It is apparent that the NaI results, when compared to those of the germanium crystal, leave something to be desired. However, from this curve and three others like it, we were able to obtain a value for the ratio

$$F=\Delta\bar{E}/\Delta\bar{E}_i=\langle v\rangle/v_i=0.65\pm0.1.$$

Copper backings were used in measurements on this level, and from the slowing-down parameters of copper listed above, and from Eq. (4), we obtain a value for the mean life of the 3.59-MeV state in B^{10} of $\tau=1.7\pm0.7\times10^{-13}$ sec. This result is in agreement with the two previous measurements.[3,4]

IV. SUMMARY

In Table III we summarize all of the measurements on mean lives of three states in B^{10}. As can be seen, all measurements now show reasonable agreement. From the mean lives for the three states shown in the table, and from the branching ratios for the 2.15-MeV and the 3.59-MeV states,[8] a total of six $M1$ transition probabilities can be obtained, and, hopefully, compared with the calculations of Cohen and Kurath.[5] However, as illustrated in the discussions of Warburton *et al.*,[4] uncertainties in the multipolarities of all but two of the transitions make such comparisons ambiguous. Of those two, the $1.74\rightarrow0.72$ MeV transition is too fast to be measured by the Doppler-shift method, a result in agreement with Cohen and Kurath's predictions. Finally, the transition probability of the $2.15\rightarrow1.74$ MeV $M1$, $\Delta T=1$, transition obtained from our measurement of the mean life and from the branching ratio of that state[8] is about one-fourth of the best value calculated for it.

TABLE III. Mean lives (seconds) of states in B^{10}.

State (MeV)	BNL[a]	Stanford[b]	Present work
1.740		$<2.8\times10^{-14}$	$<4.0\times10^{-14}$
2.154		$(4.0\pm1.0)\times10^{-12}$	$(2.1_{-0.8}^{+0.8})\times10^{-12}$
3.585	$(1.20\pm0.43)\times10^{-13}$	$(1.33\pm0.35)\times10^{-13}$	$(1.7\pm0.7)\times10^{-13}$

[a] Reference 4.
[b] Reference 3.

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Theory of the Photodisintegration of the Deuteron and of Other Nuclei*

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The interaction between the radiation field and a collection of nucleons is formulated in a manner suitable for the employment of the Franz-Stech classification of electromagnetic multipoles, and for separating the more certain parts of the interaction energy from the speculative ones, the uncertainties being concerned with exchange currents. The separation is effected by bringing the entrance of the electric charge density into evidence. The heuristic introduction of the intensity of magnetization as though it were due to magnetic moments of the fixed-magnetic-moment type is avoided, the whole interaction being expressed in terms of currents. Part of the general discussion neglects retardation so as to bring out the reasons for the particular grouping of terms, but retardation effects are included later on. It is shown that in a nonrelativistic theory the usual procedure of making calculations as though the center of mass of the p-n system were fixed may be justified as an approximation provided certain assumptions are made. The recoil of the center of mass caused by photon absorption is explicitly considered in this connection. Some limitations of the theory caused by relativistic effects are mentioned. The relationships of contributions to the electric-multipole transition amplitudes caused by the nucleon magnetic moments, as well as of related contributions of radial components of the Schrödinger current, are discussed in relation to the retardation effects. A brief review of the limitations of space-time models and of the accomplishments of the pure S-matrix approach to the $d(\gamma,n)p$ problem indicates the continued value of both approaches.

I. INTRODUCTION

IN connection with nuclear photodisintegration in general and especially that of the deuteron, it is desirable to employ a classification of electromagnetic multipoles making use of irreducible tensors. The suitability of such a treatment for the discussion of problems involving rotations of coordinate axes is clear. For this reason as well as the aesthetic appeal of a plan based on transformation properties, this classification has displaced other ways of dealing with electromagnetic multipole radiation. The well-known necessity of introducing exchange currents when exchange forces are present in the nuclear Hamiltonian makes it desirable, however, to formulate the interaction between the radiation field and the nucleons in a manner which separates

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the more certain parts of the interaction energy from the more speculative ones, the latter involving exchange currents in an essential manner. The present paper is concerned with questions arising in such a formulation.

In Sec. II the entrance of the intensity of magnetization in Maxwell's equations is discussed. This quantity, often denoted by the symbol $\mathbf{M}$, is frequently introduced heuristically as though it had its origin in magnetic moments of the fixed-magnetic-moment type. On the other hand, in applications, $\mathbf{M}$ is used for the representation of effects of magnetic moments associated with nuclear spins, which in final analysis are caused by electric currents. These are partly the currents caused by the motion of a Dirac proton as a whole and partly the currents originating in the motion of charged mesons around the nucleon. It is shown that the net effect is as though one were formally dealing with an intensity of magnetization.

Section III is concerned with a partly intuitive formulation of the interaction energy for electric multipoles in the nonretarded approximation. The electric field of the electric multipoles is expressed as the gradient of a potential, and the interaction Hamiltonian is therefore obtainable as the potential energy of the charge density $\rho(\mathbf{r},t)$ in that field. This part of the interaction energy therefore has a meaning independently of unsolved questions regarding exchange currents. This does not mean, however, that the corresponding part of the Hamiltonian is completely known. Among the uncertainties, in the first place, there is a lack of knowledge of the wave function; and secondly, the equation

$$\rho(\mathbf{r},t)=e\psi^*\psi\,, \tag{1.1}$$

commonly used for a single particle, is doubtless only an approximation, which needs refinement in view of the empirical evidence for the existence of form factors of nucleons. But to the extent to which one may formulate the theory in terms of a wave function employing only the coordinates of the centers of mass of the nucleons and their spin coordinates, and in which the electromagnetic form factors are neglected, this part of the interaction energy has an immediately obvious validity.

The magnetic multipole interaction energy is treated along somewhat similar lines in Sec. IV. The considerations regarding the meaning of $\mathbf{M}(\mathbf{r},t)$ in Sec. II are made use of, with the aid of elementary relationships between the electric current and equivalent magnetic shell formulations of magnetostatics. On account of the probable existence of exchange moments in nuclei, there is in a sense less certainty concerning these contributions than for those arising from the charge density. On the other hand, the connection between the form factor and the exchange currents must be an intimate one, and the distinction between the degrees of certainty may be more formal than actual. At the end of Sec. IV, the connection of the forms arrived at with the Hamiltonian giving the effect of radiation on matter is briefly pointed out, and the way in which equivalent operators in such a treatment may be introduced is mentioned.

The restrictions regarding the negligibility of retardation effects are removed in Sec. V. The parts of the Hamiltonian referring to electric and magnetic multipoles, having to do with the absorption of radiation, are transformed in such a way as to bring into evidence the interaction with the charge density and also the equivalent magnetic pole density. The current densities entering this discussion include effects of exchange currents. Although there is no safe prescription for calculating these, and although there is therefore no safe way of using the formulas, it is hoped that it may prove useful to have a separation of the interaction into parts containing uncertainties of different types. The information concerning electromagnetic form factors is more certain than that about exchange currents, and the contributions to photodisintegration amplitudes arising from the form factors are not expected to be as hard to arrive at with accuracy as the other contributions. One may hope, therefore, that with such a classification of terms, comparison with experimental material may lead eventually to a cleaner distinction between alternative possibilities for exchange currents. At the end of the section the transition to equivalent operators which has been mentioned at the end of the preceding section is written out.

In Sec. VI the elimination of the coordinates of the center of mass is discussed, and it is shown that under certain limitations one may treat the center of mass as fixed at a point $\mathbf{R}=0$ and take the coordinate vectors of the proton and neutron as $\mathbf{r}_p-\mathbf{R}$ and $\mathbf{r}_n-\mathbf{R}$, respectively, even though the localization of the center of mass is in contradiction with Heisenberg's uncertainty relation. The inclusion of recoil effects caused by the absorption of the photon is essential for the validity of this elimination of center-of-mass coordinates. The discussion of this elimination is nonrelativistic, and is presented in terms of the impulse approximation. Estimates indicate that both of these approximations are good in the usual range of experimental energies.

Discussions of the absence of exchange-current effects on the interaction through electric multipole effects have been given by Sachs and Austern in the old multipole classification, and by Brennan and Sachs in the modern one, making use of gauge-invariance considerations in both cases. The presence of a space gradient term in the expression for the vector potential corresponding to electric multipoles occurring in the present paper makes possible a direct transformation to the charge density. In the absence of form-factor effects, the independence of this part of the interaction energy on exchange currents is directly obvious, as discussed in Secs. III and V.

Section VII points out some of the limitations of nonrelativistic space-time models of the $d(\gamma,n)p$ reaction; discusses the relationship of the effect of magnetic mo-

ments on electric multipole transitions, and the relationship of related terms appearing in Sec. V, to the modification of electric multipole effects caused by the inclusion of retardation; briefly reviews advances made in the theory of the $d(\gamma,n)p$ reaction by dispersion-theoretical methods; and arrives at the conclusion that the space-time representation of the reaction should still be useful as a preliminary step even in the nonrelativistic local potential form. This section, and to a degree the present paper as a whole, may be considered as an introduction to a series of papers by members of a group with which the present authors have been associated.

Some of the more frequently used symbols are as follows:

M = nucleon mass.
Ψ_α = large components of the Dirac wave function ψ.
$\mathbf{X}_{lm}=\mathbf{L}Y_{lm}(\theta,\varphi)/[l(l+1)]^{1/2}$.
κ,ω = photon wave vector and angular frequency; $\kappa/2\pi$ is the photon wave number.
$\mathbf{Z}_{ls}=F_l(\kappa r)\mathbf{L}Y_{ls}/(\kappa r)$.

II. ELECTROMAGNETIC QUANTITIES

The treatment of the radiation field in this paper makes use of Franz's classification of electromagnetic multipoles[1] as presented by Blatt and Weisskopf,[2] and of many of the results obtained in these references. In the present application, the quantization of the electromagnetic field is not necessary, and the treatment in Ref. 2 can be simplified. Doing so has the additional advantage of showing in an elementary manner that exchange currents may be neglected in the evaluation of certain terms. It is also necessary to discuss a few matters of principle.

In BW use is made of the intensity of magnetization $\mathbf{M}$. While mathematically terms corresponding to $\mathbf{M}$ may be inserted in Maxwell's equations, there is no evidence for the existence of permanent magnetic dipoles, so that these terms do not have direct significance. Nevertheless it is convenient to summarize the effect of spin currents by replacing them with an equivalent intensity of magnetization $\mathbf{M}_{\text{eff}}$, which will be abbreviated as $\mathbf{M}$. In a nonrelativistic treatment by means of Dirac's equation the particle current density may be approximated as

$$\mathbf{j}=-c(\bar{\psi}^*\boldsymbol{\alpha}\psi)\approx(\hbar/2iM)(\Psi_\alpha^*\nabla\Psi_\alpha-\Psi_\alpha\nabla\Psi_\alpha^*)+(\hbar/2M)[\nabla\times\boldsymbol{\sigma}]_{\alpha\beta}\Psi_\alpha^*\Psi_\beta. \quad (2.1)$$

Here M is the mass of the particle, the nucleon in the present case, while Ψ_α ($\alpha=1, 2$) represents the "large" components of the Dirac wave function ψ. The first term on the right-hand side of this equation is the Schrödinger current $\mathbf{j}_S$; the second is the spin current. If the Dirac particle carries a charge e, the Maxwell equation expressing Ampère's law is

$$\operatorname{curl}\mathfrak{H}=\partial\mathcal{E}/c\partial t+4\pi\mathbf{J}_S/c-4\pi(e\hbar/2Mc)\times[\boldsymbol{\sigma}_{\alpha\beta}\times\nabla]\Psi_\alpha^*\Psi_\beta, \quad (2.2)$$

where

$$\mathbf{J}_S=e\mathbf{j}_S \quad (2.3)$$

is the electric current density attributable to the Schrödinger particle current density. Electrostatic units of electric charge and intensity and electromagnetic units of magnetic field strength are used. Introducing

$$\mathfrak{H}_{\text{eff}}=\mathfrak{H}-4\pi\mathbf{M};\ \mathbf{M}=(e\hbar/2Mc)\Psi_\alpha^*\boldsymbol{\sigma}_{\alpha\beta}\Psi_\beta, \quad (2.4)$$

Eq. (2.2) becomes

$$\operatorname{curl}\mathfrak{H}_{\text{eff}}=\partial\mathcal{E}/c\partial t+4\pi\mathbf{J}_S/c \quad (2.5)$$

and the remaining Maxwell equations take the form

$$\operatorname{curl}\mathcal{E}=-\partial(\mathfrak{H}_{\text{eff}}+4\pi\mathbf{M})/c\partial t, \quad (2.6)$$

$$\operatorname{div}(\mathfrak{H}_{\text{eff}}+4\pi\mathbf{M})=0,\quad \operatorname{div}\mathcal{E}=4\pi e\Psi_\alpha^*\Psi_\alpha. \quad (2.7)$$

Specializing to fields of definite frequency $\omega/2\pi$, Eqs. (2.5) and (2.6) agree with Eqs. (4.1a) and (4.1b) of Appendix B of BW, provided $\mathfrak{H}_{\text{eff}}$ is replaced by $\mathfrak{H}$. In treating nonvacuum electrodynamics it is customary to use the symbol $\mathbf{B}$ for the magnetic induction. In such treatments

$$\mathbf{B}=\mathfrak{H}+4\pi\mathbf{I},\quad \operatorname{div}\mathbf{B}=0, \quad (2.8)$$

where $\mathfrak{H}$ and $\mathbf{I}$ stand for the magnetic field and the intensity of magnetization, respectively. There is a close analogy between Eq. (2.8) and the first of the two Eqs. (2.7). In the passage from a microscopic to a macroscopic treatment of a medium consisting of moving electric charges (but not of higher-order singularities in the electric field), such as contained in the well-known development by H. A. Lorentz, the macroscopic quantity $\mathbf{B}$ is the mean of the microscopically defined magnetic field $\mathbf{h}$, and is thus more closely related to the original magnetic field than the quantity called $\mathfrak{H}$ in macroscopic field treatments. This situation is similar to $\mathfrak{H}$ being a solenoidal vector according to (2.4) and (2.7). In spite of the analogies, however, the quantity occurring in the previously quoted equations of BW is not the magnetic field but the $\mathfrak{H}_{\text{eff}}$ of Eq. (2.4).

