If we assume $\Lambda_0^2>\Lambda_c^2$, then $\bar{\Lambda}$ must be near Λ_c and we have approximately

$$\Lambda^2-\Lambda_c^2\simeq F\ln\left(\frac{4}{1-\bar{\Lambda}^2/\Lambda_c^2}\right)=F\ln 4\bar{K} \quad (6.20)$$

or

$$\bar{K}=\tfrac{1}{4}\exp\left(\frac{\Lambda_0^2-\Lambda_c^2}{F}\right). \quad (6.21)$$

Thus for power levels above threshold, i.e., $\Lambda_0^2>\Lambda_c^2$, the amplification factor $\bar{K}$ increases exponentially with Λ_0^2 but always remains finite for finite Λ_0^2.

7. DISCUSSION

The inclusion of self-consistency requirements for the pump field is thus sufficient to make the three-mode parametric model finite. However, nonlinear longitudinal mode coupling effects which we have neglected will undoubtedly reduce (perhaps greatly) the steady state amplitude which can be experimentally attained. The principal mode coupling effects are probably further three-mode couplings of the type responsible for the parametric effect itself. The parametrically excited plasmons can themselves act as a pump which couples to another plasma mode and another ion acoustic mode. These secondary processes will not be as strong as the original parametric coupling since the pump energy is now spread over a number of spatial ($\mathbf{k}$) modes and since the frequency-matching conditions for the secondary pump plasmons will not be optimum.

A complete treatment of the effect of longitudinal mode coupling on the saturation level has not yet been carried out.

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Collisionless Sound in Classical Fluids*

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The dynamic form factor $S(K,\omega)$ for a classical fluid is calculated from the linearized Vlasov equation. Following Percus or Zwanzig, the effective interatomic potential is taken as $-kTc(r)$, where $c(r)$ is the direct correlation function. The result for $S(K,\omega)$ is a simple closed expression with no free parameters except for the static structure factor $S(K)$. Using Ashcroft and Lekner's hard-sphere Percus-Yevick results for $S(K)$, we calculate the inelastic neutron scattering from liquid lead. The resulting scattering law shows a strong qualitative similarity with experiment. The narrow quasielastic peak observed experimentally is not, however, given by the calculation. The reasons for this discrepancy are discussed. An extension of the calculations to include a phenomenological collision term is also presented.

I. INTRODUCTION

RECENT slow-neutron inelastic-scattering experiments have shown a surprising persistence of phononlike excitations in the liquid state. The dynamic form factor $S(K,\omega)$ exhibits a structure associated with propagating sound waves in a variety of liquids including liquid helium above and below the lambda point[1] as well as classical liquids.[2] This naturally suggests that a mean field theory would provide a useful phenomenological description of such experiments.[3] In the present paper we present some simple calculations demonstrating that this is in fact the case for classical fluids.

The familiar classical limit of a mean field theory is the Vlasov equation. This equation has long been used[4] to calculate the dynamic form factor associated with electron density fluctuations in a plasma. To use the Vlasov equation in neutral fluids we must replace the actual interatomic potential by an appropriate effective potential. The desired substitution is $v(r)\rightarrow -kTc(r)$, where $c(r)$ is the direct correlation function. This replacement was first suggested by Percus and Yevick.[5] When used in a self-consistent way it leads to the well-known Percus-Yevick integral equation for the radial distribution function. The same replacement has been obtained by Zwanzig[6] from consideration of variational expressions for eigenfunctions of the Liouville equation. Within the context of the Vlasov equation, the replace-

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[1] A. D. B. Woods, Phys. Rev. Letters **14**, 355 (1965).
[2] P. A. Egelstaff, Rept. Progr. Phys. **29**, 333 (1966).
[3] D. Pines, in *Quantum Fluids*, edited by D. F. Brewer (John Wiley & Sons, Inc., New York, 1966).
[4] E. E. Salpeter, Phys. Rev. **120**, 1528 (1960). (There is a large body of subsequent literature on this subject.)
[5] J. K. Percus, in *The Equilibrium Theory of Classical Fluids*, edited by H. L. Frisch and J. L. Lebowitz (W. A. Benjamin, Inc., New York, 1964); see in particular Appendix A, p. II-142.
[6] Robert Zwanzig, Phys. Rev. **144**, 170 (1966).

ment of $v(r)$ by $-kTc(r)$ can be derived very simply from sum rule arguments, as we show in Appendix A of this paper.

The modified Vlasov equation leads to a form of collisionless sound which has already been discussed by Zwanzig.[6] This sound is strongly damped by thermal motion. Its dispersion relation is not sufficient to describe neutron-scattering experiments. We require an explicit solution for $S(K,\omega)$, the frequency spectrum of density fluctuations with wave number K. This solution is well known in the plasma problem,[4] and can be directly transcribed to the fluid.

