} and σ_{ba} can be calculated by means of the plane-wave Born approximation, Eq. (3) will be satisfied, independent of T invariance.

There are, however, two additional reasons why $\delta\sigma_{ab}$ may vanish or be undetectable in a particular experiment even in the absence of T invariance. First, the interaction responsible for a certain reaction may be unaffected by the T-violating part of the Hamiltonian. To give one example, it is at least conceivable that matrix elements of collective excitations in heavy nuclei might be insensitive to T-violation effects. Second, a particular experiment may satisfy Eq. (3) merely because of an "accidental" cancellation in the contributions to the antisymmetric part of the S matrix element. Such situations are expected to occur. As we shall see later, zero is an entirely possible value of $\delta\sigma_{ab}$ when H is T-violating. In the absence of a detailed dynamical theory of violation it is not possible to predict in which experiment such a zero value will occur.

In view of these considerations, a reliable test of T

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[1] E. P. Wigner and L. Eisenbud, Phys. Rev. **72**, 29 (1947).
[2] F. Coester, Phys. Rev. **89**, 619 (1953).
[3] J. H. Christenson, J. W. Cronin, V. L. Fitch, and R. Turlay, Phys. Rev. Letters **13**, 138 (1964).
[4] W. von Witsch, A. Richter, and P. von Brentano, Phys. Letters **22**, 631 (1966); Phys. Rev. Letters **19**, 524 (1967).
[5] S. T. Thornton, C. M. Jones, J. K. Bair, M. D. Mancusi, and H. B. Willard, Oak Ridge National Laboratory Report No. ORNL-4082, 1967, p. 2 (unpublished).
[6] Ernest M. Henley and Boris A. Jacobsohn, Phys. Rev. **113** 225 (1959).

violation requires several experimental measurements of $\delta\sigma_{ab}$. These could be either direct-reaction experiments or fluctuating-cross-section experiments. If direct-reaction experiments, or even average-cross-section experiments, are performed, one should choose several reactions having very different dynamical characteristics in order to avoid the above-mentioned possible T-violation insensitivity of some processes. In high resolution experiments of fluctuating cross sections, it may be assumed that several measurements at different energies in the same reaction will be sufficient. Such energies should, of course, be separated by more than the "correlation width" appropriate to the reaction.

Advantages of high resolution tests of Eq. (3) in fluctuating cross sections were pointed out by von Witsch *et al.*[4] They showed how the problems of determining the relative normalization of σ_{ab} and σ_{ba} could be solved by comparing ratios of these two cross sections measured at different energies E_1 and E_2. The quantity

$$R_{ab}(E_1,E_2)=\frac{\sigma_{ab}(E_1)}{\sigma_{ab}(E_2)}\Big/\frac{\sigma_{ba}(E_1)}{\sigma_{ba}(E_2)} \tag{5}$$

should be unity if T invariance holds. Assuming T violation, we obtain to first order in the asymmetry $\delta\sigma_{ab}$.

$$R_{ab}(E_1,E_2)-1\approx\frac{\delta\sigma_{ab}(E_1)}{\sigma_{ab}(E_1)}-\frac{\delta\sigma_{ab}(E_2)}{\sigma_{ab}(E_2)}. \tag{6}$$

To avoid effects of energy instability, it is advantageous to choose E_1 and E_2 to lie at stationary values of the cross-section energy dependence. Von Witsch *et al.* chose E_1 to be a cross-section minimum and E_2 to be a maximum. Then, if one assumes that the value of $\delta\sigma_{ab}(E)$ is uncorrelated with the value of σ_{ab}, and if the ratio of $\sigma_{ab}(E_2)/\sigma_{ab}(E_1)$ is sufficiently large, one may expect that the second term on the right-hand side of Eq. (6) is negligible and that a measurement of $R_{ab}(E_1,E_2)$ determines $\delta\sigma_{ab}/\sigma_{ab}$ at the cross-section minimum E_1.

One might be tempted on the basis of the above arguments to test T invariance in reactions whose fluctuating cross sections exhibit very large maximum-to-minimum ratios. This could be dangerous for the following reasons: Wigner has shown that a reaction in which there are only two independent competing channels has cross-section zeroes, while in the presence of three or more such channels there are no cross-section zeros.[7] If the experimental energy resolution is insufficient, very deep cross-section minima may be indicative of the presence of actual zeros and consequently of a situation in which $\delta\sigma$ must vanish because of the two-state theorem. Alternatively, the occurrence of very high cross-section maxima (compared to the average cross section) may be indicative of isolated resonance structure which, as we shall see, is ordinarily not expected to be favorable for the observation of T-violation effects.

Finally, in experiments with fluctuating cross sections, the angle of observation should be chosen so as to minimize the number of independent reaction modes (often called the number of "degrees of freedom" in cross-section-fluctuation analysis[8]). In reactions resulting from the addition of many incoherent processes, the relative asymmetry (4) will tend to be reduced by cancellations of the various contributions.

REACTION THEORY WITH T VIOLATION

In order to relate the cross-section asymmetries $\delta\sigma$ to the properties of the T-violating part of the nuclear Hamiltonian, we shall employ Wigner's R-matrix formalism.[1,9] Other methods for discussing this problem have been used by Ericson[10] and by Mahaux and Weidenmüller.[11]

As usual, we divide configuration space into an interior region where all nucleons interact strongly and an exterior one where only the nucleons within each of two fragments A and B interact strongly. In the exterior or channel region the relative motion of the two fragments is governed by a completely solvable two-body Schrödinger equation (involving ordinarily only the Coulomb interaction of A and B). We write the Schrödinger equation for fragment A as

$$(H_A-\epsilon_\alpha)\psi_{\alpha(I_\alpha,\nu_\alpha)}=0\,, \tag{7}$$

where I_α and ν_α are the spin and spin-magnetic quantum numbers of the (discrete) eigenstate ψ_α. Then the wave function describing the motion of the fragments A and B in the exterior may be written

$$\Psi_{J,M}{}^{(A,B)}=\sum_c R_c(r)\phi_c(r,\theta,\phi)\,, \tag{8a}$$

$$\phi_c(r,\theta,\phi)=\sum\,(c|JM)\frac{i^l Y_{lm}(\theta,\phi)}{r}\psi_\alpha\psi_\beta\,, \tag{8b}$$

where r, θ, ϕ are the relative spherical coordinates of fragments A and B, Y_{lm} is a spherical harmonic, and the subscript c stands for the collection of quantum numbers l, m, I_α, ν_α, I_β, and ν_β coupled to total momentum J and M by means of the angular-momentum coupling coefficient indicated by the abbreviation $(c|JM)$. The reaction amplitude, or the S matrix, is determined by the asymptotic forms of the radial functions $R_c(r)$ as $r\to\infty$. The R-matrix method is to express this asymptotic form first in terms of the value $R_c(a_c)$

[7] E. P. Wigner, Proc. Natl. Acad. Sci. U. S. 32, 302 (1946).

[8] J. Bondorf and R. B. Leachman, Kgl. Danske Videnskab. Selskab, Mat. Fys. Medd. 34, No. 10 (1965).

[9] A. M. Lane and R. G. Thomas, Rev. Mod. Phys. 30, 257 (1958).

[10] T. E. O. Ericson, Phys. Letters 23, 97 (1966).

[11] C. Mahaux and H. A. Weidenmüller, Phys. Letters 23, 100 (1966).

and the derivative $R_c'(a_c)$ of the radial function at the boundary $r=a_c$ between the exterior and interior regions. This is done by integrating the solvable Schrödinger equation for the relative motion of A and B in the exterior. This integration yields the following two complex constants:

$$L_c=S_c+iP_c \quad \text{and} \quad \Omega_c=e^{-i\phi_c}, \tag{9}$$

where S_c and P_c are known as the shift and penetration factors and ϕ_c is the hard-sphere plus Coulomb phase shift associated with the radius a_c. Next, the wave function in the interior is expanded in the discrete eigenstates of the complete Hamiltonian which satisfy real boundary conditions B_c at the channel radii a_c.

$$\Psi_{J,M}{}^{(\text{interior})}=\sum_\lambda \alpha_\lambda X_\lambda\,, \tag{10a}$$

$$(H-E_\lambda)X_\lambda=0\,,\quad \frac{a_c}{X_\lambda(a_c)}\frac{\partial X_\lambda}{\partial r}(a_c)=B_c\,. \tag{10b}$$

It is in Eq. (10) that dynamical assumptions and models are introduced.

Using the continuity conditions at $r=a_c$ and applying Green's theorem to the interior region, one may express $R_c(a_c)$ and $R_c'(a_c)$ in terms of the eigenvalues E_λ and the values of the radial parts of X_λ on the channel surfaces a_c. The latter, suitably normalized, are usually written as

$$\gamma_{\lambda c}=(\hbar^2/2M_c a_c)^{1/2}\int \phi_c{}^* X_\lambda d\mathcal{S}_{AB} \equiv(\phi_c,X_\lambda)_c\,, \tag{11}$$

where M_c is the reduced mass of fragments A and B, and the integration is carried over all coordinates except r which is set equal to a_c.

