absorption effect. Thus, it appears as though the parts of the spectrum that absorb, scan the light profile as a function of the magnetic field. When Lurio[18] encountered this effect, he applied a magnetic field to his lamp to flatten and broaden the incident light profile, and thus decrease this kind of effect.

It is easily seen that correction terms to the Hanle scattering rate must depend on an even power of the magnetic field, and that the dominant term is most likely proportional to B^2. The curves of Figs. 4(a) and 4(b) indicate that a B^2 correction term whose coefficient is temperature (optical depth) dependent gives a good first-order correction. The optical-depth dependence of the coefficient of the B^2 term as well as the dependence of the coefficients for the Hanle term and the background are to be explained in a separate paper.[17] The form of Eq. (7) fits the experimental data well.

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Relativistic Effects in the Excitation of Triplet Helium States by Electrons*

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The excitation of triplet He states is a pure rearrangement collision provided that spin-dependent potentials are neglected. The high-energy behavior of this rearrangement cross section is proportional to E^{-3}. If relativistic spin–other-orbit terms are admitted, they produce contributions to the cross section which go to a constant at moderately high energies. These new terms are incoherent with the old ones and have a relative coefficient of α_F^4, where α_F is the fine-structure constant. They become significant at an incident energy of about 8 keV and so are not presently observable. Part of the result obtained here also applies to the case where the incident particle is a proton.

I. INTRODUCTION

THE theory of rearrangement collisions is not completely understood even at high energies. To further this understanding various first Born approximations to the cross section for the excitation of triplet states of helium by electrons have been calculated and compared with experiment with somewhat inconclusive results.[1] These reactions are pure rearrangement collisions in the approximation where spin-dependent interactions are neglected. In that case, the singlet-triplet reaction can only come about through the process of the incident electron colliding with and ejecting a bound electron and then becoming bound itself. In the references cited,[1] eight different forms of the transition matrix element were tested. They gave different results but for high enough energy it can be shown that they all converge to the same results. This cross section at high energies behaves as E^{-3}.

When spin-dependent potentials are admitted, the reaction can proceed via spin flip and not by rearrangement. Spin-dependent potentials are small, but because the reaction is a direct one and not a rearrangement reaction, we may expect that this part of the cross section will not fall off as rapidly at high energies and may well be important. Indeed, we shall see that the spin-flip cross section goes to a constant at moderately high energies. This comes about from the interaction of the spin of a bound electron with the orbital motion of the projectile electron. This force is proportional to the incident velocity so that the impulse which causes the transition is energy-independent at high energies and consequently the cross section is also. There are other spin-dependent potentials (besides the spin-other orbit one) but these give contributions to the cross section which fall off with energy more rapidly than the one considered here, and so they will be neglected.

II. CALCULATION

As an example, we deal with the excitation of He (2^3S). Our starting point is the expression for the Born approximation for the T matrix for this reaction

$$T_{fi}=\langle\lambda_f(1)V_f(1)A_0\lambda_i(0)\rangle, \quad (1)$$

where the notation is that of Ref. 1. The potential may be decomposed into a spin-independent part $V_f^{(0)}(1)$ and a spin-dependent part $V_f^S(1)$. The contribution from $V_f^{(0)}(1)$ is just that obtained in Ref.

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[1] Charles J. Joachain and Marvin H. Mittleman, Phys. Rev. 140A, 432 (1965); 151, 7 (1966).

1 which we call $T^{(0)}$. It has various forms but at high energy the term I, which occurs in all of them, dominates, yielding an E^{-3} contribution to the cross section.[2] The other terms Λ_i all give E^{-5} or smaller and are dropped. We shall not be concerned with highly relativistic electrons here so that the Dirac matrices describing the electrons may be reduced to Pauli matrices in the usual way.[3] The result for the spin–other-orbit potential is

$$V_f^{(S)}(1)=V^S(1,0)+V^{(S)}(1,2), \tag{2}$$

$$V^S(1,0)=\frac{e^2h}{2m^2c^2}\boldsymbol{\sigma}_0\cdot\frac{\mathbf{r}_{01}}{r_{01}^3}\times\mathbf{p}_1=\alpha_F^2\boldsymbol{\sigma}_0\cdot\frac{\mathbf{r}_{01}\times\mathbf{p}_1}{r_{01}^3}, \tag{2'}$$

where $\boldsymbol{\sigma}_0$ is the usual Pauli matrix vector. The last expression is written in rydberg units, where α_F is the fine-structure constant. This potential may enter in two ways. The first is the way we shall treat in some detail here. It just couples the initial singlet to the final-triplet state in lowest-order perturbation theory. The spin-dependent potential will also enter by modifying the initial and final states. That is, the intial (final) singlet (triplet) state will have a small admixture of triplet (singlet) component introduced by V^S and the other spin-dependent potentials. These small admixtures will then be coupled by $V_f^{(0)}$ which will cause the transition. However, this process will have the energy dependence of any allowed transition ($E^{-1}lnE$) and will also be proportional to the small relativistic coefficient so we will neglect it here.

