and $\langle H\rangle$ is the energy expectation value calculated above for the hypothetical boson $He^3$ system. Figure 3 shows $S(k)$ obtained by SV and MW at $\rho=0.75\rho_0$, the experimental equilibrium density. Again the agreement is excellent. Table IV compares the calculated equilibrium results. Actually SV varied their $\psi_0{}^B$ in this calculation and obtain $E^{(0)}=-1.35°K/N$. What we have quoted in Table IV are the results they would have obtained had they used our approximation that $\psi_0{}^B$ is the *exact* solution of the boson $He^3$ problem. This approximation is seen to be very good. Note the dominating effect of $\langle H\rangle$ on $E^{(0)}$. Note also in our calculation that as the Fermi statistics gets "turned on," the equilibrium density drops (from $0.72\rho_0$ to $0.64\rho_0$), an obvious consequence of the Pauli repulsion.

In Ref. 1 we worked out an extension of this variational calculation to include a perturbation correction in second order. The correction depends solely on $\rho$ and $S(k)$, and amounts to $-0.33°K/N$. Since the SV calculation offers $\rho_0$ and $S(k)$ very similar to those obtained by us, we expect the second-order correction to their variational energy to be of roughly the same magnitude. In other words, starting with their $\psi_0{}^B$, we can construct a correlated basis and obtain a perturbed energy of roughly $-1.7°K/N$, in much better agreement with experiment than all previous calculations. The remaining discrepancy of about $0.8°K/N$ may again be a consequence of the absence of higher-order correlations in the trial function and the uncertainty in the two-body potential $v(r)$.

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# Order of the He⁴ II–He⁴ I Transition under Rotation

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Purely thermodynamic considerations show that the volume discontinuity at the $He^4$ II → $He^4$ I transition under rotation reported by Andronikashvili and Tsakadze cannot correspond to a transition between thermodynamic equilibrium states if $(\partial P/\partial T_\lambda)_\omega<0$. Experimental results are presented which show that at the transition $|\Delta H|\leqslant 6\times10^{-4}$ J/mole and $|\Delta V|\leqslant 3\times10^{-5}$ cc/mole at $\omega=30$ sec$^{-1}$. Thus, within these limits, the transition is of higher than first order.

## INTRODUCTION

PYCNOMETRIC measurements for rotating $He^4$ II reported by Andronikashvili and Tsakadze (AT)[1] showed that the molar volume $V(He^4\,II)$ decreases with increasing angular velocity $\omega$. In somewhat later communications,[2] AT presented measurements which indicated that $V(He^4\,I)$ is independent of $\omega$, and that at $\omega>0$, the molar volume is discontinuous at the transition temperature $T_\lambda$. Very recent attempts by Smith *et al.*,[3] and by Andelin[4] to reproduce AT's results below the transition temperature by slightly different techniques were unsuccessful. In a brief previous communication[5] this author reported on thermal measurements which revealed no dependence of the entropy of $He^4$ II on angular velocity.

No quantitative conclusions can be drawn regarding the dependence of the internal energy of $He^4$ II upon $\omega$ from AT's earlier measurements.[1] However, at the transition the Clausius-Clapeyron relation yields a large entropy discontinuity from AT's latter measurements. Therefore, the absence or presence of the reported first-order contribution to the transition is particularly important, because the corresponding entropy discontinuity indicates a dependence of the internal energy of $He^4$ II, immediately below the transition, upon $\omega$ which exceeds the angular momentum contribution expected for solid-body rotation by three orders of magnitude.

*Note added in proof.* Very recently Pobell *et al.* [Phys. Letters **25A**, 209 (1967)] reported on measurements somewhat similar to those reported here. However, they attempted to relate changes in entropy with $\omega$ to changes in volume with $\omega$ at temperatures *well below* $T_\lambda$. Therefore, it seems worthwhile to emphasize that with available experimental information,[1,2] this is possible only in the *immediate vicinity* of $T_\lambda$. The conclusion that $\Delta\rho/\rho\leq 2\times10^{-6}$ for the measurements of Pobell *et al.* at $1.80°K$ and $\omega=110$ sec$^{-1}$, which is based on thermal measurements, is thus not justified.