According to Eqs. (2.1) and (2.3) the interaction energy between radiation and matter is

$$H'=-\frac{1}{c}\int\{\mathbf{J}_S+(e\hbar/2M)[[\nabla\times\boldsymbol{\sigma}_{\alpha\beta}]\Psi_\alpha^*\Psi_\beta]\}\cdot\mathbf{A}d\mathbf{r}=-\frac{1}{c}\int([\mathbf{J}_S+c\ \operatorname{curl}\mathbf{M}]\cdot\mathbf{A})d\mathbf{r}, \quad (2.9)$$

where $\mathbf{A}$ is the vector potential of the external field, i.e., of the incident plane wave, and differential operators are supposed not to apply to quantities outside the

[1] W. Franz, Z. Physik **127**, 363 (1950); B. Stech, Z. Naturforsch. **7a**, 401 (1952).
[2] J. M. Blatt and V. F. Weisskopf, *Theoretical Nuclear Physics* (John Wiley & Sons, Inc., New York, 1952). This reference will be referred to as BW.

special brackets. Although (2.9) is clearly the *value* of the interaction energy, it is not immediately clear from the form of the equation that it gives correctly the main effect of interest in the $d(\gamma,n)p$ reaction, viz., the changes in the state of the p-n system. That this is so in the case of a single Dirac proton follows, however, from the first of the two forms $\mathbf{j}$ in (2.1) and the standard form of the interaction Hamiltonian of a Dirac particle, combined with the fact that it suffices to transform the matrix elements from the four-component ψ to the two-component Ψ forms. In (2.4), $\mathbf{M}$ includes effects of the Dirac part of the magnetic moment only. The inclusion of the pion-nucleon interactions adds in effect the "Pauli" part of the magnetic moment of the nucleon to the Bohr magneton in the equation that results from multiplying Eq. (2.1) by e. It may be verified that this addition is also correct so far as the employment of the modified $\mathbf{M}$ in (2.9) is concerned. The verification is essentially the same as that used in showing that the force exerted on a system of atomic electrons by an external magnetic field can be calculated by replacing all atomic currents with an equivalent magnetic shell, even though there are induction effects associated with the Larmor precession. The general relation thus becomes

$$\mathbf{J}=\mathbf{J}_S+c\ \mathrm{curl}\mathbf{M}. \tag{2.10}$$

The contribution of $\mathbf{M}$ to $\mathbf{J}$ can be pictured as the result of local current accumulation resulting from space nonuniformity of the intensity of Amperian whirl density. For example, if the whirls are around the z axis and if their intensity varies along the x direction, then the magnetic moment of the magnetic shell formed by area $\Delta x\Delta y$ is $J_i\Delta x\Delta y/c$, and $M_z=J_i/c$. On account of the variation of J_i with x there is a contribution to J_y,

$$J_y=[-(\partial J_i/\partial x)\Delta x]/\Delta x=-c\partial M_z/\partial x=c\ \mathrm{curl}_y\mathbf{M}.$$

It is thus clear that for a single particle, Eq. (2.9) applies, with $\mathbf{M}$ representing the intensity of magnetization that would be present had the Dirac and meson parts of the spin current been replaced by c curl$\mathbf{M}$. The result is the same as though $\mathbf{M}$ were caused by magnetic dipoles. It applies to neutrons as well as protons.

III. ELECTRIC MULTIPOLE INTERACTION ENERGY

The presence of exchange potentials brings with it[3,4] the possibility of modifications of the operators to be used for electric currents. Considerations regarding the possibilities for the absence of modifications in the calculation of $E-1$ transitions have been given by Siegert,[5] Lamb and Schiff,[6] and others.[7,8] A discussion by Sachs[7] based on the work of Austern and Sachs proves in a certain sense that "the electric multipole moment operators are independent of exchange effects." Since the multipole classifications of Sachs and of BW are not the same, since further spin-orbit potentials have assumed an added importance, and since the calculations of the $d(\gamma,n)p$ reaction are much simplified by the Franz-BW classification, it appears desirable to reexamine the question directly in connection with the BW presentation and the BW analysis of a plane electromagnetic wave into multipoles. According to Eqs. (5.4a) and (5.4b) of BW, the field strengths of a circularly polarized plane wave may be represented as

$$\mathfrak{E}_s(\mathbf{r})=\sum_l i^l[2\pi(2l+1)]^{1/2}\times\left\{\begin{matrix}s\\ -\\ \kappa\end{matrix}\mathrm{curl}\left[\frac{F_l(\rho)}{\rho}\mathbf{X}_{ls}\right]+\frac{F_l(\rho)}{\rho}\mathbf{X}_{ls}\right\}, \tag{3.1a}$$

$$\mathfrak{IC}_s(\mathbf{r})=\sum_l i^l[2\pi(2l+1)]^{1/2}\times\left\{-is\frac{F_l(\rho)}{\rho}\mathbf{X}_{ls}-\frac{i}{\kappa}\mathrm{curl}\left[\frac{F_l(\rho)}{\rho}\mathbf{X}_{ls}\right]\right\}, \tag{3.1b}$$

where

$$s=\pm1, \tag{3.1c}$$

depending on whether the wave is right or left polarized; κ is the absolute value of the propagation vector, i.e., 2π times the wave number;

$$\rho=\kappa r;$$

and

$$F_l(\rho)=(\pi\rho/2)^{1/2}J_{l+1/2}(\rho)=\rho j_l(\rho), \tag{3.1d}$$

with $J_n(x)$ denoting the Bessel function of order n and argument x. The symbols $\mathfrak{E}(r)$, $\mathfrak{IC}(r)$ are related to the

[3] H. S. W. Massey and C. B. O. Mohr, Proc. Roy. Soc. (London) **A148**, 206 (1935); Nature **133**, 211 (1934). It has been suggested by Massey and Mohr, following a related discussion by Taylor and Mott, that electric dipole radiation might disappear for systems governed by Majorana forces.

[4] G. Breit and E. U. Condon, Phys. Rev. **49**, 904 (1936). Arguments have been given in this paper for calculating the E-1 effect by the usual $-e(\mathfrak{E}\cdot\mathbf{r})$ form of interaction energy in the nonretarded approximation. It was also pointed out, shortly before Eqs. (2), that one would expect a modification of the usual expressions for the electric currents on account of the presence of exchange potentials.

[5] A. J. F. Siegert, Phys. Rev. **52**, 787 (1937).

[6] W. E. Lamb, Jr. and L. I. Schiff, Phys. Rev. **53**, 651 (1938).

[7] Robert G. Sachs, *Nuclear Theory* (Addison-Wesley Publishing Company, Inc. Cambridge, Mass., 1953). In the footnote on p. 243 of this reference the view regarding the propriety of employing $-e(\mathfrak{E}\cdot\mathbf{r})$ which was taken in Ref. 4 was attributed to Siegert.

[8] N. Austern and R. G. Sachs, Phys. Rev. **81**, 710 (1951). Cf. also R. G. Sachs, *ibid.* **71**, 457 (1947); J. G. Brennan and R. G. Sachs, *ibid.* **88**, 824 (1952). In the last-named reference, essentially the same multipole classification as in the present paper is employed, and limitations of the EEA theorem are discussed. There is also some overlap in the results and claims, especially those in the present section (Sec. III) of this paper. But in the present discussion no explicit use of gauge invariance is made. The argument is as a consequence less abstract and more pictorial. It is believed to have an advantage in applications involving an approximate treatment of form-factor effects, since gauge invariance relies on a strict point-charge interpretation of the proton, and is not truly applicable to a composite nucleon-meson system unless the constituent parts are explicitly included.

complete field strengths $\mathcal{E}(\mathbf{r},t)$, $\mathcal{H}(\mathbf{r},t)$ by

$$\mathcal{E}(\mathbf{r},t)=\mathcal{E}(\mathbf{r})e^{-i\omega t}+\mathcal{E}^*(\mathbf{r})e^{i\omega t},\quad \omega=c\kappa \tag{3.1e}$$
$$\mathcal{H}(\mathbf{r},t)=\mathcal{H}(\mathbf{r})e^{-i\omega t}+\mathcal{H}^*(\mathbf{r})e^{i\omega t};$$

and

$$\mathbf{X}_{lm}=\mathbf{L}Y_{lm}(\theta,\varphi)/[l(l+1)]^{1/2} \tag{3.2}$$

is a special case of a vector spherical harmonic. The angles θ, φ are the colatitude and azimuth of $\mathbf{r}$ for polar axis along $\boldsymbol{\kappa}$. The electric multipole part of $\mathcal{E}(\mathbf{r})$ is

$$\mathcal{E}_{Els}(\mathbf{r})=C_{ls}'\ \mathrm{curl}[F_l(\rho)\mathbf{X}_{ls}/\rho]\approx C_{ls}\ \mathrm{curl}[\mathbf{L}r^lY_{ls}], \tag{3.3}$$

where

$$C_{ls}'=i^l[2\pi(2l+1)]^{1/2},$$
$$s/\kappa=(2l+1)!![l(l+1)]^{1/2}\kappa^{-l}C_{ls}. \tag{3.3'}$$

The approximation made in Eq. (3.3) consists in keeping only the first nonvanishing power of ρ in the power-series expansion of $F_l(\rho)$. This procedure is valid in the nonretarded approximation, in which the dimensions of the deuteron are considered small in comparison with the wavelength of the γ ray. From the expansion of F_l, requiring that the first neglected term be small in comparison with the one kept, it follows that

$$\kappa^2r^2\ll 2(2l+3) \tag{3.3''}$$

must be required in order that the approximation be valid. The approximation becomes increasingly better, therefore, as l increases. It follows from (3.3) that

$$\mathcal{E}_{Els}(\mathbf{r})\approx-\nabla\varphi_{Els},\quad \varphi_{Els}=-(l+1)iC_{ls}r^lY_{ls}. \tag{3.4}$$

Here φ_{Els} is effectively the electrostatic potential for the Els part of the plane wave. The mutual energy between $\rho^*(\mathbf{r})$, defined as the $e^{i\omega t}$ part of the charge density $\rho(\mathbf{r},t)=\rho(\mathbf{r})e^{-i\omega t}+\rho^*(\mathbf{r})e^{i\omega t}$, and the Els multipole is

$$H_{Els}'=\int\rho^*(\mathbf{r})\varphi_{Els}d\mathbf{r}. \tag{3.5}$$

According to Appendix B of BW

$$\mathcal{E}_s(\mathbf{r})=(\mathbf{1}_x+si\mathbf{1}_y)e^{i\kappa z}/2^{1/2}, \tag{3.6}$$

where $\mathbf{1}_x$, $\mathbf{1}_y$ are unit vectors along the x and y axes, respectively. Hence

$$[\mathcal{E}_1(\mathbf{r})+\mathcal{E}_{-1}(\mathbf{r})]/2^{1/2}=\mathcal{E}_x(\mathbf{r}) \tag{3.6'}$$

gives a wave polarized with electric vector along the x axis of a Cartesian coordinate system, the z axis of which is the polar axis of the system used for θ, φ. The x axis corresponds to $\varphi=0$. Accordingly the mutual energy of $\rho^*(\mathbf{r})$ with a plane-polarized electromagnetic wave having for $\mathcal{E}(\mathbf{r})$ the vector

$$\mathcal{E}_x(\mathbf{r})=(1,0,0)e^{i\kappa z} \tag{3.6''}$$

is a sum of contributions from various electric multipoles El, each contributing

$$H_{Elx}'=\int\rho^*(\mathbf{r})\varphi_{Elx}d\mathbf{r}, \tag{3.7}$$

where

$$\varphi_{Elx}=\sum_s\varphi_{Els}/2^{1/2}, \tag{3.7'}$$

the sum being taken over $s=1,-1$. On making use of (3.3') and (3.4) there results the explicit form

$$H_{Elx}'=i^{l-1}\left(\frac{l+1}{l}\right)^{1/2}[\pi(2l+1)]^{1/2}\left[\frac{\kappa^{l-1}}{(2l+1)!!}\right]$$
$$\times\int\rho^*(\mathbf{r})r^l[Y_{l,1}-Y_{l,-1}]d\mathbf{r}. \tag{3.8}$$

This form has been obtained for a field which is the gradient of a potential, as seen from (3.4) and (3.7'). For such a field the force exerted on a particle is derivable from a potential energy which is the charge of the particle multiplied by the potential. One may use therefore H_{Els}', H_{Elx}' as the perturbing Hamiltonians for circularly and linearly polarized γ rays in the nonretarded approximation. It is seen that the question of exchange currents does not enter this phase of the problem. The reason for this simplification is the existence of φ_{Els}. It is a natural extension of the argument used in Ref. 4 in the special case of the electric dipole. Formally the result is the same as that of Sachs and Austern except for the fact that in the Franz classification the absence of exchange-current effects has been shown to hold only in the nonretarded approximation. The step from (3.3) to (3.4) makes use of the identity $\Delta(r^lY_{ls})=0$, which leads to the disappearance of $\mathbf{r}\Delta(r^lY_{ls})$ in the direct evaluation of (3.3). This circumstance is special to the first term of the expansion of F_l, and does not hold for the exact expression. Thus in general

$$\mathrm{curl}^2[F_l(\rho)\mathbf{X}_{ls}/\rho]\neq 0$$

and the motion of a particle is not describable by a potential.

