We have already noted that different solutions containing n vortices will occur at different H_e depending on whether they are part of the $n-1$ to n vortex mode or the n to $n+1$ mode. The current density and magnetic field plots for the two cases will look different even though both solutions fit exactly n vortices into the junction. Figure 6(d) can be compared with Fig. 7(a), which also shows a one-vortex solution but at $\mathcal{H}=0.06$. This solution is the start of the 1 to 2 vortex mode.

Solutions with a nonintegral number of vortices in the junction are also possible. In fact, since a complete vortex does not contribute to the total current I, all solutions with $I>0$ are of this type. A solution in the 1 to 2 vortex mode is shown in Fig. 7(b). A solution in the 2 to 3 vortex mode is shown in Fig. 7(c). We can see that as more vortices fit into the junction they not only have a smaller dimension, but they become more sinusoidal in shape as well. The magnetic field $H(z)$ is smoothing out as H_e is increased, since the modulating action of the tunneling currents has less effect on larger applied fields.

IV. CONCLUSIONS

The emphasis in this work has been to obtain the details of the current-magnetic-field behavior of a superconducting tunnel junction. Such junctions exhibit the phenomena of vortex structure in a particularly simple and direct way. From a theoretical point of view, exact solutions are available. From an experimental point of view, the details of the vortex structure can be probed by simply measuring the current-field behavor.[7]

In this paper we have delt with the static properties of the vortex structure. A similar detailed analysis of the dynamics of the transition between different vortex modes would be of interest. It would provide a simple model for vortex creation phenomena. Experimentally, we expect that the dynamics of the creation process are reflected in the anomalous continous voltage-magnetic-field structure observed in tunnel junctions with large L/λ_J ratios.[8] The dynamics of the transition between different vortex modes could also be investigated.

[8] D. N. Langenberg, D. J. Scalapino, B. W. Taylor, and R. E. Eck, Phys. Letters **20**, 563 (1966).

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Meissner Effect and Vortex Penetration in Josephson Junctions*

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Investigation of the magnetic field dependence of the maximum zero-voltage current of wide, high-current Josephson junctions has revealed behavior drastically different from the usual Fraunhofer pattern for narrow junctions. The experiments are interpreted as evidence for a Meissner effect for the insulating layer and adjacent penetration layers of wide junctions in low external fields, and for an eventual transition to a mixed state as the external field is increased from zero.

I. INTRODUCTION

THE most striking experimental evidence for the existence of the Josephson effect is the magnetic field dependence of the maximum zero-voltage tunneling current of narrow, weakly-coupled Josephson junctions. In a narrow junction, the dimension L of the junction perpendicular to the field is much smaller than the Josephson penetration depth λ_J.[1] We have been investigating the field dependence of the maximum zero-voltage tunneling current of wide ($L\gg\lambda_J$) high current junctions, and have observed behavior different from the Fraunhofer[2,3] pattern obtained for narrow junctions. From the measurements, it is possible to infer that wide junctions exhibit a Meissner effect at low external fields and an eventual transition to a mixed state as the external field is increased. The experiments indirectly probe field and current distributions in the insulating layer and adjacent penetration layers of the junctions. Some of the features of the observed behavior, which is a consequence of the "self"-fields generated by the tunneling currents, have already been

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[1] The quantity $\lambda_J=[\hbar c^2/8\pi eJ_1d]^{1/2}$ is the Josephson penetration depth. In the expression $d=2\lambda_L+t$, J_1 is the maximum Josephson current density, c is the velocity of light, e is the electronic charge, λ_L is the London penetration depth, and t is the oxide layer thickness.

[2] J. M. Rowell, Phys. Rev. Letters **11**, 200 (1963).

[3] M. D. Fiske and I. Giaever, Proc. IEEE **52**, 1155 (1964).

discussed in theoretical treatments of the Josephson effect.[4–10]

The experimental critical-current–applied-field curves are in qualitative agreement with the numerical computations of the field dependence of the critical current of Owen and Scalapino.[10] In their calculations, the differential equation for the pair phase jump φ across the insulator is solved. This equation is given as

$$d^2\varphi/dz^2 = 1/\lambda_J^{-2} \sin\varphi. \tag{1}$$

Boundary conditions[11] appropriate to asymmetric finite junctions of width L are then used to compute field and current distributions. In small external fields, it is found that these distributions are restricted to the junction edges, a situation resembling the Meissner effect in bulk superconductors. In large fields, the current and field distributions correspond to a one-dimensional "Abrikosov array"[4] of quantized vortices, a result similar to the mixed state of type-II superconductors. This behavior is reflected in the theoretical critical-current–field relationship which is drastically different from the Fraunhofer pattern[2,3] obtained when $L \ll \lambda_J$.