Having thus calculated $R_c(a_c)$ and $R_c'(a_c)$ in terms of the dynamical properties of the interaction embodied in Eqs. (10), we are able to determine the S-matrix elements by the procedure discussed below Eqs. (8). The resulting S matrix in the representation of Eqs. (8) has the form

$$\mathbf{S}=\mathbf{\Omega}^2+2i\mathbf{\Omega}\mathbf{P}^{1/2}\mathfrak{U}\mathbf{P}^{1/2}\mathbf{\Omega}\,, \tag{12a}$$

where

$$\mathfrak{U}=(1-\mathbf{R}\mathbf{L}^0)^{-1}\mathbf{R}\,, \tag{12b}$$

and

$$R_{cc'}=\sum_\lambda \gamma_{\lambda c}\gamma_{\lambda c'}{}^*/(E_\lambda-E)\,, \tag{13}$$

and

$$\mathbf{L}^0=\mathbf{L}-\mathbf{B}.$$

The diagonal matrices $\mathbf{\Omega}$, $\mathbf{P}$, $\mathbf{L}$, and $\mathbf{B}$ have elements Ω_c, P_c, L_c, B_c, as defined above. These expressions differ from the usual R-matrix formulas[1,9] only through the fact that the $\gamma_{\lambda c}$ are here not assumed to be real. The latter property is a result of T invariance. The evident Hermiticity of $\mathbf{R}$ still guarantees the unitarity of the S matrix (12). However, unless R is also real and symmetric, $\mathbf{S}$ will not be symmetric.

To investigate the crucial reality properties of the $\gamma_{\lambda c}$ when T is violated, we consider first the properties of the Hamiltonian H of Eq. (10b) with respect to time reversal. The antiunitary time-reversal operator θ has the properties[12] that for any two states ψ and ϕ

$$\theta^2\psi=\pm\psi \tag{14a}$$

and

$$(\theta\psi,\theta\phi)=(\psi,\phi)^*; \tag{14b}$$

and for any eigenfunction $\psi_{J,M}$ of a T-invariant scalar operator $\mathcal{H}$ with eigenvalue $\mathcal{E}$

$$(\mathcal{H}-\mathcal{E})\Psi_{J,M}=0\,,\quad [\mathcal{H},\theta]=0\,,$$

we have

$$\phi\psi_{J,M}=(-1)^{J-M}\psi_{J,-M}\,. \tag{14c}$$

We now divide the Hamiltonian H uniquely into a T-conserving part H^0 and a T-violating part H':

$$H=H^0+H'\,, \tag{15}$$

$$[H^0,\theta]=0\,,\quad \{H',\theta\}=0\,, \tag{16}$$

where H^0 is "T-even" and commutes with θ and H' is "T-odd" and anticommutes with θ.

In all that follows, we shall assume that the matrix elements of H' are small compared to the matrix elements of H^0, so that we may use first-order perturbation theory to write the eigenvalues of Eq. (10b) as

$$X_\lambda=X_\lambda{}^0+X_\lambda{}'\,, \tag{17}$$

where

$$X_\lambda{}'=\sum_{\mu\neq\lambda}\frac{H_{\mu\lambda}{}'}{E_\lambda-E_\mu}X_\mu{}^0\,. \tag{18}$$

Because they are real, the boundary conditions B_c do not affect the time-reversal properties of the X_λ.

The matrix elements $H_{\mu\lambda}{}'$ of the T-odd part of H will constitute our measure of T-violation in nuclear interactions. These matrix elements have purely imaginary values as can be seen by applying Eqs. (14b) and (16):

$$\begin{aligned}H_{\mu\lambda}{}'&\equiv(X_\mu{}^0,H'X_\lambda{}^0)=(\theta X_\mu{}^0,\theta H'X_\lambda{}^0)^*\\&=-(\theta X_\mu{}^0,H'\theta X_\lambda{}^0)^*\\&=-(-1)^{2J-2M}(X_\mu{}^0,H'X_\lambda{}^0)^*=-H_{\mu\lambda}{}'^*\,,\end{aligned} \tag{19}$$

where we have used the fact that the matrix elements of the scalar H' are independent of the sign of the magnetic quantum number M.

Applying Eqs. (14c) and (16) to an eigenvalue X_λ

[12] A full discussion of the properties of the time-reversal operator has been given by Eugene P. Wigner, *Group Theory*, translated into English by J. J. Griffin (Academic Press Inc., New York, 1959), Chap. 26.

with total angular-momentum quantum numbers J and M, we obtain the time-reversal property of the eigenfunctions of H:

$$\begin{aligned}\theta X_{\mu(J,M)}&=\theta(X_{\mu(J,M)}{}^0+X_{\mu(J,M)}{}')\\&=(-1)^{J-M}(X_{\mu(J,-M)}{}^0-X_{\mu(J,-M)}{}'). \quad (20)\end{aligned}$$

The same considerations that apply to the complete Hamiltonian H are now applied to the intrinsic Hamiltonian H_A of the fragment A and to H_B of fragment B. The Hamiltonian of the relative motion in the channels will be assumed to the T-even. We then have

$$\begin{aligned}H_A&=H_A{}^0+H_A{}',\\ [H_A{}^0,\theta]&=\{H_A{}',\theta\}=0 \quad (21)\end{aligned}$$

and similarly for H_B. Then from Eqs. (7) and (8b), we have by perturbation theory

$$\begin{aligned}\phi_c&=\phi_c{}^0+\phi_c{}',\\ \theta\phi_{c(J,M)}&=(-1)^{J-M}(\phi_{c(J,-M)}{}^0-\phi_{c(J,-M)}{}'). \quad (22)\end{aligned}$$

where

$$\begin{aligned}\phi_c{}^0&=\sum(c|JM)\frac{i^lY_{lm}(\theta,\phi)}{r}\psi_\alpha{}^0\psi_\beta{}^0,\\ \phi_c{}'&=\sum(c|JM)\frac{i^lY_{lm}(\theta,\phi)}{r}(\psi_\alpha{}^0\psi_\beta{}'+\psi_\alpha{}'\psi_\beta{}^0)\end{aligned} \quad (23)$$

and where

$$\begin{aligned}(H_A{}^0-\epsilon_\alpha)\psi_\alpha{}^0&=0,\\ \psi_\alpha{}'&=\sum_{\alpha'\neq\alpha}\frac{H_{A\alpha'\alpha}{}'}{\epsilon_\alpha-\epsilon_{\alpha'}}\psi_{\alpha'}{}^0\end{aligned} \quad (24)$$

and similar expressions for ψ_β.

Putting Eqs. (17) and (22) into Eq. (11), we find that to first order in H' and H_A'

$$\gamma_{\lambda c}=\gamma_{\lambda c}{}^0+i\gamma_{\lambda c}{}', \quad (25a)$$

where

$$\begin{aligned}\gamma_{\lambda c}{}^0&=(\phi_c{}^0,X_\lambda{}^0)_c,\\ i\gamma_{\lambda c}{}'&=(\phi_c{}^0,X_\lambda{}')_c+(\phi_c{}',X_\lambda{}^0)_c,\\ &=\sum_{\mu\neq\lambda}\frac{H_{\mu\lambda}{}'}{E_\lambda-E_\mu}\gamma_{\mu c}{}^0-\sum_{\alpha'\neq\alpha}\frac{H_{A\alpha'\alpha}{}'}{\epsilon_\alpha-\epsilon_{\alpha'}}\gamma_{\lambda c'(\alpha')}{}^0\\ &\quad-\sum_{\beta'\neq\beta}\frac{H_{B\beta'\beta}{}'}{\epsilon_\beta-\epsilon_{\beta'}}\gamma_{\lambda c'(\beta')}{}^0, \quad (25b)\end{aligned}$$

where $c'(\alpha')$ differs from $c(\alpha)$ by having fragment A in state α' instead of state α. In applying Eq. (25b), we will ordinarily assume for simplicity that only one of the fragments, say A, has internal structure and write

$$i\gamma_{\lambda c}{}'=\sum_{\mu\neq\lambda}\frac{H_{\mu\lambda}{}'}{E_\lambda-E_\mu}\gamma_{\mu c}{}^0-\sum_{c'=c}\frac{H_{Ac'c}{}'}{\epsilon_c-\epsilon_{c'}}\gamma_{\lambda c'}{}^0. \quad (25c)$$

The states c, c', etc. are the "residual states" of the reacting system.

Using the method of Eq. (19), it is easily shown that both the $\gamma_{\lambda c}{}^0$ and the $\gamma_{\lambda c}{}'$ are real. This fact is also consistent with the purely imaginary character of the matrix elements of H', H_A', and H_B'.

