PHYSICAL REVIEW VOLUME 164, NUMBER 2 10 DECEMBER 1967

Energy Loss and Straggling of High-Energy Muons in NaI(Tl)†

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(Received 28 July 1967)

Absolute values of the most probable energy loss and the energy-loss straggling for high-energy muons passing through NaI(Tl) have been measured over the muon momentum range from 0.5 to 10.5 GeV/c. The results agree, within the 1% experimental uncertainty, with the theoretical values.

INTRODUCTION

The mean ionization loss $-dE/dx$ for charged particles heavier than electrons passing through matter is given by the Bethe-Bloch formula[1–3]

$$\frac{-dE}{dx}=\frac{2\pi nz^2e^4}{mv^2}\left(\ln\frac{2mv^2W_{\max}}{I^2(1-\beta^2)}-2\beta^2-\delta-U\right),\quad(1)$$

where n = number of electrons/cc in the material, m = electron mass, z = charge of the particle in units of the electronic charge e, v = velocity of the particle, $\beta=v/c$, $W_{\max}$ = maximum energy transferable to an atomic electron in a single collision, I = mean excitation potential of the atoms of the substance, and U is the shell correction term due to the nonparticipation of the electrons in the inner atomic shells for very low velocities of the particle. U will be ignored hereafter. δ is a correction due to the density effect, arising from the polarization of the material which reduces the effect of distant collisions.

The density effect was first suggested by Swann.[4] Calculations of this effect were first made by Fermi,[5] and Halpern and Hall[6] and others. The calculations used in this paper are based on the extensive work of Sternheimer[7] on this subject. Thus, Sternheimer has expressed δ in the form

$$\delta=4.606X+C+a(X_1-X)^m,\qquad X_0<X<X_1,\quad(2)$$

$$\delta=4.606X+C,\qquad X>X_1,\quad(3)$$

where $X=\log_{10}(P/m_0c)$, P and m_0 are the momentum and rest mass of the incident particle. For NaI, Sternheimer[8] has recommended values for the constants of $I=427.1$ eV, $C=-5.95$, $a=0.3376$, $m=2.623$, $X_0=0.215$, and $X_1=3.0$.

Owing to the statistical nature of the ionization process, the ionization loss in a thin absorber is subject to large fluctuations. Further, since the probability of collisions decreases with increasing energy transfer, the energy-loss distribution is asymmetrical with a long tail on the high-energy side, corresponding to the infrequent collisions with large energy transfers. This energy-loss distribution has been calculated by Williams,[9] Landau,[10] Symon,[11] and Vavilov.[12] Modifications to Landau's theory have been made by Fano[13], Hines,[14] and Blunck and Leisegang.[15] Under the conditions of the experiment to be described, the effect of these modifications is negligible, and the theoretical distribution has been taken to be that of Landau.

The tabulation of the Landau distribution by Börsch-Supan[16] has been used. According to Landau's theory, the most probable energy loss ϵ_p for a thin absorber of thickness χ cm is given by

$$\epsilon_p=\frac{2\pi ne^4z^2\chi}{mv^2}\left(\ln\frac{2mv^2(2\pi ne^4z^2\chi/mv^2)}{I^2(1-\beta^2)}-\beta^2+0.37-\delta\right).\quad(4)$$

Measurements of both (dE/dx) and the shape of the straggling curve for high-energy heavy particles have mainly been confined to cosmic-ray muons where the accuracy of the results is affected by poor statistics, "binning" over large energy ranges and normalizing the results to the measurements at one energy.[17]

† Work supported in part by the U.S. Office of Naval Research Contract No. [Nonr 225(67)] and the U.S. Atomic Energy Commission.
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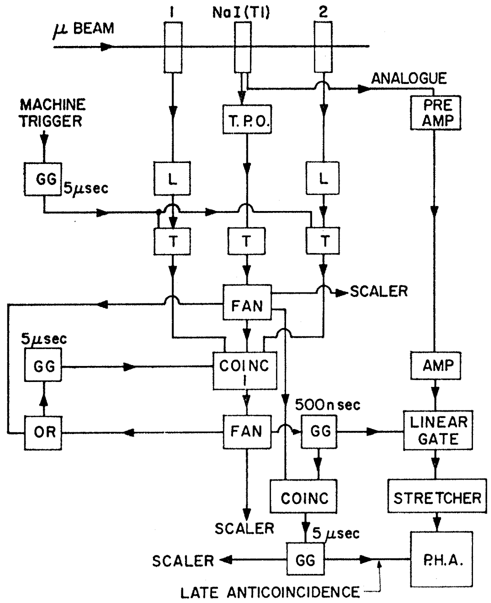

FIG. 1. Schematic diagram of the electronic system. L= limiter, T=discriminator-trigger, GG=gate generator, Fan= fan out, TPO=time pick off unit (see text).