II. EXPERIMENTAL

The techniques employed in measuring the variation of the critical Josephson current with external applied magnetic field are well known and will not be described here. Sample preparation will be described in that the production of high current junctions relatively free of metallic bridges may be related to the techniques employed. In the present work, glazed alumina substrates obtained from the American Lava Company were used in all experiments. Junctions were fabricated in the following way: Carefully outgassed, high-purity tin (Johnson-Mathey-530) was evaporated at a pressure of 5×10^{-7} Torr in an oil-free vacuum system (Ultek Corporation) to form a 2000 Å film. One atmosphere of dry oxygen was then admitted to the bell jar. The films were left in this oxygen atmosphere for 12 h at room temperature. The system was then evacuated and the upper layer was formed under the same conditions as the lower layer.

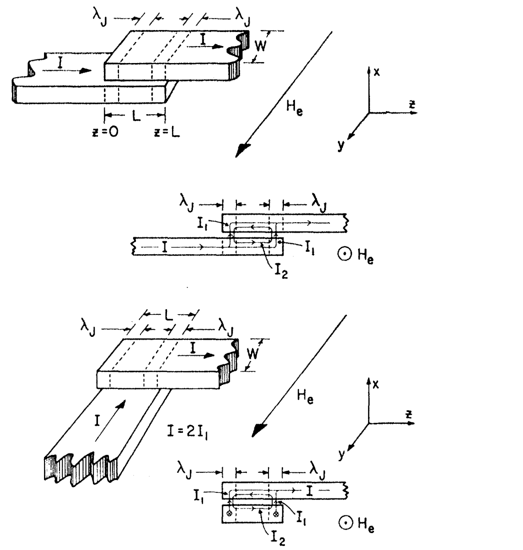

FIG. 1. Idealized drawings of the experimental geometries. The upper half of the figure depicts the symmetric geometry, the lower half the asymmetric one. I_2 is the screening current and $2I_1$ is the externally supplied current.

Idealized drawings of the symmetric and asymmetric geometries employed in the experiments are shown in Fig. 1. In all of the experiments a four-terminal technique was used to measure the current-voltage characteristic. The two voltage terminals are not shown in the drawing. All measurements were made with the sample positioned in a Mu-metal shield which reduced the ambient magnetic field to 10^{-3} G as measured with a Hewlett-Packard flux-gate magnetometer.

III. EXPERIMENTAL RESULTS

A. Symmetric Junctions

In the model of Scalapino and Owen,[10] a symmetric junction consists of two semi-infinite superconducting sheets which overlap for a distance L in the z direction, where they are separated by a thin oxide layer. Experimentally this is realized by making the y dimension W either large or small compared to λ_J[8]. For $W<\lambda_J$ and H_e, the external field parallel to y, spatial variations are not generated in the y direction. For the experiment reported here $L>\lambda_J$ and $W\sim\lambda_J$ so that only qualitative agreement with the theory is expected. In Fig. 2, we see that as the applied magnetic field increases from zero, the critical current decreases linearly for both signs of the field. This corresponds to the Meissner region in which the current is restricted to the ends of the junction, and may be understood in the following way[8]: Let I_1 be the part of the current per unit length flowing through each of the edges which is supplied by the external circuit. The total externally supplied current per unit length is then $2I_1$. In the presence of a small external field, an additional current I_2 circulates

[4] P. W. Anderson (to be published).
[5] B. D. Josephson, Rev. Mod. Phys. **36**, 216 (1964).
[6] B. D. Josephson, Advan. Phys. **14**, 419 (1965).
[7] P. G. DeGennes, *Superconductivity of Metals and Alloys* (W. A. Benjamin, Inc., New York, 1966), p. 240.
[8] I. O. Kulik, Zh. Eksperim. i Teor. Fiz. **51**, 1952 (1966) [English transl.: Soviet Phys.—JETP **24**, 1307 (1967)].
[9] R. A. Ferrell and R. E. Prange, Phys. Rev. Letters **10**, 479 (1963).
[10] C. S. Owen and D. J. Scalapino, preceding paper, Phys. Rev. **164**, 538 (1967).
[11] Yu M. Ivanchenko, A. V. Svidinskii, and V. A. Slyusarev, Zh. Eksperim. i Teor. Fiz. **51**, 494 (1966) [English transl.: Soviet Phys.—JETP **24**, 131 (1967)].

