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Retarded Interactions in Fermi Systems*

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The nature of the two-body interactions in many-fermion systems is studied from the viewpoint of meson theory. An exactly soluble model is formulated for the linear coupling of a meson field to fermion density fluctuations, in which meson degrees of freedom are treated exactly, and fermion motion is treated within the domain of the random-phase approximation (RPA). Instability conditions for the RPA ground state are established. More generally, the effective two-body interaction is deduced via a Green's-function technique by eliminating the meson degrees of freedom. This interaction is shown to be frequency-dependent, i.e., retarded in time. The resulting interaction is then applied to the calculation of the Hartree-Fock (H.F.) field and of the collective modes of the system via a generalized Landau equation. In the H.F. approximation, one obtains an unambiguous separation of renormalization (self-energy) effects and the nucleon-nucleon interactions themselves, the former reducing to the correct mass renormalization of the nucleon in the static limit. For reasonably small momenta ($p<p_F$), the retardation corrections to the H.F. field can be characterized by a small parameter $(\epsilon_F/\mu)^2$ (≈ 0.1 for actual nuclear densities), where ϵ_F = Fermi energy and μ = meson mass. The corrections become more important at high momenta and densities. In the long-wavelength limit, the frequency-dependent corrections to the collective mode energies are found to be of order $(\omega/\mu)^2$, where ω = collective mode energy. For a static Yukawa interaction, a value $\lambda^2\approx 5$ (consistent with the usual shell-model values) is found for the neutral scalar coupling constant by requiring that the giant dipole collective state appear at the experimental energy. For pseudoscalar coupling, the usual renormalized coupling constant $f^2/4\pi\approx 0.08$ is shown to yield a "breathing mode" in heavy nuclei consistent with crude estimates based on nuclear compressibilities.

1. INTRODUCTION

THE problem of trying to construct the interaction energy between two nucleons from basic principles is an old idea that dates back to the pioneering work of Yukawa[1] in 1935. In the Yukawa theory, the interaction between two nucleons is brought about by the exchange of (virtual) mesons between the two participating nucleons, which act as a source and a sink, respectively, for the exchanged meson. As is well known, this simple one-meson-exchange process can be considered to give rise to a class of interaction potentials (usually referred to as OPEP) which are instantaneous in time and thus depend only on the relative separation of the two nucleons and possibly their spin and isospin coordinates. The specific spin and isospin dependence of these potentials is determined by the type of meson that one considers to be exchanged. However, the range in coordinate space is always of order $1/\mu$, the inverse rest mass of the exchanged meson (we use units $\hbar=c=1$ throughout). A complete review of the present status of such an approach to the two-nucleon interaction is to be found, for example, in Moravcsik and Noyes.[2] For our purposes, it will be sufficient to comment that present data on two-nucleon scattering systems are consistent with interactions having a long-range character of the type produced by π-meson exchange ($1/\mu\sim 1.4$ F), but showing considerable deviations from such potentials as the nucleon separation approaches the "hard core" radius (~ 0.3 F) associated with the structure of a nucleon itself.

By contrast, the properties of interacting systems of nucleons containing many nucleons (nuclear matter) is a distinct problem and invites a different approach. Here one usually assumes the two-nucleon interaction to be given *a priori*, either by some version of the meson theory discussed above, or by a suitably parametrized two-body interaction. The force parameters are then either adjusted in an attempt to reproduce expected properties of the many-body system, e.g., saturation at observed nuclear densities, or adjusted so that characteristics of the two-nucleon system are reproduced as accurately as possible. The properties of the many-body system are then calculated in some approximation with a given two-body interaction. The latter approach has been the subject of considerable recent discussion in the

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[1] H. Yukawa, Proc. Phys. Math. Soc. Japan **17**, 48 (1935).

[2] M. J. Moravcsik and H. P. Noyes, Ann. Rev. Nucl. Sci. **11**, 95 (1961).

literature.[3–5] It has been found that multiple scattering effects in the many-body system, that modify the effective interaction of nucleons embedded in nuclear matter, have to be taken into account. The resulting "effective" two-body interactions derived in this manner using free two-body parameters are found to give very reasonable results when used to calculate spectroscopic properties of low-lying nuclear states from the nuclear shell model.

It is also clear, however, that the problem of the interaction between a pair of isolated nucleons, and a pair embedded in a nuclear system is connected in a further dynamical way in a meson-theory framework of nucleon-nucleon interactions. The mere presence of the other nucleons modifies in an essential way the motion of the pair under study. For example, since one is dealing with fermions, exclusion effects due to particle identity exclude scattering states that would otherwise be available to an isolated interacting pair, leading to modifications in the interaction between the embedded pair. One is therefore led to the alternative point of view of studying the properties of the interacting system: nucleons plus mesons without introducing the concept of nucleon-nucleon interactions directly. These interactions are now mediated entirely by the meson exchange between nucleons. Such a point of view is not new. Besides the Yukawa theory mentioned previously for two nucleon interactions, the interaction between electrons in metals is drastically modified by a similar process, the exchange of phonons (describing the motion of the ionic lattice) between electron pairs, leading as is well known to the phenomenon of superconductivity.[6]

The subject of this article is the study of Fermi systems where the interactions arise from the exchange of massive particles. For the most part the discussion will be quite general. The applications we have in mind however are to nuclear systems, in which case the fermions are nucleons, the exchanged particle one of the mesons that are thought to be associated with nucleon-nucleon interactions. We emphasize that if we are considering a nuclear system, then we certainly cannot hope to describe the actual characteristics of nucleon-nucleon interactions by the exchange of only one type of meson. For example, the "hard core" structure of the interaction at small distances cannot be reproduced in this manner. However, we can study the long-range part of the nucleon-nucleon potential (the OPEP part) by confining our attention to the exchange of π mesons which are the lightest mesons of interest for nucleon forces. Our discussion could be immediately extended to include the single quantum exchange of heavier mesons, such as the ρ, η, and ω.

In order to see what features are important in studying such systems of fermions plus exchanged particles let us couple the fermions via the exchange of a neutral scalar particle described by the real field $\phi(\mathbf{x},t)$. The simplest type of interaction in this case is (ns identifies the coupling as neutral scalar)

$$H_{\mathrm{ns}}'=\lambda\int d\mathbf{x}\,\rho(\mathbf{x},t)\phi(\mathbf{x},t)\,. \tag{1.1}$$

λ is the coupling constant and $\rho(\mathbf{x},t)$ the nucleon density at $(\mathbf{x},t)$. If the fermions are nucleons, the simplest version of Yukawa's theory considers the nucleons as sources at fixed points in space that exchange the particle described by $\phi(\mathbf{x},t)$. The nucleon motion (or lack of it) is prescribed through the density distribution $\rho(\mathbf{x},t)$ and only the field $\phi(\mathbf{x},t)$ is determined dynamically. For two fixed nucleons at a relative separation r one finds the second-order perturbation result

$$E_{\mathrm{int}}(r)=\sum_k \frac{-\lambda^2}{\mu^2+k^2}e^{-i\mathbf{k}\cdot\mathbf{r}}=-\frac{\lambda^2}{4\pi}\frac{e^{-\mu r}}{r} \tag{1.2}$$

for the interaction. The sum on $\mathbf{k}$ is over all momenta of the exchanged particle of mass μ. The result (1.2) is the well-known Yukawa interaction between two fixed nucleons.

It hardly need be pointed out that the above calculation is inconsistent for the following reasons. The motion of the nucleons is ignored in determining $E_{\mathrm{int}}(r)$, and the motion of the exchanged particle is ignored in turn when (1.2) is employed to study the nucleon motion. In principle, we have to consider the equations of motion of the coupled nucleon-meson system. Such equations are simple to obtain, but have a nonlinear structure and are consequently prohibitively difficult to use. For the two-nucleon system one therefore resorts to approximate schemes like that leading to Eq. (1.2) that neglect the nucleon recoil motion as a first approximation; this is called the static approximation. For such an approximation scheme one expects the controlling parameter to be the mass ratio μ/m, where m is the nucleon mass. Such recoil corrections have been extensively investigated for π-meson exchange between nucleons.[2] In this case $\mu/m\approx 1/7$ and the approximation appears to be reasonable. Of course this approximation worsens as the mass of the exchanged particle increases.

We now make the point that an essential simplification occurs when we use a coupling of the form (1.1) to describe the interactions in a dense system of fermions. In such systems a linear coupling to the density as given by H_{ns}' gives rise to density fluctuations about the equilibrium density distribution of the fermion system which can be regarded as small under conditions to be discussed later. The point is that the equation of motion determining the fluctuation in density can then be linearized so that linear coupled equations result for the density-fluctuations–meson system, and recoil effects

[3] K. A. Brueckner and J. L. Gammel, Phys. Rev. **109**, 1023 (1958).
[4] S. A. Moskowski and B. L. Scott, Ann. Phys. (N. Y.) **11**, 65 (1960).
[5] T. T. S. Kuo and G. E. Brown, Nucl. Phys. **85**, 40 (1966).
[6] H. Fröhlich, Phys. Rev. **79**, 845 (1950).

need not be treated perturbatively. However, another effect enters in the many-body system which is absent in two-body systems. Because of the exclusion principle, the fermions in such a system possess a zero-point motion characterized by their Fermi energy ϵ_F even when there are no interactions present. The exchange of a meson between nucleons in motion will register effects coming from the time-delay between emission and reabsorption, i.e., the interaction will no longer be instantaneous in time as in the static approximation, Eq. (1.2). The parameter controlling such time-delay effects is of order ϵ_F/μ since $1/\mu$ and $1/\epsilon_F$ are characteristic times for meson and fermion motion in the Fermi sea. However, ϵ_F depends on the fermion density; correspondingly we expect a density dependence in the effective interaction between an embedded pair that is also absent from Eq. (1.2). Further, since the average spacing between fermions decreases slower $(\sim 1/p_F)$ than ϵ_F increases with increasing density we expect the time-delay effects to become more important at higher densities.

Let us return to the question of the characteristic density fluctuations in the fermion system. One knows that such fluctuations represent excited states of interacting systems. Microscopically, this approach considers the excitation of particle-hole pairs out of the Fermi sea that characterizes the noninteracting Fermi system, and the resulting interaction of these pairs due to the exchange of a massive particle. The linearization procedure then amounts to keeping a certain class of particle-hole excitations which are then treated exactly. This is just the "random-phase approximation" (RPA) that has been widely applied in nuclear and metallic electron systems.[7,8]

Our specific problem is complicated by the time-delay effects that enter into the particle-hole interactions. Such time-dependent interactions are awkward to handle, and we will prefer to work with the Fourier transformed version of the interaction which then becomes frequency-dependent. Particle-hole systems interacting in this manner are most naturally treated by the use of Green's-function methods[9] that have been developed for similar problems in electron-phonon systems. In this manner a Bethe-Salpeter[10,11] equation, or ladder equation, is obtained for the motion of interacting particle-hole pairs.

In the following, we will restrict the discussion to fermion systems of infinite extent, which for definiteness we take to be nuclear matter. The nuclear matter approximation is not necessary. However, it does serve as a very convenient vehicle for studying the effects of particle exchange between the participating nucleons, and how these effects depend on the nuclear density etc., without getting involved with the details of nuclear structure.

For such infinite systems the Bethe-Salpeter equation is known to possess two types of solutions: single-particle scattering solutions, and "bound" solutions corresponding to collective oscillations of the medium, characterized by a frequency $\omega(k)$ with wave number k. For long wavelengths these oscillations are described by a simplified version of the ladder equation mentioned above that is identical with the Landau equation for the propagation of "zero sound" in Fermi liquids. We show that a Landau-like equation also results for the case of retarded interactions. The solutions of such equations and their ability to describe stable oscillations in nuclear matter is discussed in detail in Sec. 4. Results are presented for neutral scalar and pseudoscalar meson exchanges. In the latter case one is dealing with the more realistic situation of π-meson exchange between nucleons; the coupling constant is known and hence a comparison with effective interactions obtained by other means is possible.

The method of Green's functions is also used to study the propagation of a single nucleon in nuclear matter. In this case, the exact treatment of a certain class of excitations leads as is well known to a "Dyson equation"[12] for the single-particle propagator. The lowest-order solution of this equation allows one to identify the self-energy $\Sigma_{\mathbf{p}}$ of a nucleon of momentum $\mathbf{p}$. In the many-body system, $\Sigma_{\mathbf{p}}$ contains contributions from the mesons being emitted and reabsorbed by the same nucleon (mass renormalization effects) and contributions from emission-absorption by different nucleons (the average interaction energy). We show that in a neutral scalar theory such effects are approximately additive and suggest the point of view that nucleon "dressing" processes can be identified independently from nucleon-nucleon interactions in the Fermi sea. Since we only consider point nucleons, such renormalizations are infinite. We have not attempted to outline a "renormalization program" in these considerations beyond pointing out how such effects might occur. After removing renormalization effects, $\Sigma_{\mathbf{p}}$ is just the Hartree-Fock potential for nuclear matter. We show that the determination of $\Sigma_{\mathbf{p}}$ leads to a self-consistency problem, even in nuclear matter, contrary to the case when the interaction between nucleons is instantaneous.

2. STATEMENT OF THE PROBLEM

We consider a large system of N nucleons, mass m, interacting by the exchange of mesons of mass μ. The words "nucleon" and "meson" are used as a convenient nomenclature only. Most of the following discussion will

[7] A. M. Lane, *Nuclear Theory* (W. A. Benjamin, Inc., New York, 1964).

[8] P. Nozières and D. Pines, *Quantum Theory of Liquids* (W. A. Benjamin, Inc., New York, 1966).

[9] L. P. Kadanoff, *Lectures on the Many-Body Problem* (Academic Press, Inc., New York, 1964), Vol. II, p. 77.

[10] E. E. Salpeter and H. A. Bethe, Phys. Rev. 84, 1232 (1951).

[11] V. M. Galitskii and A. B. Migdal, Zh. Eksperim. i Teor. Fiz. 34, 139 (1958) [English transl.: Soviet Phys.—JETP 7, 96 (1958)].

