He^4 II region shows the effect of the additional mechanical heating due to rotation at 30 sec^{-1}. The transition temperature is artificially displaced by 10^{-4}°K because of the discontinuity discussed above. It was found that upon passing through the transition, thermal equilibrium becomes *extremely* poor, probably because the rotational field prevents convection. In the particular example of Fig. 2, rotation was stopped 15 sec after the transition had been reached. Heat transfer became very good, as can be seen from the dashed line in the He^4 I region. This is probably caused by rapid convection initiated by the collapse of warm (and therefore more dense) He^4 I accumulated near the outside by the rotational field. The ratio between the slopes of the dashed line above T_λ, and the solid line below T_λ, is consistent with the known heat-capacity ratio[8] and a constant background heating rate at $\omega=0$. Several determinations of T_λ, with $\omega=0$, and of the minimum temperature $T_{\min}$ when rotation at 30 sec^{-1} was stopped Δt seconds ($\Delta t=2$ to 15) after the transitions, were made. With Δt equal to 15 sec, the minimum temperature after stopping the rotation was 3×10^{-5}°K above T_λ. $T_{\min}$ as a function of Δt, extrapolated to $\Delta t=0$, agreed with T_λ ($\omega=0$) within 1×10^{-5}°K. A negative heat of transition (from a metastable state), such as implied by AT's results,[2] predicts that the minimum temperature after the transition would be 5.2×10^{-3}°K greater than T_λ. A positive enthalpy of transition would cause $T_{\min}$ to be independent of Δt, and equal to T_λ (it is assumed that the entropy of He^4 I is independent of ω). It can be concluded that any enthalpy of transition at $\omega=30$ sec^{-1}, is less than the change in enthalpy of He^4 I over a 10^{-5}°K temperature range immediately above T_λ, i.e., $|\Delta H| \leqslant 6\times10^{-4}$ J/mole.[8] It follows that any volume discontinuity is smaller than 3×10^{-5} cc/mole or 10^{-4}% of V if $(\partial P/\partial T_\lambda)_\omega=(\partial P/\partial T_\lambda)_{\omega=0}$. This is one order of magnitude smaller than the resolution of AT's measurements, and a factor of 200 smaller than the ΔV reported by AT.[2] Therefore, within the experimental resolution of this work, the He^4 II $\rightarrow$ He^4 I transition under rotation is of higher than first order.

The shift in transition temperature at $\omega=30$ sec^{-1}, is less than 10^{-5}°K, as expected from the phenomenological theory of liquid He^4 II near T_λ.[7,9]

ACKNOWLEDGMENTS

The author is grateful to F. W. Martin for the detailed design of the apparatus, and to J. H. Condon and P. C. Hohenberg for stimulating discussion. J. F. Macre designed the circuit used to measure ω.

[9] Yu. G. Mamaladze, Zh. Eksperim. i Teor. Fiz. **52**, 729 (1967).

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Superfluid Flow Transitions in Rotating Narrow Annuli*

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The attenuation of second sound has been used to observe helium-II flow transitions in five annuli in solid-body rotation at 1.40°K. The annuli were 0.6 to 1.9 mm wide, and approximately 15 mm in radius. The onset of attenuation of second sound at the angular velocity Ω_0 predicted by Fetter for the creation of a single row of vortex lines established that liquid helium in rotation is able to attain the state of lowest free energy. The onset of attenuation of the second harmonic, which has a velocity node at the middle of an annulus, showed that vortices can be detected away from the median radius at approximately 1.9 Ω_0. In the range $0.65<\Omega<0.95$ rad/sec, a critical velocity Ω_3 was observed through two effects: First, in annuli in which $\Omega_0<\Omega_3$, it was usually necessary to rotate faster than Ω_3 before the helium would display second-sound attenuation at Ω_0. Second, in the narrowest annulus, where $\Omega_3<\Omega_0$, strong attenuation occurred at Ω_3 even though the equilibrium state is vortex-free irrotational flow. The transition at Ω_3 is identified with the creation of long-lived vorticity in the helium.