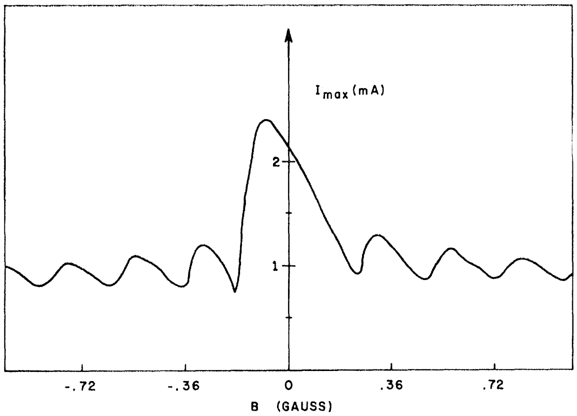

FIG. 2. Critical-current–applied-field curve for a sample that was approximately symmetric. $L=0.089$ cm, $W=0.028$ cm. Distortion is probably due to slight differences in self-inductances of upper and lower halves of the junction. $L=8.6\lambda_J$ by computation using $R_{NN}=0.1\ \Omega$ and $2\Delta=1.2$ mV. $T=1.2$°K.

around the junction screening the applied field from the interior. If Ampere's law is applied to this geometry we obtain the relationship between the screening current I_2 and the external applied field H_e:

$$I_2=(c/4\pi)H_e. \tag{2}$$

For $L\gg\lambda_J$, the critical current for the junction is reached when the sum of the currents/unit length in either edge equals $2\lambda_J J_1$.[9,10] Consequently,

$$I_1+I_2\leq 2\lambda_J J_1 \tag{3}$$

and

$$I_1-I_2\leq 2\lambda_J J_1. \tag{4}$$

are independent limiting conditions on the current through the edges. The equation which is applicable is determined by the sign of the externally applied field H_e. Combining Eq. (2) with Eqs. (3) and (4), respectively, the current I as a function of field is given by

$$I=2I_1\leq 4\lambda_J J_1-(c/2\pi)H_e \tag{5}$$

and

$$I=2I_1\leq 4\lambda_J J_1+(c/2\pi)H_e. \tag{6}$$

Equation (5) describes the decrease of current with increasing positive H_e, and Eq. (6) describes the linear decrease of the measured critical current with increasing negative H_e. The predictions of these equations are approximately realized in the experiment. A small difference in the self-inductances of the upper and lower layers of the junction probably accounts for the distortion of the experimental curve of Fig. 2. A smooth transition from this "type-I" or Meissner solution to the "type-II" solutions occurs before the critical current given by Eq. (5) falls to zero. For the mixed state, the theoretical analysis[10] predicts that for a given value of H_e there may be several different current distributions with different numbers of vortices, each capable of carrying a different critical current. The experimentally observed critical current always appears to be the highest critical current for a given value of H_e.[12]

B. Asymmetric Junctions

The asymmetric case (Fig. 3) has not been subjected to a complete analysis. However, the unusual shape of the critical-current–field curve in the Meissner regime[13] can be understood in a quantitative manner.[14] One again assumes that the current is restricted to the ends of the junction, an approximation which is quite good in this case as W, $L\gg\lambda_J$. The quantity $2I_1$ is again the externally supplied current, half of which flows through each of the two edges. The quantity I_2 is the circulating ring current which screens the external field and the field due to I_1 which flows across the top layer of the

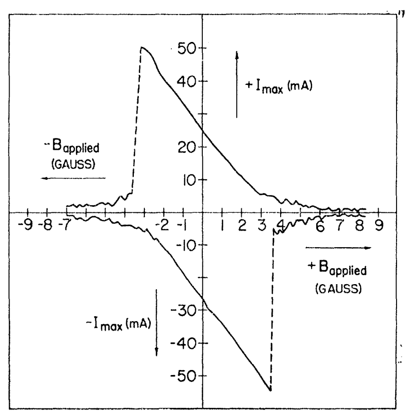

FIG. 3. Critical-current–applied-field curves for asymmetric geometry (Sn-Sn$_x$O$_y$-Sn) at 1.2°K, $W=0.010$ cm, $L=0.055$ cm. Both signs of current and field are shown. The linear portion of this curve did not show any hysteresis effects. The pattern persisted out to the critical field of tin. The critical current variation with field in the linear region is in reasonable agreement with Eqs. (5) and (6). It is easily seen from the equations that the product of the slope of critical-current–applied-field curve and the critical field should equal the critical current at zero field, a relation which is satisfied experimentally. The normal tunneling resistance $R_{NN}=4\times10^{-3}\ \Omega$. Taking $2\Delta=1.2$ mV, one can estimate $\lambda_J=1.25\times10^{-3}$ cm or $L=44.0\lambda_J$. The experimental value computed from Eq. (10) is 8.5×10^{-4} cm. It is quite probable that this junction was more strongly coupled at its edges than at its center.

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[12] This was determined by placing the junction in an externally applied magnetic field H_e and driving it beyond its critical current. The current was then reduced to zero and the external field set to a new value at which the previously measured critical current branch was no longer the highest critical current. Remeasurement of the critical current at this new field value always resulted in an observation of the highest critical current.