In Sec. II we present the basic result for $S(K,\omega)$ [Eq. (13)] and examine some of its properties. The main interest of the result is its simplicity. The only input quantity needed is the static structure factor

$$S(K)=\int_{-\infty}^{\infty} S(K,\omega)d\omega.$$

For $S(K)$ we use the analytic expression obtained by Ashcroft and Lekner[7] from the Percus-Yevick equation for hard spheres. This is known to give a fairly good fit to the measured structure factors of liquid metals. Applying our result to liquid lead at 352°C a surprisingly good qualitative description of the observed[8] scattering law $S(K,\omega)$ is obtained. The main features of the structure due to excitation of highly damped collective oscillations are given in a way similar to the experiment, but the calculation does not contain any of the observed narrow quasielastic peak.

The results have been extended to include a momentum- and energy-conserving collision term. This leads to a sensible, though not exactly correct, hydrodynamic limit. With a reasonable choice of collision frequency, however, the behavior of $S(K,\omega)$ in the regime currently explored by neutron-scattering experiments is not appreciably changed by the inclusion of collisions. The extension to include collisions is discussed in Appendix B.

We conclude in Sec. III with a discussion of the physical nature of the approximations made. In particular the choice of $-kTc(r)$ as an effective potential in the Vlasov equation is further examined.

II. VLASOV-EQUATION SOLUTION FOR $S(K,\omega)$

Through the use of linear-response theory and the fluctuation-dissipation theorem[9] the equilibrium time-dependent density fluctuations which determine slow neutron scattering are simply related to the relaxation of an externally induced density disturbance. We are thus concerned with a perturbed one-particle distribution function

$$f(\mathbf{r},\mathbf{v},t)=n_0\phi_M(v)+f^{(1)}(\mathbf{r},\mathbf{v},t), \tag{1}$$

where n_0 is the equilibrium density,

$$\phi_M(v)=(\pi v_0^2)^{-3/2}\exp(-v^2/v_0^2)$$

is the equilibrium Maxwellian velocity distribution, and $v_0^2=2kT/m$. The appropriate initial condition is

$$\tilde{f}^{(1)}(\mathbf{K},\mathbf{v},0)=\phi_M(v)S(\mathbf{K}), \tag{2}$$

where

$$\tilde{f}^{(1)}(\mathbf{K},\mathbf{v},t)=\int e^{-i\mathbf{K}\cdot r}f^{(1)}(\mathbf{r},\mathbf{v},t)d^3r, \tag{3}$$

and $S(K)$ is the structure factor of the fluid. This corresponds to an initial density disturbance $\delta(r)+n_0g(r)$ associated with localizing an atom at the origin at $t=0$. The dynamic form factor $S(K,\omega)$ is given by

$$S(K,\omega)=2\,\mathrm{Re}\lim_{\epsilon\to 0^+}Z(K,i\omega+\epsilon), \tag{4}$$

where

$$Z(K,p)=\int_0^{\infty}e^{-pt}\int\tilde{f}^{(1)}(\mathbf{K},\mathbf{v},t)d^3vdt. \tag{5}$$

As an approximation for calculating $f^{(1)}$ we use the linearized Vlasov equation. Letting u be the component of velocity along $\mathbf{K}$, and integrating out the other two components, this equation is[10]

$$\left(\frac{\partial}{\partial t}+iKu\right)F^{(1)}(K,u,t)-iKn_0V(K)\frac{dF^{(0)}}{du}\times\int_{-\infty}^{\infty}F^{(1)}(K,u,t)du=0, \tag{6}$$

where

$$F^{(1)}(K,u,t)=\int\tilde{f}^{(1)}(\mathbf{K},\mathbf{v},t)d^2v_\perp, \tag{7}$$

$$F^{(1)}(K,u,0)=F^{(0)}(u)S(K), \tag{8}$$

and

$$F^{(0)}(u)=\int\phi_M(v)d^2v_\perp=(\pi v_0^2)^{-1/2}\exp(-u^2/v_0^2).$$

In Eq. (6), $mV(K)$ is the Fourier transform of the effective interatomic potential. Following Percus[5] or Zwanzig[6] we set

$$V(K)=-\tfrac{1}{2}v_0^2C(K), \tag{9}$$

where $C(K)$ is the Fourier transform of the direct correlation function. Within the context of the Vlasov equation, Eq. (9) can be derived more simply than in the literature by considering the short time behavior of $F^{(1)}(K,u,t)$. This is done in Appendix A.