I. INTRODUCTION

IN 1946, London[1] proposed that liquid helium in a cylindrical vessel should show a rotational Meissner effect, which has recently been observed by Hess and Fairbank.[2] The flow of superfluid in a slowly rotating annulus is a more complex problem, which was first considered by Bendt and Oliphant.[3] In a recent elegant free-energy calculation, Fetter[4] determined the critical

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[1] H. London, *Report on an International Conference on Fundamental Particles and Low Temperatures*, (The Physical Society, London, 1947), Vol. 2, p. 48; F. London, *Superfluids* (John Wiley & Sons, Inc., New York, 1954), Vol. 2, p. 144.

[2] G. B. Hess and W. M. Fairbank, Phys. Rev. Letters **19**, 216 (1967).

[3] P. J. Bendt and T. A. Oliphant, Phys. Rev. Letters **6**, 213 (1961).

[4] A. L. Fetter, Phys. Rev. **153**, 285 (1967); R. J. Donnelly and A. L. Fetter, Phys. Rev. Letters **17**, 747 (1966).

velocities at which the flow patterns should change, without knowing whether the flow patterns he considered actually occur. The present experiment was undertaken to establish the relevance of Fetter's theory.

Fetter's free-energy calculations give the following expression for the angular velocity Ω_0 at which it first becomes energetically favorable to form a single row of vortex lines in the middle of an annulus:

$$\Omega_0=(\kappa/\pi d^2)\ln(2d/\pi a). \tag{1}$$

The width of the annulus is d, and a is the core radius of a quantized vortex line. The detection of the first row of vortex lines is analogous to measuring H_{c1} in a type-II superconductor. The penetration length λ for supercurrents in helium is equal to the width of the annulus d. With this substitution, the expression for H_{c1} is similar to Eq. (1):

$$H_{c1}\simeq(\Phi_0/4\pi\lambda^2)\ln(\lambda/\xi). \tag{2}$$

The coherence length ξ is analogous to the core radius a, and Φ_0 is the quantum of magnetic flux.

This paper is a more complete account of our second-sound attenuation experiments,[5] which established that under suitable conditions vortex lines appear at the angular velocity predicted by Fetter. This means that a mechanism, thought to be long-lived residual vorticity, sometimes exists which enables rotating helium to reach the lowest free-energy state.

In addition to the flow transition at Ω_0, we also studied a second transition which appeared at an angular velocity designated Ω_3, and which is characterized by the onset of strong attenuation of second sound. Since the viscous normal fluid moves with the annulus in solid-body rotation, this also must be a flow transition of the superfluid. The linear critical velocities $v_{sc}=\bar{R}\Omega_3$ ($\bar{R}$ is the mean radius) agree reasonably well with critical velocities measured in other experiments, as summarized by van Alphen *et al.*[6]

II. APPARATUS

The dimensions of the annuli are given in Table I, and a cross-section view of the part of the apparatus which was immersed in liquid helium is shown in Fig. 1. The second sound was generated and detected by Aquadag coatings on the cylindrical walls, which were anodized 6061 aluminum. A 1-mm-wide conducting strip of DuPont silver preparation was painted around the cylinders at the top and bottom of each Aquadag coating. The bottom conducting strips were grounded, and the top strips were connected to shielded wires leading to vacuum-tight coaxial connectors at the top of the cryostat. The surface roughness of the Aquadag

[5] P. J. Bendt and R. J. Donnelly, Phys. Rev. Letters **19**, 214 (1967).

[6] W. M. van Alphen, G. J. van Haasteren, R. de Bruyn Ouboter, and K. W. Taconis, Phys. Letters **20**, 474 (1966).