It is the purpose of this paper to present the results of thermal measurement, particularly in the immediate

[1] E. L. Andronikashvili, and J. S. Tsakadze, Phys. Letters **18**, 26 (1965); Zh. Eksperim. i Teor. Fiz., Pis'ma v Redaktsiyu **2**, 278 (1965) [English transl.: JETP Letters **2**, 177 (1965)].
[2] E. L. Andronikashvili and J. S. Tsakadze, Phys. Letters **20**, 446 (1966); Zh. Eksperim. i Teor. Fiz. **51**, 1344 (1966) [English transl.: Soviet Phys.—JETP **24**, 907 (1967)]; E. L. Andronikashvili, in *Proceedings of the Tenth International Conference on Low Temperature Physics* (Atomizdat, Moscow, 1966), Abstracts.
[3] E. Smith, R. Walton, H. V. Bohm, and J. D. Reppy, Phys. Rev. Letters **18**, 637 (1967); Bull. Am. Phys. Soc. **12**, 551 (1967).
[4] J. Andelin, Phys. Rev. Letters **18**, 483 (1967); Bull. Am. Phys. Soc. **12**, 552 (1967).
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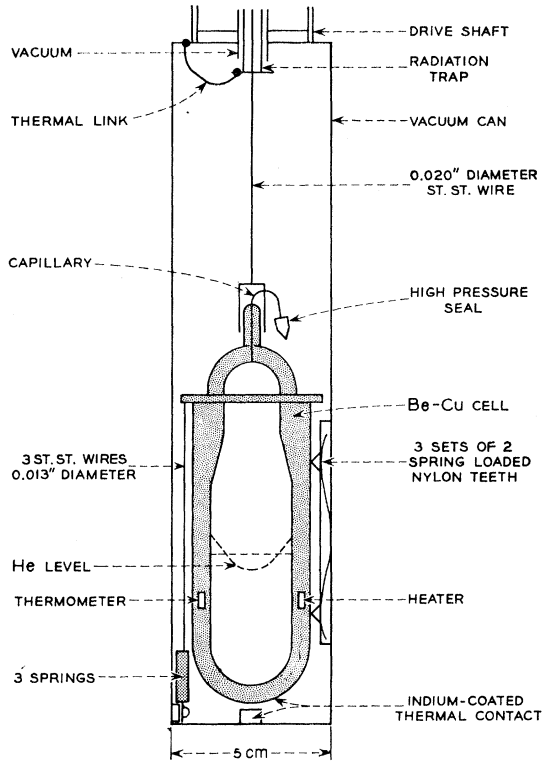


FIG. 1. Lower portion of experimental apparatus.

vicinity of the transition temperature, which show that any latent heat associated with the transition at $\omega=30$ sec$^{-1}$ is less than $6\times10^{-4}$ J/mole, and any volume discontinuity is less than $3\times10^{-5}$ cc/mole. However, first it will be shown on the basis of purely thermodynamic considerations that the discontinuous positive volume change at the transition of rotating $He^4$ II to rotating $He^4$ I reported by AT[2] is inconsistent with a transition between two thermodynamically stable phases if the pressure derivative of the transition temperature remains negative under rotation.

## THERMODYNAMIC CONSIDERATIONS

AT's measurements[2] indicate that at $T_\lambda$ $V(He^4\,II,\omega)<V(He^4\,I,\omega)$ if $\omega>0$, where $V$ is the molar volume of the indicated phase at angular velocity $\omega$. Since the pressure derivative of $T_\lambda$ is negative (it is assumed that this is at least qualitatively not altered by the rotation), it follows that, if AT's measurements were made for thermodynamic equilibrium states of the system, the change at $T_\lambda$ in the entropy $\Delta S(\omega)=S(He^4\,I,\omega)-S(He^4\,II,\omega)=\Delta V(\partial P/\partial T_\lambda)_\omega<0$. Thus, the enthalpy of transition is negative, and the enthalpy and entropy of $He^4$ I near the transition are *less* than those of $He^4$ II. This situation is not acceptable since it excludes the possibility of a quasistatic transition at constant $\omega$ from the low-temperature phase to the high-temperature phase. In fact, its acceptance would imply that $He^4$ II under rotation is not stable at $T_\lambda$ since arbitrarily small fluctuations in $T$ or $\omega$ would cause a spontaneous transition to He I at a temperature *greater* than $T_\lambda$. Further, $He^4$ II under rotation immediately below the transition would not be stable with respect to supercooled $He^4$ I. One, therfore, has to conclude that at least the low-temperature phase studied by AT was either metastable, a dynamic steady state, or an open system. In view of this, it is not surprising that the efforts of Smith *et al.*,[3] and Andelin[4] to reproduce AT's results and this author's efforts to measure a finite effect of rotation on the entropy of $He^4$ II[5] below $T_\lambda$ have not met with success. It also seems unlikely that DiCastro and Tabet's[6] model of rotating $He^4$ II is applicable to the real liquid near $T_\lambda$. Nonetheless, because a small positive $\Delta S(\omega)$ has not been ruled out, either experimentally or on the basis of thermodynamics, an investigation of the thermal properties of $He^4$ under rotation near $T_\lambda$ was undertaken.