We see therefore that the R matrix (13) can be written as the sum of a symmetric T-conserving part and an antisymmetric T-violating part, which to first order are

$$\begin{aligned}\mathbf{R}&=\mathbf{R}^S+\mathbf{R}^A,\\ \mathbf{R}^S&=\sum_\mu(\boldsymbol{\gamma}_\mu{}^0\times\boldsymbol{\gamma}_\mu{}^0)(E_\mu-E)^{-1}, \quad (26)\\ \mathbf{R}^A&=i\sum_\mu(\boldsymbol{\gamma}_\mu{}'\times\boldsymbol{\gamma}_\mu{}^0-\boldsymbol{\gamma}_\mu{}^0\times\boldsymbol{\gamma}_\mu{}')(E_\mu-E)^{-1},\end{aligned}$$

where $\boldsymbol{\gamma}_\mu{}^0$ and $\boldsymbol{\gamma}_\mu{}'$ are vectors with components $\gamma_{\mu c}{}^0$ and $\gamma_{\mu c}{}'$, respectively. Substituting this into Eq. (12), we obtain the symmetric and antisymmetric parts of the S matrix from which we can calculate the degree of violation of the reciprocity relation (3). By means of Eq. (25), we can then express this result in terms of the matrix elements of the T-violating part of the Hamiltonian and the parameters (E_λ, $\gamma_{\lambda c}{}^0$, etc.) describing the T-conserving part of the interaction.

THE ASYMMETRIC S MATRIX

We perform the calculation of the S-matrix elements by means of the level-matrix formalism,[1,9] first under the assumption that in the energy region of interest only a finite number of terms of the sums in Eq. (26) contribute to R. After that, we shall state the modifications required when this assumption is not justified.

In the usual manner, it can be shown that when

$$\mathbf{R}=\sum_{\mu=1}^N(\boldsymbol{\gamma}_\mu\times\boldsymbol{\gamma}_\mu{}^*)(E_\mu-E)^{-1} \quad (27)$$

the following identity holds:

$$(1-RL^0)^{-1}=1+\sum_{\mu\nu}\boldsymbol{\gamma}_\mu\times\mathbf{L}^0\boldsymbol{\gamma}_\nu{}^*A_{\mu\nu}, \quad (28)$$

where the level matrix $\mathbf{A}$ with elements $A_{\mu\nu}$ is given by

$$\mathbf{A}=(\mathbf{e}-\mathbf{E}-\boldsymbol{\xi})^{-1}, \quad (29)$$

where

$$e_{\mu\nu}=\delta_{\mu\nu}E_\nu,\quad E_{\mu\nu}=\delta_{\mu\nu}E,$$

and

$$\xi_{\mu\nu}=\sum_c L_c{}^0\gamma_{\mu c}{}^*\gamma_{\nu c}. \quad (30)$$

From (28) follows the relation

$$\mathbf{A}\boldsymbol{\xi}-(\boldsymbol{\xi}\mathbf{A})^{\text{tr.}}=(\mathbf{A}-\mathbf{A}^{\text{tr.}})(\mathbf{e}-\mathbf{E}),\quad \text{tr.}=\text{transpose}. \quad (31)$$

Substituting (27) into the expression (12) and using Eq. (31), we obtain the result that

$$\mathfrak{U}\equiv(1-\mathbf{RL}^0)^{-1}\mathbf{R}=\sum_{\mu\nu}\boldsymbol{\gamma}_\mu\times\boldsymbol{\gamma}_\nu{}^*A_{\mu\nu}. \quad (32)$$

The contributions of the T-conserving and T-violating parts of H to the $\gamma_{\mu c}$ have already been discussed. We treat the T-violating part of $A_{\mu\nu}$ by first-order matrix perturbation theory

$$\mathbf{A}=(\mathbf{e}-\mathbf{E}-\boldsymbol{\xi}^0-i\boldsymbol{\xi}')^{-1}=\mathbf{A}^0+i\mathbf{A}^0\boldsymbol{\xi}'\mathbf{A}^0, \quad (33)$$

where

$$\xi_{\mu\nu}{}^0=\sum_c L_c{}^0\gamma_{\mu c}{}^0\gamma_{\nu c}{}^0, \quad (34a)$$

$$\xi_{\mu\nu}{}'=\sum_c L_c{}^0(\gamma_{\mu c}{}^0\gamma_{\nu c}{}'-\gamma_{\mu c}{}'\gamma_{\nu c}{}^0), \quad (34b)$$

$$\mathbf{A}^0=(\mathbf{e}-\mathbf{E}-\boldsymbol{\xi}^0)^{-1}. \quad (34c)$$

Substituting (33) and (34) back into Eq. (32), we obtain the following first-order expression for the $\mathfrak{U}$ matrix

$$\mathfrak{U}=\mathfrak{U}^0+\mathfrak{U}', \quad (35)$$

$$\mathfrak{U}_{cc'}{}^0=\sum_{\mu\nu}\gamma_{\mu c}{}^0\gamma_{\nu c'}{}^0A_{\mu\nu}{}^0,$$

$$\mathfrak{U}_{cc'}{}'=i\sum_{\mu\nu}(\gamma_{\mu c}{}'\gamma_{\nu c'}{}^0-\gamma_{\mu c}{}^0\gamma_{\nu c'}{}')A_{\mu\nu}{}^0 +i\sum_{\mu\nu\kappa\lambda c''}\gamma_{\mu c}{}^0\gamma_{\nu c'}{}^0A_{\nu\lambda}{}^0L_{c''}{}^0 \times(\gamma_{\lambda c''}{}^0\gamma_{\kappa c''}{}'-\gamma_{\lambda c''}{}'\gamma_{\kappa c''}{}^0)A_{\kappa\nu}{}^0. \quad (36)$$

Since $\mathbf{A}^0$ is symmetric, $\mathfrak{U}^0$ is clearly symmetric and $\mathfrak{U}'$ is antisymmetric. The contributions of the first sum in $\mathfrak{U}'$ arises from the effects of T violation on $\boldsymbol{\gamma}_\mu$ and $\boldsymbol{\gamma}_\nu$ in Eq. (32), that is, from the effects on the resonance pole residues of $\mathbf{S}$. The contributions of the second sum in $\mathfrak{U}'$ arises from the effects of T violation on $\mathbf{A}$, that is, from the effects on the resonance pole positions. The distinction between these two contributions has already been noted by Mahaux and Weidenmüller.[11]

These two contributions to $\mathfrak{U}'$ are however not independent, but are in fact strongly correlated.[13] By substituting Eq. (25c) into Eq. (36), one obtains a separation of $\mathfrak{U}'$ into an "internal" part $\mathfrak{U}'^i$ which depends on the matrix elements of H' between different compound states, and an "external" part $\mathfrak{U}'^e$ which depends on the matrix elements of $H_A{}'$ between different channels or, better, between different residual states. If one is justified in assuming that the matrix elements of H' and $H_A{}'$ are dynamically independent, then $\mathbf{U}'^i$ and $\mathbf{U}'^e$ are independent parts of $\mathbf{U}'$. They are given by[13]

$$\mathfrak{U}'=\mathfrak{U}'^i+\mathfrak{U}'^e, \quad (37)$$

$$\mathfrak{U}_{cc'}{}'^i=-\sum_{\mu\nu}\gamma_{\mu c}{}^0\gamma_{\nu c'}{}^0A_{\mu\lambda}{}^0H_{\lambda\kappa}{}'A_{\kappa\nu}{}^0, \quad (38)$$

$$\mathfrak{U}_{cc'}{}'^e=\sum_{\mu\nu c''}(\gamma_{\mu c}{}^0\mathcal{H}_{Ac'c''}-\mathcal{H}_{Acc''}\gamma_{\mu c'}{}^0)\gamma_{\nu c''}A_{\mu\nu}{}^0 +\sum_{\mu\nu\kappa\lambda c''c'''}\gamma_{\mu c}{}^0\gamma_{\nu c'}{}^0A_{\mu\lambda}{}^0\gamma_{\lambda c''}{}^0\gamma_{\kappa c'''}{}^0A_{\kappa\nu}{}^0\mathcal{H}_{Ac''c'''} \times(L_{c'''}{}^0-L_{c''}{}^0), \quad (39)$$

where

$$\mathcal{H}_{Acc'}=\mathcal{H}_{Ac'c}=\frac{H_{Ac'c}{}'}{\epsilon_c-\epsilon_{c'}}, \quad c\neq c'$$
$$=0, \qquad c=c'. \quad (40)$$

The result of Eq. (38) is formally similar to the antisymmetric part of the collision matrix given by Ericson.[10] This can be seen from the fact that according to Eq. (34c), the matrix $\mathbf{A}^0$ is essentially a resonance denominator. See also Eq. (67b) below. We note, however, that while the formula in Ericson's paper[10] arises only from the effect of H' on the resonance pole positions, Eq. (38) results from the perturbation of both the pole positions and the $\gamma_{\mu c}$. In fact, the derivation of Eq. (38) involves the cancellation of large contributions from the two sums in Eq. (36).[13]