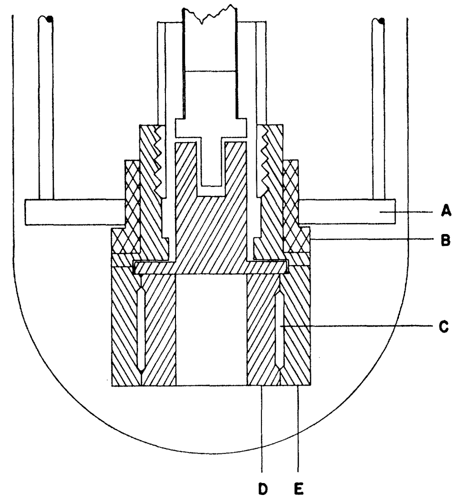

FIG. 1. Cross-section view of an annular second-sound resonant cavity. The frame A is stationary, and the bearing B is machined from Kel-F. The resonant cavity C has V-shaped top and bottom to prevent exciting axial modes. Five annulus widths were obtained by interchanging five inner cylinders D. D could be turned relative to the outer cylinder E by the center shaft and keyway. D and E were anodized 6061 aluminum.

coatings was about 2 μ, and their resistance at 1.40°K varied from 300 to 1000 Ω. The use of Aquadag to detect second sound has been described previously.[7]

The cavities were excited in their fundamental- and second-harmonic radial second-sound modes (see Table I). The second sound was usually generated at the inner cylinder. Because the annuli were quite narrow for second-sound resonances, machining precision was important in determining the Q which was obtained.[8] The tolerance specified was 2.5 μ, and the machining actually was better in some cases. Provision was made to rotate the inner cylinder relative to the outer cylinder when both were immersed in liquid helium. This was used to find the relative orientation of the cylinders which gave the highest Q; in some cases the Q could be doubled by finding the best orientation. The Q of the cavities varied between 400 and 1300. It should be emphasized that the

TABLE I. Dimensions of the annuli used in the experiment, and approximate resonant frequency of the second-sound fundamental radial mode at 1.40°K.

Annulus width (mm)	Outside radius (cm)	Inside radius (cm)	Height of straight sides (cm)	Frequency of second sound (cps)
0.62	1.560	1.498	1.6	15 700
0.82	1.560	1.478	1.6	11 900
1.04	1.560	1.456	1.6	9 400
1.40	1.560	1.420	1.6	6 950
1.90	1.560	1.370	1.6	5 130

[7] P. J. Bendt, Phys. Rev. **153**, 280 (1967).

[8] We define the Q of the cavity as $f/\Delta f$, where f is the resonant frequency and Δf is the full width of the resonance at $1/\sqrt{2}$ of the maximum amplitude. Second harmonic refers to the resonance which occurs at twice the frequency of the fundamental mode.

cylinders rotated at the same angular velocity (solid-body rotation) during all the measurements, and that turning one cylinder relative to the other was only an adjustment made prior to taking data.

The cryostat and equipment for rotating the annulus have been described previously.[7] Electrical connections to the rotating annulus were made through mercury troughs. A stable Hewlett-Packard (HP) 651A test oscillator was used to generate the second sound; the oscillator was modified so that one turn of a helipot changed the frequency by 2 cps. An HP 6204A dc power supply was used to bias the second-sound detector; the combined ac and dc power input to the cavity was never more than 10 mW. The ac signal from the second-sound detector was amplified by a Tektronix 122 preamplifier, and then sent to an HP 302A wave analyzer tuned to the second-sound frequency. The ac signal output of the wave analyzer was connected to an HP 400E ac voltmeter, which drove a strip chart recorder.

III. ATTENUATION OF SECOND SOUND

The attenuation of second sound in rotating helium II is due to mutual friction between the excitations which comprise the normal fluid and vortex lines in the superfluid. In the Feynman-Onsager model, the equilibrium density N_{eq} of vortex lines per unit area in an infinite fluid in solid-body rotation at the angular velocity Ω is given by

$$N_{eq}=2\Omega/\kappa\,, \tag{3}$$

where κ is the quantum unit of circulation ($\kappa=h/m$, where h is Planck's constant and m is the mass of a helium-4 atom). Hall and Vinen[9] have derived the following expression for the attenuation, when the direction of second-sound propagation is perpendicular to the vortex lines:

$$(A_0/A-1)=2BQ\Omega/4\pi f=BQN\kappa/4\pi f. \tag{4}$$

The second-sound amplitude when the cavity is at rest is A_0, and A is the amplitude when rotating; $(A_0/A-1)$ is called the "attenuation." B is a dimensionless parameter called the perpendicular mutual friction coefficient, Q refers to the second-sound resonant cavity, f is the frequency of the second sound, and N is the density of vortex lines. All the measurements reported here were made at 1.40°K, for which $B=1.51$.[7] Note that the attenuation is proportional to N and to Q, and inversely proportional to f.