## APPARATUS

The lower portion of the apparatus is shown in Fig. 1. The sample was permanently sealed in a beryllium-copper cell of 2.7 cm i.d. at a pressure of about 150 bar at 78°K. At low temperatures the liquid occupied about 50% of the cell volume. The amount of sample was determined to be 3.19 g by weighing the empty and filled cell. The cell was centered in the vacuum can by means of three pairs of nylon teeth which were spring loaded against their individual housings. The spring loading was necessary to avoid permanent deformation of the teeth when the vacuum can was suddenly cooled by liquid nitrogen. The interference between each tooth and the cell at 300°K was $5\times10^{-3}$ cm, and is calculated to be zero at low temperatures. The geometric axis of the cell coincided with the axis of the vacuum can within $3\times10^{-3}$ cm. The cell could be raised or lowered by a 0.051-cm-diam stainless-steel wire, and in the lowered position its indium-coated bottom was pulled with a 7 kg force by three springs against a cold indium-coated surface. Approximately 3 h were required to cool the sample from 4 to 2°K when the bath temperature was 1.3°K, provided thermal contact had been made at room temperature.

The lower assembly could be rotated by a hardened stainless-steel drive shaft. This shaft rotated in two ball bearings. The upper bearing was self-aligning and operated near 300°K. The lower bearing operated below 100°K and was not lubricated. The metal retaining ring was replaced by a heavy Teflon retaining ring. Although at the relatively low velocities a metal retainer did not cause excessive wear, it tended to vibrate, causing irreproducible heating of the sample. Initial attempts to use dry lubricants likewise resulted in mechanical noise and irreproducible heating. The shaft could be radially translated in the lower bearing. This made it possible to assure that the shaft rotated about its center at its lower end. The axis of the vacuum can could be tilted

[6] C. DiCastro and E. Tabet, Phys. Letters **23**, 675 (1966).

with respect to the shaft axis, so that it was possible to assure that the vacuum can was rotating about its axis at its bottom. Deviations of the rotation axis of the vacuum can from the geometrical axis of the system were less than $3\times10^{-3}$ cm at low speed, and less than $8\times10^{-3}$ cm at $\omega=30$ sec$^{-1}$. The shaft was driven by a nylon positive drive belt which minimized mechanical heating due to motor vibration. The angular velocity was determined by measuring electronically the time required to complete one revolution.

Eight electrical leads were brought to the top through a vacuum line located in the center of the drive shaft. The signals were brought out of the rotating system through mercury contacts. Initially, it was attempted to make dc measurements. However, thermal voltages of about $10^{-5}$ V were encountered, and these were not stable under rotation. This problem was completely overcome by using an ac bridge with a PAR model HR-8 phase-sensitive detector operating at 38 cps. The system was checked out on the temperature-independent manganin heater, which had about the same resistance (6000 Ω) as the carbon thermometer at $T_\lambda$. The rms noise was equivalent to $6\times10^{-3}$ Ω at a power dissipation (in the 6000 Ω at low temperature) of $2\times10^{-8}$ W and $\omega=0$, and increased by a factor of 2 at $\omega=20$ sec$^{-1}$. No change with $\omega$ in the measured resistance was noticeable. Equivalent performance with the thermometer in the circuit should yield a temperature resolution of $10^{-6}$°K even under rotation. However, the resolution of the measurements was limited to $10^{-5}$°K because of drifts caused by mechanical heat input to the sample during rotation.

## RESULTS

The thermometer resistance as a function of time for a typical measurement at a velocity of 42 sec$^{-1}$ is shown at 1 min intervals in Fig. 2. Prior to and after rotation, there is a small, constant drift due to imperfect thermal isolation. Within 1 sec after rotation was started, a resistance change equivalent to $+10^{-4}$°K occurred. This is believed to be an experimental problem and can be explained on the basis of an additional heat input to the thermometer of $2\times10^{-8}$ W during rotation. A similar

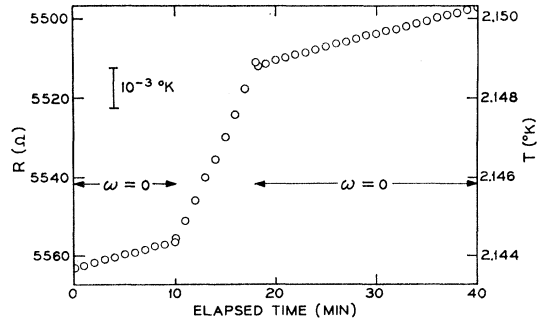


FIG. 2. Thermometer resistance as a function of time (arbitrary origin) for $He^4$ II before, during, and after rotation at 42 sec$^{-1}$.

