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Phenomenological Λ-Nucleon Potentials from S-Shell Hypernuclei. III. Dependence on Potential Shape*

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The binding-energy data of the s-shell hypernuclei and the Λ-proton scattering data are analyzed with Λ-nucleon potentials which have a hard core and an attractive part of Yukawa shape. By comparing the results with those obtained in a previous investigation where the attractive part was assumed to be of exponential shape, it is concluded that to analyze these experimental data, the choice of the spatial form for the attractive part is not a critical one.

I. INTRODUCTION

IN our previous investigations,[1,2] the binding-energy data of the s-shell hypernuclei and the total cross sections of the Λ-proton scattering have been examined with a number of two-body, spin-dependent, central Λ-nucleon potentials which have a hard core and an attractive part of exponential shape. From these investigations, it was concluded that to obtain a good agreement with these experimental data, the Λ-nucleon potential must have an intrinsic range between 1.9 and 2.3 F and a hard-core radius close to 0.6 F. In this work, we continue the study by examining the sensitivity of the results on the shape of the attractive part of the Λ-nucleon potential. What we shall do is to carry out similar calculations with a Yukawa dependence for the attractive tail, rather than an exponential dependence used in our previous calculations.

A study of possible effects due to shape dependence has been made previously by Dalitz and Downs using purely attractive Λ-nucleon potentials of intrinsic ranges equal to 0.84 and 1.48 F.[3] By examining the hypernucleus $_\Lambda H^3$, it was found by these authors that Yukawa and exponential forms for the Λ-nucleon potentials which have the same intrinsic range lead to essentially the same values for the well-depth parameter and the scattering length. In this investigation, we complement their study by considering Λ-nucleon potentials with a hard core and of longer intrinsic range. Further, we shall study not only the hypernucleus $_\Lambda H^3$ but also the hypernuclei $_\Lambda H^4$ and $_\Lambda He^5$.

Unless noted otherwise, the notations used here have the same meaning as those appearing in HTI and HTII. Also, the procedure of analysis will be the same as that described in HTII, namely, we shall employ the binding-energy data on the three- and four-body hypernuclei to obtain the charge symmetric (CS) and charge-symmetry-breaking (CSB) parts of the Λ-nucleon interactions, and then use these interactions to analyze the Λ-proton elastic-scattering data.

II. RESULTS

A. Analysis of S-Shell Hypernuclei

The nucleon-nucleon potential used here is the same as that employed in HTI and HTII. For the Λ-nucleon potential, we use

$$\begin{aligned} U_t(r) &= \infty, & (r<r_{\Lambda N}) \\ &= -U_{0t}\frac{\exp[-\lambda(r-r_{\Lambda N})]}{\lambda r}, & (r>r_{\Lambda N}), \\ U_s(r) &= \infty, & (r<r_{\Lambda N}) \\ &= -U_{0s}\frac{\exp[-\lambda(r-r_{\Lambda N})]}{\lambda r}, & (r>r_{\Lambda N}) \end{aligned} \tag{1}$$

for the CS part and

$$U_{CSB} = -\tau_3{}^N \boldsymbol{\sigma}_\Lambda\cdot\boldsymbol{\sigma}_N W_0 \frac{\exp[-\lambda(r-r_{\Lambda N})]}{\lambda r} \tag{2}$$

for the CSB part. The values of the parameters $r_{\Lambda N}$ and λ for the various Λ-nucleon potentials considered in this investigation, together with the values of the intrinsic range b, are given in Table I. As is shown in this table, these potentials will be referred to as potential EY, FY, and GY, respectively.

With trial wave functions described by Eqs. (8)–(10) of HTI, the results obtained are given in Table II.

TABLE I. Parameters of the Λ-nucleon potentials.

Potential type	b (F)	$r_{\Lambda N}$ (F)	λ (F^{-1})
EY	2.0	0.45	2.47
FY	2.0	0.60	3.62
GY	2.5	0.60	2.12

* Work performed under the auspices of the U. S. Atomic Energy Commission.

[1] R. C. Herndon and Y. C. Tang, Phys. Rev. **153**, 1091 (1967), hereafter referred to as HTI.
[2] R. C. Herndon and Y. C. Tang., Phys. Rev. **159**, 853 (1967), hereafter referred to as HTII.
[3] R. H. Dalitz and B. W. Downs, Phys. Rev. **110**, 958 (1958).