The 0.6- to 1.9-mm-wide annuli used in the present investigation were not as favorable for making accurate attenuation measurements as the 6-mm-wide annulus used previously.[7] First, the Q of the cavities was about $\frac{1}{3}$ as large, because the same absolute error in machining and alignment is a larger fraction of a second-sound wavelength. Second, the resonant frequencies f were 3 to 20 times higher. Thus the observed attenuation for the same density N of vortex lines was an order of magnitude smaller.

Equation (1) gives the expression for Ω_0 calculated by Fetter. A complete single row of vortex lines, with spacing between lines approximately equal to the annulus width d, was predicted to appear in the fluid at an angular velocity slightly above Ω_0. The range of Ω over which the row fills in is expected to be about 0.01 Ω_0, which is too narrow to observe experimentally. Taking $a=1.3\times10^{-8}$ cm,[10] the numerical value of the logarithm in Eq. (1) for the annuli we used is close to the value of 5π. Since the vortex line density N is approximately d^{-2} when the first row of lines is filled in, we have from Eqs. (1) and (3),

$$N\simeq\Omega_0/5\kappa=N_{eq}/10. \tag{5}$$

Thus the density of vortex lines slightly above Ω_0 is predicted to be an order of magnitude smaller than the equilibrium density N_{eq}. This is due to the boundary conditions in a narrow annulus. In detecting Ω_0 experimentally, the low density of vortex lines is partly compensated by the fact that the vortex lines occur where the fundamental second-sound radial mode has maximum velocity amplitude.

Since the first row of vortex lines was predicted to occur in the middle of the annulus, and since the second-sound second harmonic has a node in $(\mathbf{v}_s-\mathbf{v}_n)$ at the middle of the annulus, we expected that attenuation of the second harmonic would first occur at a value of Ω larger than Ω_0. We designated the experimental onset of attenuation of the fundamental mode as Ω' and of the second harmonic as Ω''.

We probably would not have located Ω' and Ω'' is we had not had Fetter's predicted values of Ω_0 to indicate where to look. In order to observe Ω' and Ω'', it was necessary to greatly improve our sensitivity for detecting changes in second-sound amplitude. The onset of attenuation was indicated by changes in amplitude of 0.3 to 0.8%. The precautions we took included the following:

(1) Measurements of second-sound amplitude at a given rotation speed were made by alternately starting and stopping rotation at that speed (and waiting the required 2 min for transients to decay, after each stop and start). This was done because noise and drift were often comparable to the expected change in amplitude we were looking for. Only after consistent results were obtained did we move on to the next rotation speed.

(2) Some of the time, the second-sound amplitude had a small periodicity on the strip chart recorder equal to the period of rotation. After determining the position of the annulus which gave the largest second-sound amplitude, we then always stopped the annulus in this orientation. This provided a more consistent reference amplitude.

[9] H. E. Hall and W. F. Vinen, Proc. Roy. Soc. (London) **A238**, 204 (1956).

[10] G. W. Rayfield and F. Reif, Phys. Rev. **136**, A1194 (1964).

(3) The zero of the strip chart recorder was displaced, and the signal amplified five times, so we were in effect seeing a larger change in signal amplitude. The signal-to-noise ratio was unchanged by this.

(4) The use of the wave analyzer eliminated virtually all electrical pickup and noise. The oscillator and pre-amplifier were both exceptionally stable against short term drift.