[12] P. Nozières, *Interacting Fermi Systems*, (W. A. Benjamin, Inc., New York, 1964).

apply to arbitrary fermion systems interacting via the exchange of a massive boson. We introduce the notation $a_{\mathbf{p}}^\dagger$ and $a_{\mathbf{p}}$ for nucleon creation and destruction operators in the nucleon state $\mathbf{p}$ (momentum, spin, and isospin); $B_{\mathbf{k}}^\dagger$ and $B_{\mathbf{k}}$ play the same role for a meson of momentum $\mathbf{k}$. The nucleon operators anticommute, $\{a_{\mathbf{p}}^\dagger, a_{\mathbf{p}'}\}=\delta_{\mathbf{pp}'}$, while for the meson operators we have the boson commutation relations: $[B_{\mathbf{k}}, B_{\mathbf{k}'}^\dagger]=\delta_{\mathbf{kk}'}$, all other commutators being zero.

The total Hamiltonian is taken to be the sum of three terms: the free-nucleon field, the free-meson field, and a meson-nucleon coupling term H':

$$H=\sum_{\mathbf{p}} \epsilon_{\mathbf{p}}{}^0 a_{\mathbf{p}}^\dagger a_{\mathbf{p}}+\tfrac{1}{2}\sum_{\mathbf{k}} (P_{\mathbf{k}}^\dagger P_{\mathbf{k}}+\Omega_{\mathbf{k}}^2 Q_{\mathbf{k}}^\dagger Q_{\mathbf{k}})+H'. \quad (2.1)$$

We have introduced the notation $\epsilon_{\mathbf{p}}{}^0 \equiv \mathbf{p}^2/2m$ for the kinetic energy of a nucleon and the linear combinations (canonical coordinates)

$$Q_{\mathbf{k}}=(B_{\mathbf{k}}+B_{-\mathbf{k}}^\dagger)/(2\Omega_{\mathbf{k}})^{1/2}, \quad P_{\mathbf{k}}=i(\Omega_{\mathbf{k}}/2)^{1/2}(B_{\mathbf{k}}^\dagger-B_{-\mathbf{k}})$$

for a meson of momentum $\mathbf{k}$ and energy $\Omega_{\mathbf{k}}=(\mu^2+\mathbf{k}^2)^{1/2}$.

We will only be interested in forms for H' that are linear in the meson field. Specifically, we consider (i) the neutral scalar interaction already given in Eq. (1.1) which now reads

$$H'_{\mathrm{ns}}=\lambda \sum_{\mathbf{k}} \rho_{\mathbf{k}} Q_{\mathbf{k}} \quad (2.2)$$

($\rho_{\mathbf{k}}=\sum_{\mathbf{p}} a_{\mathbf{p}+\mathbf{k}}^\dagger a_{\mathbf{p}}$ is the Fourier transform of the nucleon density), and (ii) the pseudoscalar interaction

$$H_{\mathrm{ps}}'=\frac{f}{\mu}\sum_{\mathbf{pk}} a_{\mathbf{p}+\mathbf{k}}^\dagger(\boldsymbol{\sigma}\cdot\mathbf{k})(\boldsymbol{\tau}\cdot\mathbf{Q}_{\mathbf{k}})a_{\mathbf{p}} \quad (2.3)$$

($a^\dagger$ and a are now row and column vectors in spin and isospin space) that couples the nucleon spin $\boldsymbol{\sigma}$ and isospin $\boldsymbol{\tau}$ to the meson momentum and charge states, respectively. $\mathbf{Q}_{\mathbf{k}}$ is an isovector in Eq. (2.3), and introduces mesons in three different charge states. Equation (2.3) leads in the static limit to the charge-independent coupling interaction used by Chew and Low in their discussion of pion-nucleon scattering[13] and we will therefore refer to it as the "Chew-Low interaction." The coupling constant f is dimensionless in the form (2.3). We will carry out most formal considerations using the simpler neutral scalar form for H' and simply supply the analogous results for the pseudoscalar interaction.

For the reasons given in the Introduction we will for the most part employ a Green's-function treatment of the ground and excited states of the Hamiltonian H. However, it is useful to consider the following idealized problem first in order to gain insight into some properties of systems described by H. We start with the observation that the nucleon motion is only coupled via their density $\rho_{\mathbf{k}}$ to the meson field in the neutral scalar theory. We therefore consider the equations for the motion of $Q_{\mathbf{k}}$ and the nuclear density component $\rho_{\mathbf{p},\,\mathbf{p}+\mathbf{k}} =\langle a_{\mathbf{p}+\mathbf{k}}^\dagger a_{\mathbf{p}}\rangle$ characterizing the motion of a particle-hole pair. The average is taken with respect to the exact ground state of H. Treating all operators as time-dependent, we derive the Heisenberg equations of motion

$$\left[i\frac{\partial}{\partial t}-(\epsilon_{\mathbf{p}}{}^0-\epsilon_{\mathbf{p}+\mathbf{k}}{}^0)\right]\rho_{\mathbf{p},\mathbf{p}+\mathbf{k}} =\lambda \sum_{\mathbf{k}'} Q_{-\mathbf{k}'}(\rho_{\mathbf{p}+\mathbf{k}',\,\mathbf{p}+\mathbf{k}}-\rho_{\mathbf{p},\,\mathbf{p}+\mathbf{k}-\mathbf{k}'}), \quad (2.4)$$

$$\left(\frac{\partial^2}{\partial t^2}+\Omega_{\mathbf{k}}^2\right)Q_{-\mathbf{k}}=-\lambda\rho_{\mathbf{k}}=-\lambda\sum_{\mathbf{p}}\rho_{\mathbf{p},\,\mathbf{p}+\mathbf{k}}. \quad (2.5)$$

These equations are exact. We now make the essential point: For large fermion systems a well-defined approximation exists for treating the motion of $\rho_{\mathbf{p},\,\mathbf{p}+\mathbf{k}}$ which then represents fluctuations in the nucleon density in momentum space about the Fermi distribution $n_{\mathbf{p}}$ which is established in the noninteracting ground state by the exclusion principle. If we write $\rho_{\mathbf{p},\,\mathbf{p}+\mathbf{k}}=n_{\mathbf{p}}+\rho_{\mathbf{p},\,\mathbf{p}+\mathbf{k}}{}^{(1)}$, where $\rho_{\mathbf{p},\,\mathbf{p}+\mathbf{k}}{}^{(1)}$ is the density fluctuation and ignore momentum transfers $Q_{-\mathbf{k}'}$ for $\mathbf{k}'\neq\mathbf{k}$ on the right-hand side of Eq. (2.4), we obtain

$$\left(i\frac{\partial}{\partial t}+\omega_{\mathbf{pk}}{}^0\right)\rho_{\mathbf{p},\,\mathbf{p}+\mathbf{k}}{}^{(1)}\approx\lambda(n_{\mathbf{p}+\mathbf{k}}-n_{\mathbf{p}})Q_{-\mathbf{k}}. \quad (2.4')$$

We have used $\omega_{\mathbf{pk}}{}^0=\epsilon_{\mathbf{p}+\mathbf{k}}{}^0-\epsilon_{\mathbf{p}}{}^0$ for the excitation energy of the particle-hole pair in $\rho_{\mathbf{p},\,\mathbf{p}+\mathbf{k}}{}^{(1)}$ and $n_{\mathbf{p}}$ is the Fermi distribution function $n_{\mathbf{p}}=1$ for $|\mathbf{p}|<p_F$, and $n_{\mathbf{p}}=0$ for $|\mathbf{p}|>p_F$ where p_F is the Fermi momentum. Equation (2.4′) is one version of the "random-phase approximation" (RPA) that has been extensively investigated in nuclear and electron gas problems.[7,8] In Eq. (2.4′), $\rho_{\mathbf{p},\,\mathbf{p}+\mathbf{k}}{}^{(1)}$ consists only of particle-hole excitations for which $|\mathbf{p}+\mathbf{k}|>p_F$ and $|\mathbf{p}|<p_F$ and vice versa, i.e., one index must refer to a particle state when the other index refers to a hole state. This approximation renders the pair of equations (2.5), (2.4′) linear in the unknown amplitudes $Q_{-\mathbf{k}}$ and $\rho_{\mathbf{p},\,\mathbf{p}+\mathbf{k}}{}^{(1)}$. We can therefore ask for normal modes made up of linear superpositions of these amplitudes. A normal mode of frequency ω and wave vector $\mathbf{k}$ exists if $\omega=\omega(k)$ satisfies the dispersion relation

$$1=g\frac{\lambda^2}{\Omega_{\mathbf{k}}{}^2-\omega^2}\sum_{\mathbf{p}} n_{\mathbf{p}}\frac{2\omega_{\mathbf{pk}}{}^0}{(\omega_{\mathbf{pk}}{}^0)^2-\omega^2}. \quad (2.6)$$

We have extracted the spin-isospin degeneracy of the fermions in g ($g=4$ for nucleons); the sum on $\mathbf{p}$ is therefore only over all momentum states in the Fermi sea. Apart from the frequency dependence in the coefficient in Eq. (2.6), i.e.,

$$V_{\mathbf{k}}(\omega)=-\lambda^2/(\Omega_{\mathbf{k}}{}^2-\omega^2) \quad (2.7)$$

[13] G. F. Chew and F. E. Low, Phys. Rev. **101**, 1570 (1956).

this equation is identical in form to the dispersion relation for collective oscillations in an electron gas[14] (plasmons) when $V_k(\omega)$ is the repulsive Coulomb interaction $4\pi e^2/k^2$, or to the dispersion relation for Landau's zero sound waves[15] to which it reduces when $V_k(\omega)$ is a positive, frequency-independent constant.

A brief discussion of the solutions of Eq. (2.6) is useful for our further work. As in the case of the electron gas,[16] the free particle-hole frequencies $\omega_{pk}{}^0$ are still eigenfrequencies of the coupled system, corresponding to the fact that the particle-hole pairs have no self-energy in this approximation. We can investigate the possibility of additional collective solutions of (2.6) by passing to the continuum limit and performing the sum over **p** as a principal-value integral. We obtain

$$\sum_{\mathbf{p}} n_{\mathbf{p}} \frac{2\omega_{\mathbf{pk}}{}^0}{(\omega_{\mathbf{pk}}{}^0)^2-\omega^2} = \frac{mp_F}{2\pi^2} B(k,\omega),$$

$$B(k,\omega)=\tfrac{1}{2}+\frac{p_F}{4k}\left[\left(\frac{\omega+k^2/2m}{kv_F}\right)^2-1\right] \times\ln\left|\frac{\omega+k^2/2m-kv_F}{\omega+k^2/2m+kv_F}\right| - \frac{p_F}{4k}\left[\left(\frac{\omega-k^2/2m}{kv_F}\right)^2-1\right] \times\ln\left|\frac{\omega-k^2/2m-kv_F}{\omega-k^2/2m+kv_F}\right|, \quad (2.8)$$

where $B(k,\omega)$ is a universal function for Fermi systems depending only on the fermion mass m, Fermi momentum p_F and velocity $v_F=p_F/m$. $B(k,\omega)$ is obviously a complicated function of its parameters. We will mostly be interested in $B(k,\omega)$ as a function of ω for small k at fixed density of the system. Then $B(k,\omega)$ is a smooth function, apart from a deep minimum in the vicinity of the maximum particle-hole pair excitation energy $\omega_k=kv_F+k^2/2m$. For long wavelengths, $k\to 0$, $B(k,\omega)$ becomes

$$B(k,\omega)\cong 1+\frac{\omega}{2\omega_k}\ln\left|\frac{\omega-\omega_k}{\omega+\omega_k}\right|, \quad (2.9)$$

where $\omega_k=kv_F$ gives the maximum excitation energy for small k. The minimum at $\omega=\omega_k$ now appears as a singularity in this approximate form for $B(k,\omega)$, indicating that the expansion is not valid near ω_k. A plot of $B(k,\omega)$ for various values of k is given in Fig. 1. Inserting the closed form $B(k,\omega)$ for the summation on the right-hand side of Eq. (2.6) we obtain the dispersion relation relating ω and k. The solutions of this dispersion relation that lie above the maximum particle-hole energy ω_k are interpreted as stable collective oscillations of the system.

[14] D. Bohm and D. Pines, Phys. Rev. **92**, 609 (1953).
[15] L. D. Landau, Zh. Eksperim. i Teor. Fiz. **32**, 59 (1957) [English transl.: Soviet Phys.—JETP **5**, 101 (1957)].
[16] K. Sawada, Phys. Rev. **106**, 372 (1957).

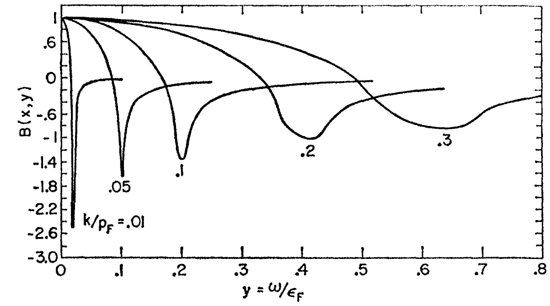

FIG. 1. Behavior of the function $B(x,y)$ defined by Eq. (2.8) as a function of $y=\omega/\epsilon_F$ for various values of $x=k/p_F$.

The graphical solution of this dispersion relation proceeds as in Fig. 2, where we qualitatively plot $B(k,\omega)$ and $f(k,\omega)=2\pi^2/g\lambda^2 mp_F(\Omega_k{}^2-\omega^2)$ as a function of ω^2 for fixed k. The intersections of $B(k,\omega)$ and $f(k,\omega)$ give the solutions of (2.6). Values $\omega^2<0$ correspond to collective modes whose amplitude grows exponentially with time, i.e., the system exhibits an instability. The threshold for instability is obtained by requiring that $\omega=0$ be a solution of (2.6) corresponding to point B in Fig. 2. We find that for $g\lambda^2\leq 2\pi^2\mu^2/mp_F$, there exist no instabilities for any value of k. For $g\lambda^2>2\pi^2\mu^2/mp_F$, an instability appears for $k<k_c$ where the critical momentum k_c is given by

$$\Omega_{k_c}{}^2=\frac{g\lambda^2 mp_F{}^2}{4\pi^2 k_c}\left\{\frac{k_c}{p_F}+\left(1-\frac{k_c{}^2}{4p_F{}^2}\right)\times\ln\left|\frac{1+k_c/2p_F}{1-k_c/2p_F}\right|\right\}. \quad (2.10)$$

The threshold k_c is plotted in Fig. 3 as a function of λ^2. We conclude that instability is a phenomenon peculiar to long wavelengths and strong coupling constants for neutral scalar coupling. In the event of instability, the assumed ground state will evolve spontaneously into some other state with a lower energy. This modified

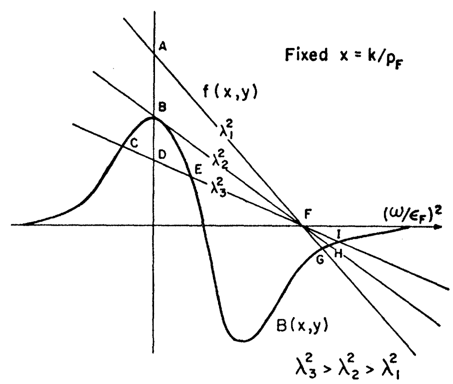

FIG. 2. Graphical solution of Eq. (2.6) for the collective mode energies of the coupled meson-nucleon system. The lines AF, BF, and DF correspond to successively increasing values of the neutral scalar coupling constant λ^2.