IV. MAGNETIC MULTIPOLE INTERACTION ENERGY

The effect of $\mathbf{M}$ on magnetic multipoles may be obtained on noting that the last term in curly braces in (3.1b) takes the place of the first term in curly braces of (3.1a) when one goes from electric to magnetic multipoles and makes use of the following consideration regarding the similarity of electric and magnetic quantities. From (2.5) and (2.6),

$$c\ \mathrm{curl}\mathcal{H}_{\mathrm{eff}}(\mathbf{r})=-i\omega\mathcal{E}(\mathbf{r})+4\pi\mathbf{J}_S(\mathbf{r}), \tag{4.1a}$$
$$c\ \mathrm{curl}\mathcal{E}(\mathbf{r})=i\omega[\mathcal{H}_{\mathrm{eff}}(\mathbf{r})+4\pi\mathbf{M}(\mathbf{r})], \tag{4.1b}$$
$$\mathrm{div}\mathbf{J}_S(\mathbf{r})=i\omega\rho(\mathbf{r}), \tag{4.1c}$$

where the original $\mathbf{J}_S(r,t)$ is represented by

$$\mathbf{J}_S(\mathbf{r},t)=\mathbf{J}_S(\mathbf{r})e^{-i\omega t}+\mathbf{J}_S^*(\mathbf{r})e^{i\omega t}. \quad (4.1d)$$

The first three equations are very similar to Eqs. (4.1a), (4.1b), and (4.1c) in Appendix B of BW, but differ from them through the occurrence of $\mathfrak{H}\mathfrak{C}_{\text{eff}}$ in place of $\mathfrak{H}\mathfrak{C}$, and of $\mathbf{J}_S$ in place of $\mathbf{j}$. It follows from (4.1a) that

$$\text{div}\mathcal{E}(\mathbf{r})=4\pi\rho(\mathbf{r}) \quad (4.2a)$$

and from (4.1b) that

$$\text{div}\mathfrak{H}\mathfrak{C}_{\text{eff}}(\mathbf{r})=-4\pi\ \text{div}\mathbf{M}(\mathbf{r}). \quad (4.2b)$$

Comparison of (4.2b) with (4.2a) indicates that the replacement $\rho(\mathbf{r})\rightarrow-\text{div}\mathbf{M}(\mathbf{r})$ combined with $s\rightarrow-i$, the latter arising from the comparison of the relevant terms in (3.1a) and (3.1b), should give the contribution of $\mathbf{M}$ to the magnetic multipole interaction. This procedure gives

$$\varphi_{Mls}=-(l+1)sC_{ls}r^lY_{ls}$$

$$=-i^lk^{l-1}\left(\frac{l+1}{l}\right)^{1/2}\frac{[2\pi(2l+1)]^{1/2}}{(2l+1)!!}r^lY_{ls}, \quad (4.3)$$

$$H_{Mls}'=-\int[\text{div}\mathbf{M}^*(\mathbf{r})]\varphi_{Mls}d\mathbf{r}. \quad (4.4)$$

The meaning of s is here the same as previously, i.e., $s=+1$ and $s=-1$ give, respectively, right- and left-handed polarizations of the incident plane wave. From (4.4) it follows that the interaction energy for a plane wave polarized along the x axis is

$$H_{Mlx}'=i^lk^{l-1}\left(\frac{l+1}{l}\right)^{1/2}\frac{[\pi(2l+1)]^{1/2}}{(2l+1)!!}$$

$$\times\int[\text{div}\mathbf{M}^*(\mathbf{r})]r^l(Y_{l1}+Y_{l,-1})d\mathbf{r}. \quad (4.5)$$

The somewhat formal procedure giving H_{Mlx}' through a comparison with H_{Elx}' may appear unsatisfactory because in the photodisintegration process the production of the field by the fictitious magnetic charge density $-\text{div}\mathbf{M}$ does not enter directly. The expansion of the plane wave into multipoles as in (3.1a) and (3.1b) should suffice for a complete treatment of first-order effects, and the comparison of (4.2a) with (4.2b) appears at first sight irrelevant. The inverse reaction $p(n,\gamma)d$, however, is concerned with field production, and the substitution $\rho(\mathbf{r})\rightarrow-\text{div}\mathbf{M}(\mathbf{r})$ is in this case directly applicable. According to reciprocity, correct results for transition matrix elements are therefore obtainable by the procedure used. The result of a direct transformation of the original interaction energy which agrees with Eq. (4.4) is given in the next section. Independently of this consideration, a photodisintegration reaction such as $d(\gamma,n)p$, when considered directly, can be treated by means of Eq. (2.9), employing for $\mathbf{A}$ a vector potential describing the incident plane wave. The part of H' corresponding to $\mathbf{M}$ is

$$H_{\mathbf{M}}'=-\int(\mathbf{A}\cdot\text{curl}\mathbf{M}^*)d\mathbf{r}=-\int(\mathbf{M}^*\cdot\mathfrak{H}\mathfrak{C})d\mathbf{r}. \quad (4.6)$$

If, within the physically important region, $\mathfrak{H}\mathfrak{C}$ is derivable from a potential as

$$\mathfrak{H}\mathfrak{C}=-\nabla\varphi_{\mathfrak{H}\mathfrak{C}}, \quad (4.7)$$

then the nonretarded approximation to $H_{\mathbf{M}}'$ is

$$H_{\mathbf{M}}'=\int(\mathbf{M}^*\cdot\nabla\varphi_{\mathfrak{H}\mathfrak{C}})d\mathbf{r}=-\int\varphi_{\mathfrak{H}\mathfrak{C}}\ \text{div}\mathbf{M}^*d\mathbf{r}. \quad (4.8)$$

The replacement of $\rho(r)$ by $-\text{div}\mathbf{M}(\mathbf{r})$ is thus justifiable.

The magnetic multipole transitions are also affected by the Schrödinger current density $\mathbf{J}_S$. The magnetic field caused by this current can be described by a vector potential. Employing the transformation from the curl form to the gradient form in the same way as in going from Eq. (3.3) to Eq. (3.4) and treating the magnetic field produced by $\mathbf{J}_S$ as in magnetostatics, one finds

$$\mathfrak{H}\mathfrak{C}_{\text{eff}}\approx\text{curl}\mathbf{A}_{\text{mgst}},\quad \mathbf{A}_{\text{mgst}}=(-i/s)C_{ls}\mathbf{L}(r^lY_{ls}) \quad (4.9)$$

and hence

$$H_{\mathbf{J}_S}'=-(1/sc)C_{ls}\int(r^lY_{ls})\ \text{div}[\mathbf{J}_S^*\times\mathbf{r}]d\mathbf{r}, \quad (4.10)$$

which, combined with (4.8), gives

$$(H_{\mathbf{M}}'+H_{\mathbf{J}_S}')_s=\frac{l+1}{s}C_{ls}\int r^lY_{ls}$$

$$\times\left\{\frac{\text{div}[\mathbf{r}\times\mathbf{J}_S^*]}{c(l+1)}+\text{div}\mathbf{M}^*\right\}d\mathbf{r}, \quad (4.11)$$

where

$$\frac{l+1}{s}C_{ls}=\frac{[2\pi(2l+1)]^{1/2}i^lk^{l-1}}{(2l+1)!!}\left(\frac{l+1}{l}\right)^{1/2}. \quad (4.11')$$

For linear polarization of the incident wave along the x axis, employing (3.6) as in obtaining (3.8),

$$(H_{\mathbf{M}}'+H_{\mathbf{J}_S}')_x=\frac{l+1}{\sqrt{2}s}C_{ls}\int r^l(Y_{l,1}+Y_{l,-1})$$

$$\times\left\{\frac{\text{div}[\mathbf{r}\times\mathbf{J}_S^*]}{c(l+1)}+\text{div}\mathbf{M}^*\right\}d\mathbf{r}. \quad (4.12)$$

In the above discussion no clear distinction between the meaning of H' as a part of the Hamiltonian and its meaning as a contribution to the total energy has been drawn. The expressions obtained may nevertheless be used as contributions to the Hamiltonian in the photo-

disintegration problem, provided they are suitably interpreted. This follows from the fact that if in the formulas for $\mathbf{J}_S$ and $\mathbf{M}$ the replacement

$$\Psi_\alpha^* \rightarrow \Psi_\alpha^\dagger \tag{4.13}$$

is made, and Ψ_α, $\Psi_\alpha^\dagger$ are interpreted as the Jordan-Wigner quantized wave functions, then the transformations used are still valid. On the other hand, Ψ_α, $\Psi_\alpha^\dagger$ are connected with the quantized wave functions ψ_μ, $\psi_\mu^\dagger$ of the original Dirac formulation by the same equations as connect the unquantized Ψ_α, Ψ_α^* with the unquantized ψ_μ, ψ_μ^*, provided the replacement

$$\psi_\mu^* \rightarrow \psi_\mu^\dagger \tag{4.13'}$$

is made. Since the original interaction energy, expressed in terms of ψ_μ, $\psi_\mu^\dagger$, has also the significance of the interaction Hamiltonian representing the coupling of radiation and matter, the necessary matrix elements may be calculated by regarding H' as the interaction Hamiltonian also in the formulation employing two components per particle. The quantities $\mathbf{J}^*(\mathbf{r})$, $\mathbf{M}^*(\mathbf{r})$, $\mathbf{J}_S^*(\mathbf{r})$, and $\rho^*(\mathbf{r})$ are readily identified with expressions involving the initial and final states of the material system, either by means of quantized amplitudes or directly in spin-coordinate space. They correspond to the employment of operators $\mathbf{J}^{\mathrm{Op}}$, $\mathbf{M}^{\mathrm{Op}}$, $\mathbf{J}_S^{\mathrm{Op}}$, and ρ^{Op} such that, taking $\mathbf{M}^*(\mathbf{r})$ as an example,

$$(\Psi_f, \mathbf{M}^{\mathrm{Op}}\Psi_i)_{\mathrm{Sp}} = \mathbf{M}^*(\mathbf{r}), \tag{4.14}$$

where the inner product is taken over spin coordinates only, as indicated by the subscript Sp. In (4.14), Ψ_i and Ψ_f are the wave functions of the system of particles in the initial and final states, respectively.

V. EFFECT OF RETARDATION AND ADDITIONAL TERMS

The formulas discussed so far have been obtained by neglecting in each $F_l(\kappa r)$, which originally entered through the electromagnetic field expressions (3.1a) and (3.1b), all but the lowest part of κr. The approximate results obtained in this way can be made to include the effect of all multipoles, but the evaluation of the effect of any one multipole is not exact. A complete evaluation requires a knowledge of the exchange currents, which is not available. For the present it will be supposed, however, that the exchange-current operators will eventually become ascertained, and the expressions for the interaction energy will be written out in such a way that the terms in which exchange-current effects can appear are obvious.

The electric multipole effects are caused by the first two terms in curly braces in (3.1a) and (3.1b). They may be represented by means of the four-potential having for its space part

$$\mathbf{A}_{Els}^{(0)}(\mathbf{r}) = \mathbf{A}_{Els}(\mathbf{r}) - \nabla\Phi_{Els}(\mathbf{r})/(i\kappa), \tag{5.1}$$

and 0 for the time part. Here one may take

$$\mathbf{A}_{Els}(\mathbf{r}) = C_{ls}'\hat{r}F_l(\kappa r)Y_{ls}/[l(l+1)]^{1/2} \tag{5.1'}$$

and

$$\Phi_{Els}(\mathbf{r}) = -iC_{ls}'[dF_l(\kappa r)/d(\kappa r)]Y_{ls}/[l(l+1)]^{1/2}. \tag{5.1''}$$

The field strengths are

$$\mathfrak{E}_{Els}(\mathbf{r}) = -\partial\mathbf{A}_{Els}(\mathbf{r})/c\partial t - \nabla\Phi_{Els}(\mathbf{r}), \tag{5.2}$$

where $\partial/c\partial t = -i\kappa$ for electromagnetic field quantities in accordance with (3.1e), and

$$\mathfrak{H}_{Els}(\mathbf{r}) = \mathrm{curl}\mathbf{A}_{Els}(\mathbf{r}). \tag{5.3}$$

Equations (5.2), (5.3) have the same form as though one used $(\mathbf{A}_{Els}(r), \Phi_{Els}(\mathbf{r}))$ as the vector and scalar potentials, respectively. Whether this is done in place of using $(\mathbf{A}_{Els}^{(0)}(\mathbf{r}), 0)$ does not matter, but it may be observed that the $(\mathbf{A}_{Els}(\mathbf{r}), \Phi_{Els}(r))$ do not satisfy the Lorentz condition. This, however, is immaterial to the application of the four-potential. In fact, from (5.1) and the equation of continuity

$$\mathrm{div}\mathbf{J}^*(\mathbf{r}) + ic\kappa\rho^*(\mathbf{r}) = 0, \tag{5.4}$$

together with the assumption $\mathbf{j}(|\mathbf{r}| = \infty) = 0$, it follows that

$$H_{Els}' = -\frac{1}{c}\int (\mathbf{A}_{Els}^{(0)}(\mathbf{r}) \cdot \mathbf{J}^*(\mathbf{r}))d\mathbf{r}$$

$$= \int\left[\Phi_{Els}(\mathbf{r})\rho^*(\mathbf{r}) - \frac{1}{c}\mathbf{A}_{Els}(\mathbf{r}) \cdot \mathbf{J}^*(\mathbf{r})\right]d\mathbf{r}. \tag{5.5}$$

This interaction energy follows directly, employing either of the two gauges already mentioned. Substitution of $\Phi_{Els}(\mathbf{r})$, $\mathbf{A}_{Els}(\mathbf{r})$ gives

$$H_{Els}' = \frac{C_{ls}'}{[l(l+1)]^{1/2}}\int Y_{ls}\left[-i\frac{dF_l(\kappa r)}{d(\kappa r)}\rho^*(\mathbf{r}) - \frac{1}{c}(\hat{r} \cdot \mathbf{J}^*(\kappa r))F_l(\kappa r)\right]d\mathbf{r}. \tag{5.6}$$

The first term does not contain the current density, the equation of continuity having been used in the form of Eq. (5.4) to express this part in terms of $\rho^*(\mathbf{r})$. The question of exchange currents therefore does not affect the contribution of the term containing $\rho^*(\mathbf{r})$ in the bracket in Eq. (5.6). This contribution is thus analogous to the electric multipole contribution in the classification of R. G. Sachs for which there are proofs[7,8] of the independence of electric multipoles on exchange current effects.