IV. CRITICAL VELOCITIES

Fetter[4] also calculated a critical velocity Ω_c, which is the velocity of the annulus at which, assuming the fluid to be at rest, it becomes possible to create vortices at a distance Ca from the inner wall, where C is a constant of order unity:

$$\Omega_c = (\kappa/4\pi\bar{R}d)\ln(2C). \qquad (6)$$

The mean radius of the annulus is $\bar{R}$. Note that Ω_c is of the order $d/4\bar{R}$ smaller than Ω_0. It is presumed Ω_c is the velocity at which the superfluid first interacts with the walls,[11] and thus is brought into rotation. At angular velocities $\Omega_c < \Omega < \Omega_0$, the equilibrium state of rotating superfluid was predicted to be quantized circulation around the inside cylinder; i.e., an irrotational $1/R$ velocity field.

Since irrotational flow ordinarily does not attenuate second sound, we were unable to determine whether or not superfluid circulation around the inside cylinder occurs when $\Omega > \Omega_c$ (this point is discussed further in Sec. V). The critical velocity effects we observed were at much larger angular velocities than Ω_c, and were in fact larger than Ω_0, except in the narrowest annulus.

On the first run after cooling the helium below T_λ, attenuation usually did not occur at Ω_0 in the 1.04-mm and wider channels. When we increased Ω to considerable larger values, strong attenuation finally set in at the critical velocity we have designated Ω_3. We then observed attenuation until Ω was decreased below Ω_0 (see the middle panel in Fig. 2). In later runs, attenuation always appeared at Ω_0. We conclude that when we first increased the angular velocity through Ω_0, we were unable to nucleate the row of vortex lines. On one occasion, after taking Ω above Ω_3 and later observing the transition at Ω_0, we then raised the temperature above T_λ for 5 min, and cooled below T_λ again. We found that the mechanism for nucleating vortex lines at Ω_0 had been destroyed, and it was necessary to increase Ω to Ω_3 again, before attenuation occurred.

Consider now the few occasions when we did not have to increase Ω to Ω_3 before observing attenuation at Ω_0. We believe that in these cases, the mechanism for nucleating the vortex lines was established before the measurements were begun, by turning the inside cylinder. This was done prior to taking data, in order to find the orientation with the highest Q.

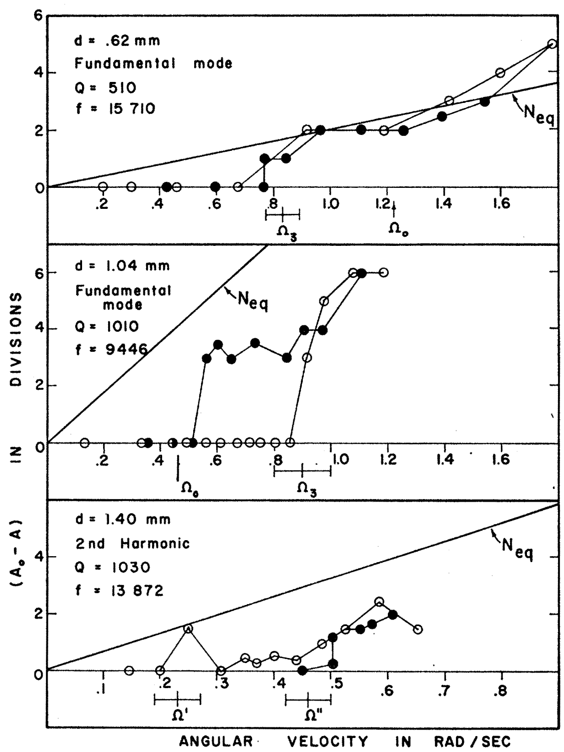

FIG. 2. Experimental curves, showing the reduction in amplitude of the second sound in divisions on the strip chart recorder, as a function of angular velocity. Each division corresponds to a reduction of 1 part in 350 in the second-sound amplitude. The open circles were taken while increasing the angular velocity, and the solid circles while decreasing the angular velocity. The straight lines labeled N_{eq} are approximately the attenuation from an equilibrium density N_{eq} of vortex lines, and their slopes are uncertain by $\pm 20\%$. The experimental error on the points is ± 0.5 divisions. The error limits on Ω', Ω'', and Ω_3 are taken from Table I. The frequencies f are in cps.