The assumption (27) is generally not satisfactory. Rather one must write

$$\mathbf{R}=\mathbf{R}^{(1)}+\mathbf{R}^{(\infty)}, \quad (41)$$

where $\mathbf{R}^{(1)}$ contains the contributions of nearby states E_μ that strongly affect the energy dependence of $\mathbf{R}$ in the energy region of interest

$$R_{cc'}{}^{(1)}=\sum_{\mu=1}^{N}\gamma_{\mu c}\gamma_{\mu c'}{}^*(E_\mu-E)^{-1}, \quad (42)$$

while $\mathbf{R}^{(\infty)}$ contains the contribution of more distant poles which do not contribute appreciable energy variations in the limited energy region of interest but which nevertheless have an appreciable effect on the R matrix. The contribution of these distant poles is particularly important for the diagonal elements of the R matrix.[14] Their contribution can in general be evaluated by means of the principal-value integral

$$R_{cc'}{}^{(\infty)}=\mathrm{Pr}\int_{-\infty}^{\infty}dE'\,\rho(E')\frac{\langle\gamma_{\mu c}\gamma_{\mu c'}{}^*\rangle_{E'}}{E'-E}, \quad (43)$$

where $\rho(E')$ is the density of states E_μ at the energy E' and the symbol $\langle\ \ \rangle_{E'}$ indicates an average over indices μ for which E_μ is the vicinity of E'. Clearly $\mathbf{R}^{(\infty)}$ will consist of a real symmetric part $\mathbf{R}^{(\infty)S}$ and a purely imaginary antisymmetric part $\mathbf{R}^{(\infty)A}$

$$R_{cc'}{}^{(\infty)S}=\mathrm{Pr}\int_{-\infty}^{\infty}dE'\,\rho(E')\langle\gamma_{\mu c}{}^0\gamma_{\mu c'}{}^0\rangle_{E'}(E'-E)^{-1}, \quad (44a)$$

$$R_{cc'}{}^{(\infty)A}=\mathrm{Pr}\int_{-\infty}^{\infty}dE'\,\rho(E')i\langle\gamma_{\mu c}{}'\gamma_{\mu c'}{}^0-\gamma_{\mu c}{}^0\gamma_{\mu c'}{}'\rangle_{E'} \times(E'-E)^{-1}. \quad (44b)$$

The resulting $\mathfrak{U}$ matrix consists of a smooth part $\mathfrak{U}^{(s)}$ and a fluctuating part $\mathfrak{U}^{(f)}$, where the latter contains

[13] The correlation between the two sums in the second Eq. (36) and the resulting simplification of Eq. (38) were pointed out by T. Ericson (private communication).

[14] For a recent example showing the errors produced by ignoring $R^{(\infty)}$ in elastic scattering, see C. Mahaux and H. A. Weidenmüller, Nucl. Phys. **A97**, 378 (1967).

the explicit pole terms

$$\mathfrak{U}=\mathfrak{U}^{(s)}+\mathfrak{U}^{(f)}. \tag{45}$$

The off-diagonal elements of $\mathfrak{U}^{(s)}$, when substituted into the S matrix (12), provide the proper description of direct-reaction amplitudes within the framework of the R-matrix formalism. Ordinarily this description of direct reactions is not very useful except insofar as it provides a unified picture of the correlations and interferences of resonance and direct-reaction amplitudes.[15] We shall, however, be able to draw useful conclusions about T-violation effects in direct reactions from the R-matrix formalism.

To first order in the T-odd part of the Hamiltonian, $\mathfrak{U}^{(s)}$ has the symmetric and antisymmetric parts

$$\mathfrak{U}^{(s)0}=(\mathbf{1}-\mathbf{R}^{(\infty)S}\mathbf{L}^0)^{-1}\mathbf{R}^{(\infty)S}, \tag{46a}$$

$$\mathfrak{U}^{(s)\prime}=(\mathbf{1}-\mathbf{R}^{(\infty)S}\mathbf{L}^0)^{-1}\mathbf{R}^{(\infty)A}(\mathbf{1}-\mathbf{L}^0\mathbf{R}^{(\infty)S})^{-1}. \tag{46b}$$

By means of the level matrix inversion method, $\mathfrak{U}^{(f)}$ consists, to first order, of one symmetric and two antisymmetric terms analogous to those of Eq. (36).

$$\mathfrak{U}^{(f)0}=\sum_{\lambda\mu}\boldsymbol{\alpha}_\lambda{}^0\times\boldsymbol{\alpha}_\mu{}^0A_{\lambda\mu}{}^0, \tag{46c}$$

$$\mathfrak{U}^{(f)\prime}=i\sum_{\lambda\mu}(\boldsymbol{\alpha}_\lambda{}'\times\boldsymbol{\alpha}_\mu{}^0-\boldsymbol{\alpha}_\lambda{}^0\times\boldsymbol{\alpha}_\mu{}')A_{\lambda\mu}{}^0 +i\sum_{\lambda\mu\kappa\nu}\boldsymbol{\alpha}_\lambda{}^0\times\boldsymbol{\alpha}_\mu{}^0A_{\lambda\kappa}{}^0\xi_{\kappa\nu}{}''A_{\nu\mu}{}^0, \tag{46d}$$

where

$$\boldsymbol{\alpha}_\mu{}^0=(\mathbf{1}-\mathbf{R}^{(\infty)S}\mathbf{L}^0)^{-1}\boldsymbol{\gamma}_\mu{}^0, \tag{47a}$$

$$\boldsymbol{\alpha}_\mu{}'=(\mathbf{1}-\mathbf{R}^{(\infty)}\mathbf{L}^0)^{-1}(\boldsymbol{\gamma}_\mu{}'+\mathbf{R}^{(\infty)A}\mathbf{L}^0\boldsymbol{\alpha}_\mu{}^0), \tag{47b}$$

$$\xi_{\mu\nu}{}^0=\boldsymbol{\gamma}_\mu{}^0\cdot\mathbf{X}\boldsymbol{\gamma}_\nu{}^0, \tag{47c}$$

$$\xi_{\mu\nu}{}''=i(\boldsymbol{\gamma}_\mu{}^0\cdot\mathbf{X}\boldsymbol{\gamma}_\nu{}'-\boldsymbol{\gamma}_\mu{}'\cdot\mathbf{X}\boldsymbol{\gamma}_\nu{}^0)+\boldsymbol{\gamma}_\mu{}^0\cdot\mathbf{X}\mathbf{R}^{(\infty)A}\mathbf{X}\boldsymbol{\gamma}_\nu{}^0. \tag{47d}$$

Here $\mathbf{A}^0$ is as given in Eq. (34c) and with ξ^0 as given by Eq. (47c), and the symmetric matrix $\mathbf{X}$ is defined as

$$\mathbf{X}=\mathbf{L}^0(\mathbf{1}-\mathbf{R}^{(\infty)S}\mathbf{L}^0)^{-1}=(\mathbf{1}-\mathbf{L}^0\mathbf{R}^{(\infty)S})^{-1}\mathbf{L}^0. \tag{48}$$

The first-order expression for $\mathfrak{U}$ consists of the sum of the five contributions in Eqs. (46). The contribution $\mathfrak{U}^{(s)0}$ is the usual direct-reaction amplitude as derived in R-matrix theory, and $\mathfrak{U}^{(s)\prime}$ is the antisymmetric part of the direct amplitude due to T violation. The expressions for $\mathfrak{U}^{(f)}$ are entirely analogous to the resonance amplitudes of Eq. (36), except for the effect of the background matrix $\mathbf{R}^{(\infty)S}$ on the resonance terms. This illustrates the correlation that always exists between the parameters specifying the direct- and resonance-reaction amplitudes.[15,16] The antisymmetric part of the resonance amplitude also has contributions from $\mathbf{R}^{(\infty)A}$, the antisymmetric part of the background R matrix.

As in the case of Eq. (36), we eliminate the correlation between the two sums in Eq. (46d) by using Eqs. (25c) and (47) to express $\mathfrak{U}^{(f)\prime}$ in terms of the matrix elements of H' and $H_A{}'$. This time we obtain three independent contributions: an "internal" part $\mathfrak{U}^{(f)\prime i}$ which depends on H', and "external" part $\mathfrak{U}^{(f)\prime e}$ which depends on $H_A{}'$, and a "direct" part $\mathfrak{U}^{(f)\prime d}$ which depends on $R^{(\infty)A}$. These are given by the following expressions:

$$\mathfrak{U}^{(f)\prime}=\mathfrak{U}^{(f)\prime i}+\mathfrak{U}^{(f)\prime e}+\mathfrak{U}^{(f)\prime d}, \tag{49a}$$

$$\mathfrak{U}^{(f)\prime i}=-\sum_{\mu\nu}\boldsymbol{\alpha}_\mu{}^0\times A_{\mu\lambda}{}^0H_{\lambda\kappa}{}'A_{\kappa\nu}{}^0\boldsymbol{\alpha}_\nu{}^0, \tag{49b}$$

$$\mathfrak{U}^{(f)\prime e}=\sum_{\mu\nu}A_{\mu\nu}{}^0(\boldsymbol{\alpha}_\mu{}^0\times\mathcal{H}_A\boldsymbol{\alpha}_\nu{}^0-\mathcal{H}_A\boldsymbol{\alpha}_\mu{}^0\times\boldsymbol{\alpha}_\nu{}^0)+\sum_{\mu\nu\kappa\lambda}\boldsymbol{\alpha}_\mu{}^0\times\boldsymbol{\alpha}_\nu{}^0 \times[A_{\mu\kappa}{}^0\boldsymbol{\alpha}_\kappa{}^0\cdot\mathbf{L}^0(\mathbf{X}^{-1}\mathcal{H}_A-\mathcal{H}_A\mathbf{X}^{-1})\mathbf{L}^0\boldsymbol{\alpha}_\lambda{}^0A_{\mu\nu}{}^0], \tag{49c}$$

$$\mathfrak{U}^{(f)\prime d}=\sum_{\mu\nu}A_{\mu\nu}{}^0(\mathbf{L}^{0-1}\mathbf{X}\mathbf{R}^{(\infty)A}\mathbf{L}^0\boldsymbol{\alpha}_\mu{}^0\times\boldsymbol{\alpha}_\nu{}^0 -\boldsymbol{\alpha}_\mu{}^0\times\mathbf{L}^{0-1}\mathbf{X}\mathbf{R}^{(\infty)A}\mathbf{L}^0\boldsymbol{\alpha}_\nu{}^0)+\sum_{\mu\nu}\boldsymbol{\alpha}_\mu{}^0\times\boldsymbol{\alpha}_\nu{}^0 \times(A_{\mu\kappa}{}^0\boldsymbol{\alpha}_\kappa{}^0\cdot\mathbf{L}^0\mathbf{R}^{(\infty)A}\mathbf{L}^0\boldsymbol{\alpha}_\lambda{}^0A_{\lambda\nu}{}^0). \tag{49d}$$

The matrix $\mathcal{H}_A$ has been defined in Eq. (40).