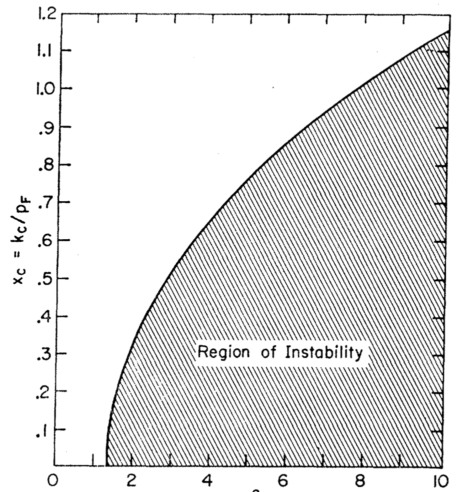

FIG. 3. Critical momentum k_c for the onset of instability in the RPA ground state (permanent density fluctuations) as a function of the neutral scalar coupling strength λ^2.

ground state will contain permanent fluctuations corresponding to the type of collective mode we are considering (in this case density fluctuations). In such a situation the RPA is no longer appropriate for a description of the ground state, since it assumes small deviations from the unperturbed Fermi surface.

For the spin- and isospin-dependent Chew-Low coupling (2.3), one can also formulate an exactly soluble model as in the neutral scalar case. The dispersion relation analogous to (2.6) becomes ($g=4$ now, since spin and isospin have to be considered)

$$1=\frac{8f^2}{\mu^2}\frac{k^2}{\omega^2-\Omega_{\mathbf{k}}^2}\sum_{\mathbf{p}} n_{\mathbf{p}}\frac{\omega_{\mathbf{pk}}^0}{\omega^2-(\omega_{\mathbf{pk}}^0)^2}. \qquad (2.11)$$

The instability threshold k_c is found to be

$$\Omega_{k_c}^2=\frac{f^2mp_F^2}{\pi^2\mu^2}k_c\left\{\frac{k_c}{p_F}+\left(1-\frac{k_c^2}{p_F^2}\right)\times\ln\left|\frac{1+k_c/2p_F}{1-k_c/2p_F}\right|\right\}. \qquad (2.12)$$

The region of instability is shown graphically in Fig. 4. As for the neutral scalar case, the instability is a strong-coupling phenomenon. However, the instability is most probable at intermediate momenta for pseudoscalar coupling, and vanishes in the long-wavelength limit ($k\to 0$), in striking contrast to the neutral scalar case. Physically, this is due to the fact that the Chew-Low interaction vanishes at $k=0$, and hence the instability must also disappear.

From Fig. 2, we also note the existence of solutions for $\omega^2>0$ corresponding to points G, H, and I. The behavior of these solutions for $k\to 0$ can be obtained by a straightforward expansion of $B(k,\omega)$ in a power series in k:

$$\omega^2\approx\Omega_k^2(1+N\lambda^2k^2/m\mu^4),$$

where $N=gp_F^3/6\pi^2$ is the particle density. Since $\omega^2\to\Omega_k^2$ as $k\to 0$, we refer to this solution as the "dressed meson." Such a mode has appeared because we have included meson degrees of freedom explicitly, and does not appear if one starts with some given direct interaction.

For short-range forces, we are interested in the existence of collective modes with $\omega\to 0$ as $k\to 0$. For these modes to be undamped, the energy ω must exceed the maximum particle-hole energy of ω_k. If $\omega<\omega_k$, the mode will be strongly damped into the particle-hole continuum (Landau damping).[15] It can be shown that the minimum of $B(k,\omega)$ corresponds to an energy $\omega<\omega_k$. From Fig. 2, we then see that the only undamped mode supported by a neutral scalar coupling is the "dressed meson." The mode corresponding to point E, for example, will be strongly Landau damped. We can understand this result physically by noting that our simple model includes only the direct matrix element of a neutral scalar interaction, which is attractive for the $S=T=0$ channel (S and T are the total spin and isospin of the collective mode). Hence the energy of this mode is pushed down below ω_k. Thus to explore collective modes, we must extend the simple model to include exchange matrix elements. In this case, one no longer obtains a simple dispersion relation like (2.6), but must solve an integral equation. This program will be carried out in Sec. 4.

3. EFFECTIVE INTERACTIONS AND THE HARTREE-FOCK FIELD

We now turn to the general problem of identifying the effective interaction in the many-body system, when these interactions arise from the exchange of a meson. In the previous section we were able to identify the effec-

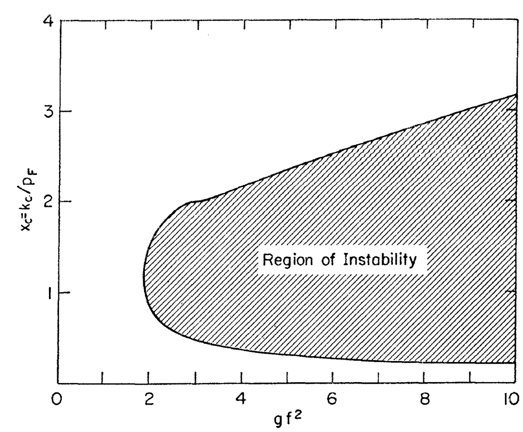

FIG. 4. Critical momentum for instability of the RPA ground state for a pseudoscalar meson theory (coupling constant f^2).

tive interaction by simply looking at the form of the dispersion relation (2.6) that provided the eigenfrequencies of the problem, and reading off the matrix element (2.7) of the interaction. We cannot do this in general, however, since the dispersion relation will no longer be a closed-form algebraic equation. Instead, we must consider the equations of motion of some quantity related to the motion of a particle and identify the form of the effective interaction determining the variation of this quantity by eliminating explicitly all reference to the meson variables. It will be convenient for reasons that will become apparent later to consider the single-particle Green's function $G_{p\nu}(t-t')$, defined below in Eq. (3.1), as the quantity related to the motion of a particle. For a linear meson-nucleon density coupling of the type we are considering, the elimination of the meson field can produce terms in the equation of motion of $G_{p\nu}(t-t')$ that are at most quartic in the nucleon creation and destruction operators $a_p^\dagger$ and a_p, i.e., give rise to effective interactions that have a two-body structure.

Consider, then, a nucleon of momentum $\mathbf{p}$ moving in an infinite Fermi sea. Its Green's function, or propagator, is supplied by the time-ordered expression

$$G_{p\nu}(t-t')=i\langle T\{a_{p\nu}(t)a_{p\nu}^\dagger(t')\}\rangle, \tag{3.1}$$

where the angular brackets denote an average over the exact ground state of H in Eq. (2.1) and T the time-ordering operator. We note for later use that $G_{p\nu}(t-t')$ is only a function of this time difference $(t-t')$ and vanishes if the momentum and spin labels $\mathbf{p}$ and ν are different on a and $a^\dagger$. The equation of motion that determines $G_{p\nu}(t-t')$ is readily found to be

$$\left(i\frac{\partial}{\partial t}-\epsilon_p{}^0\right)G_{p\nu}(t-t')=-\delta(t-t')+i\lambda\sum_{\mathbf{k}}\langle T\{Q_{-\mathbf{k}}(t)a_{\mathbf{p+k},\nu}(t)a_{p\nu}^\dagger(t')\}\rangle \tag{3.2}$$

for $H'=H'_{\rm ns}$. The δ function discontinuity in $t-t'$ arises because of the anticommutation of $a_{p\nu}^\dagger$ and $a_{p\nu}$ at equal time. We now proceed to eliminate explicit reference to the meson field operators by examining the equation of motion for the quantity $\langle T\{Q_{-\mathbf{k}}(t)a_{\mathbf{p+k},\nu}(t'')a_{p\nu}^\dagger(t')\}\rangle$. Since the $Q_{-\mathbf{k}}(t)$ satisfy

$$\ddot{Q}_{-\mathbf{k}}+\Omega_k{}^2Q_{\mathbf{k}}=-\lambda\rho_{\mathbf{k}}(t) \tag{3.3}$$

from the form of H in Eq. (2.1), we also have

$$[(\partial^2/\partial t^2+\Omega_k{}^2]\langle T\{Q_{-\mathbf{k}}(t)a_{\mathbf{p+k},\nu}(t'')a_{p\nu}^\dagger(t')\}\rangle=-\lambda\langle T\{\rho_{\mathbf{k}}(t)a_{\mathbf{p+k},\nu}(t'')a_{p\nu}^\dagger(t')\}\rangle. \tag{3.4}$$

The fermion and meson operators commute and so carrying out a time-ordering does not introduce a discontinuity in the right-hand side of Eq. (3.4).

Now we wish to insert Eq. (3.4) for the quantity of interest. We do so by introducing the meson Green's function $D_{\mathbf{k}}(t-t')$ which is a solution of

$$\left(\frac{\partial^2}{\partial t}+\Omega_k{}^2\right)D_{\mathbf{k}}(t-t')=\delta(t-t'). \tag{3.5}$$

Since we will be interested in virtual meson absorption and emission we choose boundary conditions in time such that

$$\lim_{t\to\pm\infty}D_{\mathbf{k}}(t)=0,$$

i.e., no real mesons present at $t\to\pm\infty$. The solution for $D_{\mathbf{k}}(t-t')$ corresponding to these boundary conditions is

$$D_{\mathbf{k}}(t-t')=(1/2i\Omega_k)\exp[i(\Omega_k+i\eta)|t-t'|],$$

where $\eta\to0^+$.

We may now solve Eq. (2.7) for $\langle T\{Q_{-\mathbf{k}}(t)a_{\mathbf{p+k},\nu}(t'')\times a_{p\nu}^\dagger(t')\}\rangle$ and substitute the result into the right hand side of (3.2) after letting $t''\to t$. The result is an equivalent equation of motion for $G_{p\nu}(t-t')$:

$$\left(i\frac{\partial}{\partial t}-\epsilon_p{}^0\right)G_{p\nu}(t-t')=-\delta(t-t')-i\lambda^2\sum_{\mathbf{k}}\int_{-\infty}^{+\infty}dt_1\,D_{\mathbf{k}}(t-t_1)\times\langle T\{\rho_{\mathbf{k}}(t_1)a_{\mathbf{p+k},\nu}(t)a_{p\nu}^\dagger(t')\}\rangle. \tag{3.6}$$

This equation avoids explicit reference to the meson field variables $Q_{-\mathbf{k}}$. On the other hand a nonlocal operator in time has appeared on the right-hand side of this equation to replace the somewhat simpler structure of Eq. (3.2). Comparing the structure of Eq. (3.6) with the corresponding equation when an instantaneous two-body force $V(|\mathbf{r}_1-\mathbf{r}_2|)$ between particles is considered, i.e.,

$$\left(i\frac{\partial}{\partial t}-\epsilon_p{}^0\right)G_{p\nu}(t-t')=-\delta(t-t')+i\sum_{\mathbf{k}}V_{\mathbf{k}}\langle T\{\rho_{\mathbf{k}}(t)a_{\mathbf{p+k},\nu}(t)a_{p\nu}^\dagger(t')\}\rangle \tag{3.7}$$

[$V_{\mathbf{k}}$ is the momentum space matrix element of $V(|\mathbf{r}_1-\mathbf{r}_2|)$], we can identify the momentum-space matrix elements of an equivalent "effective interaction"

$$V_{\mathbf{k}}(t-t')=-\lambda^2D_{\mathbf{k}}(t-t') \tag{3.8}$$

for scalar meson exchange. The corresponding expression for pseudoscalar meson exchange is found to be

$$V_{\mathbf{k}}(t-t')=-(f/\mu)^2(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)(\boldsymbol{\sigma}_1\cdot\mathbf{k})(\boldsymbol{\sigma}_2\cdot\mathbf{k})D_{\mathbf{k}}(t-t'). \tag{3.8$'$}$$

Notice that this interaction is no longer instantaneous in time, since $D_{\mathbf{k}}(t-t')$ exhibits contributions from all times. This circumstance just reflects the fact that, physically, a nucleon can change the meson field by emitting a meson. The resulting variation in meson field propagates in space and time and hence can influence the motion of the same nucleon (or another one) at a later time. The price we pay for eliminating explicit reference to the meson variables is the appearance of a retarded interaction in time between participating nucleons.

Such interactions are well known in the theory of superconductivity where they arise due to the exchange of lattice phonons between electrons.[6] As in the case of superconductivity it is more convenient to work with a particular Fourier component of $V_{\mathbf{k}}(t-t')$, instead of a time-dependent interaction directly. We write

$$V_{\mathbf{k}}(\omega)=\int_{-\infty}^{+\infty} dt\, e^{i\omega t}V_{\mathbf{k}}(t)\,, \tag{3.9}$$

where boundary conditions in time on $V_{\mathbf{k}}(t)$ like those given below Eq. (3.5) are supposed to ensure the convergence of the integral. We find

$$V_{\mathbf{k}}(\omega)=-\lambda^2 D_{\mathbf{k}}(\omega)\,;\quad D_{\mathbf{k}}(\omega)=[(\Omega_{\mathbf{k}}+i\eta)^2-\omega^2]^{-1}, \tag{3.10}$$

where $D_{\mathbf{k}}(\omega)$ is the transform of the meson propagator $D_{\mathbf{k}}(t-t')$ in Eq. (3.8) that satisfies the boundary conditions

$$\lim_{t\to\pm\infty} D_{\mathbf{k}}(t)=0.$$

This result agrees with the identification of the effective interaction already given in Eq. (2.7).