The antisymmetrization operator occurring in (1) is

$$A_0=1-P_{01}-P_{02}=2-P_{01}, \tag{3}$$

where P exchanges spin and space coordinates. The last step comes about from the symmetry of the final state and the interaction. The factor two in A_0 combined with $V_f^S(1)$ gives an exchange reaction (electron zero is free in the initial state, electron one is free in the final state). This is a rearrangement collision brought about by a spin-dependent force. It has the small coefficient of the relativistic potential and the rapid-energy decay characteristic of a rearrangement collision so it is neglected here. The term $-P_{10}$ in A_0 combined with V_f^S gives the spin-flip term described in the Introduction. The processes described by $T^{(0)}$ and the remaining part of $T^{(S)}$ are incoherent. This is most easily seen from the fact that the final-spin state of the electron is different in the two processes. Thus, the cross sections for the two processes may be added. The rearrangement cross section has been obtained previously.[1] The high-energy part in units of Bohr radii squared is

$$\sigma^{(0)}=\frac{3}{8\pi}\int_{-1}^{1}d\mu|I|^2, \tag{4}$$

where

$$I=\int(d^3r)^3e^{-i\mathbf{p}_f\cdot\mathbf{r}_1}\phi_{2S}(\mathbf{r}_0,\mathbf{r}_2)\frac{2}{r_{10}}e^{i\mathbf{p}_i\cdot\mathbf{r}_0}\phi_{1S}(\mathbf{r}_1,\mathbf{r}_2). \tag{5}$$

We are interested only in the high-energy limit of (4) and (5) in which case a method for evaluating I due to Ochkur[4] is applicable. The result is

$$I=\frac{8\pi}{p^2}\int d^3r_1d^3r_2e^{-i\Delta\cdot\mathbf{r}_1}\phi_{2S}(\mathbf{r}_1,\mathbf{r}_2)\phi_{1S}(\mathbf{r}_1,\mathbf{r}_2)+O(p^{-4}) \tag{6}$$

$$=(8\pi/p^2)\Sigma(x),$$

where $\boldsymbol{\Delta}$ is the momentum transfer and $x=\Delta^2$. Note that Σ is the generalized oscillator strength and that it vanishes when its argument is zero.

The spin-flip part of the cross section is obtained from the T matrix

$$T^{(S)}=-2\langle\lambda_f(1)V^{(S)}(1,0)\lambda_i(1)\rangle, \tag{7}$$

where the factor of two arises from the fact that the two terms in Eq. (2) contribute identically. Upon performing the spin algebra and some simple integration, the result is

$$\sigma^{(S)}=16\pi\alpha_F^4p^4\int_{-1}^{1}d\mu(1-\mu^2)(1/x^2)[\Sigma(x)]^2. \tag{8}$$

Combining Eqs. (4) and (8), the total cross section is

$$\sigma=8\pi\int_{-1}^{1}d\mu\{(3/p^4)+2\alpha_F^4(p^4/x^2)(1-\mu^2)\}[\Sigma(x)]^2. \tag{9}$$

The integration in (9) is facilitated by using x, the momentum transferred squared, instead of μ, the cosine of the scattering angle. When this is done and higher-order terms in p^{-2} are dropped, the result for ρ in units of squared Bohr radii and E in units of rydbergs is

$$\sigma=12\pi\int_0^{\infty}dx\{1/E^3+\tfrac{2}{3}\alpha_F^41/x\}|\Sigma(x)|^2. \tag{10}$$

At this point, an explicit evaluation of Σ becomes necessary. In order to do this, we use the simple expressions for the wave functions which were used in Ref. 1.

$$\phi_{1S}(r_1,r_2)=(\alpha^3/\pi)e^{-\alpha(r_1+r_2)}, \tag{11a}$$

$$\phi_{2S}(r_1,r_2)=\frac{(\beta\gamma)^{3/2}}{4\pi N}(1-P_{12})e^{-\beta r_1}e^{-\gamma/2r_2}(r_2\gamma/2-1), \tag{11b}$$

[2] The term I arises from a direct Coulomb interaction between the two electrons which exchange places.