Formally, the procedure used here appears at first sight to be very different from that used by Sachs. However, the difference is not great. In the Sachs-

Austern proof, the independence of

$$D_l=\sum_j e_j(\mathbf{u}\cdot\mathbf{r}_j)(\hat{\kappa}\cdot\mathbf{r}_j)^{l-1}$$

on exchange currents is an essential ingredient, but the primary cause for the success of the proof is that the electric multipole effect is expressible as a gradient of a suitable function. The employment of gauge invariance shows that there exists such a function. But the demonstration of its existence and its determination do not require the direct employment of gauge invariance. The occurrence of the gradient becomes evident in the present case in the second term of Eq. (5.1) and is responsible for the appearance of div$\mathbf{J}^*(\mathbf{r})$ in (5.6), which brings in $\rho^*(\mathbf{r})$ according to (5.4). The questions arising about the correctness of the usual calculation of $\rho^*(\mathbf{r})$ are related, however, to the questions about the correctness of calculating $\mathbf{J}^*(\mathbf{r})$ without taking account of exchange currents. The least that may be expected is an inaccuracy in the employment of $e\psi^*\psi$ for a charged particle in the computation of its charge density, the recoil action of the meson emission providing some smearing in the effective position of the nucleon charge. Some of these form-factor effects are present, however, even if the interaction with the second nucleon is very weak, so that they cannot be blamed completely on meson exchange between the nucleons. Following current custom, however, these effects will be neglected below, and it will be assumed that $\rho^*(\mathbf{r})$ can be obtained from $\psi^*\psi$ in the usual manner.

The second term in square brackets in (5.6) contains in $J^*(r)$ the effects of $\mathbf{M}^*$. These have been deliberately left out in some earlier calculations.[9] The character of the omitted effect is seen from (2.10), according to which the contribution of $\mathbf{M}$ to $\mathbf{J}$ is c curl$\mathbf{M}$. The effect of nucleon structure can conceivably be large for this term, because the contribution to $\mathbf{J}$ is a differential effect between the intensities of $\mathbf{M}$ at neighboring points. The evaluation of effects of this term appear to be insufficiently reliable to justify its inclusion when other effects such as retardation have been omitted, but it is inconsistent to omit this term if retardation is taken into account.

The effect of $\mathbf{M}$ on magnetic multipole transitions is less delicate, depending on $\mathbf{M}$ directly rather than on curl$\mathbf{M}$. Neglecting the effect of curl$\mathbf{M}$ on the El transitions would be inconsistent with taking retardation into account, and is to some extent inconsistent with the inclusion of effects of many multipoles.

Magnetic multipole effects can be similarly treated. The part of the vector potential corresponding to magnetic multipole radiation for a magnetic 2^l pole is

$$\mathbf{A}_{Mls}=\frac{C_{ls}'F_l(\kappa r)}{is\kappa r}\mathbf{X}_{ls} \tag{5.7}$$

and the corresponding magnetic field is

$$\mathfrak{H}_{Mls}=\text{curl}\mathbf{A}_{Mls}, \tag{5.8}$$

since these equations clearly account for the second term in curly braces in (3.1a) and the first in (3.1b). The 2^l-pole contribution to the interaction energy may be put in the form

$$H_{ls}'=H_{Els}'+H_{Mls}', \tag{5.9}$$

with

$$\begin{aligned}H_{Mls}'&=-\frac{C_{ls}'}{[l(l+1)]^{1/2}}\int Y_{ls}\frac{F_l(\kappa r)}{sc\kappa r}(\mathbf{r}\cdot\text{curl}\mathbf{J}^*(\mathbf{r}))d\mathbf{r}\\&=\frac{C_{ls}'/s}{[l(l+1)]^{1/2}}\int Y_{ls}\left\{\frac{dF_l(\kappa r)}{d(\kappa r)}\,\text{div}\mathbf{M}^*\right.\\&\quad\left.-[(\mathbf{r}\cdot\text{curl}\mathbf{J}_S^*)+c\kappa^2(\mathbf{r}\cdot\mathbf{M}^*)]\frac{F_l(\kappa r)}{\kappa rc}\right\}d\mathbf{r}.\end{aligned} \tag{5.10}$$

In the second of the above two forms of H_{Mls}', the indication of the argument $\mathbf{r}$ in $\mathbf{M}^*$ and $\mathbf{J}_S^*$ has been omitted so as to shorten the formula. Combining this form with (5.6), (5.9), and (3.3′),

$$\begin{aligned}H_{ls}'=&\frac{i^l[2\pi(2l+1)]^{1/2}}{\kappa[l(l+1)]^{1/2}}\int Y_{ls}\\&\times\left\{\frac{s}{c\kappa}\left[(\text{div}\mathbf{J}^*)\frac{dF_l}{d(\kappa r)}-\kappa(\hat{r}\cdot\mathbf{J}^*)F_l\right]+(\text{div}\mathbf{M}^*)\frac{dF_l}{d(\kappa r)}\right.\\&\left.-\kappa(\hat{r}\cdot(\mathbf{M}^*+\text{curl}\mathbf{J}_S^*/c\kappa^2))F_l\right\}d\mathbf{r}.\end{aligned} \tag{5.11}$$

In this form the symmetry between electric and magnetic quantities is apparent. The bracket inside the curly braces originated in the electric multipole part of the plane wave. The last two terms correspond to magnetic multipoles. Leaving s out of consideration, the magnetic quantity div$\mathbf{M}^*$ corresponds to the electric quantity div$\mathbf{J}^*/(c\kappa)$, and the magnetic quantity $\mathbf{M}^*+\text{curl}\mathbf{J}_S^*/(c\kappa^2)$ corresponds to $\mathbf{J}^*/(c\kappa)$. Both statements could be covered by regarding $\mathbf{J}^*/(c\kappa)$ as replaced by $\mathbf{M}^*+\text{curl}\mathbf{J}_S^*/(c\kappa^2)$. The current density $\mathbf{J}^*(\mathbf{r})$ entering the electric quantities contains the Schrödinger current $\mathbf{J}_S^*(\mathbf{r})$ and c curl$\mathbf{M}^*(\mathbf{r})$. From (3.3), it is seen that

$$\begin{aligned}-\frac{1}{c}\mathbf{A}_{Els}^0(\mathbf{r})\cdot\mathbf{J}^*(\mathbf{r})=&i^{l+1}\left[\frac{2\pi(2l+1)}{l(l+1)}\right]^{1/2}\\&\times\frac{s}{c\kappa^2}\mathbf{J}^*(\mathbf{r})\cdot\text{curl}\mathbf{Z}_{ls},\end{aligned} \tag{5.12}$$

where

$$\mathbf{Z}_{ls}\equiv F_l(\kappa r)\mathbf{L}Y_{ls}/(\kappa r). \tag{5.12′}$$

[9] M. L. Rustgi, W. Zernik, G. Breit, and D. J. Andrews, Phys. Rev. **120**, 1881 (1960).

Substitution in (5.5) with the aid of (2.10) and of

$$\int \mathbf{B}(\mathbf{r})\cdot\mathrm{curl}\mathbf{M}^*(\mathbf{r})d\mathbf{r}=\int \mathbf{M}^*(\mathbf{r})\cdot\mathrm{curl}\mathbf{B}(\mathbf{r})d\mathbf{r}, \quad (5.13)$$

which follows for the boundary conditions at infinity previously used, with $\mathbf{B}(\mathbf{r})$ standing for any of the field strengths used here, gives

$$H_{El_s}'=K_l h_{El_s}, \quad K_l=i^{l+1}\left[\frac{2\pi(2l+1)}{l(l+1)}\right]^{1/2}, \quad (5.14)$$

$$h_{El_s}=\int\left\{\frac{s}{c\kappa^2}\mathbf{J}_S^*(\mathbf{r})\cdot\mathrm{curl}\mathbf{Z}_{l_s}+s\mathbf{M}^*(\mathbf{r})\cdot\mathbf{Z}_{l_s}\right\}d\mathbf{r}. \quad (5.14')$$

Hence the calculation of H_{El_s}' may be performed employing the operator

$$H_{El_s}^{\mathrm{Op}}=K_l h_{El_s}^{\mathrm{Op}}, \quad (5.15)$$

$$h_{El_s}^{\mathrm{Op}}=(s/c\kappa^2)\mathbf{J}_S^{\mathrm{Op}}\cdot\mathrm{curl}\mathbf{Z}_{l_s}+s\mathfrak{y}^{\mathrm{Op}}\cdot\mathbf{Z}_{l_s}, \quad (5.15')$$

where $\mathbf{J}_S^{\mathrm{Op}}$ and $\mathfrak{y}^{\mathrm{Op}}$ are the operators giving, respectively, the Schrödinger current and the magnetic moment of the system. The operators are meant to be used in the sense of Eq. (4.14). The justification for including all the effects of all the particles by summing the operators for the individual particles can be formulated, of course, by means of quantized amplitudes along the lines mentioned in connection with Eq. (4.13). For the Schrödinger current operator one may use

$$\mathbf{J}_S^{\mathrm{Op}}=(\hbar/i)\sum_j (e_j/M_j)\delta(\mathbf{r}_j-\mathbf{r})\nabla_j, \quad (5.16)$$

where the summation is taken over all the particles. Their charges and masses are denoted by e_j and M_j, respectively, the subscript j being used as a particle label. Equation (5.16) follows from the fact that

$$\tfrac{1}{2}\int \mathbf{C}(\mathbf{r}_j)\cdot[\varphi^*\nabla_j\psi-\psi\nabla_j\varphi^*]d\mathbf{r}_j = \int \mathbf{C}(\mathbf{r}_j)\cdot\varphi^*\nabla_j\psi d\mathbf{r}_j, \quad (5.16')$$

which holds provided the otherwise arbitrary vector $\mathbf{C}(\mathbf{r}_j)$ satisfies

$$\int \mathrm{div}_j[\varphi^*\psi\mathbf{C}(\mathbf{r}_j)]d\mathbf{r}_j=0. \quad (5.16'')$$

This relation is satisfied for the $\mathbf{C}(\mathbf{r}_j)$ occurring in the present problem whenever at least one of the states φ, ψ represents a bound state, as is seen from Eq. (5.14').

According to the discussion of the meaning of $\mathbf{M}$ following Eq. (2.10), it is permissible to use

$$\mathfrak{y}^{\mathrm{Op}}=\sum_j (e\hbar/2Mc)\mu_j\sigma_j\delta(\mathbf{r}_j-\mathbf{r}), \quad (5.17)$$

where the μ_j and σ_j are, respectively, the magnetic moments in Bohr magnetons and the Pauli spin matrices of the particles. A short calculation making use of (5.7) shows that

$$H_{M l_s}'=-\frac{1}{c}\int \mathbf{A}_{M l_s}\cdot(\mathbf{J}_S^*+c\,\mathrm{curl}\mathbf{M}^*)d\mathbf{r}=K_l h_{M l_s}, \quad (5.18)$$

where

$$h_{M l_s}=\frac{1}{\kappa}\int[(1/c)\mathbf{J}_S^*\cdot\mathbf{Z}_{l_s}+\mathbf{M}^*\cdot\mathrm{curl}\mathbf{Z}_{l_s}]d\mathbf{r}. \quad (5.19)$$

Hence the calculation of $H_{M l_s}'$ may be performed by means of the operator

$$H_{M l_s}^{\mathrm{Op}}=K_l h_{M l_s}^{\mathrm{Op}}, \quad (5.20)$$

where

$$h_{M l_s}^{\mathrm{Op}}=(1/\kappa c)\mathbf{J}_S^{\mathrm{Op}}\cdot\mathbf{Z}_{l_s}+(1/\kappa)\mathfrak{y}^{\mathrm{Op}}\cdot\mathrm{curl}\mathbf{Z}_{l_s} \quad (5.20')$$

and the operators used in the sense of Eq. (4.14) are again as in (5.16) and (5.17).

In Eqs. (5.14') and (5.18) the vector densities $\mathbf{J}_S^*(\mathbf{r})$, $\mathbf{M}^*(\mathbf{r})$ may have to be supplemented by exchange currents. The employment of the spin–coordinate-space operators $\mathbf{J}_S^{\mathrm{Op}}$, $\mathfrak{y}^{\mathrm{Op}}$ in the form of Eqs. (5.16) and (5.17) does not take these contributions into account, and caution must be expressed regarding the literal use of these expressions.