Our further discussions will for the most part be based on the form (3.10) for the effective interaction, which is now frequency-dependent. However, to get some feeling for the effects introduced by treating the meson exchange between nucleons explicitly, we consider the coordinate space version of the effective interaction, $V(\mathbf{r},t)$, that obeys the causal boundary condition $V(\mathbf{r},t)=0$ for $t<0$ where the time t is interpreted as the time-delay between emission and reabsorption of the meson. Introducing the transform $D_{\mathbf{k}}^{(c)}(\omega)$ of $D_{\mathbf{k}}^{(c)}(t)$ again we find

$$V(\mathbf{r},t)=-\lambda^2\int\frac{d\mathbf{k}}{(2\pi)^3}\int\frac{d\omega}{2\pi}D_{\mathbf{k}}^{(c)}(\omega)e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)} \tag{3.11}$$

provided $D_{\mathbf{k}}^{(c)}(t)$ satisfies the same boundary condition as $V(\mathbf{r},t)$, i.e., $D_{\mathbf{k}}^{(c)}(t)=0$, for $t<0$. This requires that $D_{\mathbf{k}}^{(c)}(\omega)$ be given by

$$D_{\mathbf{k}}^{(c)}(\omega)=[\Omega_{\mathbf{k}}^2-(\omega+i\eta)^2]^{-1} \tag{3.12}$$

instead of the expression in Eq. (3.10). We may now evaluate (3.11) by contour integration. The ω integration only contributes for $t>0$ because of the form of $D_{\mathbf{k}}^{(c)}(\omega)$ in (3.12), which only has poles in the lower-half ω plane. In this case, we can close the contour in the lower-half ω plane and get

$$V(\mathbf{r},t)=\frac{\lambda^2}{2\pi^2 r}\int_0^\infty\frac{kdk}{\Omega_k}\sin kr\,\sin\Omega_k t \tag{3.13}$$

after also performing the angular integration over all directions of the variable $\mathbf{k}$. Since $\Omega_{\mathbf{k}}=(k^2+\mu^2)^{1/2}$, integral (3.13) is a special case of the Sonine-Gegenbauer integral[17] which yields

$$\begin{aligned} V(\mathbf{r},t)&=-\frac{\mu\lambda^2}{4\pi}\frac{J_1[\mu(t^2-r^2)^{1/2}]}{(t^2-r^2)^{1/2}} &&\text{for}\quad t>r\\ &=0 &&\text{for}\quad t<r \end{aligned} \tag{3.14}$$

where $J_1(x)$ is a Bessel function of order 1.

Equation (3.14) has the typical structure of a retarded interaction. The field $V(\mathbf{r},t)$ sweeps past a point distance r from its source in a time $\Delta t\approx 1/\mu$ determined by the width of the first oscillation of $J_1(x)$. This in turn depends on the value of r in the argument of J_1 in Eq. (3.14). For a typical nucleon separation in nuclear matter $r\approx 1.4/\mu$ F, one has $\Delta t\approx 2.4/\mu$, which should be small compared to the single-particle lifetime in a heavy nucleus. Finally, integrating over all time in (3.14) we recover the Yukawa interaction form

$$\int_0^\infty dt\, V(\mathbf{r},t)=-\frac{\lambda^2}{4\pi}\frac{e^{-\mu r}}{r}$$

already introduced in connection with Eq. (1.2).

Turning now to the properties of $V_{\mathbf{k}}(\omega)$ as a function of frequency, we see from Eq. (3.10) that $V_{\mathbf{k}}(\omega)$ is attractive for all frequencies $\omega<\Omega_{\mathbf{k}}$ and repulsive for $\omega>\Omega_{\mathbf{k}}$. Since the momentum transfer and energy k and ω in $V_{\mathbf{k}}(\omega)$ are controlled by conservation laws, we will see later that $V_{\mathbf{k}}(\omega)$ in fact remains attractive for frequencies of interest in nuclear structure problems, the change-over occurring at rather high frequencies. The situation in electron-phonon systems is reversed because of the low value $\Omega_{\mathbf{k}}$ assumes in this case, and the change of sign occurs at low frequencies. For single-particle motion within the Fermi sea the frequencies of interest are expected to be $\omega\approx\epsilon_F$, where ϵ_F is the Fermi energy. The parameter determining the importance of frequency-dependent effects is correspondingly $\epsilon_F/\mu\approx 2/7$. Since this parameter enters into the effective interaction as $(\epsilon_F/\mu)^2$, we expect such effects to be quite small. Notice also that, in a many-body system, we can look upon the frequency dependence as giving rise to a density-dependent interaction since $\omega\approx\epsilon_F$, so that the frequency-dependent effects increase with increasing density. This fact can easily be understood if we observe that the average nucleon velocity p_F/m increases with density like $\rho_0^{1/3}$ so that the retardation effects show a corresponding increase. Finally, as $\omega\to\infty$ we note that $V_{\mathbf{k}}(\omega)$ vanishes; the nucleons are not able to follow the rapid variations in the meson field, and the effective in-

[17] W. Magnus and F. Oberhettinger, *Formulas and Theorems for the Special Functions of Mathematical Physics* (Chelsea Publishing Co., New York, 1949).

teraction becomes negligible. In the other limit $\omega\to 0$ the effective interaction reduces to the Yukawa interaction

$$V_{\mathbf{k}}(0)=-\lambda^2/\Omega_{\mathbf{k}}^2=-\lambda^2/(\mu^2+k^2) \tag{3.15}$$

which has been given in Eq. (1.2) in coordinate space.

Since the range of frequencies ω that is important in (3.10) is determined by what physical property of the many-body system is under consideration, let us consider two separate situations: (i) the Hartree-Fock potential for nuclear matter generated by the interaction (3.10) and (ii) properties of collective excited states of nuclear matter.

Let us first consider the Hartree-Fock potential. We return to Eq. (3.6) which for the neutral scalar interaction given by Eq. (3.8) can be written in the form

$$\left(i\frac{\partial}{\partial t}-\epsilon_{\mathbf{p}}^0\right)G_{\mathbf{p}\nu}(t-t')=-\delta(t-t')-\sum_{\mathbf{k}\mathbf{p}'\nu'}\int_{-\infty}^{+\infty}dt_1\,V_{\mathbf{k}}(t-t_1)\times K(\mathbf{p}'\nu't_1,\ \mathbf{p}'+\mathbf{k}\nu't_1^+;\ \mathbf{p}+\mathbf{k},\ \nu t,\ \mathbf{p}\nu t') \tag{3.16}$$

after introducing the two-particle Green's function

$$K(1234)=i\langle T\{a_1(t_1)a_2^\dagger(t_2)a_3(t_3)a_4^\dagger(t_4)\}\rangle \tag{3.17}$$

using the definition of Galitskii and Migdal,[11] and writing t_1^+ for times infinitesimally greater than t_1 in order to reproduce the time-ordering appropriate for the right-hand side of Eq. (3.6).

Equation (3.16) is exact, and represents the first of an infinite chain of equations coupling 1-particle, 2-particle, 3-particle, etc. Green's functions. The Hartree-Fock (HF) or self-consistent field approximation consists in truncating this chain of equations at stage (3.16) by introducing the approximation

$$iK(1234)\approx G(1,2)G(3,4)-G(1,4)G(3,2)\,, \tag{3.18}$$

i.e., replacing the exact two-particle Green's function by an antisymmetrized product of one-particle Green's functions $G(i,j)$ as defined in Eq. (3.1). This amounts to replacing the propagator of two interacting particles by the product of "free" propagators of two noninteracting particles. We thus replace iK in Eq. (3.16) by

$$iK(\mathbf{p}'\nu't_1,\ \mathbf{p}'+\mathbf{k}\nu't_1';\ \mathbf{p}+\mathbf{k}\nu t,\ \mathbf{p}\nu t')\approx G_{\mathbf{p}'\nu'}(t_1-t_1^+)G_{\mathbf{p}\nu}(t-t')\delta_{\mathbf{k}0}-G_{\mathbf{p}\nu}(t_1-t')\times G_{\mathbf{p}+\mathbf{k},\,\nu}(t-t_1^+)\delta_{\mathbf{p}\mathbf{p}'}\delta_{\nu\nu'}\,, \tag{3.19}$$

where we have made use of the invariance of the one-particle Green's functions under spatial and temporal translations in infinite systems to reduce the number of variables on the right-hand side. In this approximation, Eq. (3.16) for $G_{\mathbf{p}\nu}(t-t')$ becomes

$$\left(i\frac{\partial}{\partial t}-\epsilon_{\mathbf{p}}^0\right)G_{\mathbf{p}\nu}(t-t')=-\delta(t-t')+\sum_{p'\nu'}\int dt''\times\{n_{\mathbf{p}'\nu'}V_0(t-t'')G_{\mathbf{p}\nu}(t-t')-i\delta_{\nu\nu'}G_{p'\nu'}(t-t''^+)\times V_{\mathbf{p}-\mathbf{p}'}(t-t'')G_{\mathbf{p}\nu}(t''-t')\}\,, \tag{3.20}$$

where we have introduced the relation

$$\lim_{t'\to t^+} iG_{\mathbf{p}\nu}(t-t')=\langle a_{\mathbf{p}\nu}^\dagger(t)a_{\mathbf{p}\nu}(t)\rangle=n_{\mathbf{p}\nu} \tag{3.21}$$

between the particle density distribution $n_{\mathbf{p}\nu}$ and the Green's function at equal times.

We first observe that Eq. (3.20) reduces to the ordinary HF equation for $G_{\mathbf{p}\nu}(t-t')$ if $V_{\mathbf{k}}(t-t')$ is replaced by an instantaneous interaction, $V_{\mathbf{k}}\delta(t-t')$. Then, using Eq. (3.21) once more, we find $G_{\mathbf{p}\nu}(t-t')$ satisfies

$$\left(i\frac{\partial}{\partial t}-\epsilon_{\mathbf{p}}^0\right)G_{\mathbf{p}\nu}(t-t')=-\delta(t-t')+\mathcal{V}_{\mathbf{p}\nu}G_{\mathbf{p}\nu}(t-t')\,, \tag{3.22}$$

where $\mathcal{V}_{\mathbf{p}\nu}$ is the HF potential

$$\mathcal{V}_{\mathbf{p}\nu}=\sum_{\mathbf{p}'\nu'}[V_0-\delta_{\nu\nu'}V_{|\mathbf{p}-\mathbf{p}'|}]n_{\mathbf{p}'\nu'} \tag{3.23}$$

which is actually independent of ν in the present case.

Returning to the general case of time-dependent interactions, Eq. (3.20), we introduce the Fourier transforms $V_{\mathbf{k}}(\omega)$ and $G_{\mathbf{p}\nu}(\omega)$ of $V_{\mathbf{k}}(t-t')$ and $G_{\mathbf{p}\nu}(t-t')$, defined as in Eq. (3.9). Then we find an equation very similar to (3.22), viz.,

$$\left(i\frac{\partial}{\partial t}-\epsilon_{\mathbf{p}}^0\right)G_{\mathbf{p}\nu}(t-t')=-\delta(t-t')+\int\frac{d\omega}{2\pi}\mathcal{V}_{\mathbf{p}\nu}(\omega)G_{\mathbf{p}\nu}(\omega)e^{-i\omega(t-t')} \tag{3.24}$$

for $G_{\mathbf{p}\nu}(t-t')$. The potential $\mathcal{V}_{\mathbf{p}\nu}(\omega)$ is given by

$$\mathcal{V}_{\mathbf{p}\nu}(\omega)=\sum_{\mathbf{p}'\nu'}\int_C\frac{d\omega'}{2\pi}\times[V_0(0)-\delta_{\nu\nu'}V_{|\mathbf{p}-\mathbf{p}'|}(\omega-\omega')]iG_{\mathbf{p}'\nu'}(\omega') \tag{3.25}$$

and is a frequency-dependent generalization of the HF field given by Eq. (3.23). In deriving this result, we have used the relation

$$\int_C iG_{\mathbf{p}\nu}(\omega)\frac{d\omega}{2\pi}=n_{\mathbf{p}\nu}\,, \tag{3.26}$$

where the contours C in integrals (3.25) and (3.26) run along the real ω axis and close in the upper half ω plane. Taking the time Fourier transform on both sides of Eq. (3.24) we finally obtain

$$[\omega-\epsilon_{\mathbf{p}}^0-\mathcal{V}_{\mathbf{p}\nu}(\omega)]G_{\mathbf{p}\nu}(\omega)=-1 \tag{3.27}$$

or

$$G_{\mathbf{p}\nu}(\omega)=[\epsilon_{\mathbf{p}}^0+\mathcal{V}_{\mathbf{p}\nu}(\omega)-\omega]^{-1} \tag{3.28}$$

for the frequency-dependent Green's function $G_{p\nu}(\omega)$ in the HF approximation.

One immediately recognizes Eq. (3.28) as a solution of the Dyson integral equation[12]

$$G_{p\nu}(\omega)=G_{p\nu}^{(0)}(\omega)+G_{p\nu}^{(0)}(\omega)\sum_{p\nu}(\omega)G_{p\nu}(\omega) \quad (3.29)$$

for the Green's function, where

$$G_{p\nu}^{(0)}(\omega)=(\epsilon_p^0-\omega-i\delta)^{-1};\quad \begin{array}{l}\delta=0^+,\ |\mathbf{p}|>p_F\\ \quad =0^-,\ |\mathbf{p}|<p_F\end{array} \quad (3.30)$$

is the free-particle Green's function, and $\sum_{p\nu}(\omega)$ the nucleon self-energy. The HF approximation (3.28) is then seen to correspond to calculating the self-energy to first order in the interaction, but using the Green's function $G_{p\nu}(\omega)$ for intermediate states, also calculated in the HF approximation. A self-consistency condition thus arises: $\mathcal{V}_{p\nu}(\omega)$ and $G_{p\nu}(\omega)$ must be determined simultaneously from Eqs. (3.25) and (3.28). Later on, we try to meet this condition very approximately by introducing an effective mass approximation. We also remark that in the context of perturbation theory graphs, the HF approximation to $\sum_p(\omega)$ given by Eq. (3.25) amounts to allowing only one meson at a time in intermediate states but summing over all such states. It is interesting to note the close analogy between the expressions (3.23) and (3.25) for the HF potential for static and nonstatic two-body interactions. In (3.23) we sum over all momenta in the Fermi sea according to their distribution $n_{p'}$ over allowed momentum states. Equation (3.25) has a very similar structure. We make the analogy complete by noting that, according to Eq. (3.26), the quantity $iG_{p\nu}(\omega)=n_{p\nu}(\omega)$ can be interpreted as the distribution function of states labeled by $\mathbf{p}\nu$ and ω.