Absolute measurements were made using cosmic-ray muons by Hudson and Hofstadter.[18] Bowen[19] has made absolute measurements with accelerator produced pions and muons and cosmic-ray muons with average energies of 0.37, 0.76, 1.47, and 5.23 GeV in a NaI(T1) crystal and Millar *et al.*[20] have used a large liquid scintillator to study cosmic-ray muons of 0.30 and 2.2 GeV with high statistical accuracy. These authors found essential agreement with the theoretical predictions.

EXPERIMENTAL METHOD

The existence of a high-resolution, high-energy muon beam at the Stanford Linear Accelerator Center afforded a convenient opportunity to make accurate absolute measurements of the variation of energy loss with energy for muons in a well-defined (1%) energy interval, and to study the straggling in this energy loss.

The apparatus is shown schematically in Fig. 1. A 3-in.-diam NaI(Tl) crystal 0.245±0.001 in-thick was mounted perpendicular to the beam and viewed with an RCA 8054 photomultiplier. Plastic scintillators of 2 in. diameter (counters 1 and 2) were placed on either side of the NaI(Tl) crystal and viewed with 56 AVP photomultipliers. The thickness of the NaI(Tl) crystal was chosen so that the most probable energy loss (~3 MeV) for fast muons was close to, and bracketed by, the energy calibrations obtainable from γ-ray sources.

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Coincidences between counters 1 and 2 were used to open a 500-nsec linear gate to pass the amplified analog pulses from the NaI(Tl) crystal into a 256-channel pulse-height analyzer. A logic pulse was obtainable from the NaI crystal via a time pick-off (TPO) unit which gave a trigger pulse off the rising edge of the amplified dynode signal. In order to avoid recording pulse heights where more than one pulse was present in the NaI crystal, the circuitry of Fig. 1 was used to veto the pulse if a TPO signal occurred either (a) within the 5 μsec immediately preceding the coincidence or (b) following the coincidence (resolving time 50 nsec), but within the 500-nsec gate. Counting rates were also limited to one particle per twenty machine pulses (1.6 μsec long). Measurements were made at various muon momenta between 0.5 and 11 GeV/c, each run being made for from 30 000 to 50 000 muons through the crystals. Before and after each run, the crystal was calibrated using γ rays from Cs^{137} (0.662 MeV), Co^{60} (1.173 and 1.332 MeV), and a Pu–Be source (double escape peak 3.412 MeV). The calibrations were done using the same electronics, but with the coincidence unit 1 switched to operate on singles from the time pick-off unit instead of coincidences from 1 and 2. This ensured that any gate pedestal or dependence of the over-all gain on the position in time with respect to the various gates, were included in the calibrations. Calibrations bracketing each run usually agreed to 0.1–0.5%. Although all these γ-ray peaks were used to check the linearity of the apparatus, the actual energy calibration of the energy-loss peak was essentially determined by the 3.41-MeV γ-ray peak. A few runs were also made with $\frac{1}{2}$- and 3-in. NaI(Tl) crystals and a $\frac{1}{4}$-in. CsI(Na) crystal. The results from these were in essential agreement with the more extensive runs with the $\frac{1}{4}$-in. NaI(Tl) crystal. Electron and muon peaks were also compared using a NaI(Tl) crystal 0.030 in. thick.