CROSS-SECTION ASYMMETRIES

Denoting the symmetric and antisymmetric parts of the $\mathbf{S}$ and $\mathfrak{U}$ matrices by $\mathbf{S}^S$, $\mathbf{S}^A$ and $\mathfrak{U}^S$, $\mathfrak{U}^A$, respectively, we see from Eqs. (2) and (4) that the cross-section asymmetry is given to first order by

$$\begin{aligned}\delta\sigma_{cc'}&=\sigma_{cc'}-\sigma_{c'c}=4\ \mathrm{Re}(S_{cc'}{}^SS_{cc'}{}^{A*})\\&=16P_cP_{c'}\ \mathrm{Re}(\mathfrak{U}_{cc'}{}^S\mathfrak{U}_{cc'}{}^{A*}).\end{aligned} \tag{50}$$

We shall compare this asymmetry to the mean cross section which is given to first order by

$$\begin{aligned}\sigma_{cc'}{}^0&=\tfrac{1}{2}(\sigma_{cc'}+\sigma_{c'c})=|S_{cc'}{}^S|^2\\&=4P_cP_{c'}|\mathfrak{U}_{cc'}{}^S|^2,\\&c\neq c'.\end{aligned} \tag{51}$$

We consider in turn the cross-section asymmetries in three different kinds of situations: direct reactions with no resonances, isolated resonances, and fluctuating cross sections arising from contributions of many resonance pole terms with no competing direct-reaction amplitude. Other cases can be discussed equally well but tend to lead to complications without further clarification of the essential features of the results.

Direct Reactions

In the direct-reaction case with no nearby contributing pole terms, we take

$$\mathfrak{U}^S=\mathfrak{U}^{(s)0},\quad \mathfrak{U}^A=\mathfrak{U}^{(s)\prime} \tag{52}$$

as defined in Eqs. (46a) and (46b), and we evaluate $\mathbf{R}^{(\infty)}$ by means of Eqs. (44a) and (44b). From Eq.

[15] P. A. Moldauer, Phys. Rev. **157**, 907 (1967).

[16] K. F. Ratcliff and N. Austern, in *Perspectives in Modern Physics*, edited by R. E. Marshak (Interscience Publishers, Inc., New York, 1966), p. 57 and Ann. Phys. (N. Y). **42**, 185 (1967).

(25c), we have the result that

$$i\langle\gamma_{\mu c}'\gamma_{\mu c'}{}^0\rangle_{E'}=\sum_{\lambda\neq\mu}\left\langle\frac{H_{\lambda\mu}'}{E_\mu-E_\lambda}\gamma_{\lambda c}{}^0\gamma_{\mu c'}{}^0\right\rangle_{E'}-\sum_{c''\neq c}\frac{H_{Ac''c}'}{\epsilon_c-\epsilon_{c''}}\langle\gamma_{\mu c''}{}^0\gamma_{\mu c'}{}^0\rangle_{E'}. \quad (53)$$

The first term on the right-hand side is expected to vanish for a variety of reasons: Each of the factors $H_{\lambda\mu}'$, $(E_\mu-E_\lambda)^{-1}$, and $\gamma_{\lambda c}{}^0\gamma_{\mu c'}{}^0$ $(\mu\neq\lambda)$ has an expectation value of zero, and there is no reason to expect the signs or values of any of these factors to be correlated. Therefore only the second term on the right of Eq. (53) contributes, and we have by Eqs. (44a) and (44b)

$$\mathbf{R}^{(\infty)A}=[\mathbf{R}^{(\infty)S},\mathcal{H}_A]. \quad (54)$$

Putting Eq. (54) into (46b), we obtain

$$\mathfrak{U}^{(s)\prime}=[\mathfrak{U}^{(s)0},\mathcal{H}_A]+\mathfrak{U}^{(s)0}[\mathcal{H}_A,\mathbf{L}^0]\mathfrak{U}^{(s)0}, \quad (55)$$

which is to be inserted into Eqs. (50) and (51) in accordance with the definitions (52) in order to obtain the symmetric and antisymmetric parts of the direct-reaction cross section. We note the following properties of this result.

1. The asymmetry $\delta\sigma_{cc'}$ of a direct-reaction cross section depends linearly on the matrix elements of the T-odd part of the Hamiltonian between states of the *residual* nucleus, divided by the energy separation of these states.

2. The asymmetry $\delta\sigma_{cc'}$ depends in a complicated way upon all matrix elements $H_{Ac''c'''}'$ between residual states c'' and c''' that are coupled to c or c' by direct-reaction amplitudes such as $\mathfrak{U}_{cc''}{}^{(s)0}$. In fact, $\delta\sigma_{cc'}$ may be nonvanishing even though the T-odd matrix element $H_{Acc'}'$ between c and c' vanishes. Therefore the dynamical interpretation of direct-reaction cross-section asymmetries may be somewhat involved.

3. It goes without saying that as indicated in Eq. 50 the existence of an asymmetry $\delta\sigma_{cc'}$ depends on a nonvanishing direct-reaction amplitude $\mathfrak{U}_{cc'}{}^{(s)0}$. Also the two-state theorem holds. On the other hand $\mathfrak{U}_{cc'}{}^{(s)\prime}$ can have a nonvanishing value even if $\mathfrak{U}_{cc'}{}^{(s)0}$ is diagonal. For example, in the case of a single nonvanishing T-odd matrix element $\mathcal{H}_{Acc'}$ we have

$$\mathfrak{U}_{cc'}{}^{(s)\prime}=\mathcal{H}_{Acc'}[\mathfrak{U}_{cc}{}^{(s)0}-\mathfrak{U}_{c'c'}{}^{(s)0}+(L_{c'}{}^0-L_c{}^0)\mathfrak{U}_{cc}{}^{(s)0}\mathfrak{U}_{c'c'}{}^{(s)0}]. \quad (56)$$

The possibility exists that such an amplitude can be observed through interference effects in angular correlation or polarization experiments.

In the presence of nearby resonance pole terms contributing local energy variations in $\mathfrak{U}$, the concept of a direct-reaction amplitude is not clearly defined but rather depends on the particular type of direct reaction one has in mind. The two simplest definitions of the direct-reaction amplitude are first, the above definition in terms of the contributions of distant R-matrix pole terms and second, as the contribution of the energy average of the $\mathfrak{U}$ matrix, which includes an average over nearby pole terms. It is easy to see that the relation (54) will also hold for the second definition of a direct-reaction amplitude, and therefore the same conclusions follow.

Isolated Resonances

A commonly discussed simple situation is the case where the energy dependence of $\mathfrak{U}$ is affected appreciably only by one pole E_0 of the R matrix (13). In that case, Eq. (42) reduces to a single term

$$R_{cc'}{}^{(1)}=\gamma_{0c}\gamma_{0c'}{}^*/(E_0-E), \quad (57)$$

and therefore

$$A^0=(E_0-E+S_0-\tfrac{1}{2}i\Gamma_0)^{-1}, \quad (58)$$

where

$$-S_0+\tfrac{1}{2}i\Gamma_0=\xi_{00}{}^0.$$

In the absence of competing direct reactions, that is when $\mathbf{R}^{(\infty)}=\mathbf{R}^{(\infty)S}$ is diagonal, the expression (47d) vanishes, and we have from Eq. (46)

$$\mathfrak{U}_{cc'}=\frac{-\alpha_{0c}{}^0\alpha_{0c'}{}^0+i(\alpha_{0c}{}^0\alpha_{0c'}{}'-\alpha_{0c}{}'\alpha_{0c'}{}^0)}{E-E_0-S_0+\frac{1}{2}i\Gamma_0}. \quad (59)$$