[7] N. W. Ashcroft and J. Lekner, Phys. Rev. **145**, 83 (1966).
[8] P. D. Randolph and K. S. Singwi, Phys. Rev. **152**, 99 (1966).
[9] L. Kadanoff and P. C. Martin, Ann. Phys. (N. Y.) **24**, 419 (1963).
[10] D. C. Montgomery and D. A. Tidman, *Plasma Kinetic Theory* (McGraw-Hill Book Company, Inc., New York, 1964), Chap. 5.

To calculate $S(K,\omega)$ we first calculate $Z(K,p)$ and then use Eq.* (4). Taking the Laplace transform of Eq. (6), dividing by $(p+iKu)$, and integrating over u gives

$$Z(K,p)=\frac{\pi^{1/2}}{Kv_0}\frac{S(K)w(z)}{[1/S(K)-i\pi^{1/2}n_0C(K)zw(z)]}, \quad (10)$$

where

$$w(z)=i\pi^{-1}\int_{-\infty}^{\infty}\exp(-t^2)(z-t)^{-1}dt \quad (11)$$

and

$$z=ip/Kv_0.$$

In obtaining Eq. (10) we have used the initial condition of Eq. (2), and the defining relation for the direct correlation function[5]

$$n_0C(K)=\frac{S(K)-1}{S(K)}. \quad (12)$$

Using Eq. (4) and the familiar result[4,11]

$$\lim_{\epsilon\to 0^+} w(x+i\epsilon)=\exp(-x^2)\left[1+2\pi^{-1/2}i\int_0^x \exp(t^2)dt\right],$$

we finally obtain

$$S(K,\omega)=\frac{2}{Kv_0}\times\frac{A(x)}{[1/S(K)+n_0C(K)xB(x)]^2+[n_0C(K)xA(x)]^2}, \quad (13)$$

where $x=\omega/Kv_0$, and

$$A(x)=\pi^{1/2}\exp(-x^2),$$

$$B(x)=2\exp(-x^2)\int_0^x \exp(t^2)dt. \quad (14)$$

To use Eq. (13) we need an expression for $C(K)$. Ashcroft and Lekner[7] have pointed out that the analytic solution of the Percus-Yevick equation[5] for rigid spheres gives a simple expression for $S(K)$ which fits the measured structure of liquid metals fairly well with an appropriate choice of parameters. Their result is

$$n_0C(K)=-24\eta y^{-3}[\alpha(\sin y-y\cos y)+\beta y^{-1}\{2y\sin y-(y^2-2)\cos y-2\}+\gamma y^{-3}\{(4y^3-24y)\sin y-(y^4-12y^2+24)\cos y+24\}], \quad (15)$$

where $y=K\sigma$, σ is the hard-core diameter,

$$\eta=\tfrac{1}{6}n_0\pi\sigma^3,$$

and

$$\alpha=(1+2\eta)^2/(1-\eta)^4,$$
$$\beta=-6\eta(1+\tfrac{1}{2}\eta)^2/(1-\eta)^4,$$
$$\gamma=\tfrac{1}{2}\eta\alpha.$$

The hard-core diameter σ and the packing fraction η can be chosen to fit the experimental structure factor. For liquid lead at 352°C the choice of $\eta=0.45$ and $\sigma=3.0$ Å gives a reasonably good fit.

With this choice for $C(K)$, the values of $S(K,\omega)$ given by Eq. (13) are plotted in Figs. 1 and 2. In Fig. 1, $S(K,\omega)$ is plotted as a function of K with the dimensionless parameter $\beta=\hbar\omega/kT$ labelling the energy transfer $\hbar\omega$. This is the "scattering-law" form in which neutron inelastic-scattering data are usually presented. Comparing with Fig. 7 of Ref. 8 we find a striking qualitative similarity with the experiments in liquid lead. For small β there is a sharp maximum at $K=2.2$ Å^{-1}, which is the location of the principal maximum in $S(K)$. As β increases, this maximum becomes broadened and is eventually replaced by a minimum at nearly the same K value. This feature of the experiments has not been easily obtained from other phenomenological theories. It is somewhat exaggerated in the present calculation.