In the frequency-dependent case (3.25) the first term represents the forward scattering of the particle $\mathbf{p}$, with particles distributed according to $n_{p\nu}(\omega)$, by exchanging a meson. Since there is no momentum or energy transfer, only $V_0(0)$ appears. This term therefore exactly equals the direct term in Eq. (3.23). In the second term $\delta_{\nu\nu'}V_{|p-p'|}(\omega-\omega')$ of Eq. (3.25) $\mathbf{p}$ exchange scatters with particles distributed according to $n_{p'\nu'}(\omega')$. The meson carries the momentum and energy transfer $\mathbf{p}-\mathbf{p}'$ and $\omega-\omega'$ and we must sum over all $\mathbf{p}'\nu'$ and ω' consistent with the distribution $n_{p'\nu'}(\omega')$; explicit effects of the frequency dependence in the two-body interaction now appear in the calculation.

Notice also that the exclusion principle is also carried at each stage of the calculation; the structure of $n_{p\nu}(\omega)$ depends explicitly on the presence of the other fermions. By analogy with Eq. (3.30) we have

$$G_{p\nu}(\omega)=[\epsilon_p^0+\mathcal{V}_{p\nu}(\omega)-\omega-i\delta]^{-1};\quad \begin{array}{l}\delta=0^+,\ |\mathbf{p}|>p_F\\ \quad =0^-,\ |\mathbf{p}|<p_F\end{array} \quad (3.31)$$

since the noninteracting Fermi distribution is not disturbed in the HF approximation.

Now let us calculate $\mathcal{V}_{p\nu}(\omega)$ explicitly from Eq. (3.25). We are interested in the potential seen by a nucleon in a momentum state $\mathbf{p}$. The energy of this state is given by the poles of $G_{p\nu}(\omega)$, i.e., by $\omega=\epsilon_p$, where

$$\epsilon_p^0+\sum_p(\epsilon_p)-\epsilon_p=0 \quad (3.32)$$

which then defines the single-particle energies ϵ_p. When $\sum_{p\nu}(\omega)$ is calculated to first order one knows that $G_{p\nu}(\omega)$ has unit residues at the poles $\omega=\epsilon_p$ and that the ϵ_p are real. We can now evaluate (3.25) by contour integration. There are contributions from (i) poles of $G_{p\nu}(\omega)$ which are at $\epsilon_p\pm i\delta$ and (ii) poles of $V_k(\omega)$ which are at $\pm(\Omega_k+i\eta)$. The contribution from (i) gives

$$\mathcal{V}_{p\nu}(\epsilon_p)=-\lambda^2\sum_{p'\nu'}\left[\frac{1}{\mu^2}-\frac{\delta_{\nu\nu'}}{\Omega_{p-p'}{}^2-(\epsilon_p-\epsilon_{p'})^2}\right]n_{p'\nu'} \quad (3.33)$$

after inserting the $V_k(\omega)$ explicitly from Eq. (3.10), while the contribution from (ii) is

$$\sum_{p'\nu'}\delta_{\nu\nu'}\frac{\lambda^2}{2\Omega_{p-p'}}G_{p'\nu'}(\epsilon_p+\Omega_{p-p'}). \quad (3.34)$$

Equation (3.33) is the HF potential for the frequency-dependent interaction given by the one-meson exchange we have considered. To interpret (3.34), we turn to the static neutral scalar theory for meson exchange between two nucleons. In this static approximation one can identify the mass renormalization of a nucleon represented by its transformation from a bare to a physical nucleon surrounded by a cloud of virtual mesons. The physical mass m is related to the bare mass m_0 which appears in the original Hamiltonian by $m=m_0+\delta m$, where

$$\delta m=-\lambda^2\sum_{p'}\frac{|f(\mathbf{p}')|^2}{2\Omega_{p'}{}^2} \quad (3.35)$$

and $f(\mathbf{p})$ is a nucleon form factor in momentum space. For $f=1$ (point nucleons), expression (3.35) is identical with the static limit of Eq. (3.34), and is of course divergent. We interpret the contribution (ii) from the poles of the meson propagator as a mass renormalization of the particles in the Fermi sea. We see that the expression for this is different from the case of free nucleons (except in the static limit) and should include the effects of the exclusion principle on the formation of virtual meson clouds around each nucleon in the Fermi sea. We have not investigated this point in detail.

It is of course no surprise that $\sum_p(\omega)$ should contain both interaction and renormalization effects. There is nothing in Eq. (3.25) that distinguishes the nucleon in state $\mathbf{p}'$ from the "probe" nucleon $\mathbf{p}$. The emitted meson can be absorbed by the probe nucleon again (mass renormalization) or another nucleon of the Fermi sea (interaction). This point becomes obvious if we note that, in the system composed of one nucleon plus mesons, there is no Fermi sea, so that poles of type (i) do not appear. Only the mass renormalization remains.

Let us illustrate the quantitative effects of the frequency dependence by evaluating the HF field for both the neutral scalar and pseudoscalar interactions given by Eqs. (3.8) and (3.8′), respectively. The HF field for the former is given by (3.33) with $V_{\mathbf{k}}(\omega)=-\lambda^2 D_{\mathbf{k}}(\omega)$. For the pseudoscalar interaction (3.8′), we obtain by similar reasoning

$$\mathcal{V}_{\mathbf{p}\sigma\tau}(\omega)=\left(\frac{f}{\mu}\right)^2\sum_{\mathbf{k}\sigma'\tau'}\int_C\frac{d\omega'}{2\pi}D_{\mathbf{k}}(\omega-\omega')\times iG_{\mathbf{p}+\mathbf{k},\,\sigma'\tau'}(\omega')(\sigma\sigma'|\boldsymbol{\sigma}_1\cdot\mathbf{k}\;\boldsymbol{\sigma}_2\cdot\mathbf{k}|\sigma'\sigma)\times(\tau\tau'|\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2|\tau'\tau) \quad (3.36)$$

for the H.F. potential, where we have supplied the spin σ and isospin τ indices explicitly. If the Green's function $G_{\mathbf{p}+\mathbf{k},\,\sigma\tau}(\omega)$ does not depend on the spin and isospin indices (a consistent assumption for "normal" fermion systems), then the HF field is also independent of these indices, and we have (the factor $3k^2$ arises from performing the trace over spin and isospin variables)

$$\mathcal{V}_{\mathbf{p}}(\omega)=3\left(\frac{f}{\mu}\right)^2\sum_{\mathbf{k}}\int_C\frac{d\omega'}{2\pi}k^2D_{\mathbf{k}}(\omega-\omega')iG_{\mathbf{p}+\mathbf{k}}(\omega')=3\left(\frac{f}{\mu}\right)^2\sum_{\mathbf{p}'}n_{\mathbf{p}'}\frac{|\mathbf{p}-\mathbf{p}'|^2}{\Omega_{\mathbf{p}-\mathbf{p}'}{}^2-(\omega-\epsilon_{\mathbf{p}'})^2} \quad (3.37)$$

after a change of variable from $\mathbf{k}$ to $\mathbf{p}'=\mathbf{p}+\mathbf{k}$. The last form of this result is similar in structure to Eq. (3.33) except that there is no direct term; the pseudoscalar field cannot be transferred without momentum change due to its intrinsic odd-parity nature.

In both expressions (3.33) and (3.37), the effects of frequency dependence in the two-body interaction are confined to the exchange contribution to the HF potential. We can evaluate these terms explicitly if the dependence of the single-particle energies $\epsilon_{\mathbf{p}}$ on the momentum $\mathbf{p}$ is known. This dependence is given implicitly by Eq. (3.32), through the dependence of $\sum_{\mathbf{p}}(\epsilon_{\mathbf{p}})$ on $\mathbf{p}$. For small momenta $p/p_F\ll 1$ we can solve this equation for $\epsilon_{\mathbf{p}}$ by expanding the HF potential in powers of $|\mathbf{p}|$ and keeping only the lowest-order correction $\sim p^2$. This leads to an effective-mass approximation $\epsilon_{\mathbf{p}}=p^2/2m^*+$constant, where the constant term is independent of momentum, and m^* is the "effective mass."[18] For our further discussion we assume this form for $\epsilon_{\mathbf{p}}$ to be valid for all $\mathbf{p}$. Then, we have for the neutral scalar (ns) and pseudoscalar (ps) interactions, respectively,

$$V_{\mathbf{p}}^{ex}(\epsilon_{\mathbf{p}})=\lambda^2\frac{p_F}{8\pi^2}\frac{1}{y}I_{\text{ns}}(y)\,,\qquad V_{\mathbf{p}}^{ex}(\epsilon_{\mathbf{p}})=3\left(\frac{f}{\mu}\right)^2\left[\frac{p_F{}^3}{6\pi^2}-\frac{\mu^2 p_F}{8\pi^2 y}I_{\text{ps}}(y)\right], \quad (3.38)$$

[18] V. Weisskopf, Nucl. Phys. 3, 423 (1957).

where the summations in Eqs. (3.33) and (3.37) have been converted into integrals over the Fermi sphere. We have

$$I_{\text{ns}}(y)=\int_0^1 dx\times x\ln\left|\frac{1+(p_F/\mu)^2(x+y)^2-(\epsilon_F{}^*/\mu)^2(x^2-y^2)^2}{1+(p_F/\mu)^2(x-y)^2-(\epsilon_F{}^*/\mu)^2(x^2-y^2)^2}\right|,$$

$$I_{\text{ps}}(y)=\int_0^1 dx\, x\left[1-\left(\frac{\epsilon_F{}^*}{\mu}\right)^2(x^2-y^2)^2\right]\times\ln\left|\frac{1+(p_F/\mu)^2(x+y)^2-(\epsilon_F{}^*/\mu)^2(x^2-y^2)^2}{1+(p_F/\mu)^2(x-y)^2-(\epsilon_F{}^*/\mu)^2(x^2-y^2)^2}\right|, \quad (3.39)$$

where $y=p/p_F$, $\epsilon_F{}^*=(m/m^*)\epsilon_F$, and the integrals are over the variable $x=p'/p_F$. In the static limit, which corresponds to taking the limit $m\rightarrow\infty$, i.e., nucleon mass≫meson mass, both I_{ns} and I_{ps} tend to the same expression

$$I(y)=\frac{2\mu y}{p_F}\left\{\frac{p_F}{\mu}+\frac{1+(p_F/\mu)^2(1-y^2)}{4(p_F/\mu)y}\times\ln\left|\frac{1+(p_F/\mu)^2(1+y)^2}{1+(p_F/\mu)^2(1-y)^2}\right|-\tan^{-1}[(p_F/\mu)(1+y)]-\tan^{-1}[(p_F/\mu)(1-y)]\right\}. \quad (3.40)$$

The structure of I_{ns} and I_{ps} again shows that the frequency-dependent effects enter through the parameter ϵ_F/μ as was pointed out in the Introduction. The potentials in Eq. (3.38) are also density-dependent through their dependence on p_F/μ, and ϵ_F/μ. The dependence of the potentials (3.38) on the recoil parameter μ/m is also of interest. This ratio formally enters via the parameter $\epsilon_F/\mu=\frac{1}{2}(\mu/m)(p_F/\mu)^2$; thus Eqs. (3.38) carry nucleon recoil corrections to the potential exactly within the framework of the H.F. approximation.

In Figs. 5 and 6 we plot the potentials for neutral scalar and pseudoscalar interactions, respectively, and compare these with the corresponding static limit where I_{ns} and I_{ps} are replaced by I. Figures 7 and 8 show the fractional change $\Delta V/V_{\text{st}}=[\mathcal{V}(\epsilon_p)-\mathcal{V}_{\text{st}}]/\mathcal{V}_{\text{st}}$ for both cases. We have taken the coupling constants $\lambda^2\approx 5$ and $f^2\approx 1$, a nominal value $m^*=0.8m$, and assumed a Fermi momentum $p_F=270$ MeV/c, which corresponds to that in nuclear systems at normal density.

One observes that the frequency-dependent Hartree-Fock field deviates progressively from the static limit as momentum increases. However, for single-particle motion within the Fermi sea ($p<p_F$), the frequency effects remain very small at physical densities ($\eta=1$). At very high momenta ($\approx 6p_F$ for $\eta=1$) an additional physical

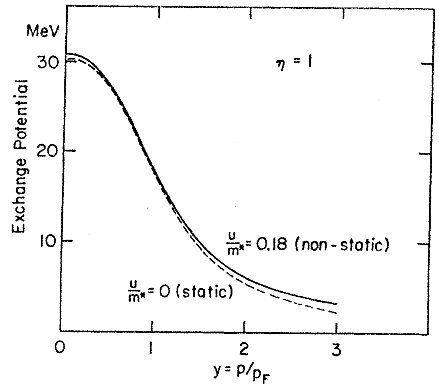

FIG. 5. Effects of the frequency dependence of the effective interaction on the single-particle Hartree-Fock potential at normal density ($\eta=1$). The solid curve represents the exchange potential $\mathcal{V}_p^{ex}(\epsilon_p)$ of Eq. (3.38) for neutral scalar coupling, while the dashed curve corresponds to the static limit (3.40).

effect appears, that is, the production of real mesons. This effect arises from the pole in the integrand (3.33) for $\epsilon_p-\epsilon_{p'}=\Omega_{p-p'}$. Note that since we have momentum and energy conservation in the one-meson-exchange process, the above pole corresponds to a meson on the energy shell; i.e., the meson cannot be produced at rest, but must also provide the correct momentum transfer $\mathbf{p}-\mathbf{p}'$. Thus in the vicinity of the meson production threshold, the effects of the frequency dependence are particularly important. However, at these high momenta, the Hartree-Fock approximation we have used is certainly not adequate. In particular, contributions from multiparticle excitations (not included in HF) will produce additional frequency dependence that has been ignored in the present calculation.

Figures 7 and 8 also illustrate the increasing importance of frequency effects at higher densities. However, even at $\eta=2$ (8 times normal density), the effects are still less than 10% for momenta $p<p_F$.