Using Eqs. (25c) and (47b), we find that

$$\alpha_{0c}{}'=q_c\alpha_{0c}{}^0, \quad (60)$$

where

$$iq_c=\sum_{\mu\neq0}\frac{H_{\mu0}'}{E_0-E_\mu}\frac{\gamma_{\mu c}{}^0}{\gamma_{0c}{}^0}-\sum_{c'\neq c}\frac{H_{Ac'c}'}{\epsilon_c-\epsilon_{c'}}\frac{\gamma_{0c'}{}^0}{\gamma_{0c}{}^0}. \quad (61)$$

In consequence of the purely imaginary values of $H_{\mu\nu}'$ and $H_{Acc'}'$, the q_c are real and

$$\mathfrak{U}_{cc'}=-\frac{\alpha_{0c}{}^0\alpha_{0c'}{}^0}{E-E_0-S_0+\frac{1}{2}i\Gamma_0}(1+iq_c-iq_{c'}). \quad (62)$$

From this, it follows that

$$|\mathfrak{U}_{cc'}|^2=|\mathfrak{U}_{c'c}|^2, \quad \delta\sigma_{cc'}=0. \quad (63)$$

We see, therefore, that in the absence of a competing direct reaction an isolated resonance cross section does not violate the reciprocity relation to first order in the matrix elements of the T-odd part of the Hamiltonian. This result has already been stated by Mahaux and Weidenmüller.[11] The conclusion (63) is, of course, not valid in the presence of any competing reaction mode such as a direct component or the overlapping "tail" of a distant resonance. The cross-section asymmetries for such cases can be calculated by means of the above formulas. They involve a large number of parameters, including, of course, parameters which refer to at least one third competing channel as in the case of Eq. (55) and as required by the two-state theorem.[17]

Fluctuating Cross Sections

In order to discuss the effects of several or many R-matrix poles upon the energy dependence of a cross section, it is useful to express the S matrix as a series of resonance pole terms of the type given in Eq. (59). The general method for doing this has been discussed in detail elsewhere.[9,18] In the present application, we diagonalize the complex symmetric matrix $\mathbf{A}^0$ by means of the complex orthogonal matrix T,

$$T_{\mu\nu}A_{\nu\kappa}{}^0T_{\lambda\kappa}=\delta_{\mu\lambda}(\mathcal{E}_\mu-\tfrac{1}{2}i\Gamma_\mu-E)^{-1}, \tag{64}$$

and assume $\mathbf{R}^{(\infty)}$ to be diagonal (no direct reactions) so that $\mathbf{R}^{(\infty)A}$ vanishes. Then, if we define

$$g_{\mu c}=(2P_c)^{1/2}\exp(-i\phi_c)\sum_\nu T_{\mu\nu}\alpha_{\nu c}{}^0, \tag{65}$$

we find from Eq. (46c) that

$$S_{cc'}{}^S=-i\sum_\mu\frac{g_{\mu c}g_{\mu c'}}{E-\mathcal{E}_\mu+\frac{1}{2}i\Gamma_\mu},\quad c\neq c', \tag{66}$$

and from Eq. (49) we see that the antisymmetric part of the S matrix contains an internal and an external contribution.

$$\mathbf{S}^A=\mathbf{S}^{Ai}+\mathbf{S}^{Ae}, \tag{67a}$$

$$S_{cc'}{}^{Ai}=-i\sum_{\mu\nu}\frac{g_{\mu c}\hat{H}_{\mu\nu}{}'g_{\nu c'}}{(E-\mathcal{E}_\mu+\frac{1}{2}i\Gamma_\mu)(E-\mathcal{E}_\nu+\frac{1}{2}i\Gamma_\nu)}, \tag{67b}$$

$$S_{cc'}{}^{Ae}=i\sum_\mu\frac{g_{\mu c}\mathcal{H}_{Ac'c''}g_{\mu c''}-\mathcal{H}_{Acc''}g_{\mu c''}g_{\mu c'}}{E-\mathcal{E}_\mu+\frac{1}{2}i\Gamma_\mu}$$
$$+i\sum_{\mu\nu c''c'''}\frac{g_{\mu c}g_{\nu c'}g_{\mu c''}\mathcal{H}_{Ac''c'''}\mathcal{L}_{c''c'''}g_{\nu c'''}}{(E-\mathcal{E}_\mu+\frac{1}{2}i\Gamma_\mu)(E-\mathcal{E}_\nu+\frac{1}{2}i\Gamma_\nu)}, \tag{67c}$$

where we have used the abbreviations

$$\hat{H}_{\mu\nu}{}'=T_{\mu\kappa}H_{\kappa\lambda}{}'\tilde{T}_{\lambda\nu} \tag{68}$$

and

$$\mathcal{L}_{cc'}=L_{c'}{}^0-L_c{}^0+L_c{}^0L_{c'}{}^0(R_{c'c'}{}^{(\infty)S}-R_{cc}{}^{(\infty)S}). \tag{69}$$

In the case of overlapping resonances, that is when the Γ_μ are larger than the spacings of the $\mathcal{E}_\mu$, the energy variations of cross sections are conveniently discussed in statistical terms such as their energy averages, average squares, correlations, etc. We shall calculate such averages by the method of the statistical S matrix which was defined in Ref. 18 and which represents the S matrix in a finite energy interval ΔE by a uniform random function of the type given in Eqs. (66) and (67). These random functions are specified by an appropriate ensemble of resonance parameters $g_{\mu c}$, $g_{\mu c'}$, $\cdots$, Γ_μ, $\mathcal{E}_\mu$. We shall assume that the ensemble averages over the index μ of $g_{\mu c}$ vanish.

$$\langle g_{\mu c}\rangle_\mu=0,\quad \text{all } c \tag{70}$$

and that all channel-channel correlations vanish

$$\langle g_{\mu c}g_{\mu c'}\rangle_\mu=\langle g_{\mu c}{}^*g_{\mu c'}\rangle_\mu=0,\quad c\neq c'. \tag{71}$$

This latter assumption is possible only as long as all off-diagonal elements of $R^{(\infty)}$ vanish.[15,16]

Average S-Matrix Asymmetry

We first calculate the energy averages of Eqs. (66) and (67) by means of Eqs. (B3) and (B11b) of Ref. 18 and obtain, using Eqs. (70) and (71),

$$\bar{S}_{cc'}{}^S=0,\quad c\neq c' \tag{72}$$

$$\bar{S}_{cc'}{}^{Ai}=0, \tag{73}$$

$$\bar{S}_{cc'}{}^{Ae}=(\pi/\mathfrak{D})\mathcal{H}_{Acc'}(\langle g_{\mu c}{}^2\rangle_\mu-\langle g_{\mu c'}{}^2\rangle_\mu)$$
$$-i\pi^2\mathfrak{D}^{-2}\langle g_{\mu c}{}^2\rangle_\mu\langle g_{\nu c'}{}^2\rangle_\nu\mathcal{H}_{Acc'}\mathcal{L}_{cc'}, \tag{74}$$

where $\mathfrak{D}$ is the mean spacing of the $\mathcal{E}_\mu$. We see that in general the antisymmetric part of the average S-matrix element $\bar{S}_{cc'}$ vanishes only if $H_{Ac'c}{}'$ vanishes or if both $\langle g_{\mu c}{}^2\rangle_\mu$ and $\langle g_{\mu c'}{}^2\rangle_\mu$ vanish. Of course, the nonvanishing of $\bar{S}_{cc'}{}^A$ has no important consequences, since according to Eq. (72) $\bar{S}_{cc'}{}^S$ does vanish according to our assumption, and hence the contribution of the average S matrix to the cross-section asymmetry vanishes.

Average Cross-Section Asymmetry

The simplest observable statistical property is the average cross section. The symmetric part of the average cross section has been calculated in Ref. 18, where it was shown that in the limit $\bar{\Gamma}\equiv\langle\Gamma_\mu\rangle_\mu\gg\mathfrak{D}$

$$\sigma_{cc'}{}^0=(2\pi/\mathfrak{D})\langle|g_{\mu c}|^2|g_{\mu c'}|^2/\Gamma_\mu\rangle_\mu. \tag{75}$$

The assumptions (70) and (71) imply that the energy variations of the antisymmetric part (67) of the S matrix are uncorrelated with those of the symmetric part (66) and that therefore the energy average of the cross-section asymmetry (50) vanishes:

$$\langle\delta\sigma_{cc'}\rangle=0,\quad \text{(no direct reactions)}. \tag{76}$$

This result may be confirmed by performing the average with the help of the methods of Ref. 18 and applying the conditions (70) and (71).

Cross-Section Asymmetry Fluctuations

We must therefore study the fluctuations in $\delta\sigma_{cc'}$. The simplest description of the magnitude of these fluctuations is given by the normalized mean square

[17] We note that the calculation of Mahaux and Weidenmüller (Ref. 11) for the case of two interfering resonances yields an expression for the cross-section asymmetry that is not clearly consistent with the requirements of the two-state theorem.

[18] P. A. Moldauer, Phys. Rev. 135, B642 (1964).

value

$$\Delta_{cc'}=\langle\delta\sigma_{cc'}{}^2\rangle/\bar{\sigma}_{cc'}{}^2\,, \tag{77}$$

which is useful for the interpretation of the cross-section ratios measured in the experiment of von Witsch *et al.*[4]; see Eq. (6).