The critical velocity Ω_3 appears to be smaller than Ω_0 in the 0.62-mm annulus. Since this is the narrowest annulus, we may be seeing a transition set off by the relatively larger perturbing effects of mechanical imperfections. It seems more reasonable to us that the strong attenuation which occurs before Ω_0 is reached is due to the same phenomena which occur at Ω_3 in the wider annuli.

van Alphen *et al.*[6] have reviewed superfluid critical velocity measurements in channels varying in width from 10^{-7} cm (film flow) to ~ 1 cm, and find they can be correlated by the empirical relation

$$v_{sc} = C' d^{-1/4}, \qquad (7)$$

where C' is a constant of order 1 cm$^{5/4}$/sec, and d is in cm. We refer to Eq. (7) as the Leiden critical velocity, and we believe $v_{sc} = \bar{R}\Omega_3$ is an example of it. The Leiden critical velocity for "wide channels" is based on those experiments in which the normal fluid component did

[11] This is sometimes referred to as the "Feynman critical velocity," since the idea was suggested by R. P. Feynman; see *Progress in Low Temperature Physics*, edited by C. J. Gorter (North-Holland Publishing Company, Amsterdam, 1955), Vol. I, p. 17.

TABLE II. Flow transitions in five narrow annuli, at 1.40°K. The outside radius was 1.56 cm. Values of Ω_0 were calculated by Eq. (1) using $a=1.3\times10^{-8}$ cm. The numbers in parentheses are the number of times the transition was observed. The experimental errors indicate reproducibility. The critical velocity Ω_3 was observed using the second-sound fundamental mode; an error of ±0.1 rad/sec was assigned when Ω_3 was measured only once. The two values for Ω' in the 0.82-mm annulus were both observed on the same runs; the high value is presumed due to Ω_3.

Annulus width (mm)	Ω_0 (rad/sec)	Ω' (rad/sec)	Ω'' (rad/sec)	Ω_3 (rad/sec)	Ω''/Ω'
0.62	1.22	0.83±0.06(4)	0.84±0.06(4)	0.83±0.08(4)	
0.82	0.72	0.74±0.08(3)	0.95±0.06(4)	0.94±0.1 (1)	
		0.94±0.06(4)			
1.04	0.46	0.49±0.05(4)	0.91±0.05(4)	0.90±0.1 (1)	1.9±0.3
1.40	0.25	0.23±0.04(7)	0.46±0.04(6)	0.74±0.1 (1)	2.0±0.4
1.90	0.14	0.14±0.04(7)	0.26±0.04(7)	0.65±0.1 (3)	1.8±0.5

not move relative to the walls. This is also true of our rotating annulus, since, because of viscosity, the normal fluid was in solid-body rotation with the annulus.

V. DISCUSSION OF THE MEASUREMENTS

The top panel in Fig. 2 shows attenuation we observed on one run with the 0.62-mm annulus. In addition to occurring at angular velocities less than Ω_0, the attenuation is approximately equal to the amount expected from an equilibrium density of lines N_{eq}, and more than expected from a single row of vortex lines. If superfluid turbulence occurred at Ω_3, the density of vortex lines must have been larger than N_{eq}, since the vortex lines in turbulence are not oriented perpendicular to the direction of second-sound propagation, and the attenuation is smaller for all other orientations.[12]

The middle panel in Fig. 2 is an example of a first run after cooling below T_λ. It shows that attenuation, which first occurs at Ω_3 with increasing Ω, continues to be observed when Ω is decreased below Ω_3. Since Ω'' is approximately twice Ω' for this annulus width, we suppose the solid circles above Ω_0 show attenuation by a single row of vortex lines.