[3] H. A. Bethe and E. E. Salpeter, *Quantum Mechanics of One and Two Electron Atoms* (Academic Press Inc., New York, 1957).

[4] V. I. Ochkur, Zh. Experim. i Teor. Fiz. 45, 734 (1963) [English transl.: Soviet Phys.—JETP 18, 503 (1964)].

where

$$N^2=1-\frac{8(\beta\gamma)^3(\gamma-\beta)^2}{(\beta+\gamma/2)^8},$$

and

$$\alpha=27/16,\quad \beta=2.01,\quad \gamma=1.53.$$

The result is

$$\Sigma(x)=\frac{4(\beta\gamma)^{3/2}\alpha^3}{N}\left\{\frac{(\gamma-\alpha)(\alpha+\beta)}{(\alpha+\gamma/2)^4}\frac{1}{[x+(\alpha+\beta)^2]^2}-\frac{[(\gamma-\alpha)(\alpha+\gamma/2)^2-(\alpha+\gamma)x]}{(\alpha+\beta)^3[x+(\alpha+\gamma/2)^2]^3}\right\}. \quad (12)$$

The remaining integral in (10) could be evaluated analytically but the result is cumbersome. Instead, numerical integration results in

$$\sigma=\frac{3.02}{E^3}(1+0.49\times10^{-9}E^3). \quad (13)$$

Ochkur and Brattsev[5] give values for the 2^3S excitation cross section up to 500 eV. Their cross section is proportional to E^{-3} at the upper end of their energy range with a coefficient of 4.68 which compares with our value of 3.02. They use Hartree-Fock wave functions for their calculations so that their value is expected to be somewhat better than the result obtained here from the crude wave functions of Eq. (11). The magnitude of the change is, however, somewhat surprising. It is reasonable to suppose that the ratio of the two integrals in Eq. (10) is not as critically dependent upon the wave functions as are the individual integrals, so that the coefficient of E^3 in the bracket of Eq. (13), which is the main result of this paper, is probably accurately obtained here.

This cross section has recently been measured[6] in the energy range 100–225 eV, where within experimental error it behaves as E^{-3}. The experimental results fall well below Eq. (13). The fact that the cross section has attained its (nonrelativistic) asymptotic behavior at such a low energy is surprising. Before concluding that this really is an asymptotic result, the experiments should be extended to higher energies. The second term in Eq. (13) is certainly not observable at these energies. Indeed, it is only a 10% correction at about 8 keV. Note that the total cross section depends upon two numbers involving the function $\Sigma(x)$ in different ways so that an explicit representation for the wave functions is necessary in order to construct Σ. However, at any given angle Σ^2 enters only as a factor so that the ratio of the two parts of the cross section in simply obtained

$$\left(\frac{d\sigma^{(S)}}{d\Omega}\right)\Big/\left(\frac{d\sigma^{(0)}}{d\Omega}\right)=\tfrac{2}{3}\alpha_F{}^4\frac{E^4(1-\mu^2)}{x^2}=\frac{\alpha_F{}^4}{6}E^2\frac{1-\mu^2}{[1-\mu+\mathcal{E}^2/8E^2]^2}, \quad (14)$$

where $\mathcal{E}=1.457$ the excitation energy of the 2^3S state in rydbergs.

This ratio maximizes at an angle given by

$$\mu=\frac{1}{1+\mathcal{E}^2/8E^2}\simeq1-\frac{\mathcal{E}^2}{8E^2}, \quad (15)$$

that is, almost in the forward direction where the value is given by

$$\left(\frac{d\sigma^{(S)}}{d\Omega}\right)\Big/\left(\frac{d\sigma^{(0)}}{d\Omega}\right)_{\max}=\frac{2}{3}\frac{(\alpha_F E)^4}{\mathcal{E}^2}. \quad (16)$$

This is about 3% for 1-keV incident electrons and rises rapidly for higher energies. Thus, the relativistic effect may be more readily observable as a narrow anomolous bump in the angular distribution at an angle given by Eq. (15).

At the energies in question here protons and electrons give the same cross section when they are incident at the same center-of-mass velocity except of course that exchange effects are absent in the proton case. Therefore, we may obtain the excitation cross section for protons by simply dropping the first term of Eq. (13). The result is

$$\sigma/a_0{}^2=1.5\times10^{-9}.$$

[5] V. I. Ochkur and V. F. Brattsev, Opt. i Spektroskopiya **19**, 490 (1965) [English transl.: Opt. Spectry. (USSR) **19**, 274 (1965)].

[6] The experiment is by C. E. Kuyatt *et al.* I am indebted to Dr. Kuyatt for sending me their results prior to publication.