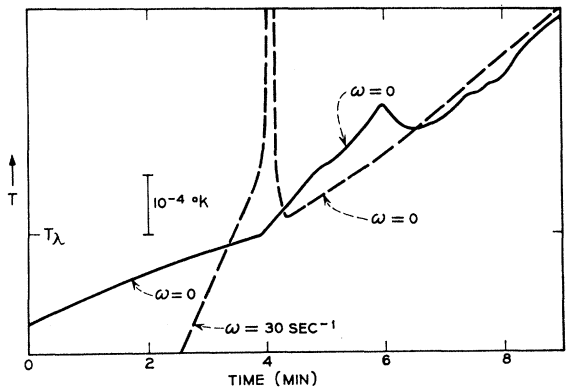


FIG. 3. Heating curves through $T_\lambda$. Solid line-$\omega=0$. Dashed line-$\omega=30$ sec$^{-1}$ until 15 sec after $T_\lambda$ was reached, and $\omega=0$ thereafter. See text for discussion.

change in the opposite direction appeared when rotation was stopped. It seems unlikely that this effect reflects the properties of the sample because of the very short time constant. Time constants for setting the liquid into rotation should be of the order of 1 or 2 min, this estimate being based on measurements by others at similar velocities.[7] During rotation, the temperature rose at a rate corresponding to a constant heat input because of additional mechanical heating. No time-dependent cooling or heating was observed. After rotation was stopped, the drift corresponded within $2\times10^{-5}$°K over a 10 min period to the same constant heat input that was observed before rotation was started. These observations were made at several temperatures between 1.8°K and the transition temperature, and at several angular velocities up to 42 sec$^{-1}$. For some runs, rotation was continued for 20 min and terminated when the temperature was within $10^{-4}$°K of $T_\lambda$. Any measurable nonzero value of $(\partial T/\partial\omega)_S$ would have resulted in thermal effects over the first 1 or 2 min after rotation of the cell was started and stopped. From the linearity of the drift after rotation was stopped, it can be concluded that $|(\Delta T/\Delta\omega)_S| \leqslant 0.5\times10^{-6}$°K sec. The known heat capacity $C$,[8] and the relation $(\partial S/\partial\omega)_T = -(C/T)(\partial T/\partial\omega)_S$, yield $|(\Delta S/\Delta\omega)_T| \leqslant 10^{-5}$ J sec/mole°K. Thus, the corresponding contribution to the internal energy is quite small over the entire temperature range between 1.8°K and $T_\lambda$, although, on the basis of the present resolution, it could still be slightly greater than the angular momentum contribution, assuming solid-body rotation.

Heating curves were also taken through the transition temperature. The solid line in Fig. 3 was taken with $\omega=0$. It shows the typical effect of poor convective heat transfer in the $He^4I$ region. The dashed curve in the

[7] For a review, see E. L. Andronikashvili and Yu. G. Mamaladze, Rev. Mod. Phys. 38, 567 (1966).

[8] W. M. Fairbank, M. J. Buckingham, and C. F. Kellers, in *Proceedings of the Fifth International Conference on Low Temperature Physics and Chemistry*, edited by J. R. Dillinger (University of Wisconsin Press, Madison, Wisconsin, 1958), p. 50.