The substitution of expressions (66) and (67) into the square of $\delta\sigma_{cc'}$ as given in Eq. (50) yields a great many terms which must be averaged. By considering again the limiting case $\bar{\Gamma}\gg\mathfrak{D}$ and applying the assumptions (70) and (71), the number of terms contributing to the average of $(\delta\sigma_{cc'})^2$ is reduced to the following:

$$8\sum_\mu\left|\frac{g_{\mu c}g_{\mu c'}}{E-\mathcal{E}_\mu+\frac{1}{2}i\Gamma_\mu}\right|^2\Bigg[\sum_\nu\left|\sum_{c''}\frac{\mathcal{H}_{Acc''}g_{\nu c''}g_{\nu c'}-g_{\nu c}\mathcal{H}_{Ac'c''}g_{\nu c''}}{E-\mathcal{E}_\nu+\frac{1}{2}i\Gamma_\nu}\right|^2$$
$$+\sum_{\nu\kappa}\left|\frac{g_{\nu c}(\hat{H}_{\nu\kappa}'-\sum_{c''c'''}g_{\nu c''}\mathcal{H}_{Ac''c'''}\mathfrak{L}_{c''c'''}g_{\kappa c'''})g_{\kappa c'}}{(E-\mathcal{E}_\nu+\frac{1}{2}i\Gamma_\nu)(E-\mathcal{E}_\kappa+\frac{1}{2}i\Gamma_\kappa)}\right|^2\Bigg]. \tag{78}$$

To average this expression, we employ the result of Eq. (B17) in Ref. 18 and the following generalization which is derivable in the same way:

$$\left\langle\prod_{i=1}^N\sum_\mu\left|\frac{a_\mu{}^{(i)}}{E-\mathcal{E}_\mu+\frac{1}{2}i\Gamma_\mu}\right|^2\right\rangle=2N\left(\frac{\pi}{\mathfrak{D}}\right)^N\left\langle\prod_{i=1}^N\frac{|a_{\mu_i}{}^{(i)}|^2}{\Gamma_{\mu_i}}\right\rangle_{\mu_j\neq\mu_k}\,,\quad\bar{\Gamma}\gg\mathfrak{D}. \tag{79}$$

We obtain then the result

$$\langle\delta\sigma_{cc'}{}^2\rangle=\frac{48\pi^3}{\mathfrak{D}^3}\left\langle\frac{G_{\mu c}G_{\mu c'}G_{\nu c}G_{\kappa c'}}{\Gamma_\mu\Gamma_\nu\Gamma_\kappa}\Big(|\hat{H}_{\nu\kappa}'|^2+\sum_{c''c'''}G_{\nu c''}G_{\kappa c'''}|\mathcal{H}_{Ac''c'''}\mathfrak{L}_{c''c'''}|^2\Big)\right\rangle_{\mu\neq\nu\neq\kappa\neq\mu}$$
$$+\frac{32\pi^2}{\mathfrak{D}^2}\left\langle\frac{G_{\mu c}G_{\mu c'}}{\Gamma_\mu\Gamma_\nu}\Big[\sum_{c''}G_{\nu c''}(|\mathcal{H}_{Acc''}|^2G_{\nu c'}+|\mathcal{H}_{Ac'c''}|^2G_{\nu c})-|\mathcal{H}_{Acc'}|^2(g_{\nu c}{}^2g_{\nu c'}{}^{*2}+g_{\nu c}{}^{*2}g_{\nu c'}{}^2)\Big]\right\rangle_{\mu\neq\nu}\,,$$
$$\bar{\Gamma}\gg\mathfrak{D}\,,\quad\text{no direct reactions}\,, \tag{80}$$

where

$$G_{\mu c}\equiv|g_{\mu c}|^2. \tag{81}$$

To obtain $\Delta_{cc'}$, we must divide (76) by

$$(\sigma_{cc'}{}^0)^2=\frac{4\pi^2}{\mathfrak{D}^2}\left\langle\frac{G_{\mu c}G_{\mu c'}}{\Gamma_\mu}\right\rangle_\mu^2. \tag{82}$$

Assuming the number of independent open channels to be large, we can neglect the effects of correlations between the values of the $G_{\mu c}$ and the Γ_μ and replace the averages of the functions of these resonance parameters in Eqs. (80) and (82) by the functions of their averages. For example,

$$(\sigma_{cc'}{}^0)^2\cong\frac{4\pi^2}{\mathfrak{D}^2}\frac{\bar{G}_c{}^2\bar{G}_{c'}{}^2}{\bar{\Gamma}^2}\quad\text{(many open channels)}. \tag{82$'$}$$

With this assumption, we find that after separating the internal and external contributions $\Delta_{cc'}$ becomes

$$\Delta_{cc'}=\Delta_{cc'}{}^i+\Delta_{cc'}{}^e\,, \tag{83a}$$
$$\Delta_{cc'}{}^i=12\pi\langle|\hat{H}_{\nu\kappa}'|^2\rangle_{\nu\kappa}/\bar{\Gamma}\mathfrak{D}\,, \tag{83b}$$
$$\Delta_{cc'}{}^e=6\sum_{c''c'''}\bar{\sigma}_{c''c'''}{}^0|\mathcal{H}_{Ac''c'''}\mathfrak{L}_{c''c'''}|^2$$
$$+8\Big[\sum_{c''}\langle G_{\nu c''}(|\mathcal{H}_{Acc''}|^2/G_{\nu c}+|\mathcal{H}_{Ac'c''}|^2/G_{\nu c'})\rangle_\nu$$
$$-|\mathcal{H}_{Acc'}|^2\langle(g_{\nu c}{}^2g_{\nu c'}{}^{*2}+g_{\nu c}{}^{*2}g_{\nu c'}{}^2)/G_{\nu c}G_{\nu c'}\rangle_\nu\Big] \tag{83c}$$

(no direct reactions, many open channels, $\bar{\Gamma}/\mathfrak{D}\gg1$).

In order to estimate the magnitude of $\Delta_{cc'}{}^i$, we assume that the values of the matrix elements $H_{\mu\nu}'$ are random in μ and ν and have zero means. Then from the definition (68), we find that

$$|\hat{H}_{\nu\kappa}'|^2=\sum_{\mu\lambda}(|T_{\nu\mu}|^2|T_{\kappa\lambda}|^2-T_{\nu\mu}T_{\nu\lambda}{}^*T_{\kappa\lambda}T_{\kappa\mu}{}^*)|H_{\mu\lambda}'|^2$$
$$\approx N_\nu N_\kappa\langle|H_{\mu\lambda}'|^2\rangle_{\mu\lambda}\,, \tag{84}$$

where we have made use of the expected zero average of the $T_{\mu\nu}$ and the definition of the important resonance normalization factor N_μ[9,18]:

$$\sum_\nu|T_{\mu\nu}|^2\equiv N_\mu\geq1. \tag{85}$$

The average value of N_μ is close to unity when the transmission coefficients T_c of all competing channels is less than about 0.3. But this average may become very large when one or more transmission coefficients approach unity. In the single-channel case, a lower limit is given by[15,19]

$$N\equiv\langle N_\mu\rangle_\mu\geq T(1-T)^{-1/2}|\ln(1-T)|\,, \tag{86}$$

where T is the transmission coefficient. In multichannel cases the estimation of N still depends on numerical statistical-model calculations.[19]

[19] P. A. Moldauer, Bull Am. Phys. Soc. **12**, 27 (1967); (to be published).

We find then that

$$\Delta_{cc'}{}^{i}\approx 12\pi N^2\langle|H'|^2\rangle/\bar{\Gamma}\mathfrak{D}, \tag{87}$$

where $\langle|H'|^2\rangle$ is the mean absolute square value of the matrix elements of the T-violating part of the Hamiltonian taken between different compound nuclear states (or more precisely, between R-matrix states).

To estimate the value of $\Delta_{cc'}{}^{e}$, we note first that in the many-channel case the sum $\sum_{c'}\bar{\sigma}_{cc'}{}^{0}$ is approximately equal to the transmission coefficient T_c and that $\sum_c T_c$ can be written as $n\bar{T}$, where n is an effective number of competing open channels and $\bar{T}$ is the average transmission coefficient for these channels.

The sum $\sum_{c''}\langle G_{\nu c''}/G_{\nu c}\rangle_\nu$ may be estimated to have the value $n\bar{T}/T_c$,[20] and the last term in Eq. (83c) may be neglected, as it may be either positive or negative and involves no sum over channels. As a result, we obtain the estimate

$$\Delta_{cc'}{}^{e}\approx\frac{8\pi^2}{3}n\bar{T}\left(\frac{1}{T_c}+\frac{1}{T_{c'}}+\tfrac{3}{4}\langle|\mathfrak{L}|^2\rangle\right)\langle|H_A'|^2\rangle/D_{\mathrm{res}}{}^2. \tag{88}$$

Here D_{res} is the mean spacing of residual nucleus levels, $\langle|H_A'|^2\rangle$ and $\langle|\mathfrak{L}|^2\rangle$ are the averages over residual levels of the absolute squares of the matrix elements of H_A' and of $\mathfrak{L}$, respectively. We have assumed that the values of the $H_{Acc'}{}'$ are uncorrelated with either the residual level spacings $\epsilon_c-\epsilon_{c'}$ or with the $\mathfrak{L}_{cc'}$. The numerical factor in Eq. (88) corresponds to the case where the ϵ_c are equally spaced. It will differ slightly for more realistic spacing distributions.