4. COLLECTIVE EXCITATIONS

We turn now to the study of excited states of the many-fermion system with interactions of the type given by Eqs. (3.8) and (3.8′). We have already studied the case of particle-hole excitations within the frame work of the simple model of Sec. 2. There, the solubility rested on the simplicity of Eq. (2.4′) when the exchange of mesons with momentum different from the particle-hole pair momentum is ignored. In this section, we will restore the exchange terms, and at the same time derive the equations of motion for the density fluctuations (particle-hole excitations) that contain the effective interactions $V_{\mathbf{k}}(t-t')$, instead of the explicit reference to the meson field $Q_{-\mathbf{k}}$, as in Eq. (2.4′).

It is well known[11] that the density fluctuations bear a close connection with the two-particle Green's function $K(1234)$ defined in Eq. (3.17) for the special time ordering $(t_1,t_2)>(t_3,t_4)$. For this time ordering one can introduce a complete set of states $|n\rangle$ of the many-body system and write

$$K(1234)=i\sum_n \chi_n(1,2)\tilde{\chi}_n(3,4) \tag{4.1}$$

without changing the value of K. Here, the functions

$$\chi_n(1,2)=\langle 0|T\{a_1(t_1)a_2^\dagger(t_2)\}|n\rangle \tag{4.1′}$$

and their adjoint functions $\tilde{\chi}_n$ measure the amplitude of particle-hole excitations in the state $|n\rangle$. If $|n\rangle$ refers to a bound state formed by the interaction of many particle-hole pairs, the associated amplitude $\chi_n(1,2)$ satisfies a homogeneous Bethe-Salpeter[10] equation

$$\chi_n(1,2)=i\int\sum_{5678} G(1,5)G(6,2)\Gamma(56;78) \times\chi_n(7,8)dt_5dt_6dt_7dt_8. \tag{4.2}$$

The function $\Gamma(56;78)$ is a vertex function, or "compact four-pole diagram" in field theory language.[11] We will always replace Γ by the antisymmetric matrix elements of the two-body force, which is valid to first order in the interaction between particle-hole pairs. This corresponds to treating the motion of such pairs in the RPA approximation. The functions G in Eq. (4.2) are the exact one-particle Green's functions introduced in Eq. (3.1). In keeping with the approximations for Γ, we replace them by free one-particle Green's functions, or by Green's functions describing the motion of the particle in the H.F. field of the system.

Equation (4.2) simplifies considerably for an infinite system. Using the invariance of such systems under the translation of coordinates and time, we pass at once to relative and total momentum and frequency variables for a particle-hole excitation and write

$$\chi(\mathbf{p}'\nu\epsilon',\mathbf{p}\nu\epsilon)=\delta(\epsilon'-\epsilon-\omega)f_{\mathbf{k}\omega\nu}(\mathbf{p},\epsilon) \tag{4.3}$$

for a particle-hole excitation of energy ω and momen-

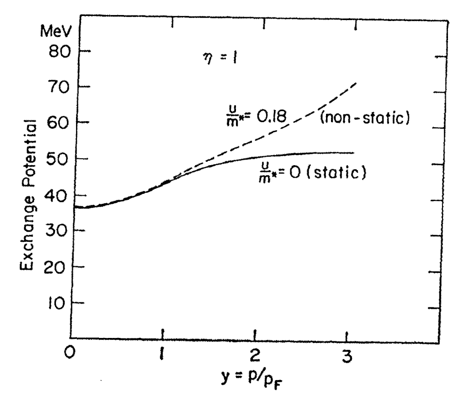

FIG. 6. Effects of the frequency dependence of the effective inreaction on the Hartree-Fock potential at normal density ($\eta=1$). The dashed curve represents $\mathcal{V}_p^{ex}(\epsilon_p)$ of Eq. (3.38) for pseudoscalar coupling, while the solid curve corresponds to the static Bhew-Low interaction.

tum **k**. Then making the replacements for Γ and G as discussed above, Eq. (4.2) becomes

$$f_{\mathbf{k}\omega\nu}(\mathbf{p},\epsilon)=-iG_{\mathbf{p}+\mathbf{k},\,\nu}(\epsilon+\omega)G_{\mathbf{p}\nu}(\epsilon)\sum_{\mathbf{p}'\nu'}\int\frac{d\epsilon'}{2\pi}\times[V_{\mathbf{k}}(\omega)-\delta_{\nu\nu'}V_{\mathbf{p}-\mathbf{p}'}(\omega-\epsilon')]f_{\mathbf{k}\omega\nu'}(\mathbf{p}',\epsilon') \quad (4.4)$$

for the neutral scalar form of the interaction in Γ.

The physical content of this equation is clear: The interaction between particles is not instantaneous in time, which means that particle-hole pairs are not destroyed and recreated simultaneously. Therefore, the particle-hole amplitudes $f_{\mathbf{k}\omega}$ at different times (and therefore frequencies) are coupled. If the matrix elements in Eq. (4.4) were independent of frequency, we can integrate over frequency on the right-hand side and obtain

$$f_{\mathbf{k}\omega\nu}(\mathbf{p})=\frac{n_{\mathbf{p}}-n_{\mathbf{p}+\mathbf{k}}}{\omega-\omega_{\mathbf{pk}}}\sum_{\mathbf{p}'\nu'}[V_{\mathbf{k}}-\delta_{\nu\nu'}V_{|\mathbf{p}-\mathbf{p}'|}]f_{\mathbf{k}\omega\nu'}(\mathbf{p}') \quad (4.5)$$

after setting

$$f_{\mathbf{k}\omega\nu}(\mathbf{p})=\int d\epsilon\; f_{\mathbf{k}\omega\nu}(\mathbf{p},\epsilon)$$

and using the fact that

$$i\int_C G_{\mathbf{p}+\mathbf{k},\,\nu}(\epsilon+\omega)G_{\mathbf{p}\nu}(\epsilon)\frac{d\epsilon}{2\pi}=\frac{n_{\mathbf{p}}-n_{\mathbf{p}+\mathbf{k}}}{\omega_{\mathbf{pk}}-\omega} \quad (4.6)$$

which is valid for both the free form Eq. (3.30) and HF form Eq. (3.28) for $G_{\mathbf{p}\nu}(\epsilon)$. The only difference is in the value of the particle-hole excitation energy $\omega_{\mathbf{pk}}$ that is used: $\omega_{\mathbf{pk}}=\epsilon_{\mathbf{p}+\mathbf{k}}^0-\epsilon_{\mathbf{p}}^0$ for the free case and $\omega_{\mathbf{pk}}=\epsilon_{\mathbf{p}+\mathbf{k}}-\epsilon_p$, a difference of HF energies in the second case. Equation (4.5) is precisely the RPA equation[7] derived in Sec. 2 for the density fluctuations $\rho_{\mathbf{p},\mathbf{p}+\mathbf{k}}^{(1)}$. Equation (4.5), and its generalization, Eq. (4.4), to frequency-dependent interactions, describe collective excitations of a particle-hole nature of an infinite Fermi system.

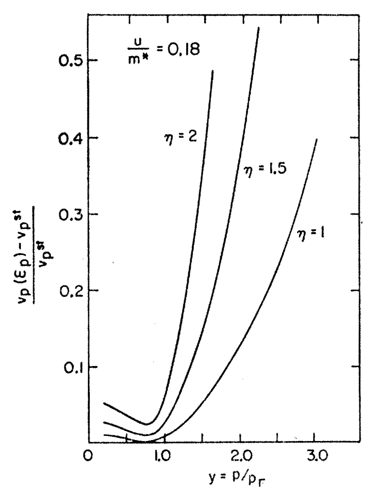

FIG. 7. The relative change in the Hartree-Fock field due to the frequency dependence of the effective interaction for various values of the density (neutral scalar coupling). $\mathcal{V}_{\mathbf{p}}(\epsilon_{\mathbf{p}})$ is given by Eq. (3.38) and the static limit $\mathcal{V}_{\mathbf{p}}^{st}$ corresponds to Eq. (3.40).

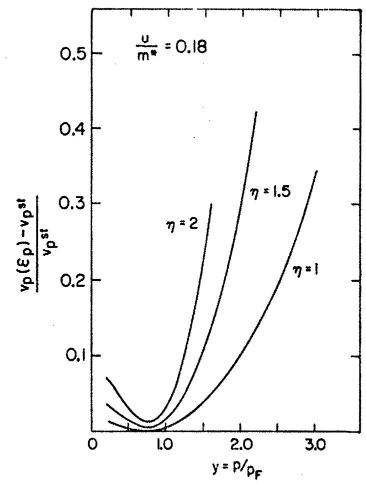

FIG. 8. The relative change in the Hartree-Fock field due to the frequency dependence of the interaction for various densities (pseudoscalar coupling). $\mathcal{V}_{\mathbf{p}}(\epsilon_{\mathbf{p}})$ and the static limit $\mathcal{V}_{\mathbf{p}}^{st}$ correspond to Eqs. (3.38) and (3.40), respectively.

5. THE LANDAU EQUATION

A. Static Interactions

We expect the effects of frequency dependence to be controlled by the parameter ω/μ, where ω is the collective frequency. For nuclear systems this parameter is small ($\approx 10^{-1}$); the slow vibrations in the collective mode cannot follow the rapid changes in the meson field providing the interaction. Let us therefore concentrate on the static case first. For convenience we replace the amplitudes $\rho_{\mathbf{p},\,\mathbf{p}+\mathbf{k}}^{(1)}$ in Eq. (2.4′) [or $f_{\mathbf{k}\omega\nu}(\mathbf{p})$ in Eq. (4.5)] by linear combinations of a given total spin S and isospin T, $f_{\mathbf{k}\omega}^{ST}(\mathbf{p})=\sum_\nu(-1)^{\delta_\nu}f_{\mathbf{k}\omega\nu}(\mathbf{p})$. There are four possibilities depending on the phase δ_ν:

(i) $S=0$, $T=0$ for $\delta_\nu=0$, (density oscillations).

(ii) $S=0$, $T=1$ for $\delta_\nu=\tau$, (isospin density oscillations).

(iii) $S=1$, $T=0$ for $\delta_\nu=\sigma$, (spin density oscillations).

(iv) $S=1$, $T=1$ for $\delta_\nu=\sigma+\tau$, (spin-isospin density oscillations).

Furthermore, we restrict ourselves to the long-wavelength limit of Eq. (4.5), which then becomes identical in form with Landau's equation for zero sound propagation in a Fermi liquid.[15] The restriction to long wavelengths is suggested by the fact that $k\ll p_F$ for collective oscillations in nuclei (when viewed as a sample of nuclear matter). We will see that the mathematical complexity of the problem is also greatly reduced by this assumption.

For $k \to 0$ the particle-hole excitations are restricted to the vicinity of the Fermi surface. This is obvious, but can also be seen from the form assumed by the difference in momentum distributions: $n_{\mathbf{p}+\mathbf{k}} - n_{\mathbf{p}} \approx -[(\mathbf{k}\cdot\mathbf{p})/p_F] \times\delta(|\mathbf{p}|-p_F)$. Only the direction (θ,ϕ) of $\mathbf{p}$ remains unrestricted. It is convenient to introduce an amplitude $U(\theta,\phi)\delta(p-p_F)$ instead of $f_{\mathbf{k}\omega}{}^{ST}(\mathbf{p})$ which reflects this fact. Then Eq. (4.5) is replaced by

$$(\cos\theta-s)U(\theta,\phi)+\frac{mp_F}{(2\pi)^3}\cos\theta\int d\Omega'\ V^{ST}U(\theta',\phi')=0 \tag{5.1}$$

after converting the sum on p' into an integral, where V^{ST} is the particle-hole matrix element

$$V^{ST}=(4V_k\delta_{S0}\delta_{T0}-V_{|\mathbf{p}-\mathbf{p}'|})_{|\mathbf{p}|=|\mathbf{p}'|=p_F} \tag{5.2}$$

for neutral scalar coupling. We have put $\omega_{\mathbf{pk}}=\omega_k\cos\theta$ in Eq. (5.1) where $\omega_k=kv_F$ is the maximum particle-hole energy in a noninteracting Fermi gas, and set $\omega=s\omega_k$. Equation (5.1) is the Landau equation; $U(\theta,\phi)$ measures the displacement of the Fermi surface along the direction (θ,ϕ) relative to the direction $\mathbf{k}$. These amplitudes are of course labeled by S and T as well.

The structure of Eq. (5.1) suggests an expansion in spherical harmonics. We write

$$U_m(\theta,\phi)=\cos\theta\sum_n a_{nm}(2n+1)P_n{}^m(\cos\theta)e^{im\phi}, \tag{5.3}$$

where $P_n{}^m(\cos\theta)$ are Legendre functions and a_{nm} is an expansion coefficient. We have labeled each eigensolution by an azimuthal index m, classifying the type of symmetry in ϕ measured about $\mathbf{k}$ ($m=0$ for longitudinal mode, $m=1$ for transverse mode, etc.) and explicitly extracted a $\cos\theta$ factor from the expansion, since Eq. (5.1) shows that $U_m(\frac{1}{2}\pi,\phi)$ must vanish for $s\neq 0$. The coefficients a_{nm} satisfy a three term recursion relation

$$\frac{2n+1}{g_n}sa_{nm}=(n-m)a_{n-1,\,m}+(n+m+1)a_{n+1,\,m} \tag{5.4}$$

which holds for $n\geq m$, with the convention that $a_{m-1,\,m}=0$. The multipole strengths g_n are related to the expansion coefficients of the interaction V^{ST} in multipoles:

$$g_n=1+f_n{}^{ST}, \tag{5.5}$$

$$V^{ST}=\frac{2\pi^2}{mp_F}\sum_n(2n+1)f_n{}^{ST}P_n(\cos\vartheta), \tag{5.6}$$

where ϑ is the angle between $\mathbf{p}$ and $\mathbf{p}'$ which are both on the Fermi surface.