$He^4$ II region shows the effect of the additional mechanical heating due to rotation at 30 $sec^{-1}$. The transition temperature is artificially displaced by $10^{-4}$°K because of the discontinuity discussed above. It was found that upon passing through the transition, thermal equilibrium becomes *extremely* poor, probably because the rotational field prevents convection. In the particular example of Fig. 2, rotation was stopped 15 sec after the transition had been reached. Heat transfer became very good, as can be seen from the dashed line in the $He^4$ I region. This is probably caused by rapid convection initiated by the collapse of warm (and therefore more dense) $He^4$ I accumulated near the outside by the rotational field. The ratio between the slopes of the dashed line above $T_\lambda$, and the solid line below $T_\lambda$, is consistent with the known heat-capacity ratio[8] and a constant background heating rate at $\omega=0$. Several determinations of $T_\lambda$, with $\omega=0$, and of the minimum temperature $T_{\min}$ when rotation at 30 $sec^{-1}$ was stopped $\Delta t$ seconds ($\Delta t=2$ to 15) after the transitions, were made. With $\Delta t$ equal to 15 sec, the minimum temperature after stopping the rotation was $3\times10^{-5}$°K above $T_\lambda$. $T_{\min}$ as a function of $\Delta t$, extrapolated to $\Delta t=0$, agreed with $T_\lambda$ ($\omega=0$) within $1\times10^{-5}$°K. A negative heat of transition (from a metastable state), such as implied by AT's results,[2] predicts that the minimum temperature after the transition would be $5.2\times10^{-3}$°K greater than $T_\lambda$. A positive enthalpy of transition would cause $T_{\min}$ to be independent of $\Delta t$, and equal to $T_\lambda$ (it is assumed that the entropy of $He^4$ I is independent of $\omega$). It can be concluded that any enthalpy of transition at $\omega=30$ $sec^{-1}$, is less than the change in enthalpy of $He^4$ I over a $10^{-5}$°K temperature range immediately above $T_\lambda$, i.e., $|\Delta H| \leqslant 6\times10^{-4}$ J/mole.[8] It follows that any volume discontinuity is smaller than $3\times10^{-5}$ cc/mole or $10^{-4}$% of $V$ if $(\partial P/\partial T_\lambda)_\omega=(\partial P/\partial T_\lambda)_{\omega=0}$. This is one order of magnitude smaller than the resolution of AT's measurements, and a factor of 200 smaller than the $\Delta V$ reported by AT.[2] Therefore, within the experimental resolution of this work, the $He^4$ II $\rightarrow$ $He^4$ I transition under rotation is of higher than first order.

The shift in transition temperature at $\omega=30$ $sec^{-1}$, is less than $10^{-5}$°K, as expected from the phenomenological theory of liquid $He^4$ II near $T_\lambda$.[7,9]

## ACKNOWLEDGMENTS

The author is grateful to F. W. Martin for the detailed design of the apparatus, and to J. H. Condon and P. C. Hohenberg for stimulating discussion. J. F. Macre designed the circuit used to measure $\omega$.

[9] Yu. G. Mamaladze, Zh. Eksperim. i Teor. Fiz. **52**, 729 (1967).

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# Superfluid Flow Transitions in Rotating Narrow Annuli*

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The attenuation of second sound has been used to observe helium-II flow transitions in five annuli in solid-body rotation at 1.40°K. The annuli were 0.6 to 1.9 mm wide, and approximately 15 mm in radius. The onset of attenuation of second sound at the angular velocity $\Omega_0$ predicted by Fetter for the creation of a single row of vortex lines established that liquid helium in rotation is able to attain the state of lowest free energy. The onset of attenuation of the second harmonic, which has a velocity node at the middle of an annulus, showed that vortices can be detected away from the median radius at approximately 1.9 $\Omega_0$. In the range $0.65<\Omega<0.95$ rad/sec, a critical velocity $\Omega_3$ was observed through two effects: First, in annuli in which $\Omega_0<\Omega_3$, it was usually necessary to rotate faster than $\Omega_3$ before the helium would display second-sound attenaution at $\Omega_0$. Second, in the narrowest annulus, where $\Omega_3<\Omega_0$, strong attenuation occurred at $\Omega_3$ even though the equilibrium state is vortex-free irrotational flow. The transition at $\Omega_3$ is identified with the creation of long-lived vorticity in the helium.

## I. INTRODUCTION

IN 1946, London[1] proposed that liquid helium in a cylindrical vessel should show a rotational Meissner effect, which has recently been observed by Hess and Fairbank.[2] The flow of superfluid in a slowly rotating annulus is a more complex problem, which was first considered by Bendt and Oliphant.[3] In a recent elegant free-energy calculation, Fetter[4] determined the critical

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[1] H. London, *Report on an International Conference on Fundamental Particles and Low Temperatures*, (The Physical Society, London, 1947), Vol. 2, p. 48; F. London, *Superfluids* (John Wiley & Sons, Inc., New York, 1954), Vol. 2, p. 144.

[2] G. B. Hess and W. M. Fairbank, Phys. Rev. Letters **19**, 216 (1967).

[3] P. J. Bendt and T. A. Oliphant, Phys. Rev. Letters **6**, 213 (1961).

[4] A. L. Fetter, Phys. Rev. **153**, 285 (1967); R. J. Donnelly and A. L. Fetter, Phys. Rev. Letters **17**, 747 (1966).