In (5.20') the position of $\mathfrak{y}^{\mathrm{Op}}$ corresponds to the mutual energy of a magnetic doublet and the external magnetic field. In the absence of exchange current and associated exchange-moment effects, this form has been derived above—even though the origin of $\mathbf{M}$ is in the electric currents, as follows from Sec. II and considerations in connection with Eq. (4.13). Similarly, the position of $\mathbf{J}_S^{\mathrm{Op}}$ in (5.15') and (5.20') corresponds to interactions with the nonrelativistic momentum, although the actual current has a more complicated structure.

VI. ELIMINATION OF THE DEUTERON'S CENTER OF MASS

On account of the large ratio of the nuclear to the electronic mass, the motion of the center of gravity of the atom is relatively unimportant in the interaction of atoms with the electromagnetic field. In the case of the deuteron the masses of the proton and neutron are practically equal, and greater care must therefore be exercised regarding the distinction between the motion of either particle around the common center of mass and their relative motion. Taking the proton and neutron masses to be equal, there is the temptation to write $\mathbf{r}_p=-\mathbf{r}_n=\frac{1}{2}\mathbf{r}$, where $\mathbf{r}=\mathbf{r}_p-\mathbf{r}_n$ is the relative coordinate vector of the two particles, while $\mathbf{r}_p$ and $\mathbf{r}_n$ are, respectively, the coordinate vectors of the proton and neutron in the center-of-mass system. Actually, however,

$$\mathbf{r}_p=\mathbf{R}+\tfrac{1}{2}\mathbf{r}, \quad \mathbf{r}_n=\mathbf{R}-\tfrac{1}{2}\mathbf{r}, \quad (6.1)$$

where $\mathbf{R}=\frac{1}{2}(\mathbf{r}_p+\mathbf{r}_n)$ is the coordinate vector of the center of mass. To set $\mathbf{R}=0$ implies perfect definition of the position of the center of gravity and therefore an infinite spread in the uncertainty of its momentum $\mathbf{P}$. Such a spread leads to a large spread in the apparent frequency of the incident radiation as it appears to the deuteron, and hence to the inapplicability of usual theory. If one of the center-of-mass coordinates were defined to within $\Delta X=e^2/mc^2$, the associated minimum uncertainty in the kinetic energy of the momentum of the deuteron would be $\Delta P_x\approx(\hbar c/e^2)mc=137mc$, and the corresponding uncertainty in the energy of the deuteron would be

$$\Delta E_d=(\Delta P_x)^2/(4M)$$
$$=(137)^2(m/4M)mc^2\approx 1.2\text{ MeV}. \quad (6.2)$$

Here m, M are, respectively, the masses of the electron and nucleon. For high E this uncertainty is not very serious, but for γ-ray energies in the 1–20-MeV range it is too large to make ordinary theory applicable. Furthermore, $\Delta X=2.8$ F is not small compared with the dimensions of the deuteron, $\frac{1}{3}$ or $\frac{1}{10}$ of this value being presumably necessary in the definition of its position if details of the angular distribution were to be guaranteed from the viewpoint of accurate localization of every point. On the other hand, the localization of the center of mass is not directly related to the angular distribution of the disintegration products. It is necessary therefore to consider the situation more completely.

The discussion presented below is closely related to Sec. 3 of Chap. VII of Ref. 10. The presence of many final states in the continuum of the p-n system and of just one plane electromagnetic wave initially are the main reasons for the impossibility of direct employment of results in the reference cited. The initial and final proton-neutron wave functions will be taken to have the form

$$\Psi_i(\mathbf{R},\mathbf{r})=\psi_i(\mathbf{r})\exp(-i\omega_i t)\int C^i(\mathbf{K})e^{i\mathbf{K}\cdot\mathbf{R}}d\mathbf{K},$$
$$\Psi_f(\mathbf{R},\mathbf{r})=\frac{1}{(2\pi)^3}\int C^f(\mathbf{K},\mathbf{k})e^{i\mathbf{K}\cdot\mathbf{R}}\psi_{\mathbf{k}}^{(-)}(\mathbf{r})d\mathbf{K}d\mathbf{k}. \quad (6.3)$$

In the first of these equations $\psi_i(\mathbf{r})$ is the space factor of the initial deuteron wave function, the energy of the deuteron is $\hbar\omega_i$, and the integral is the wave function of the center of mass. The second equation gives the expansion of the final-wave function in products of plane waves for the center of mass with the functions $\psi_{\mathbf{k}}^{(-)}(\mathbf{r})$, each of which is the ingoing-wave modification caused by the p-n interaction of a plane wave of unit density in the space of relative coordinates $\mathbf{r}$ with momentum $\hbar\mathbf{k}$ at infinite r. The coefficients C^i and C^f are time-dependent. The spin coordinates of the proton and neutron enter ψ_i and the $\psi_{\mathbf{k}}$, but are not explicitly indicated. The deuteron is actually bound in a molecule or a similar structure. A complete treatment would include the consideration of forces that keep the deuteron in place before the collision with the photon, and would therefore be complicated, especially because the forces may be different in the initial and final states. The problem is therefore first considered here only in the impulse approximation, i.e., as though the forces binding the deuteron to the molecule were switched off at the time $t=0$ and the interaction with the incident electromagnetic wave were switched on at the same instant. The relative smallness of the molecular forces suggests that this approximation is good enough. Arguments to be mentioned at the end of this section show that conclusions arrived at here employing the impulse approximation should hold also if this approximation is not made.

The wave packet represented by the first of the two equations (6.3) spreads with time. This spread is caused by the time dependence of the $C^i(\mathbf{K})$, which is not explicitly indicated. Since the deuteron has only one stable state, the whole wave function may be represented as $\Psi_i+\Psi_f$. Substitution of this expression into the Schrödinger equation gives, on making use of

$$\int\psi_{\mathbf{k}'}{}^{(-)*}(\mathbf{r})\psi_{\mathbf{k}}{}^{(-)}(\mathbf{r})d\mathbf{r}=(2\pi)^3\delta(\mathbf{k}-\mathbf{k}'), \quad (6.4)$$

the result

$$\left(\frac{\hbar d}{idt}+\frac{\hbar^2K^2}{4M}+\frac{\hbar^2k^2}{2\mu}\right)C^f(\mathbf{K},\mathbf{k})=-\frac{\exp(-i\omega_i t)}{(2\pi)^3}$$
$$\times\int(e^{i\mathbf{K}\cdot\mathbf{R}}\psi_{\mathbf{k}}{}^{(-)}(\mathbf{r}),H'e^{i\mathbf{K}'\cdot\mathbf{R}}\psi_i(\mathbf{r}))$$
$$\times C^i(\mathbf{K}')d\mathbf{K}'d\mathbf{R}d\mathbf{r}, \quad (6.5)$$

where $\mu=\frac{1}{2}M$ is the reduced mass for relative p-n motion. The Hamiltonian is supposed to have the form

$$H=H_0+H', \quad (6.6)$$

with H_0 taking account of the kinetic energies of the proton and neutron and of their interaction energy. The interaction energy of the two particles with the incident photon wave is represented by H'. The absorption of the photon is obtained to within first-order terms of H' by solving Eq. (6.5) for C^f, employing the unperturbed values of the C^i on the right-hand side. The inner product under the integral sign is meant to apply to spin coordinates only.

In the special case of the interaction of the Schrödinger current with a transverse electromagnetic wave, the part of H' responsible for photodisintegration has the form

$$H_1'=-\frac{he}{2iMc}(\mathbf{a}\cdot[\nabla_p-(\leftarrow)\nabla_p])\exp[i(\boldsymbol{\kappa}\cdot\mathbf{r}_p-\omega t)], \quad (6.7)$$

[10] G. Breit, Rev. Mod. Phys. 5, 91 (1933).

with the understanding that the arrow pointing to the left indicates the application of the differentiations to the first rather than the second factor of the inner product. Substitution into (6.5) gives for this part of C^f the equation

$$\left[\frac{\hbar d}{idt}+\frac{\hbar^2(\mathbf{K}+\boldsymbol{\kappa})^2}{4M}+\frac{\hbar^2k^2}{2\mu}\right]C_1{}^f(\mathbf{K}+\boldsymbol{\kappa},\,\mathbf{k})\cong\frac{he}{2iMc}C^i(\mathbf{K})$$

$$\times\exp[-i(\omega_i+\omega)t]\int\{(\psi_{\mathbf{k}}{}^{(-)}(\mathbf{r}),[\mathbf{a}\cdot(\nabla_r-(\leftarrow)\nabla_r)]$$

$$\times e^{i\boldsymbol{\kappa}\cdot\mathbf{r}/2}\psi_i(\mathbf{r})\rangle+i(\mathbf{a}\cdot\mathbf{K})(\psi_{\mathbf{k}}{}^{(-)}(\mathbf{r}),e^{i\boldsymbol{\kappa}\cdot\mathbf{r}/2}\psi_i(\mathbf{r}))\}d\mathbf{r}. \quad (6.8)$$

The vector $\mathbf{a}$ is defined by

$$\mathbf{A}(\mathbf{r},t)=\mathbf{a}e^{-i(\omega t-\boldsymbol{\kappa}\cdot\mathbf{r})}+\mathbf{a}^*e^{i(\omega t-\boldsymbol{\kappa}\cdot\mathbf{r})}. \quad (6.7')$$

Only the first part on the right-hand side is kept, since the second does not contribute to absorption. Summations over spin indices are understood but not indicated in the inner products of Eq. (6.8).

The second term in curly braces can easily be missed in an "intuitive" consideration. It will be seen that it may be assumed to vanish in the nonretarded approximation unless an energy dependence of the p-n potential is postulated. This dependence must be of a special type, however, if a nonvanishing result is desired in the present treatment, because in the proof of Eq. (6.4) it is assumed[11] that the radial functions for uncoupled states entering the $\psi_{\mathbf{k}}{}^{(-)}$ satisfy the orthogonality relation

$$\int\mathfrak{F}_L(kr)\mathfrak{F}_L(k'r)dr=\tfrac{1}{2}\pi\delta(k-k'), \quad (6.9)$$

which holds for energy-independent potentials, but may be violated otherwise. Since the bound state does not enter Eq. (6.4), it is conceivable that

$$\int(\psi_{\mathbf{k}}{}^{(-)}(\mathbf{r}),\psi_i(\mathbf{r}))d\mathbf{r}\neq0$$

without violation of (6.4), provided that the potential for the deuteron ground state differs from the potential in unbound states, and that the potential in unbound states of the same symmetry is the same. This possibility is very improbable, however. Furthermore, the values of K corresponding to nonvanishing $C^i(\mathbf{K},t)$ in the photodisintegration problem are small. Thus assuming that the deuteron is bound in a molecule within a distance $\frac{1}{5}a_H\approx10^{-9}$ cm, where a_H is the Bohr radius, the value of $\kappa/K\approx54(\hbar\omega)_{\text{MeV}}>118$. Furthermore, if the molecule is at rest, $\mathbf{K}$ has an angular distribution with the same probability along $\mathbf{a}$ as in the opposite direction. The nonretarded effect of the term in $(\mathbf{a}\cdot\mathbf{K})$ is therefore negligibly small.

[11] G. Breit, Ann. Phys. (N. Y.) **16**, 346 (1961).

The effect of the factor $\exp(i\boldsymbol{\kappa}\cdot\mathbf{r}/2)$ is to give, at low energies, matrix elements of the same type as for $E1$ transitions. In this approximation the orthogonality of different states illustrated in Eq. (6.4) does not make the result vanish. The randomness of directions of $\mathbf{K}$ with respect to $\mathbf{a}$, combined with the abovementioned relative smallness of the important range of values of $\mathbf{K}$, makes it improbable that the effect of these terms need be taken into account. This is especially true because even apart from the randomness of the directions of $\mathbf{K}$, the effect of the terms is very small at low energies on account of the smallness of κ; whereas at high energies the accuracy of the measurements is small, and the presence of effects of many electromagnetic multipoles introduces other uncertainties in the calculations. The terms in $(\mathbf{a}\cdot\mathbf{K})$ will therefore be omitted, even though they are present in principle.