TABLE II. Results of the variational calculation for the s-shell hypernuclei.[a]

Hypernucleus ${}_{\Lambda}Z^A$	Potential type	U_{0A} (MeV)	E_A (MeV)	B_Λ (MeV)	$\langle r_{NN}^2\rangle^{1/2}$ (F)	a_A (MeV)	b_A (MeV)$^{1/2}$
	EY	690.0	-2.32 ± 0.07	0.09 ± 0.07	3.24 ± 0.07	667.6	74.1
		710.0	-2.55 ± 0.06	0.33 ± 0.06	3.15 ± 0.07		
${}_{\Lambda}H^3$	FY	2310.0	-2.41 ± 0.07	0.18 ± 0.07	3.27 ± 0.07	2216.5	219.8
		2350.0	-2.59 ± 0.06	0.37 ± 0.06	3.21 ± 0.07		
	GY	555.0	-2.43 ± 0.07	0.20 ± 0.07	3.24 ± 0.07	524.3	68.6
		570.0	-2.67 ± 0.06	0.45 ± 0.06	3.21 ± 0.07		
	EY	665.0	-9.15 ± 0.14	1.73 ± 0.15	2.59 ± 0.07	567.6	74.1
		680.0	-9.72 ± 0.15	2.30 ± 0.16	2.58 ± 0.07		
${}_{\Lambda}H^4$	FY	2260.0	-9.22 ± 0.15	1.80 ± 0.16	2.65 ± 0.07	1998.6	194.6
		2310.0	-9.98 ± 0.16	2.56 ± 0.17	2.64 ± 0.07		
	GY	550.0	-9.23 ± 0.19	1.81 ± 0.20	2.60 ± 0.07	437.5	83.7
		565.0	-9.74 ± 0.20	2.32 ± 0.21	2.59 ± 0.07		
	EY	622.0	-30.97 ± 0.52	2.66 ± 0.55	2.23 ± 0.05	529.2	56.9
${}_{\Lambda}He^5$		654.0	-33.12 ± 0.47	4.81 ± 0.51	2.22 ± 0.05		
	FY	2165.0	-31.05 ± 0.54	2.74 ± 0.57	2.23 ± 0.05	1933.9	139.6
		2255.0	-33.60 ± 0.48	5.29 ± 0.52	2.22 ± 0.05		

[a] The statistical accuracy in the value of E_A is obtained with 200 000 estimates in the Monte-Carlo calculation.

Using the values of a_A and b_A listed in this table and the experimental values of the hypernuclear binding energies given by Eq. (1) of HTII,[4] we obtain the values of U_{0t}, U_{0s}, W_0, U_{0t}^p, and U_{0s}^p for the various potentials; these are listed in Table III.

From Table III, it is seen that, for potential GY, the value of U_{0t}^p is greater than that of U_{0s}^p. As was explained in HTII, this indicates that potential GY cannot be a candidate to represent the effective Λ-nucleon interaction. In the following, we shall, therefore, omit potential GY from most of our discussions.

In Table IV, the values of the well-depth parameters (s_t^p and s_s^p) and effective-range parameters (a_t^p, r_{0t}^p, a_s^p, and r_{0s}^p) for potentials EY and FY are compared with the corresponding values for potentials E and F of HTII. From this table, we see that, regardless of whether the attractive tail has an exponential or a Yukawa dependence, Λ-nucleon potentials which have the same hard-core radius and intrinsic range have also nearly the same values for all the quantities under comparison. Thus, even though the present investigation is only concerned with particular forms for the attractive part of the Λ-nucleon interaction, we do get a strong indication that for the analysis of the binding-energy data of the s-shell hypernuclei and the low-energy data of Λ-proton scattering the choice of the spatial dependence for the potential is not a critical one.

Using the values of U_{0t} and U_{0s} given in Table III, the values of $B_\Lambda({}_{\Lambda}He^5)$ can be computed. For potentials EY and FY, these turn out to be equal to 5.0 and 5.6 MeV, respectively. Comparing with the experimental value of 3.09 ± 0.03 MeV, we find these calculated values are clearly too large. Thus, as in HTII, we conclude that the isospin and tensor suppression effects[5,6] in ${}_{\Lambda}He^5$ may be quite important. It is possible, of course, that this discrepancy may actually arise from the fact that we have not included a possible three-body ΛNN potential in our calculation.[7] If this latter potential does exist with a significant strength, then an inclusion of it could reconcile the binding-energy data of all the s-shell hypernuclei. At present, we are making a detailed calculation to take this type of potential into account and the result should be available shortly.