In order to estimate the relative magnitude of the external and internal contributions to $\Delta_{cc'}$, we assume that the magnitudes of the matrix elements of H' do not depend strongly on the excitation energies of the states and that hence $\langle|H'|^2\rangle\approx\langle|H_A'|^2\rangle$.

We also take $T_c\approx T_{c'}\approx\bar{T}$ and estimate $\langle|\mathfrak{L}|^2\rangle$ to be of the order of unity. Then

$$\frac{\Delta_{cc'}{}^{e}}{\Delta_{cc'}{}^{i}}\approx\frac{2n\bar{\Gamma}\mathfrak{D}}{3N^2D_{\mathrm{res}}{}^2}. \tag{89}$$

At low energies where there are no strongly absorbed channels, the ratio (89) is of the order of $(n^2/3\pi)\times(\mathfrak{D}/D_{\mathrm{res}})^2$, which in medium- and heavy-weight nuclei is of the order of 10^{-4} to 10^{-6}. At higher energies this ratio may increase, but it is still expected to be small at energies sufficiently low that resonance fluctuations can be observed experimentally.

We conclude that in the absence of direct reactions and in the limit $\bar{\Gamma}\gg\mathfrak{D}$ the normalized mean-square cross-section asymmetry $\Delta_{cc'}$ is given by Eq. (87).[20]

Asymmetry–Cross-Section Correlations

One can go on to calculate more complicated coefficients describing the distribution and correlations of the cross-section asymmetry $\delta\sigma_{cc'}$. We have calculated the correlation coefficient between the cross section $\sigma_{cc'}$ and the square of the asymmetry $\sigma\delta_{cc'}$, neglecting the effects of H_A'. With the help of Eq. (79) we find that

$$\langle(\delta\sigma_{cc'})^2\sigma_{cc'}\rangle/\langle(\delta\sigma_{cc'})^2\rangle\langle\sigma_{cc'}\rangle-1=\tfrac{1}{3},\quad H_A'=0. \tag{90}$$

The brackets indicate energy averages. The small positive correlation is not surprising since the cross-section asymmetry arises from the interference of the antisymmetric and symmetric parts of the S matrix. It indicates that the magnitude of the asymmetry has a very small tendency to be larger at cross-section maxima than at minima. The correlation is not large enough to affect the argument of von Witsch *et al.*[4] in applying Eq. (6) to the measurement of the cross-section asymmetries at cross-section minima.

Discussion

We have shown that cross-section asymmetries $\delta\sigma_{cc}{}'=\sigma_{cc'}-\sigma_{c'c}$ measure different aspects of the T-violating part of the Hamiltonian, depending on the mechanism responsible for the reaction.

In direct reactions, the asymmetry depends on those matrix elements of the T-odd part of H that connect different states of the residual nuclei divided by the energy separation of these states. Moreover, matrix elements involving all possible competing residual states contribute to every direct cross-section asymmetry. Since compound-nucleus effects were shown to contribute no asymmetry to average cross sections, we conclude that only the above direct-reaction effects contribute to asymmetries of cross sections measured with energy resolutions that are broad compared to fluctuation intervals.

An isolated compound-nucleus resonance should display no cross-section asymmetry except insofar as the resonance interferes with a direct-reaction background. In that case, the magnitude of the asymmetry is again governed by the same matrix elements and energy separations that govern the direct-reaction cross-section asymmetry.

Cross sections which fluctuate with energy because of the simultaneous energy-dependent contributions of a large number of compound resonance poles ($\bar{\Gamma}\gg\mathfrak{D}$) exhibit a fluctuating asymmetry whose average value vanishes in the absence of direct-reaction competition and whose rms value is estimated to be

$$\mathrm{rms}(\delta\sigma)/\bar{\sigma}=2(3\pi)^{1/2}N\langle|H'|^2\rangle^{1/2}/(\bar{\Gamma}\mathfrak{D})^{1/2} \tag{91}$$

in the case when many competing decay channels are open. In Eq. (91), $\langle|H'|^2\rangle^{1/2}$ is the root mean absolute square of the matrix elements of the T-odd part of H

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[20] Actually the relationship between $\langle G_{\nu c}\rangle_\nu$ and T_c is quite nonlinear, as indicated by the inequality $2\pi\langle G_{\nu c}\rangle_\nu/D\geq T_c(1-T_c)^{-1/2}$, see Refs. 18, 19. This means that a considerable enhancement of $\Delta_{cc'}{}^{e}$ compared to $\Delta_{cc'}{}^{i}$ can be obtained in experiments where $\bar{T}$ is large (many strongly absorbed competing channels), but either T_c or $T_{c'}$ is small.

that connect different *compound states* (as defined in R-matrix theory), $\bar{\Gamma}$ is the mean width, and $\mathfrak{D}$ the mean spacing of resonance poles. The average resonance normalization factor N is unity when all competing open channels are weakly absorbed (have small transmission coefficients). In the presence of strongly absorbed channels ($T_c \approx 1$), the factor N can become very large, thus enhancing the observable cross-section asymmetry. Estimates of the values of N can be obtained from Eq. (86) and Refs. 15 and 19. Except for the factor N, the result (91) is formally almost identical to that of Ericson.[10]

We do not go on to interpret Eq. (91) in terms of a "fraction of time-reversal-odd force,"[6] or of a relative "strength" of T-violation.[10] These concepts involve the comparison of the matrix elements of H' with those of another measurable but T-even part of the Hamiltonian, say $H^{(0)}$. Such a comparison would only be useful if these matrix elements were proportional to one another except for easily ascertained kinematical factors, for otherwise the value of such a "strength" or "fraction" might vary strongly from one experiment to another. But the selection, or even the existence, of such a dynamically proportional $H^{(0)}$ depends on properties of H' which are presently unknown, that is on whether H' is related to the strong interaction, to the electromagnetic interaction, to a super-weak interaction, or to none of these. The disagreement between Ericson[10] and Mahaux and Weidenmüller[11] in their estimates of an "enhancement factor" originates in their different choices of $H^{(0)}$, neither of which appears presently to be justified as clearly superior.

In order to clarify these remarks, we point out that the "strength" of the parity-violating interaction has a fixed value relative to the strength of the parity-conserving part of the weak interaction because they are related by a simple kinematical proportionality operation (i.e., a constant times γ_5).[21] Had parity violation been discovered in a way that did not link it with the weak interaction, no particular insight would have been gained by comparing the amounts of parity violation with, say, the magnitudes of the shell-model residual interaction matrix elements in a variety of experiments involving respectively weak, electromagnetic, and strong interactions. The deduced value of the "strength" would have varied over many orders of magnitude.

A second reason for avoiding the use of ratios of the matrix elements of H' to those of an appropriate H^0 to parametrize the magnitudes of reciprocity violation effects in nuclear reactions is the fact that measured cross-section ratios tend to be complicated functions of such matrix-element ratios and depend on other relevant parameters besides. Thus, as we have seen, the order of magnitude of the effect in direct reactions depends also on the spacings of residual levels, and in fluctuating reactions, it depends also on the spacings and widths of compound levels.

Thus, in the absence of any clearly superior and meaningful dimensionless parameter, we prefer to leave the result in the form of Eq. (55) for direct reactions and Eq. (91) for fluctuating cross sections. The problems to be investigated experimentally are then first, whether reciprocity violations can be observed and second, what the magnitudes of the matrix elements of H' are in different cases of reciprocity violation. The latter information would then have to be interpreted in terms of models and theories of T violation and of nuclear structure.

We conclude that the best way to *detect* cross-section asymmetries due to T violation is in high-resolution experiments of fluctuating cross sections, though cross sections with very drastic fluctuations require further checking for possible applicability of the two-state theorem or isolated resonance difficulties. Particularly advantageous are reactions having many strongly absorbed competing channels which must not, however, give rise to direct reactions and which can be measured under conditions where very few independent alternatives, or "degrees of freedom," contribute to the measured cross section. Combining all favorable aspects would be a reaction with small residual nucleus spins that can be measured at forward or backward angles at moderate energies where, however, many different competing composite particle channels have energies above their Coulomb barriers.

On the other hand, the *interpretation* of cross-section asymmetries in terms of the dynamical properties of the T-odd part of H would probably be easier in the case of direct reactions, particularly if it should be possible to measure the asymmetries in many of the reactions coupling one set of competing direct-reaction channels. In that case, it might be possible to solve the set of simultaneous equations which result from substituting Eqs. (55) into (50) and thus obtain the values of the various matrix elements of H_A'—the T-odd part of the Hamiltonian of the residual nuclei.

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[21] In the case of T violation, it is not even clear that one can find an analogous operator α which does not depend on relative momenta or other dynamical variables and which transforms a Hermitean T-even interaction into a Hermitean T-odd one by $H' = \alpha H^{(0)}$.