If instead of using Eq. (6.3) the motion of the p-n center of mass is disregarded, and the problem is treated in the fixed-center-of-mass procedure (FCOMP) by setting the p-n wave function equal to

$$\psi=c^i\psi_i(\mathbf{r})e^{-i\omega_it}+\int c^f(\mathbf{k})\psi_{\mathbf{k}}{}^{(-)}(\mathbf{r})d\mathbf{k}/(2\pi)^3, \quad (6.10)$$

then a calculation similar to that which gave (6.5) starting with (6.3) gives

$$\left(\frac{\hbar d}{idt}+\frac{\hbar^2k^2}{2\mu}\right)c^f(\mathbf{k})=-\frac{\exp(-i\omega_it)}{(2\pi)^3}c^i$$

$$\times\int(\psi_{\mathbf{k}}{}^{(-)}(\mathbf{r}),H'\psi_i(\mathbf{r}))d\mathbf{r}. \quad (6.11)$$

Employment of the part H_1' of H' and the assumption that the calculation of first-order effects suffices makes c^i time-independent and gives for the first-order effect on c^f the equation

$$\left(\frac{\hbar d}{idt}+\frac{\hbar^2k^2}{2\mu}\right)c_1{}^f(\mathbf{k})=\frac{he}{2iMc}c^i\exp[-i(\omega_i+\omega)t]$$

$$\times\int(\psi_{\mathbf{k}}{}^{(-)}(\mathbf{r}),[\mathbf{a}\cdot(\nabla_r-(\leftarrow)\nabla_r)]e^{i\boldsymbol{\kappa}\cdot\mathbf{r}/2}\psi_i(\mathbf{r})\rangle d\mathbf{r}. \quad (6.12)$$

On account of the transverse nature of the incident wave, the factor $\exp(i\boldsymbol{\kappa}\cdot\mathbf{r}/2)$ can be taken out of the ( ) in both (6.8) and (6.12). For $K=0$ these two equations are seen to be very similar. To make the correspondence more complete one may set

$$C^i(\mathbf{K})=c^i\delta(\mathbf{K}). \quad (6.13a)$$

The first line of Eq. (6.3) gives then $\Psi_i=c_i\psi_i\exp(-i\omega_it)$, which is the same initial state as in (6.10). Equation (6.8) shows that $C_1{}^f$ is of the form

$$C_1{}^f(\mathbf{K}+\boldsymbol{\kappa},\,\mathbf{k})=\mathfrak{C}_1{}^f(\mathbf{K}+\boldsymbol{\kappa},\,\mathbf{k})\delta(\mathbf{K}), \quad (6.13b)$$

which according to the second line of Eq. (6.3) gives

$$\Psi_{f1}(\mathbf{R},\mathbf{r})=\frac{e^{i\boldsymbol{\kappa}\cdot\mathbf{R}}}{(2\pi)^3}\int \mathcal{C}_1{}^f(\boldsymbol{\kappa},\mathbf{k})\psi_{\mathbf{k}}{}^{(-)}(\mathbf{r})d\mathbf{k} \quad (6.13c)$$

and according to (6.8)

$$\left[\frac{\hbar d}{idt}+\frac{\hbar^2\kappa^2}{4M}+\frac{\hbar^2k^2}{2\mu}\right]\mathcal{C}_1{}^f(\boldsymbol{\kappa},\mathbf{k})=\frac{he}{2iMc}c^i\exp[-i(\omega_i+\omega)t]$$

$$\times\int\{\psi_{\mathbf{k}}{}^{(-)}(\mathbf{r}),[\mathbf{a}\cdot(\nabla_r-(\leftarrow)\nabla_r)]e^{i\boldsymbol{\kappa}\cdot\mathbf{r}/2}\psi_i(\mathbf{r})\}d\mathbf{r}. \quad (6.13d)$$

This equation is similar to Eq. (6.12), but differs from it by the presence of the term $\hbar^2\kappa^2/4M$ in the factor multiplying $\mathcal{C}_1{}^f(\boldsymbol{\kappa},\mathbf{k})$. If it were not for that term, $\mathcal{C}_1{}^f(\boldsymbol{\kappa},\mathbf{k})$ would be equal to $c_1{}^f$, the right-hand sides of (6.12) and (6.13d) as well as the initial conditions for $\mathcal{C}_1{}^f$ and $c_1{}^f$ being the same. The quantity $\hbar^2\kappa^2/4M$ is the kinetic energy of a particle with mass $2M$, which is the sum of the proton and neutron masses, and momentum $\hbar\boldsymbol{\kappa}$, which is the momentum of the absorbed photon. A consideration of the well-known way in which the energy $\hbar^2k^2/2\mu$ becomes selected if the electromagnetic wave acts for a long time shows that for (6.12)

$$\hbar^2k^2/2\mu=\hbar\omega+\hbar\omega_i \quad (6.12')$$

and for (6.13d)

$$\hbar^2k^2/2\mu+\hbar^2\kappa^2/4M=\hbar\omega+\hbar\omega_i. \quad (6.13e)$$

In addition to this difference between the FCOMP and the more realistic consideration, the factor $\exp(i\boldsymbol{\kappa}\cdot\mathbf{R})$ in (6.13c) modifies the final-state contribution to ψ, represented by the second term in (6.10). This modification corresponds to the transfer of the photon momentum to the center of mass of the p-n system.

The interaction energy associated with the action of the magnetic field on the nucleon magnetic moments gives similar results regarding the role of the recoil of the center of mass. Setting

$$\mathfrak{h}=i[\boldsymbol{\kappa}\times\mathbf{a}], \quad (6.14)$$

this part of H' for the absorption problem may be taken to be

$$H_2'=-(e\hbar/2Mc)\{\mu_p(\mathfrak{h}\cdot\boldsymbol{\sigma}_p)\exp[i(\boldsymbol{\kappa}\cdot\mathbf{r}_p-\omega t)]$$
$$+\mu_n(\mathfrak{h}\cdot\boldsymbol{\sigma}_n)\exp[i(\boldsymbol{\kappa}\cdot\mathbf{r}_n-\omega t)]\}, \quad (6.15)$$

where the magnetic moments of the proton and neutron in units of the nuclear Bohr magneton are μ_p and μ_n, respectively, and the corresponding Pauli spin operators are $\boldsymbol{\sigma}_p$ and $\boldsymbol{\sigma}_n$. From Eq. (6.5) it follows that

$$\left[\frac{\hbar d}{idt}+\frac{\hbar^2(\mathbf{K}+\boldsymbol{\kappa})^2}{4M}+\frac{\hbar^2k^2}{2\mu}\right]C_2{}^f(\mathbf{K}+\boldsymbol{\kappa},\mathbf{k})$$
$$=(e\hbar/2Mc)C^i(\mathbf{K})\exp[-i(\omega_i+\omega)t]\left\{\mathfrak{h}\cdot\int\{\psi_{\mathbf{k}}{}^{(-)*}(\mathbf{r}),\right.$$
$$\left.\times[\mu_p\boldsymbol{\sigma}_pe^{i\boldsymbol{\kappa}\cdot\mathbf{r}/2}+\mu_n\boldsymbol{\sigma}_ne^{-i\boldsymbol{\kappa}\cdot\mathbf{r}/2}]\psi_i(r)\}d\mathbf{r}\right\} \quad (6.16)$$

and from (6.11)

$$\left(\frac{\hbar d}{idt}+\frac{\hbar^2k^2}{2\mu}\right)c_2{}^f(\mathbf{k})=\frac{e\hbar}{2Mc}c^i\exp[-i(\omega_i+\omega)t]\{\ \}, \quad (6.17)$$

the expression inside the curly braces being identical in the two cases. Comparing the relationship of (6.17) to (6.16) with that of (6.12) to (6.8), it is seen that if $\mathcal{C}_2{}^f(\mathbf{K}+\boldsymbol{\kappa},\mathbf{k})$ is introduced by changing subscripts 1 to subscripts 2 on both sides of (6.13b), then the relationship of $\mathcal{C}_2{}^f(\boldsymbol{\kappa},\mathbf{k})$ to $c_2{}^f(\mathbf{k})$ is the same as that of $\mathcal{C}_1{}^f(\boldsymbol{\kappa},\mathbf{k})$ to $c_1{}^f(\mathbf{k})$. Thus for H_2', as was already discussed for H_1', the modification regarding energy given by Eq. (6.13e) as compared with Eq. (6.12'), and that regarding total momentum caused by the factor $\exp(i\boldsymbol{\kappa}\cdot\mathbf{R})$ in (6.13c), describe the effect of considering the system in the impulse approximation as compared with the FCOMP.

The two special cases corresponding to H_1' and H_2' show the typical forms that enter the comparison of $\mathcal{C}^f(\mathbf{K}+\boldsymbol{\kappa},\mathbf{k})$ with $c^f(\mathbf{k})$. In the case of H_1' the presence of $\mathbf{r}_p$ in $\mathbf{R}$ caused the appearance of the term containing $(\boldsymbol{\alpha}\cdot\mathbf{K})$ as a factor inside the curly braces in Eq. (6.8). The reason for its appearance is that the Schrödinger current operator contains the differential operator ∇_p. The extra term turns out to be small. Aside from it, the operations contained in H'; are concerned with the presence of $\mathbf{r}_p$ and $\mathbf{r}_n$ in $\mathbf{r}$ rather than $\mathbf{R}$. For this reason the parts of the right-hand sides of both (6.8) and (6.16) are the same as the corresponding ones of (6.12) and (6.17), respectively. These features of the relationship to the FCOMP of the result of eliminating the c.m. do not depend on the details of the form of the interaction energy, since they simply amount to neglecting in the answer terms arising from $[H',e^{i\mathbf{K}\cdot\mathbf{R}}]$.

The change from (6.12') to (6.13e) accompanied by the inclusion of the factor $\exp(i\boldsymbol{\kappa}\cdot\mathbf{R})$ in (6.13c) is equivalent to a Galilean transformation of the final-state wave function. Stated more precisely, the relationships are as follows. For the same value of $\mathbf{k}$, the two relationships mean

$$\Psi_f=(\psi^f)_{\text{FCOMP}}\exp\left\{\frac{i}{\hbar}\left[\hbar\boldsymbol{\kappa}\cdot\mathbf{R}-\frac{(\hbar\kappa)^2}{4M}t\right]\right\}. \quad (6.18)$$

The Galilean transformation connecting the wave functions ψ and ψ' describing two particles in the reference systems K and K', respectively, is

$$\psi(\mathbf{r}_1,\mathbf{r}_2,t)=\psi'(\mathbf{r}_1',\mathbf{r}_2',t')$$
$$\times\exp\{(i/\hbar)(\bar{M}v_0X-\tfrac{1}{2}\bar{M}v_0{}^2t)\}, \quad (6.19)$$

where the coordinates in the two systems are related by

$$x=x'+v_0t',\quad y=y',\quad z=z',\quad t=t', \quad (6.19')$$

and where

$$\bar{M}=M_1+M_2,\quad X=(M_1x_1+M_2x_2)/\bar{M} \quad (6.19'')$$

are the total mass and the x coordinate of the COM.

Comparing with Eq. (6.18), agreement is obtained if Ψ_f is identified with ψ, $(\psi^f)_{\text{FCOMP}}$ with ψ', $\hbar\kappa$ with $(Mv_0,0,0)$, and $2M$ with $\bar{M}$. Since the coordinate transformation (6.19′) means that system K' has the velocity $(v_0,0,0)$ with respect to K, the above identification means that $(\psi^f)_{\text{FCOMP}}$ is the result of looking, in the system moving with the recoiling center of mass K', at the state that is described by Ψ_f in the laboratory system K. A property of the Galilean transformation is that if $V(\mathbf{r}_1,\mathbf{r}_2,t)$ is the potential energy describing the system in K, and if $V'(\mathbf{r}_1',\mathbf{r}_2',t')$ is the potential energy describing it in K', then

$$V(\mathbf{r}_1,\mathbf{r}_2,t)=V'(\mathbf{r}_1',\mathbf{r}_2',t'). \qquad (6.19''')$$

This property is in agreement with the second line of Eq. (6.3), since the $\psi_{\mathbf{k}}^{(-)}(\mathbf{r})$ correspond to wave functions in different systems, and since they are taken to be independent of the momentum $\hbar\kappa$ of the c.m. as viewed from the laboratory system K. It is nevertheless desirable to remember that (6.19‴) is correct only in a nonrelativistic model. The vector fields which are supposed to be partly responsible for nucleon-nucleon interactions transform relativistically in such a way that the effective potential replacing the V of Eq. (6.19‴) is not a function of $\mathbf{r}_1-\mathbf{r}_2$ only. This complication can be taken care of by replacing the Galilean with a Lorentz transformation. The replacement of the steps from Eq. (6.3) to Eq. (6.5) or Eq. (6.16) involves the employment of relativistic dynamics of the *p-n* problem and is not attempted in the present paper.

The possibility of describing the nonrelativistic calculation of Ψ_f by a Galilean transformation does not mean that the calculation could have been made more simply in the system K'. In that system the deuteron is not at rest initially, and the photon wave has a frequency different from that in K. On account of the initial motion of the deuteron, it is not possible to apply the FCOMP in K' directly, since in that procedure the initial and final *p-n* functions are treated as though they were referred to the same coordinate system. A partial explanation of the agreement of FCOMP with the calculations made on the basis of (6.3) is obtained from the consideration of the Doppler effect. On account of it, the apparent value of ω in K' is

$$\omega'=\omega[1-\hbar\kappa/2Mc]=\omega-\hbar\kappa^2/2M. \qquad (6.20)$$

Combining this with (6.13e) gives

$$\hbar\omega'=-\frac{\hbar^2\kappa^2}{4M}+\left(\frac{\hbar^2k^2}{2\mu}-\hbar\omega_i\right), \qquad (6.20')$$

which on combining with (6.13) again gives

$$\tfrac{1}{2}(\hbar\omega+\hbar\omega')=\hbar^2k^2/2\mu-\hbar\omega_i. \qquad (6.20'')$$

The right-hand side of the last equation is the photon energy corresponding to the value of k on the FCOMP. This energy is thus the arithmetic mean of the energies in the laboratory system K and in the recoiling c.m. system K'. Introducing a system K'' in which the systems K and K' appear to move with equal and opposite velocities, the apparent photon energy $\hbar\omega''=\hbar\omega-\hbar^2\kappa^2/4M$. In K'' the energy of the center of mass does not change during the collision. Therefore

$$\hbar\omega-\hbar^2\kappa^2/4M=\hbar\omega''=\hbar^2k^2/2\mu-\hbar\omega_i, \qquad (6.20''')$$

which reproduces (6.13e). The reference system K'' thus corresponds to that used in the FCOMP. It will be noted that in K'' the centers of mass of the ground and excited states move with respect to each other in opposite directions. This interrelationship of the initial and final states is not a part of the FCOMP. The justification of the latter cannot be made therefore by simple identification of the fixed c.m. system with K'', but requires an additional dynamical consideration. This conclusion is in agreement with the fact that in order to obtain agreement with the FCOMP, it was necessary to discard the $(\mathbf{a}\cdot\mathbf{K})$ term in (6.8). This was done, not as a matter of principle, but because estimates indicate that in applications this term has relatively little effect.