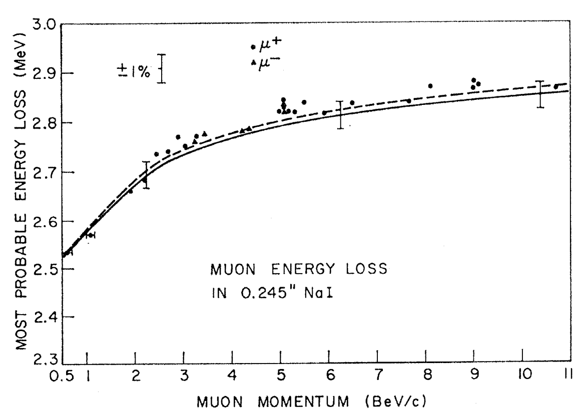

FIG. 2. Energy loss of muons in passing through 0.245-in. NaI(Tl) crystal. The solid line is the corrected theoretical curve (see text). The dotted line is the corrected theoretical curve with the density correction reduced by 3%.

The muon beam was described by Barna *et al.*[21] Electrons and strongly interacting particles were removed by placing the counters behind 4 ft 8 in. of iron. To obtain electrons, this iron was removed as was the Pb radiator at the first beam focus.

RESULTS

A. Most Probable Energy Loss

The most probable energy loss for each run at a given momentum setting was determined by curve fitting to the peak of the Landau distributions and to the calibration peaks obtained on the pulse-height analyzer. These energy losses are plotted directly in Fig. 2. The error bars on the two lowest momenta points indicate the spread in momentum introduced in passing through the Fe absorber. Comparison with a smooth curve drawn through these points shows that the statistical deviation of an individual measurement is less than 0.5%. These values of energy loss should not be directly compared with the most probable energy loss given by Eq. (4) until various corrections have been made. The theoretical curve shown in Fig. 2 is obtained by evaluating Eq. (4) and then applying the various corrections and considering the errors enumerated below:

(1) The light output dL/dE in the NaI crystal is not independent of dE/dx (Refs. 22–24). Normalizing to dE/dx at 1.18 MeV/g this correction amounts to $-1.3\pm0.2\%$ at 0.5 GeV/c increasing to $+0.8\%$ at 10 GeV/c.

(2) A similar correction must be applied to the γ-ray calibration point amounting to $-1.5\pm0.5\%$.

(3) A correction must be made for the Čerenkov light emitted by the muons both in the NaI(Tl) and in the glass window (0.120-in. thick), with allowance for the Čerenkov light emitted by the electron-positron pair produced in forming the 3.412-MeV double escape peak. This correction is $0.25\pm0.25\%$.

(4) There is a shift in the position of the maximum of the 3.412-MeV γ-ray peak due to the nonuniform background, giving a correction of $+1.5\pm0.5\%$.

(5) The resolution of the apparatus has to be folded into the asymmetrical Landau distribution giving a correction of $+1.7\pm0.2\%$.

(6) The thickness of the NaI(Tl) crystal is known to ±0.001-in. (i.e., 0.4%).

The theoretical line in Fig. 2 has had corrections 1–5 applied and the error bars of $\pm1\%$ indicated at three places in this curve also include the uncertainty in thickness (correction 6). In addition, there is an uncertainty in the density correction (δ) of about 3%[25] which gives an additional error in dE/dx of $\pm0.5\%$ at 10 GeV/c and $\pm0.1\%$ at 1.0 GeV/c. The effect on the shape of the curve of reducing δ by 3% is shown by the dotted line in Fig. 2. A similar change of shape would occur if dL/dE varied more rapidly with changes in dE/dx.

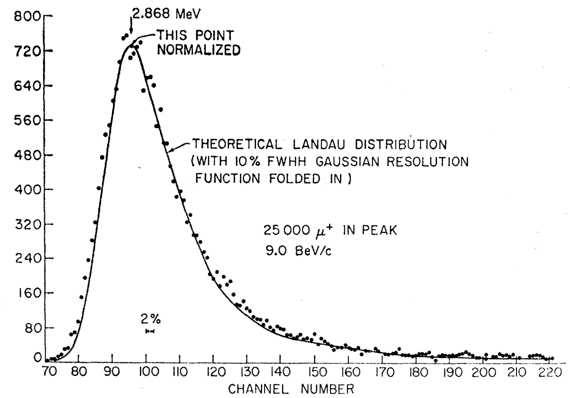

Fig. 3. Comparison of a typical energy-loss distribution with the Landau distribution. (FWHH=full width at half-height.)