The recursion relations (5.4) constitute an infinite set of homogeneous linear equations for the coefficients a_{nm} and possess a nontrivial solution if the corresponding determinant vanishes, i.e.,

$$\begin{vmatrix} s/g_0 & 1 & 0 & 0 & \cdots \\ 1 & 3s/g_1 & 2 & 0 & \cdots \\ 0 & 2 & 5s/g_2 & 3 & \cdots \\ \vdots & \vdots & & \vdots & \end{vmatrix}=0 \tag{5.7}$$

where, for simplicity, we specialize to the longitudinal mode $m=0$. This is the most interesting case for nuclear excitations. Modes of higher m represent more complicated distortions of the Fermi surface and require very strong coupling between particles to be formed.[19]

Infinite tri-diagonal determinants of the form (5.7) are familiar from the theory of Fredholm integral equations of the second kind,[20] of which (5.1) is an example. The determinantal condition (5.7) can be rewritten in terms of the infinite continued fraction

$$\frac{s}{g_0}=\cfrac{1}{(3s/g_1)-\cfrac{2^2}{(5s/g_2)-\cfrac{3^2}{(7s/g_3)-\cfrac{4^2}{\cdots}}}} \tag{5.8}$$

In principle we have now solved the integral equation (5.1), since we have constructed a dispersion relation for the eigenvalues s. Knowing the value of s, we can deduce the set of coefficients a_n and hence build the complete eigenfunction $U(\theta,\phi)$ given by (5.3).

For an arbitrary set of multipoles $\{g_n\}$, it does not seem possible to display the continued fraction (5.8) in terms of a function of s in closed form. For purposes of application it will be sufficient to consider several exactly soluble models which do reduce the eigenvalue condition (5.8) to a closed form.

(i) $g_0\neq 1$, all other coefficients $g_n=1$. This corresponds to assuming a constant interaction in momentum space and is equivalent to the model proposed by Landau for zero sound waves,[15] who gives simple expressions for s and U. We rederive his results from the continued fraction expression (5.8). Setting $g_n=1$ on the right-hand side of Eq. (5.8) and employing the result[21]

$$\tanh^{-1}(1/x)=\cfrac{1}{x-\cfrac{1^2}{3x-\cfrac{2^2}{5x-\cdots}}} \tag{5.9}$$

[19] C. B. Dover, Ph.D. thesis, Massachusetts Institute of Technology, 1967 (unpublished).

[20] P. M. Morse and H. Feshbach, *Methods of Theoretical Physics* (McGraw-Hill Book Co., New York, 1952), Vol. I.

[21] I. Khovanskii, *The Application of Continued Fractions and their Generalizations to Problems in Approximation Theory* (Noordhoff, Groningen, 1963).

one finds, after some algebra

$$\frac{s}{g_0}=s-\frac{1}{\tanh^{-1}(1/s)}=\frac{Q_1(s)}{Q_0(s)}, \tag{5.10}$$

where the last form follows from the relations ($s>1$)

$$\tanh^{-1}(1/s)=\tfrac{1}{2}\ln\left|\frac{s+1}{s-1}\right|=Q_0(s),$$

$$sQ_0(s)=1+Q_1(s),$$

and $Q_0(s)$, $Q_1(s)$ are Legendre functions of the second kind.[20] Equation (5.10) is equivalent to Landau's result. To find the coefficients a_n, we note that

$$(s/g_0)a_0=a_1; \tag{5.11}$$
$$(2n+1)sa_n=na_{n-1}+(n+1)a_{n+1},\quad n\geq 1$$

so that a_n for $n\geq 1$ satisfies the recursion relation for Legendre functions. Therefore $a_n=Q_n(s)$ for $n\geq 1$ [the other possibility, $P_n(s)$, leads to the trivial solution]. The eigenvalue condition (5.10) shows that this solution also holds for $n=0$, i.e., $a_0=Q_0(s)$, and so

$$U(\theta)=\cos\theta\sum_{n=0}^{\infty}(2n+1)Q_n(s)P_n(\cos\theta)=\frac{\cos\theta}{s-\cos\theta} \tag{5.12}$$

provided $s>1$. This is the result given by Landau.[15]

(ii) The simple Landau model suggests the generalization where a finite number of force multipoles are kept, i.e., g_0, g_1, g_2, $\cdots$ g_N, $g_{N+1}=g_{N+2}=\cdots=1$. We find

$$a_n=Q_n(s)\quad\text{for}\quad n\geq N \tag{5.13}$$

as before. However, the lower coefficients $a_n(n<N)$ are modified; we obtain

$$U(\theta)=\frac{\cos\theta}{s-\cos\theta}+\cos\theta\sum_{n=0}^{N-1}(2n+1)\times[a_n-Q_n(s)]P_n(\cos\theta), \tag{5.14}$$

where the finite sum appears as a correction to the simple monopole result of Eq. (5.12) as more multipoles are added. The dispersion relation (5.10) also changes. For example, we find

$$(s/g_0)[Q_0(s)+3s(1/g_1-1)Q_1(s)]=Q_1(s)\quad\text{for}\quad N=1,$$
$$(s/g_0)[Q_0(s)+3s(1/g_1-1)Q_1(s)+5/2g_1(3s^2-g_0g_1)\times(1/g_2-1)Q_2(s)]=Q_1(s),\quad\text{for}\quad N=2\text{ etc.} \tag{5.15}$$

which gives an indication of the general pattern. The associated eigenfunctions for the eigenvalue equations shown in Eq. (5.15) turn out to be

$$U(\theta)=\frac{\cos\theta}{s-\cos\theta}+\begin{cases}3s(1/g_1-1)Q_1(s)\cos\theta & \text{for } N=1\\ 3s(1/g_1-1)Q_1(s)\cos\theta \\ \quad +15s/2(1/g_2-1)Q_2(s) \\ \quad \times(s/g_1+\cos\theta)\cos\theta & \text{for } N=2,\text{ etc.}\end{cases} \tag{5.16}$$

(iii) A third soluble model that is complementary to (i) and (ii) assumes a multipole pattern $g_0g_1g_2\cdots g_N,g,g,g\cdots$, i.e., the first $N+1$ multipoles arbitrary and all multipoles equal to g thereafter. By considering the recursion relation satisfied by the functions $Q_n(s/g)$, one immediately obtains the following results for such cases:

$$\frac{s}{g}=\frac{Q_1(s/g)}{Q_0(s/g)}\quad\text{for the pattern }(g_0=g_1=\cdots=g),$$

$$\frac{s}{g_0}=\frac{Q_1(s/g)}{Q_0(s/g_0)}\quad\text{for the pattern }(g_0,\,g_1=g_2=\cdots=g), \tag{5.17}$$

$$\frac{s}{g_0}=Q_1(s/g)[3s(1/g_1-1/g)Q_1(s/g)+Q_0(s/g)]^{-1}\quad\text{for the pattern }(g_0,\,g_1,\,g_2=g_3=\cdots=g). \tag{5.18}$$

For instance, the first case in Eq. (5.17) (all multipole strengths equal) has the eigenvalue and eigenfunction

$$s=g;\quad U(\theta)=\cos\theta/(1-\cos\theta) \tag{5.19}$$

which becomes singular at $\cos\theta=1$ (infinite distortion of the Fermi surface) in contrast to the solution (5.12). This is not surprising since the interaction in momentum space corresponding to equal multipoles is proportional to $\delta(1-\cos\vartheta)$, i.e., there is only an interaction when $\mathbf{p}$ and $\mathbf{p}'$ are parallel, whereas the interaction leading to Eq. (5.12) is independent of angle.

To illustrate the usefulness of the solutions we have obtained for special choices of the interaction let us consider the solutions of Eq. (5.1) in the channel $S=0$, $T=1$ (isospin density oscillations). This mode is identified with the giant dipole states that are excited by γ-ray absorption in nuclei. The matrix elements V^{ST} are given by Eq. (5.2) with $V_{\mathbf{p}-\mathbf{p}'}=-\lambda^2/(\mu^2+(\mathbf{p}-\mathbf{p}')^2)$, i.e., the static Yukawa interaction. Then we can calculate that

$$g_n=1+\frac{m\lambda^2}{4\pi^2p_F}Q_n(\beta);\quad \beta=1+\frac{\mu^2}{2p_F{}^2}. \tag{5.20}$$

We observe that $g_0>g_1>g_2>\cdots>1$ in this case and that

$$\lim_{n\to\infty}g_n=1.$$

Model (ii), which includes only a finite number of multipoles in the force, will supply a converging sequence of approximations to the eigenvalue s for a given coupling strength associated with the full Yukawa interaction. We will rather regard s as being given by the observed excitation energy of the giant dipole state in nuclei and determine λ^2. We notice from Eq. (5.20) that all multipoles are repulsive. Thus the value of λ^2 we obtain from model (ii) provides an upper bound for λ^2. Alternatively, model (iii) with the pattern $g_0, g_1, g_2 \cdots g_N, g, g \cdots$ yields a set of lower bounds on λ^2.

The results in Table I were obtained by solving numerically dispersion relations of the type (5.15) and

TABLE I. Pattern of multipoles and corresponding value of neutral scalar coupling constant λ^2 that yield the eigenvalue $s=1.125$ for the giant dipole state ($S=0$, $T=1$) in nuclear matter.

Multipoles	λ^2
$g_0g_0g_0g_0\cdots$	1.0
$g_0g_1g_1g_1\cdots$	2.38
$g_0g_1g_2g_2\cdots$	4.36
$g_0g_1g_2g_3g_3\cdots$	4.74
$g_0g_1g_2g_3g_4g_4\cdots$	4.88
$g_0g_1g_2g_3g_4 11\cdots$	4.98
$g_0g_1g_2g_3 11\cdots$	5.02
$g_0g_1g_2 11\cdots$	5.2
$g_0g_1 11\cdots$	6.14
$g_0 111\cdots$	13.48

(5.17) for a prescribed value of s. We present results for the value $s=\omega/\omega_k=1.125$ which is obtained from the estimates $\omega\approx 80A^{-1/3}$ (MeV), $k\approx\pi/2R$ for the dipole state energy and its effective wave number[22] k in a nucleus of radius R.

Using $R=1.25A^{1/3}$ we find $k/p_F=0.16$ in a heavy nucleus like Pb($A=208$), so that the long-wavelength limit is probably applicable. The assumed A dependence of ω and k actually makes our estimate of $s=\omega/\omega_k$ independent of A, but the identification of the giant dipole mode with a solution of the Landau equation is only justified for large A. We see from Table I that a coupling constant given by $4.88<\lambda^2<4.98$ will give a collective mode at the observed excitation of the dipole state. This value agrees well with the strength of Yukawa interactions used in shell-model calculations of this state.[23]

Now let us look at the static limit of the Chew-Low interaction (3.8′). Its particle-hole matrix elements turn out to be attractive in $S=0$ $T=1$, so the pseudoscalar coupling in our model will not support an isospin wave. This is just one more symptom of the fact that pseudoscalar meson exchange alone cannot give the entire two-body interaction. However, the pseudoscalar theory gives repulsive particle-hole matrix elements in the "breathing mode" $S=T=0$. Let us therefore compare our excitation frequencies for such a mode, using the known (renormalized) value[13] of $f^2\approx 1$, with the estimates of this frequency based on nuclear compressibility. Such a comparison is expected to be very crude and we will be interested in orders of magnitude only.

In the $S=T=0$ channel one finds that the multipole strengths are

$$g_n=1+(3/2\pi^2)mp_F(f/\mu)^2[(1-\beta)Q_n(\beta)+\delta_{n0}], \quad (5.21)$$

where β has the same definition as before. Using the dispersion relations (5.15) with g_n given by (5.21) and $f^2=1.09$, $\mu=135$ MeV. $p_F=270$ MeV/c, we find the values of s shown in Table II. The frequencies ω refer to

TABLE II. Energies of $S=T=0$ "breathing mode" in Pb^{208} corresponding to the renormalized pseudoscalar coupling constant $f^2\approx 1.09$ for various multipole patterns. $s=\omega/\omega_k$ where $\omega_k=12$ MeV.

Multipoles	s	ω (MeV)
g_0	1.15	13.8
g_0g_1	1.07	12.8
$g_0g_1g_2$	1.05	12.6

a nucleus the size of Pb^{208}, for which we assume $\omega_k=12$ MeV. We note that the calculation of s converges very rapidly, the addition of g_2 having very little effect on the value of s. Nuclear compressibilities[24,25] indicate that the breathing mode energy lies somewhere between 8 and 16 MeV in a nucleus like Pb. Our estimate, based on the Landau equation and the known pseudoscalar coupling constant, is not inconsistent with this result. However, we emphasize again that our simple model only contains a part of the two-body interaction, so this result is only of qualitative interest.

B. Nonstatic Interactions

We now discuss the case where the full frequency dependence of Eq. (4.4) comes into play. As with the static case above, we consider only the long-wavelength limit of Eq. (4.4), where both the particle and the hole states are on the Fermi surface. In the long-wavelength limit we have

$$iG_{\mathbf{p}+\mathbf{k},\nu}(\epsilon+\omega)G_{\mathbf{p}\nu}(\epsilon)\approx\pi\frac{\mathbf{k}\cdot\mathbf{p}}{p_F}\delta(p-p_F)\frac{1}{\omega_{\mathbf{p}\mathbf{k}}-\omega}\times[\delta(\epsilon_{\mathbf{p}+\mathbf{k}}-\omega-\epsilon)+\delta(\epsilon_{\mathbf{p}}-\epsilon)], \quad (5.22)$$

where the $\epsilon_{\mathbf{p}+\mathbf{k}}$, $\epsilon_{\mathbf{p}}$ and $\epsilon_{\mathbf{p}+\mathbf{k}}-\epsilon_{\mathbf{p}}=\omega_{\mathbf{p}\mathbf{k}}$ are either HF energies and energy differences if the HF Green's functions are used, or kinetic energies and their differences if free-particle Green's functions are used. If we introduce amplitudes $U(\theta,\phi,\epsilon)\delta(p-p_F)$ on the Fermi surface for $f_{\mathbf{k}\omega}{}^{ST}(\mathbf{p},\epsilon)$, we have

$$(\cos\theta-s)U(\theta,\phi,\epsilon)+\pi[\delta(\epsilon_F+\omega_k\cos\theta-\omega-\epsilon)+\delta(\epsilon_F-\epsilon)]\times\frac{mp_F}{(2\pi)^3}\cos\theta\int d\Omega'\int\frac{d\epsilon'}{2\pi}V^{ST}U(\theta'\phi'\epsilon')=0, \quad (5.23)$$

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where V^{ST} stands for the matrix elements

$$V^{ST}=[4V_{\mathbf{k}}(0)\delta_{S0}\delta_{T0}-V_{|\mathbf{p}-\mathbf{p}'|}(\epsilon-\epsilon')]_{|\mathbf{p}|=|\mathbf{p}'|=p_F} \quad (5.24)$$

for the neutral scalar case and $\omega_k=kv_F$ is the maximum particle-hole energy in the long-wavelength limit as before. Suppression of the frequency dependence in V^{ST} and subsequent integration of Eq. (5.23) over frequency reproduces the static case, Eq. (5.1) (π times the two δ functions just cancels the 2π in the energy integral over ϵ').