The lowest panel in Fig. 2 is an example of what we often observed with the second harmonic. At Ω' we observed one point which shows attenuation; this may be due to transient phenomena associated with the creation of the first row of vortices. At larger angular velocities, the attenuation is within experimental error of being zero; above 0.45 rad/sec, it increases steadily. Here the second harmonic appears to be gradually attenuated as additional vortex lines are established away from the median radius. The value of Ω'' is obtained by extrapolation back to the onset of this effect.

The results of our measurements, all at 1.40°K, are tabulated in Table II, and shown in Fig. 3. The curves in Fig. 2 show 6 of the 64 observations of flow transitions used to compile Table II. There were two breaks in the attenuation curve for the fundamental mode for the 0.82-mm annulus. The much stronger break occurs at 0.94 rad/sec, and we identify it with Ω_3. After careful searching, we detected a weak break at 0.74 rad/sec, which we identify with Ω_0. The onset of attenuation of the second harmonic in the 0.82- and 1.04-mm annuli is probably due to the transition at Ω_3. As previously noted, we explain both Ω' and Ω'' in the 0.62-mm annulus as due to the transition at Ω_3.

The good agreement between Ω' and Ω_0 in the four widest annuli is evident in Fig. 3, and shows that a mechanism sometimes exists by which the liquid helium can reach the lowest free-energy state. We suggest that this mechanism is residual vorticity, and if the liquid has been undisturbed, this vorticity is not generated until Ω equals Ω_3. As we observed in an earlier experiment,[13] the residuum of the vorticity generated at Ω_3 has a long lifetime, which is of the order of an hour in an annulus at rest, and probably indefinite in an annulus intermittently rotated above Ω_0.

In the three widest annuli, Ω'' was approximately equal to 1.9 Ω', as reported in Table I. This shows that the vortex lines which attenuate the fundamental mode at Ω' are located near the *median radius* of the annulus, where the second harmonic has a velocity node, and where Fetter predicted they would occur. Because the attenuation of the second harmonic increases gradually with angular velocity, a more sensitive experiment might show smaller values of Ω''.[14]

In Fetter's theory,[4] Ω_0 does not depend on ρ_s/ρ, so the measurements reported here are not expected to be temperature-dependent, except possibly close to T_λ. Some of our early measurements of Ω' in the 1.40-mm annulus were made at five temperatures between 1.25 and 1.85°K. We observed no change in Ω' with temperature, but this negative result may have been due to the lower sensitivity we had before we improved our experimental technique.

In Fig. 3 we show with a dotted line a Leiden critical velocity Ω_L, for comparison with Ω_3. The constant C' in Eq. (7) was treated as a free parameter, and the value 0.7 gave the best fit to the data. The equation for Ω_L is

$$\Omega_L=0.7/\bar{R}d^{1/4}, \tag{8}$$

where d and $\bar{R}$ are in centimeters. The agreement with van Alphen *et al.*[6] is considered satisfactory, since C' was

[12] H. A. Snyder and Z. Putney, Phys. Rev. **150**, 110 (1966).

[13] P. J. Bendt, Phys. Rev. **127**, 1441 (1962).

[14] Stauffer and Fetter have recently calculated that $\Omega''/\Omega'\approx 1.85$, in agreement with our experimental result. See D. Stauffer and A. L. Fetter, Phys. Rev. (to be published).

not accurately determined by the experiments they considered.

The attenuation of second sound at a given angular velocity was determined by alternately observing the amplitude when the annulus was rotating, and when it was at rest. Transient attenuation was usually observed each time rotation was started and stopped; the transients decayed away in about 2 min. At angular velocities larger than Ω', attenuation of the fundamental mode was present when the transient disappeared. When we did not see attenuation after the transient decayed, we continued to observe the amplitude in steady rotation for at least 10 min. There was a small amount of hystersis in Ω'; smaller than the experimental errors quoted in Table I. There was large transient attenuation when $\Omega > \Omega_3$, and the attenuation was still large after 10 min, even when $\Omega_3 < \Omega_0$, as shown in the top panel of Fig. 2. We did not undertake a systematic study of time-dependent or transient effects.