The energy loss of 10-GeV/c electrons in a NaI(Tl) crystal of thickness 0.030-in. was measured using the energy loss of 10-GeV/c muons for calibration. A thin crystal was used to minimize the effects of bremsstrahlung radiation in the crystal. Tsytovich[26,27] has predicted a decrease in ionization loss of 5–10% at very high values of $\gamma=E/mc^2$ for NaI over the value calculated from Eq. (5) (or its equivalent for electrons). The energy-loss distribution for electrons was wider than and had a different shape from the Landau distribution obtained for the muons, and was peaked about 7% above the theoretical energy neglecting the Tsytovich effect. By making plausible assumptions about the γ-ray background in the electron beam, a good fit to the shape of the energy-loss distribution could be made and the most probable energy loss was found to be 1.01 ± 0.03 times the theoretical value, giving no support to the Tsytovich theory. This is in agreement with the measurements of Ashton and Simpson[28] on cosmic-ray muons, but not with those of Zhdanov *et al.*,[29] and Alekseeva *et al.*,[30] on electrons in emulsions. Owing to

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the assumption of the γ-ray background to explain the present results, the value of energy loss obtained should be treated with caution.

B. Energy-Loss Straggling

The energy-loss distribution obtained for 9.0-GeV/c muons in the 0.245-in.-thick NaI(Tl) crystal is shown in Fig. 3. The points shown are the experimental counts in each channel. No counts were observed in channels 0–70. The full line is the calculated Landau distribution with an experimental width of $\pm 5\%$ at half-height folded in. This was the experimental width obtained for the Pu–Be 3.412-MeV calibration peak. The theoretical width at $\frac{1}{2}$ height is about 1% less than the experimental width. Allowance for variation of dL/dE with dE/dx would broaden the theoretical curve by perhaps 3%, but it must be noted that such broadening effects are already present in the calibration peak. Further, since the broadening in the curve is mainly due to the effect of δ rays, the dependence of dL/dE on dE/dx under these conditions is reduced.[31] The 1% discrepancy in width shown in Fig. 3 is less than the theoretical and experimental error.

CONCLUSIONS

The theoretical predictions both on energy loss and straggling have been verified to quite a high degree of accuracy, and no direct evidence has been found for the reduction in energy loss at very high energies predicted by Tsytovich.

ACKNOWLEDGMENT

Our thanks are due to the Harshaw Chemical Company for providing CsI(Na) crystals and the 0.030-in.-thick NaI(Tl) crystal.

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Dynamical Spin Hamiltonian and the Anisotropy of Spin-Lattice Relaxation for the Kramers Doublets. I. General Considerations

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(Received 20 March 1967)

The limitations of a previous treatment of the dynamical spin Hamiltonian in relation to Kramer's doublets have been pointed out. It is shown that the momentum operators in addition to the symmetry coordinates must be included in the dynamical spin Hamiltonian for such systems. The most important effect of this inclusion is the appearance of the Zeeman-field-independent Van Vleck two-phonon terms which account for most of the spin-lattice relaxation in such systems at high temperature. The anisotropy of the spin-lattice relaxation expected from such a term for crystals of various symmetries has been derived from straightforward symmetry considerations.

I. INTRODUCTION

IN a previous paper[1] the terms in the dynamical spin Hamiltonian in paramagnetic crystals were generated from symmetry arguments. It was argued there that the basic interactions that cause the dynamical spin Hamiltonian involves the operators $S, I, H,$ and Q, the symmetry coordinate of the complex formed around the magnetic ion site, and so this Hamiltonian must be the function of these operators. Only those functions are allowed to be nonvanishing which transform as the identity representation of the point group at the site of the paramagnetic ion and which are invariant to time-inversion operation.

These considerations are strictly valid for the case of non-Kramers ions, but the dynamical spin Hamiltonian for Kramers conjugate states will be shown to have more terms than those for the non-Kramers case. This is due to the fact that P, the momentum operator[2] which is present in the expression for the $\mathcal{H}_{\text{lattice}}$, generates terms in the dynamical spin Hamiltonian containing both P and Q. Perturbation derivation and group theoretic justifications of such terms will be discussed in the next section. In Sec. 3, the anisotropy of spin-lattice relaxation expected from such terms for Kramers doublets will be discussed for crystals of various point symmetries.

II. DERIVATION OF THE MOMENTUM-DEPENDENT TERMS IN THE DYNAMICAL SPIN HAMILTONIAN

The terms in the dynamical spin Hamiltonian depending linearly on the Zeeman field are already given

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