We also refer to Eq. (5.23) as a Landau equation. Its solutions are complicated by the appearence of δ function singularities multiplying the interaction term. In fact, we may extract these singularities as follows: Consider again the case of longitudinal distortions about $\mathbf{k}$ so that ω is independent of ϕ and write

$$U(\theta,z)=f_1(\theta)\delta(z)+f_2(\theta)\delta(z+\omega_k\cos\theta-\omega), \quad (5.25)$$

where we measure the energy $z=\epsilon_F-\epsilon$ from the Fermi energy. Then $U(\theta)=\int U(\theta,z)d\epsilon$ measures the distortion of the Fermi surface. The amplitudes $f_1(\theta)$ and $f_2(\theta)$ satisfy coupled equations. We write down these equations for the neutral scalar coupling in the channel $S=0$, $T=1$, i.e., the isospin wave channel that we considered in the static case. Then, from Eqs. (5.24) and (3.8),

$$V^{ST}=-V_{\mathbf{p}-\mathbf{p}'}(\omega)=\frac{\lambda^2}{\mu^2+(\mathbf{p}-\mathbf{p}')^2-\omega^2}, \quad (5.26)$$

and the amplitudes f_1 and f_2 obey

$$(x-s)f_1(x)=\frac{mp_Fx}{8\pi^2}\int_{-1}^{+1}dx'\times[V_{|\mathbf{p}-\mathbf{p}'|}(0)f_1(x')+V_{|\mathbf{p}-\mathbf{p}'|}(\omega-\omega_kx')f_2(x')], \quad (5.27)$$

$$(x-s)f_2(x)=\frac{mp_Fx}{8\pi^2}\int_{-1}^{+1}dx'\,[V_{|\mathbf{p}-\mathbf{p}'|}(\omega_kx-\omega_kx')\times f_2(x')+V_{|\mathbf{p}-\mathbf{p}'|}(\omega-\omega_kx)f_1(x')],$$

after introducing the variable $x=\cos\theta$.

It is clear from the structure of the matrix elements in these equations that frequency-dependent effects are of order $(\omega/\mu)^2\approx(\omega_k/\mu)^2$ in the limit of long wavelengths. For nuclear systems $(\omega_k/\mu)^2\approx10^{-2}$ so that the effects are very small. We saw that the situation was different for single-particle motion in the HF field where this effect was larger by an order of magnitude.

We expect to find solutions to Eqs. (5.27) that are close to the static solutions given for Eq. (5.1). We consider the particular case where the dependence of $V_{|\mathbf{p}-\mathbf{p}'|}(\omega)$ on the angle between $\mathbf{p}$ and $\mathbf{p}'$ is suppressed [this is similar to soluble model (i) of the static equation]. Then the equation for $f_1(x)$ has the solution

$$f_1(x)=x/(x-s) \quad (5.28)$$

apart from a normalizing constant. If we eliminate f_1 from the second equation in (5.27) by using (5.28), and replace the matrix elements $V(\omega_kx-\omega_kx')$ and $V(\omega-\omega_kx)$ by

$$V(\omega_kx-\omega_kx')\approx-\frac{\lambda^2}{\mu^2}\left[1+\left(\frac{\omega_k}{\mu}\right)^2(x-x')^2\right],$$
$$V(\omega-\omega_kx')\approx-\frac{\lambda^2}{\mu^2}\left[1+\left(\frac{\omega_k}{\mu}\right)^2(s-x)^2\right], \quad (5.29)$$

valid to order $(\omega_k/\mu)^2$, then the integral equation for f_2 has the exact solution

$$f_2(x)=[x/(x-s)](a_0+a_1x+a_2x^2), \quad (5.30)$$

where the coefficients a_0, a_1, a_2 are determined by direct substitution. The eigenvalue condition is then a consistency condition that f_1 and f_2 as given above satisfy Eq. (5.27). This leads to the complicated relation

$$1=4CQ_1(s)+2C^2\left(\frac{\omega_k}{\mu}\right)^2\times[(8/3+3s^2/C)Q_1(s)-10s^2Q_1^2(s)-1/C+\tfrac{2}{3}], \quad (5.31)$$

where $C=\lambda^2mp_F/8\pi^2\mu^2$. The second term on the right-hand side of this dispersion relation for s exhibits the frequency-dependent effects to order $(\omega_k/\mu)^2$. If we solve the static dispersion relation $1=4CQ_1(s)$ for C (and hence λ^2) to place the eigenvalue at $s=1.125$ for normal density $\sim p_F^3$ as before, one finds $\lambda^2=2.412$. With the second term in Eq. (5.31) taken into account we obtain $\lambda^2=2.406$ for the same s. The ratios a_1/a_0 and a_2/a_0 can likewise be expressed as functions of the eigenvalue s and constants C and ω_k/μ, but the resulting expressions are unwieldy and are not displayed here.[19] We only quote the corrections to $f_2(x)$ evaluated for $s=1.125$,

$$f_2(x)=[x/(x-s)]a_0(1+0.043x+0.003x^2), \quad (5.32)$$

showing that the distortion $U(x)=f_1(x)+f_2(x)$ of the Fermi surface is close to that of the static case. We see then that using the static approximation for the one-pion-exchange interactions between nucleons participating in low-frequency oscillations of the many-body system introduces only very small errors.

6. SUMMARY

The results we have developed are to a large extent self-explanatory and hardly require further comment. We simply summarize the main consequences and implications of our approach:

(i) In the one-meson-exchange model of nucleon-nucleon interactions that we have used, it is simpler to obtain solutions for the nucleon motion in infinite nuclear matter than for two isolated nucleons. This feature is evident both from the simple model of Sec. 2 for the coupled meson-nucleon system as well as from the extended treatment in Secs. 3, 4, and 5 of the Hartree-Fock potential and collective excitations by means of Green's-function techniques. The simplicity arises because of the zero-point motion of the nucleons in the Fermi sea. This motion is fairly adequately described by the independent-particle model, and can be treated rather well in the Hartree-Fock approximation. The presence of the unperturbed Fermi sea also provides the basis for a discussion of the characteristic particle-hole excitations of the system via the random-phase approximation.

(ii) In the Hartree-Fock approximation one obtains a clean separation of renormalization effects and nucleon-nucleon interactions in the single-particle self-energy function $\sum_{\mathbf{p}}(\omega)$ that appears in the Dyson equation.[12] We noted that the renormalization contribution to $\sum_{\mathbf{p}}(\omega)$ came entirely from the poles of the meson Green's function $D_{\mathbf{p}}(\omega)$. The resulting expression reduces to the familiar results of the neutral scalar theory in the static limit, $\omega\rightarrow 0$ and $p\rightarrow 0$ where it can be interpreted as a mass renormalization of the nucleon. There is no renormalization of the coupling constant in the HF approximation. Intuitively,[26] one expects renormalization effects associated with nucleons developing a "meson cloud" to differ for isolated nucleons and nucleons in a Fermi sea. These differences are contained in formula (3.34) as compared with (3.35) for an isolated nucleon. A more complete treatment of the problem is required to analyze these differences.

(iii) Frequency-dependent effects in the effective two-body interaction are characterized by the parameter ϵ_F/μ for single-particle motion, or ω/μ for collective excitations (ω = collective frequency). They are therefore much more important in the former case. We notice that the effects increase with increasing Fermi energy ϵ_F showing that the effective interaction is density-dependent.

(iv) We showed in Sec. 3 that the Hartree-Fock potential derived from pseudoscalar one pion exchange (OPEP) is in fact repulsive in nuclear matter (except at very high momenta p). This is just a symptom of the fact that a one-pion-exchange model for the interaction between nucleons is too simple; for instance, the OPEP does not support the giant dipole excitation that is well established experimentally. In nuclear matter, the one-pion-exchange model excludes the possibility of direct matrix elements that are attractive and can produce a net binding of the system. One could go beyond OPEP by introducing more meson fields in the model, to simulate the nucleon-nucleon interaction at shorter ranges.[27] However, since a possible frequency dependence in the resulting effective interaction becomes less important as the range decreases, it would be reasonable to treat the short-range character of the nucleon-nucleon interaction phenomenologically (by introducing hard cores, etc.) while still treating the exchange of π mesons in the manner we have done. This would allow for a more realistic discussion of the saturation problem and how saturation is effected by the density dependence, which is introduced through the frequency dependence of the long-range part $V_{\mathbf{k}}(\omega)$ of the interaction.

(v) It is clear from the form of our effective interaction $V_{\mathbf{k}}(\omega)$ that frequency-dependent effects should increase with increasing nucleon energy. Unfortunately the expression $\sum_{\mathbf{p}}(\omega)$ for the Hartree-Fock field is modified at higher energies by higher-order contributions from multiparticle excitations out of the Fermi sea. These also introduce a frequency dependence into $\sum_{\mathbf{p}}(\omega)$, even if the two-nucleon interaction is static. This suggests that processes involving meson production or absorption could perhaps provide information on the importance of the one-pion-exchange effects in nuclear structure. For example, one can derive a meson optical potential from the equations of Sec. 3 by examining the full Green's function for meson propagation instead of nucleons. It would be interesting to compare the results obtained in this manner with calculations of the meson optical potential based on other methods, such as multiple scattering theory.[28] In general, we would maximize the effects of frequency dependence by looking at processes involving high momentum transfers. For example, one could consider high-energy inelastic electron scattering at backward angles or nuclear reactions such as $(p,2p)$.

(vi) For low lying excited states such as the collective excitations in heavy nuclei, the effects of frequency dependence in $V_{\mathbf{k}}(\omega)$ are inessential. The static solution of the meson-nucleon coupled system is completely adequate.

We may thus regard the interaction $V_{\mathbf{k}}(\omega)$ as the frequency-dependent generalization of the ordinary static one-pion-exchange potential (OPEP). However, the Green's-function method we used for the identification of $V_{\mathbf{k}}(\omega)$ does not depend on the use of perturbation theory. A more sophisticated calculation would attempt to include the strong short-range repulsion necessary to explain saturation and to fit high-energy nucleon-nucleon scattering data. Since methods based on meson theory are unambiguous only for the long-range OPEP part of the interaction, the short-range repulsion is probably best included by means of a phenomenological hard core. In the case of the Landau equation, only the long-range part of the interaction is sampled in the $k\rightarrow 0$ limit, and hence the collective mode energies should be insensitive to the presence of the hard core. In the case of the Hartree-Fock field, we do not expect the OPEP to produce the correct magnitude of the single-particle potential. However the effects of frequency dependence should be most important for the OPEP, and hence we

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expect that the modification of the static Hartree-Fock field due to retardation will be well represented in our approach.

We view the calculations with the frequency-dependent OPEP as only the first step in the program of studying meson exchange between nucleons in a nucleus. The missing pieces of the interaction, such as the effects of multimeson exchange, could be inserted phenomenologically to give a quantitatively more useful theory. In spite of its limitations we have seen that the simple theory can be carried quite far. It also provides a compact characterization of the order of magnitude of retardation effects in the Hartree-Fock field and collective modes [the small quantities $(\epsilon_F/\mu)^2$ and $(\omega/\mu)^2$]. Our calculations have been performed for infinite nuclear matter. The translational invariance of the system led to considerable calculational simplicity, and many results could be obtained analytically. The formulation for a finite system is simply obtained by replacing the linear momentum by angular-momentum quantum numbers $\{l,m\}$. However, so long as we are not interested in specific nuclear-structure effects, the nuclear-matter approximation is probably sufficient for a discussion of meson-nucleon interactions in heavy nuclei.

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Modification of the Spectroscopic Factor in (^{3}He,d) Reactions due to the t·T Interactions*

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A correction term to the usual distorted-wave Born-approximation amplitude of the (^{3}He,d) reaction is derived by using Lane's **t**·**T** interaction. This interaction changes the ^{3}He into t and the target into its analog, and thus gives rise to a (t,d) reaction from this analog channel; the deuteron produced in this way is coherent with the deuteron produced by the straightforward (^{3}He,d) reaction. It is shown that this correction term changes the value of the derived spectroscopic factor by an important amount, and makes it agree well, if not completely, with the value predicted by the shell-model calculation.

A RECENT paper[1] showed that it is possible to remove a large discrepancy[2] between the experimental and theoretical spectroscopic factors $S_>$ in the ^{9}Be$(d,n)^{10}$B ($T=1$, 1.74-MeV state) reaction, if the **t**·**T** interaction[3] is considered between the final ^{10}B$+n$ channel and its analog, i.e., the ^{10}Be$+p$ channel. More specifically, it was known[2] that if the experimental data for the above process for $E_d\approx 5$-MeV were analyzed by the usual distorted-wave Born-approximation (DWBA) calculation, one gets $S_>(d,n)\approx 1.0$, while the corresponding theoretical value $S_>^{\text{th}}$ of Kurath[4] is 1.96. However, consideration of the **t**·**T** interaction gives rise to a contribution to the above (d,n) process from a new process, in which the ^{9}Be$(d,p)^{10}$Be reaction occurs first and then charge exchange follows. Our calculation showed[1] that this contribution was rather large (interfering destructively), and made $S_>(d,n)\approx 2$, in very good agreement with $S_>^{\text{th}}$.

In Ref. 2 it was also pointed out that $S_>(^3\text{He},d)$ of the ^{9}Be$(^3$He$,d)^{10}$B process for $E_{^3\text{He}}=10\sim 25$ MeV ranged from 3.35 to 2.65, if the usual DWBA was used in analyzing the data. This value disagrees with $S_>^{\text{th}}$ [and thus with our new value of $S_>(d,n)$], and also disagrees very badly with the DWBA value of $S_>(d,n)\approx 1$. The purpose of the present paper is to show that a technique similar to that used previously[1] can be used here again, and it works to remove the above discrepancy to a large extent, if not completely.

The way the **t**·**T** interaction comes into our present calculation, however, is not exactly the same as it did in the (d,n) reaction. There, the **t**·**T** interaction worked in the final channel, while in the present case it works in the incident channel. That is, the incident ^{3}He$+^9$Be channel changes, because of the **t**·**T** interaction, into a $t+^9$B channel and in this new channel a (t,d) reaction can occur. The deuteron produced in this way is coherent, and thus interferes with the deuteron pro-

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