[13] For the runs reported here, it was not possible to trap flux in what has been called the Meissner regime by cycling the external field or temperature in any manner. This was not true for all experiments, however.

[14] The authors are indebted to C. S. Owen for clarification of this point.

junction.[15] (See Fig. 2.) Ampere's law then gives

$$(4\pi/c)=H_e+(4\pi/c)I_1. \tag{7}$$

Combining Eq. (7) with Eq. (3) we obtain an equation which determines current I through the junction:

$$I=2I_1\leq 2\lambda_J J_1-(c/4\pi)H_e. \tag{8}$$

Combining Eqs. (7) and (4), we obtain the additional condition:

$$-(c/4\pi)H_e\leq 2\lambda_J J_1. \tag{9}$$

Equation (5) with the equality sign describes the linear *decrease* in the critical current to zero with increasing positive H_e and a linear *increase* of critical current with increasing negative H_e. Equation (9) gives the most negative H_e for which the Meissner solution applies. Experimentally, an abrupt transition to the vortex state is observed for the negative-field (increasing current) case whereas this transition is continuous for the case of positive field (decreasing current). (See Fig. 3.) Further examination of Fig. 3 indicates that the critical-current–field curve for the mixed state of asymmetric junctions is qualitatively similar to the results of calculations[10] for the symmetric case.

C. Temperature Dependence of the Critical Field of Asymmetric Junctions

It should be noted that the maximum field at which the Meissner solution is possible, which Anderson[2] has called H_{c_1}, is given by

$$H_{c_1}=\frac{8\pi}{c}\lambda_J J_1=\frac{\phi_0}{2\pi\lambda_J ed}\,. \tag{10}$$

In this equation ϕ_0 is the flux quantum, and $d=2\lambda_L+t$, where t is the oxide layer thickness. The temperature dependence of this transition field can be calculated using $\lambda_J=[\hbar c^2/8\pi J_1 ed]^{1/2}$, $J_1=\pi\Delta/2R_{NN}A$, and the temperature dependences of Δ,[16] λ_L,[17] and J_1.[18] In these equations, J_1 is the maximum Josephson current density, R_{NN} is the normal tunneling resistance, Δ is the energy-gap parameter, and λ_J is the London penetration depth. The results of experiments and calculations which are shown in Fig. 4 appear to be in good agreement.

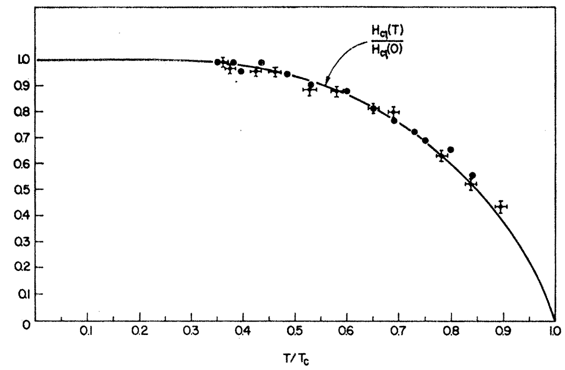

FIG. 4. Temperature dependence of critical fields of junctions of Fig. 3 (dots) and another sample not shown (crosses). Solid line is computed from temperature dependence of Eq. (10) as indicated in the text. Points for each sample are fitted to calculated curve at lowest temperature.

IV. DISCUSSION

The experiments indicated here, which are typical results of a set of 15 samples, demonstrate the existence of a Meissner state at small magnetic fields in high current junctions and are in qualitative agreement with the existence of an "Abrikosov array" of quantized vortices at higher fields. It has not been possible with present techniques to obtain quantative agreement with the theory, as the high current junctions employed in the experiments are usually not uniform and may have small metallic bridges. It would be of interest to extend this work to study the interaction of electromagnetic waves with the "Abrikosov array" of vortices in a junction. A detailed analysis of this problem, which is similar to the problem of the resistive behavior of type-II superconductors, has been made by Kulik.[8]

ACKNOWLEDGMENTS

The authors would like to thank C. S. Owen and D. J. Scalapino for many informative discussions and for permission to use the results of their calculations prior to publication.

[15] In the symmetric case, the field due to I_1 in the upper layer was cancelled by the equal and opposite field due to the parallel current I_1 flowing across the lower layer. In the present situation the current in the lower layer flows parallel to the applied field and does *not* produce a field that is screened in the Meissner regime.

[16] B. Muhlschlegel, Z. Physik **155,** 313 (1959).

[17] $\lambda_L(T)=\lambda_L(0)[1-(T/T_c)^4]^{-1/2}$.

[18] V. Ambegaokar and A. Baratoff, Phys. Rev. Letters **10,** 486 (1963); **11,** 104(E) (1963).