The validity of the impulse approximation was presupposed in this section without detailed justification. In the final state the energies of the proton and neutron are usually very high compared to molecular binding. There is little doubt, therefore, regarding the propriety of neglecting the molecular effects on these states. In the initial state, however, the deuteron is bound to the molecule of which it is a part. In this respect there might perhaps be more doubt regarding the applicability of the impulse approximation. But although the picture implied by the impulse approximation does not hold for the initial state, the effect of this violation of assumptions on the equations derived above is small. In fact, the Fourier integral in the first line of Eq. (6.3) is still adequate, provided the time-dependent factor $\exp(-i\hbar K^2t/4M)$ in $C^i(K)$ is replaced by $\exp(-i\epsilon t/\hbar)$, where ϵ is the energy of the deuteron in the field of the molecule. The original time factor disappeared in the calculation after Eq. (6.8) because K was set equal zero, the distribution of energies $\hbar^2K^2/4M$ in a wave packet of molecular dimensions being confined to small values of the general order of electron volts. The energy ϵ is negative, rather than positive like $\hbar^2K^2/4M$, but they are of the same order of magnitude. The presence of ϵ therefore affects Eq. (6.13e) to a negligible extent. At very low energies the final state is affected by the interaction of the proton with the charged particles surrounding the deuteron, but these effects can hardly affect the $\psi_{\mathbf{k}}(\mathbf{r})$ at the deuteron or modify the factor $\exp(i\mathbf{K}\cdot\mathbf{R})$ in the second line of Eq. (6.3) appreciably, since in collisions on an atomic scale the proton interaction energies are of the order of 10 eV, and these interactions affect the energies at the deuteron only indirectly. For proton energies above 100 keV these effects may be expected to be negligible.

When deuteron photodisintegration data are obtained employing the bremsstrahlung spectrum, the γ-ray energy cannot be measured directly, but has to be inferred for each disintegration process from the measured energy of a disintegration product—usually the proton. Graphs and tables for this purpose employing relativistic kinematics have been provided by Wiener,[12] and such calculations were made previously on a nonrelativistic basis by Scharff-Goldhaber.[13] Conversion of cross sections from the laboratory to the center-of-mass system is also provided in these references. The center-of-mass system under consideration is the system in which the total momentum is zero, i.e., the center-of-mass system of the disintegration products. Wiener's opinion that most theoretical calculations are made in the center-of-mass system, and that therefore experimental results should be presented in that system in order to be theoretically meaningful, is seen not to be justified if calculations made by the FCOMP (which constituted the majority at the time of his paper) are to be included. In fact, the system K' used above is the nonrelativistic c.m. system. The nonrelativistically calculated photon frequency in it is $1-\hbar\kappa/2Mc$ times the photon frequency of the laboratory system, in contrast with the photon energy $\hbar\omega-\hbar^2\kappa^2/4M$ which must be used in the FCOMP in order to give the correct value of k. This energy corresponds to a frequency equal to $1-\hbar\kappa/4Mc$ times the laboratory frequency rather than $1-\hbar\kappa/2Mc$ times that frequency. However, it is convenient to think of wave functions as associated with K' because they have to do only with the relative n-p motion. It is furthermore clear that the angular distribution in one frame of reference can be used to calculate the angular distribution in another frame. There is therefore no question of one or another frame providing more meaningful results.

Since the justification for the FCOMP given in this section is nonrelativistic, and since there is no available relativistic justification, it may be suspected that relativistic calculations will provide a definite improvement over older calculations[13] (except in a literally interpreted kinematical problem) whenever the FCOMP procedure remains in error even after the recoil corrections are made. For this reason it is desirable to examine the relationship of the nonrelativistic[13] treatment to the relativistic[12] one.

Neglecting the proton-neutron mass difference and denoting each of these masses by M, the relativistic expression for the sum of the kinetic energies of the disintegration products available in the center-of-mass system (the system of zero total momentum) is readily shown to be

$$2(E_p'-Mc^2)=\frac{\hbar\omega}{(1+2\gamma)^{1/2}}-|\epsilon_d|+M_dc^2\left[\frac{1+\gamma}{(1+2\gamma)^{1/2}}-1\right]$$
$$=\hbar\omega-|\epsilon_d|-\frac{(\hbar\omega)^2}{2M_dc^2}+\frac{\hbar\omega}{2}\gamma^2+\cdots, \quad (6.21)$$

where

$$\gamma=\hbar\omega/M_dc^2=\beta/(1-\beta), \quad \beta=v/c, \quad (6.22)$$

with v denoting the transformation velocity, and with the deuteron mass expressed as

$$M_d=2M-|\epsilon_d|. \quad (6.23)$$

The first three terms in the second form for $2(E_p'-Mc^2)$ are essentially the nonrelativistic ones occurring in (6.13e), but with $|\epsilon_d|=-\hbar\omega_i$, and with $4M$ replaced by $2M_d$. Since $(M_d/2M)-1\approx-0.001$, the latter replacement usually matters little. Otherwise, the first nonvanishing relativistic correction $\hbar\omega\gamma^2/2$ is about 0.14 and 0.7 MeV at $\hbar\omega=100$ and 170 MeV, respectively. The kinetic energy $2(E_p'-Mc^2)$ is the important quantity for the calculation of the relativistic reduced wave number k. In this respect the nonrelativistic calculations appear to be good enough kinematically.

The relation between the angles made by the proton with the incident photon direction in the two systems is

$$\tan\theta=\frac{(1-\beta^2)^{1/2}\sin\theta'}{\cos\theta'+(\beta E'/cp')}, \quad (6.24)$$

all quantities being meant for the proton. The quantity occurring in the denominator is explicitly calculable from $\hbar\omega$ as

$$\frac{\beta E'}{cp'}=\frac{[\gamma(1+2\gamma)]^{1/2}}{2^{1/2}(1+\gamma)\{1+[1-(2M/M_d)^2]/(2\gamma)\}^{1/2}}. \quad (6.24')$$

Expanding in powers of γ and neglecting in numerator and denominator terms $O(\gamma^3)$,

$$\tan\theta=\frac{(1-\gamma^2/2+\cdots)\sin\theta'}{\cos\theta'+(\frac{1}{2}\gamma)^{1/2}\{1-\frac{1}{2}\gamma^2+[(2M)+M_d]|\epsilon_d|/(4\gamma M_d^2)+\cdots\}}. \quad (6.25)$$

The term containing $|\epsilon_d|$ explicitly in the denominator is usually small, because

$$|\epsilon_d|[(2M)+M_d]/2M_d^2\approx 0.001.$$

Excluding this small effect and the $(-\frac{1}{2}\gamma^2)$ correction terms in the coefficient of $\sin\theta'$ in the numerator and in that of the coefficient of $(\frac{1}{2}\gamma)^{1/2}$ in the denominator, there results the classical-mechanics relation between θ and θ', except for the factor $(2M/M_d)^{1/2}$ in the term added to $\cos\theta'$ in the denominator. This effect is usually very small. The relativistic effects on the angles are

[12] Martin Wiener, Natl. Bur. Std. (U. S.) Circ. 5/5 (1951).
[13] G. Scharff-Goldhaber, Brookhaven National Laboratory Report No. I-3 August 1, 1948 (unpublished).

therefore mainly those caused by the change from 1 to $1-\frac{1}{2}\gamma^2$, and are of the same relative order of magnitude as the effects in the energy transformation. Thus at incident γ-ray energies up to about 200 MeV, neither the transformation of kinetic energy nor that of angles indicates, through failure of nonrelativistic kinematics, that serious errors in the dynamics are introduced by the nonrelativistic dynamics of the discussion from Eq. (6.3) on to Eq. (6.20). On the other hand, the absence of major kinematical effects cannot be regarded as a proof of the absence of nonnegligible dynamical effects.

The recoil corrections obtained in this section consist in: (a) calculating the difference between the final and initial energies of relative motion as

$$E_f-E_i=\hbar^2k^2/2\mu-\hbar\omega_i=\hbar\omega-\hbar^2\kappa^2/4M\,, \quad (6.26)$$

and (b) leaving ω at its value in the laboratory system in all parts of the work not concerned with final-state wave functions. The two interaction energies H_1' and H_2' of (6.7) and (6.15) have in common the occurrence of the combinations $\exp[i(\boldsymbol{\kappa}\mathbf{r}_p-\omega t)]$ or $\exp[i(\boldsymbol{\kappa}\mathbf{r}_n-\omega t)]$ in every term. The proof, as carried out, depends on this feature. Part (a), i.e., Eq. (6.26), is of course a consequence of the conservation of energy and momentum. The simplicity of part (b) may perhaps be destroyed by the introduction of exchange-current modifications. If, for instance, such a modification is made in the $\mathbf{A}_{El_s}$ but not in the Φ_{El_s} part of H_{El_s}' in Eq. (5.5) ,then the sum of the contributions to the total perturbation energy H' from all l for $s=1$ is

$$\Phi_E(\mathbf{r})=\sum_{l=1}^{\infty}\Phi_{El,s}=\frac{e^{i\varphi}}{2i\kappa r\sin\theta}\times[ze^{i\kappa z}-re^{i\kappa r}+(r-z)\cos(\kappa r)]$$
$$=-(x+iy)/\sqrt{2}+\cdots. \quad (6.27)$$

The last of the above forms of Φ_E is the result of expanding the form listed just before it in powers of r, using r and θ as variables. In that limit the nonretarded approximation is thus reproduced. But in general there occur to both $\exp(i\kappa r)$ and $\cos(\kappa r)$ which are not immediately related to $\exp(\pm i\kappa z)$. It is not claimed therefore that feature (b) of the recoil correction recipe is valid for any interaction energy. A violation of feature (b) is furthermore expected on a composite nucleon model, because the acceleration of the deuteron associated with its recoil implies accelerations of the proton and neutron additional to those taking place on account of the force acting between them. The associated temporary change of the relationship between the location of a nucleon and that of its charge-current system may also interfere with the validity of feature (b). Since, however, the recoil effect on the cross section is of the order of only about 10% at a γ-ray energy of 177 MeV and is much smaller at lower energies, and for polarization, it appears good enough to use the simple result for the intermediate range of energies.

VII. DISCUSSION

The considerations presented above have been devised partly for applications to calculations with nucleon-nucleon potentials. As seen in the discussion following Eq. (5.6), there is no complete certainty regarding the accuracy of the final results, even in the case of the simplest terms originating in the charge density. It is nevertheless hoped that by classifying the contributions into more and less certain ones, comparison with experimental data—including those on nuclear structure—will provide sufficient information regarding the relationship between $e\psi^*\psi$ and $\rho^*(\mathbf{r})$ to improve the meaningfulness of present analyses of the data.

The representation of nucleon-nucleon interactions by means of static potentials does not pretend to give a fundamentally meaningful description. The actual situation cannot be represented in its entirety by a mathematical two-body problem. There have, however, been partial successes of such simplified replacements, both in the theory of nuclear matter and in that of effective interactions in finite nuclei. The static potentials are apparently capable of furnishing some of the off-energy-shell matrix elements approximately correctly. The uncertainty regarding the correctness of off-energy-shell matrix elements enters photodisintegration calculations as well, but in this case there are also questions regarding nucleon electromagnetic factors and their possible dependence on the interaction of a nucleon with other nucleons, and related questions regarding the influence of exchange currents. Since most of the relations of Secs. II, III, IV, and V refer to the connection between the electromagnetic field and electromagnetic charge and current densities, the difficulties just mentioned enter mainly in establishing the connection between these quantities and the nucleon dynamics. However, definite forms of the Schrödinger current $\mathbf{J}_S$ and of $\mathbf{M}$ of Eq. (2.4) imply approximations which may not be valid, quite apart from the difficulties arising from insufficient knowledge concerning the electromagnetic structure of the nucleon.

One of these approximations arises from the relativistic effect of the appearance of electric dipole moments when a magnetic moment executes a uniform motion of translation. The hard-core potentials in use for nucleon-nucleon scattering are large in absolute value and attractive in some states. It is questionable, therefore, whether these motional effects are negligible. The Pauli part μ_P of the nucleon magnetic moment in interaction with a static electric field gives rise[14] to an interaction energy

$$H_P'\cong(\mu_P/2Mc)\times\{-\hbar\,\mathrm{div}\boldsymbol{\mathcal{E}}+[\mathbf{p}\times\boldsymbol{\mathcal{E}}]\cdot\boldsymbol{\sigma}-[\boldsymbol{\mathcal{E}}\times\mathbf{p}]\cdot\boldsymbol{\sigma}\}\,, \quad (7.1)$$

[14] G. Breit, Proc. Natl. Acad. Sci. U. S. 37, 837 (1951).

which may also be written

$$H_P' = (\hbar\mu_P/2Mc) \times\{-\mathrm{div}\mathcal{E} - i\boldsymbol{\sigma}\cdot\mathrm{curl}\mathcal{E} - 2i[\mathcal{E}\times\boldsymbol{\sigma}]\cdot\nabla\}. \quad (7.1')$$

If this result is applied to the case of incident electromagnetic wave, the term containing curl$\mathcal{E}$ can be expressed in terms of the magnetic field $\mathfrak{H}$ of the wave and gives a correction of the order of $(\hbar\omega/2Mc^2)\mu_P$ to the effective value of the magnetic moment of the particle. The term involving the ∇ in Eq. (7.1′) produces roughly similar effects, but involves the space properties of the wave functions differently. For $\hbar\omega = 170$ MeV the fraction $\hbar\omega/2Mc^2 \approx 1/11$. The effects are not definitely negligible, therefore.