When the electronics were sufficiently stable, we observed another phenomena with the 1.04- and 1.40-mm annuli. The second-sound amplitude when the annulus was stopped was two or more divisions larger than the attenuated amplitude when the annulus was rotating at $\Omega > \Omega'$. When we reduced Ω below Ω', we observed equal amplitudes, but this constant amplitude was approximately midway between the previous "stopped" and "rotating" amplitudes. In addition, no transients were observed when the rotation was started and stopped. Upon increasing Ω through Ω', we again observed the transients, and we observed the former "stopped" and "rotating" amplitudes. Since this effect could be due to electronic drift, we are less confident about the observation of this effect than we are of the transitions shown in Fig. 3.

We now speculate on a possible explanation of the above effect. We suggest that above Ω', the row of vortex lines provides coupling between the annulus and the superfluid, so that when the annulus stops rotating, the superfluid is also stopped. Below Ω', a long-lived superfluid current exists, which does not decay appreciably within 10 min after stopping the annulus. This long-lived current is assumed to be irrotational flow around the inner cylinder with initial circulation approximately equal to the value predicted by Fetter[4] for the equilibrium state when $\Omega_c < \Omega < \Omega_0$. The irrotational flow will not attenuate second sound; however, the superfluid velocity of the long-lived current relative to the walls of the annulus was greater than $\Omega_c \bar{R}$ when the annulus was at rest, and also when the annulus was rotating. This latter situation is due to the fact that the wall velocity while rotating is proportional to R, while the irrotational flow is proportional to $1/R$. Therefore, vortex lines could be generated at the walls by the long-lived current, and could account for the small attenuation of second sound which distinguished the current from the superfluid at rest.

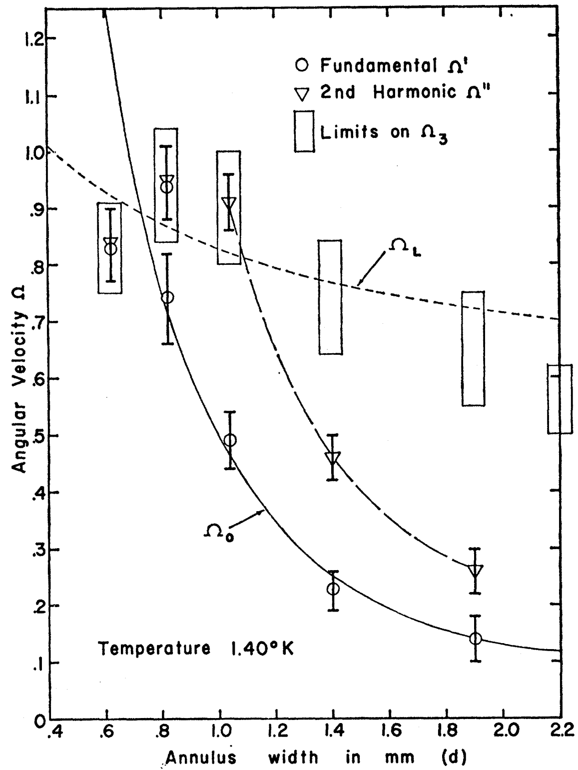

FIG. 3. Angular velocities of the flow transitions in rad/sec, as a function of the width of the annuli in mm. The outside radius of all the annuli was 1.56 cm. The values of Ω_0 were calculated from Eq. (1), using $a = 1.3 \times 10^{-8}$ cm. The Leiden critical velocity Ω_L was calculated from Eq. (8). The value of Ω_3 for a 2.2-mm annulus corresponds to the linear critical velocity observed in an earlier experiment [Phys. Rev. **127**, 1441 (1962)].

We speculate further and consider the situation where the annulus is rotating at Ω, but the superfluid is at rest. We suggest that the difference in flow transitions at Ω_c and Ω_3 is that, while vortex lines can be generated at the walls at Ω_c, superfluid turbulence is generated at Ω_3. We further suggest that the much greater vorticity present in turbulence is necessary to provide sufficient residual vorticity to nucleate the row of vortex lines which appear at Ω_0.

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