TABLE III. Values of the potential depths U_{0t}, U_{0s}, W_0, U_{0t}^p, and U_{0s}^p.

Potential type	U_{0t} (MeV)	U_{0s} (MeV)	W_0 (MeV)	U_{0t}^p (MeV)	U_{0s}^p (MeV)
EY	634.7 ± 15.2	719.4 ± 20.7	7.0	641.6 ± 15.2	698.4 ± 20.7
FY	2243.5 ± 38.3	2328.3 ± 56.7	18.4	2261.9 ± 38.3	2273.3 ± 56.7
GY	578.2 ± 15.8	544.0 ± 20.0	7.9	586.1 ± 15.8	520.3 ± 20.0

TABLE IV. Comparison of well-depth and effective-range parameters for Λ-nucleon potentials with exponential and Yukawa dependence.

	Potential type		Potential type	
	EY	E	FY	F
s_t^p	0.715 ± 0.017	0.712 ± 0.015	0.815 ± 0.014	0.804 ± 0.013
s_s^p	0.778 ± 0.023	0.761 ± 0.025	0.819 ± 0.020	0.820 ± 0.021
a_t^p (F)	-1.60 ± 0.16	-1.60 ± 0.15	-2.01 ± 0.23	-1.84 ± 0.20
r_{0t}^p (F)	3.70 ± 0.20	3.61 ± 0.17	3.25 ± 0.17	3.34 ± 0.17
a_s^p (F)	-2.38 ± 0.37	-2.16 ± 0.36	-2.08 ± 0.37	-2.09 ± 0.37
r_{0s}^p (F)	3.08 ± 0.20	3.15 ± 0.21	3.20 ± 0.24	3.15 ± 0.23

[4] Upon the completion of this work, we have noticed that W. Gajewski *et al.* [Nucl. Phys. **B1**, 105 (1967)] have published new results on the binding-energy values of the s-shell hypernuclei. These newer values are, however, only very slightly different from the values used in this analysis; hence, all the conclusions mentioned here and in HTII are still perfectly valid.

[5] A. R. Bodmer, Phys. Rev. **141**, 1387 (1966).

[6] R. H. Dalitz, in Proceedings of the Conference on the use of Elementary Particles in Nuclear Structure Studies, Brussels, 1965 (unpublished).

[7] Calculations with ΛNN potential have been made by A. R. Bodmer and S. Sampanthar [Nucl. Phys. **31**, 251 (1962)]; A. R. Bodmer and J. W. Murphy [*ibid.* **64**, 593 (1965)]; and A. Gal [Phys. Rev. **152**, 975 (1966)].

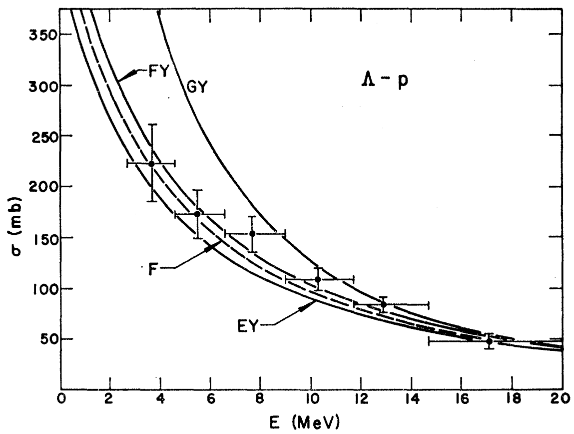

FIG. 1. Total Λ-proton elastic-scattering cross section as a function of c.m. energy for potentials EY, FY, GY, and F. The experimental data are from Table VII of HTII.