The employment of a nonrelativistic Schrödinger equation with a hard-core potential, such as has often been made for the $d(\gamma,n)p$ reaction, is also subject to criticism because of the errors which it may introduce in the calculation of the Schrödinger current, as well as because of the questionable meaning of the charge density derived from it. At the surface of the hard core, the Yale and the Hamada-Johnston potentials give for singlet even states attractive interaction energies of the order of 1 BeV. The nonrelativistic Schrödinger equation is not a good approximation under such conditions. On the other hand, the absolute value of the potential drops very rapidly with distance when it is so large, the decrease being by a factor $\frac{1}{2}$ in a distance of roughly $\hbar/2m_\pi c$. Since the wave function vanishes at the core, this particular difficulty is not as large as the potential depth at the core might suggest.

Approaches to the $d(\gamma,n)p$ problem minimizing the role of effective potentials have been used. Without attempting an exhaustive survey, a few characteristic contributions may be mentioned. Donnachie and O'Donnell[15] employ a deuteron ground-state wave function of the Hulthén-Sugawara type. Their final-state wave functions are solutions of the field-free equation from a certain small radius on, the linear combination of the regular and irregular functions being made to correspond to the phenomenological phase shifts. Inside that distance the regular radial functions for the field-free equation are used. Agreement with experiment up to 120 MeV is satisfactory, especially with 6% D-state probability and with inclusion of retardation effects. Although it is instructive to know that a reasonably successful representation of the data can be obtained by obtained by this procedure, there remains the question of what happens to the agreement if one uses wave functions that satisfy a Schrödinger equation capable of representing the phenomenological phase shifts. Employment of free-field wave functions with phase shifts amounts to assuming that the interaction is confined to very short distances, a view for which there is little support.

Lomon and Feshbach[16] use potentials of a semitheoretical type outside the energy-independent boundary condition radius, and calculate the matrix elements employing the same region. Their success in giving a decidedly better than qualitative representation of nucleon-nucleon phase shifts employing the same model speaks in favor of the approach. However, the energy independence of the boundary condition has been justified only qualitatively, and the role of the region inside the boundary-condition radius needs further elucidation.

Le Bellac, Renard, and Tran Thanh Van[17] give a relativistic dispersion-theoretical treatment and obtain agreement with experiment of about the same quality as that with nucleon-nucleon-potential treatments. Without claiming decided superiority for their approach, the authors point out that it has the advantage of being relativistic to begin with, so that relativistic corrections need not be made. They also emphasize that their theory makes use of the phase shifts directly, and does not depend on the more or less arbitrary choice of a nucleon-nucleon potential. There is no doubt regarding the power of the dispersion-theoretical approach. It may be remarked, however, that the simpler problem of the two-nucleon interaction has not been solved in a quantitative sense so far. It may be argued that the employment of phenomenological phase shifts brings in the effect of the nucleon-nucleon interaction to a sufficient degree to make its further examination unnecessary. In other words, it is conceivable that if a detailed theory based on the consideration of charge and current distributions were carried through, the formulas for the photodisintegration amplitudes would reduce to expressions containing only phase shifts (speaking more precisely, phase parameters) and some properties of the ground state. But it would be surprising if the description of the deuteron ground state by means of a wave function referring to two nucleons were sufficient, especially if the wave function does not originate in a relativistic formulation but rather in a procedure involving an adjustable cutoff. This starting point does not appear to be any more fundamental than that of a theory employing static local potentials. A cancellation of final-state interaction effects, as *might* occur if the properties of the ground state were fully used, would not be destroyed by the simplified treatment of the ground state. The fit to the $d(\gamma,n)p$ data is obtained by adjusting the two free parameters of the four-pole wave function. There is no other control on these parameters in the work cited. In this respect the employ-

[15] A. Donnachie and P. J. O'Donnell, Nucl. Phys. **53**, 128 (1964).

[16] H. Feshbach and E. Lomon, Phys. Rev. **102**, 891 (1956); H. Feshbach, E. Lomon, and A. Tubis, Phys. Rev. Letters **6**, 635 (1961). Cf. also G. Breit and W. G. Bouricius, Phys. Rev. **74**, 1546 (1948); **75**, 1029 (1949); a comprehensive report, including mention of agreement with $d(\gamma,n)p$, is to be found in an invited paper read in March, 1967 at the International Conference on Nucleon-Nucleon Interaction, Gainesville, Florida. E. Lomow and H. Feshbach, Rev. Mod. Phys. **39**, 611 (1967).

[17] M. Le Bellac, F. M. Renard, and J. Tran Thanh-Van, Nuovo Cimento **33**, 594 (1964); **34**, 450 (1964).

ment of potentials is less arbitrary, since any adjustment usually produces an effect on scattering. The usual procedure is to fit the latter first. In such cases no adjustment to $d(\gamma,n)p$ data takes place at all. It is conceivable, however, that some combinations of changes in potential parameters do not affect scattering, but do change the $d(\gamma,n)p$ observables. It is believed by Bellac *et al.* that since they are working with a gauge-invariant theory, they are taking exchange currents into account. As has been shown by Osborn and Foldy,[18] however, securing gauge invariance in a theory does not determine the exchange currents uniquely, so that from a purely phenomenological viewpoint there is no assurance that the exchange currents used in Ref. 17 are the actual ones. Furthermore, the consideration of exchange currents by means of gauge invariance, taken literally, presupposes that the particles are strictly points. The error caused by spreading the particle to take into account its structure is thus not part of the usual consideration. The electromagnetic size of the nucleon is, on the other hand, comparable with (though smaller than) the p-n separation distance in the deuteron. It appears, therefore, that the exchange currents will have to be determined either from the phenomenology of $d(\gamma,n)p$ and possibly of p-p bremstrahlung, or else from the theory of nucleon-nucleon scattering. As the authors point out, there are disagreements in their fit to low-energy $d(\gamma,n)p$ data. It appears therefore that even though the dispersion-theoretical approach may eventually furnish a much superior treatment of the deuteron photodisintegration problem, it has not reached that stage so far.

Employing a dispersion-theoretical consideration, Skolnick[19] claims to have removed most of the discrepancy between the expected and observed values of the $p(n,\gamma)d$ capture cross section at a neutron laboratory velocity of 2200 m/sec—the "interaction effect" of Austern and Rost[20] which at the time of publication of the last reference became apparent in the comparison of a calculated value of 0.303 ± 0.012 b with the experimental value of 0.3315 ± 0.0017 b. A reconsideration of this problem by Noyes[21] is in sharp disagreement with Skolnick's conclusions, and reduces the correction to a much smaller value, on the grounds that Skolnick's assumption regarding the shape-independent approximation was made without sufficient justification. Here again the S-matrix approach has not eliminated the desirability of calculations made with models employing a space-time description.

Such models, as well as the larger part of the present paper, are not limited to the employment of potentials. These provide however a possible starting point for a more realistic attack. Since exchange currents, relativistic effects, nonlocality of potentials, electromagnetic nucleon structure, and related complications affect contributions from various parts of expressions occurring in Eq. (5.11) differently, it is desirable to arrange the calculations in such a way as to be able to study the effect of their contributions separately. A way of doing so is to deal directly with the amplitudes corresponding to different relative spin orientations of the final states,[22] and in the more complicated cases to obtain from these the values of the observables. Other essentially equivalent arrangements of the calculations have been used by various authors, notably Bethe and Longmire, Austern, and Nicholson and Brown. References to the papers by these and many other authors may be found in Ref. 9. Potentials similar to those in Ref. 22 have been used by de Swart and Marshak[23] and more recently by Partovi.[24] Extensive use is made of Wigner's 3-j, 6-j, and 9-j symbols by the latter author. If, however, the amplitudes are left in their original form, as in the papers listed in Ref. 22, the calculation of any observable is quite simple, especially if a highspeed digital computing machine is available. An example of such an application is the calculation of the photodisintegration of polarized and aligned deuterons by Zickendraht, Andrews, and Rustgi.[25] The direct employment of amplitudes makes it easy to take into account all effects of additional multipole or final states without calculating many cross-product terms. Their occasional omission[24] is unnecessary in the amplitude method. Since matrix elements corresponding to various final states and multipoles may be affected differently by the pions and vector mesons responsible for the p-n interactions, a beginning of a systematic study intended to provide an insight into the relative importance of such contributions has been made. The results will be described in three companion papers to the present one.

In the paper by Zickendraht, Rustgi, and Brandt[26] the calculation of amplitudes is arranged in a form convenient for use with an electronic digital computer. The method employed makes use of the representation coefficients of the rotation group, having this in common with the work of Partovi,[24] but has been developed and used before the appearance and without the knowledge of Partovi's papers. The employment of the representation coefficients is convenient for a digital machine program. The formulas do not take retardation into account. The effects of the second term in Eq. (5.6) are not included, for reasons mentioned in the discussion following Eq. (5.10) above. To these it may be added that the simplest form of exchange currents expected

[18] R. K. Osborn and L. L. Foldy, Phys. Rev. **79**, 795 (1950).
[19] M. H. Skolnick, Phys. Rev. **136**, B1493 (1964).
[20] N. Austern and E. Rost, Phys. Rev. **117**, 1506 (1959).
[21] H. Pierre Noyes, Nucl. Phys. **74**, 508 (1965).
[22] W. Zernik, M. L. Rustgi, and G. Breit, Phys. Rev. **114**, 1358 (1959); W. Zickendraht, D. J. Andrews, M. L. Rustgi, W. Zernik, A. J. Torruella, and G. Breit, Phys. Rev. **124**, 1538 (1961). Cf. also Ref. 9.
[23] J. J. deSwart and R. E. Marshak, Phys. Rev. **111**, 272 (1958); Physica **25**, 1001 (1959).
[24] F. Partovi, Ann. Phys. (N. Y.) **27**, 79 (1964); **27**, 114 (1964).
[25] W. Zickendraht, D. J. Andrews, and M. L. Rustgi, Phys. Rev. Letters **7**, 252 (1961).
[26] W. Zickendraht, M. L. Rustgi, R. G. Brandt, and G. Breit, Bull. Am. Phys. Soc. **10**, 447 (1965).

from considerations of gauge invariance[27] involves currents along the line joining the interacting nucleons. Since there is some uncertainty regarding these currents as a result of uncertainties in the exchange character of the p-n interaction, the numerical treatment of these terms may not be very reliable. Comparing orders of magnitude of the contributions of the Schrödinger-current part in the second term in square brackets of Eq. (5.6) with that of the term containing the charge density $\rho^*(\mathbf{r})$, and neglecting all but the lowest-order terms in κr, the order of magnitude of the ratio is

$$|(J_S \text{ part})/(\rho \text{ part})| \approx (\kappa r)^2/(l+1). \quad (7.2)$$

The same power of the parameter κr appears here as in bringing in effects of retardation for the ρ part by expanding $F_l(\kappa r)$. At $\hbar\omega=100$ MeV, $r=e^2/mc^2$, and $l=1$, the above ratio is about 1.1; but at $\hbar\omega=10$ MeV, it is about 0.011. Comparing similarly the nucleon-magnetic-moment part with the ρ part, and employing a gyromagnetic ratio 3 for the combined effect of μ_p and μ_n, one obtains for the ratio

$$|(\mu_p,\mu_n \text{ part})/(\rho \text{ part})| \approx 1.5\hbar\omega/(l+1)Mc^2. \quad (7.3)$$

For $\hbar\omega=100$ MeV this is $\approx 0.16/(l+1)$, and at 2.2 MeV it is $\approx 0.004/(l+1)$. Formally the nucleon-magnetic-moment effects are of a lower order than the corresponding Schrödinger current terms. The presence of the small length $\hbar/Mc$, however, makes the effects of the two kinds of terms on the integrand of (5.6) comparable, except for energies close to the reaction threshold, where the effects are quite small. In the paper by Botzian, Rustgi, and Torruella,[28] a different and in some respects simpler derivation of the same formulas is described, making use of operators similar to those in Eqs. (5.15), (5.15′), (5.16), (5.16′), (5.17), (5.20), and (5.20′). This derivation has been made after the one in Ref. 26.

The equations arrived at in the work referred to have been applied by Brandt, Zickendraht, Torruella, Schrils, and Breit[29] to the evaluation of the differential cross section and of the proton polarization, tracing the effect of including additional multipoles. The Yale and the Hamada-Johnston potentials were used in these calculations. A procedure taking into account changes in phenomenological fits to nucleon-nucleon scattering without refitting the potential will also be found in that paper. This way of dealing with the final-state interaction will be seen to be much less arbitrary than the prescription used in Ref. 15.

Calculations including effects of retardation and of terms omitted in Eq. (5.6) are expected to be prepared for publication in the near future, together with an examination of effects of successive additions of electromagnetic multipoles supplementary to that in Ref. 29, and an examination of fits to additional low-energy data.

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[27] R. G. Sachs, Phys. Rev. **74**, 433 (1948). It may be noted that in the first section of that paper the author states that one expects the exchange current to consist of two parts, one depending in some way (as in the paper by Villars) on the details of the field describing the interaction between nucleons.

[28] R. Botzian, M. L. Rustgi, and A. J. Torruella, Bull. Am. Phys. Soc. **10**, 448 (1965).

[29] R. G. Brandt, W. Zickendraht, A. J. Torruella, R. Schrils, and G. Breit, Bull. Am. Phys. Soc. **10**, 448 (1965). The contents of this reference as well as of Refs. 26 and 28 are being prepared for more complete publication in the near future. The present paper is an expanded form of that described in G. Breit and M. L. Rustgi, Bull. Am. Phys. Soc. **10**, 447 (1965).