B. Analysis of Λ-Proton Scattering Data

In Fig. 1, the solid lines show the behavior of the total Λ-proton scattering cross section σ as a function of the c.m. energy E for potentials EY, FY, and GY, while the dashed line shows the behavior for potential F. The values of x, defined by Eq. (14) in HTII, are equal to 0.2 for potentials EY, FY, and F, and 0.3 for potential GY. As was explained in HTII, these values represent the best values of x determined by using the information on the total cross sections in the c.m. energy region 20–40 MeV and the forward-to-backward ratios F/B. From this figure, it is evident that the cross sections calculated using potentials FY and F are nearly the same, being different by less than 10% for all energy values of interest. A similar conclusion has also been reached when the cross sections obtained for potentials EY and E are compared. Thus, we conclude that, even in the medium-energy region, the Λ-proton scattering cross sections are not sensitive to the spatial form of the attractive part of the Λ-proton interaction.

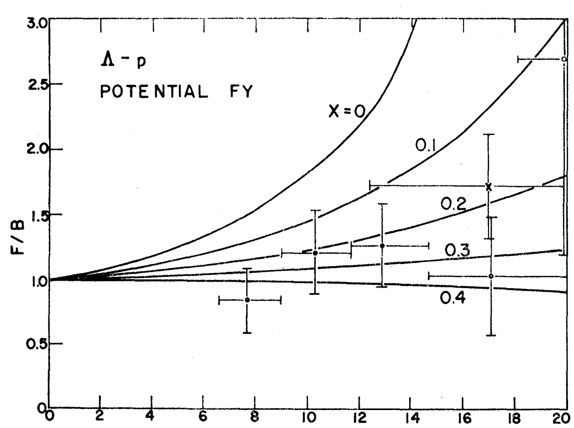

FIG. 2. Λ-proton F/B ratio as a function of c.m. energy for potential FY and various values of x. The experimental data are from Refs. 3, 4, and 5 of HTII.

The behavior of the F/B ratio as a function of E is illustrated for potential FY in Fig. 2. Here again, as in the case of potential F of HTII, it is seen that the fit to the experimental data is very poor for x equal to zero, but becomes quite acceptable when x is in the range from 0.15–0.30.

The values of χ^2, defined by Eq. (15) in HTII, are equal to 2.6, 0.6, and 6.4 for potentials EY, FY, and GY, respectively. The small value of 0.6 for potential FY indicates that this potential yields a good fit to the binding-energy data of the three- and four-body hypernuclei and the Λ-proton scattering data. Thus, together with potential H proposed in HTII, it can be used in other problems where an effective Λ-nucleon interaction is required, such as the calculation of the binding energy of a Λ particle in nuclear matter.

III. CONCLUSION

The results of this calculation using Λ-nucleon interactions of Yukawa spatial dependence show that for the analysis of the binding-energy data of the s-shell hypernuclei and the Λ-proton scattering data in the low- and medium-energy region, the choice of the spatial form for the attractive part of the Λ-nucleon potential is not a critical one. Thus, all the conclusions which have been made in HTII can also be made here without modification.

To summarize, this series of investigations indicates that if a central, two-body potential could be used to represent the Λ-nucleon interaction, then it should have the following properties: (i) It has an intrinsic range of about 2 F and a hard core with a radius close to 0.6 F; (ii) its degree of spin dependence is rather small, with both the triplet and singlet well-depth parameters equal to about 0.8; and (iii) its strength in odd-parity states is only about half as much as that in even-parity states.

Also, it should be mentioned that with this Λ-nucleon potential, the hypernucleus $_{\Lambda}H^3$ will have a slightly bound or a slightly unbound excited state of $J=\frac{3}{2}$ and $T=0$. This is interesting, since it means that the Λ-d scattering system would have large cross sections at low energies.

Our previous conclusion about the nonexistence of $_{\Lambda\Lambda}H^4$ in either $T=0$ or $T=1$ configuration also needs to be reexamined.[8] This is so, since the energy of this system depends rather sensitively on the intrinsic range of the Λ-nucleon potential and in our previous calculation a value of 1.5 F has unfortunately been used. Thus, with the longer intrinsic range found here, the conclusion about the $_{\Lambda\Lambda}H^4$ system may be quite different. In fact, a crude calculation shows that with the Λ-nucleon potential found in this series of investigations (potential H or FY), $_{\Lambda\Lambda}H^4$ will be particle-stable in the $T=0$ configuration, but is still unlikely to be bound in the $T=1$ configuration.

[8] Y. C. Tang and R. C. Herndon, Phys. Rev. Letters **14**, 991 (1965).