In Fig. 2, $S(K,\omega)$ is plotted as a function of $x=\omega/Kv_0$ with K as a parameter. A well-defined side peak for $K=1.6$ Å^{-1} is clearly seen. This peak moves to smaller frequency as K increases towards the principal diffraction maximum and disappears for K greater than 2 Å^{-1}. This is qualitatively consistent with the ob-

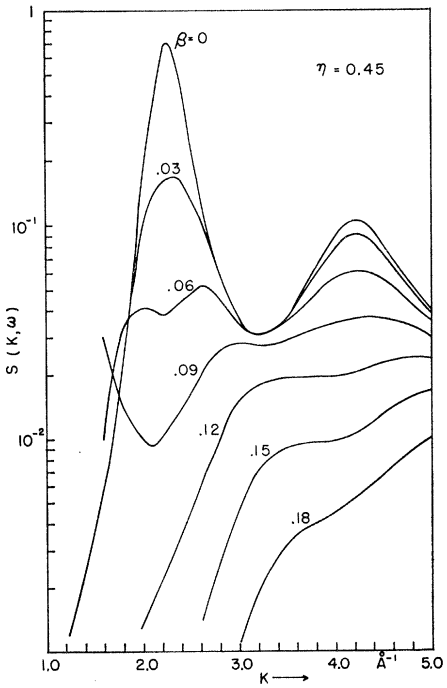

FIG. 1. The response function $S(K,\omega)$ is plotted against the wave number K for the Vlasov equation. β is the dimensionless parameter $(=\hbar\omega/kT)$, where $\hbar\omega$ is the energy transfer.

[11] V. N. Faddeyeva and N. M. Terent'ev, *Tables of Probability Integral for Complex Arguments* (Pergamon Press, Ltd., London, 1961).

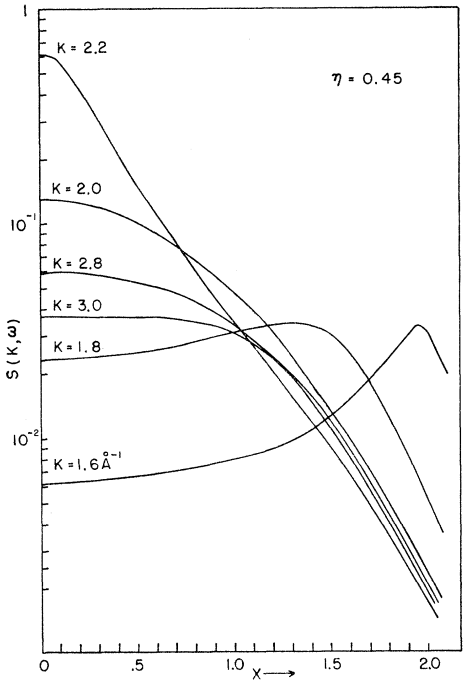

FIG. 2. $S(K,\omega)$ is plotted against x $(=\omega/Kv_0)$ for various values of K.

served[2,8] "dispersion relations" for collective modes in simple liquids. It should be recalled in this connection that neutron time-of-flight experiments[2] greatly exaggerate the peaks associated with these modes compared to the rather modest protuberances that appear in plots of $S(K,\omega)$ versus ω.

One feature of Fig. 2 is in gross qualitative disagreement with experiment. For values of K where the inelastic peak is prominent our calculated $S(K,\omega)$ has a broad minimum near $\omega=0$. Experimentally there is a narrow quasielastic peak for small ω which is completely missed by the present calculation.

The present calculation can be made more realistic by the phenomenological addition of a collision term to Eq. (6). The results of an extension to include collisions are presented in Appendix B. The behavior of $S(K,\omega)$ at small K is qualitatively changed by collisions. The familiar hydrodynamic limit of the Landau-Placzek theory of light scattering is obtained. This contains a quasielastic peak associated with the diffusion of entropy fluctuations at constant pressure. With a reasonable choice of collision frequency, however, the effects of collisions on $S(K,\omega)$ in the region of existing neutron-scattering experiments is small. We must therefore look elsewhere for the quasielastic peak missing from Eq. (13).

III. DISCUSSION

In the preceding calculation we have assumed that the effect on the fluid of localizing an atom at the origin is small. This shows up clearly in Percus's[5] derivation of Eq. (9), which asks "for that potential $v^*(r)$ whose effect at the origin *on a linear basis* is to produce the same forced density fluctuation $n_0g(r)-n$ which the true interparticle potential $v(r)$ due to the arrival of a particle would produce." In Appendix A we see that the same result is obtained by requiring a linearized Vlasov equation to give the correct short time behavior to order t^2. In both derivations the essential approximation is to neglect the large *dynamical* effects associated with the motion of the originally localized atom. At $t=0$ the structure of the fluid is taken into account properly through the construction of the effective potential $v^*(r)=-kTc(r)$, but at later times, the structure is not taken into account correctly.

We thus ignore the fact that the density disturbance whose propagation we are studying is to a substantial extent tied to the originally localized atom. The single-particle diffusive motion known to be important in inelastic neutron scattering is entirely omitted. This is probably the reason that we do not obtain a quasi-elastic peak.

Although the true equation of evolution for $f_1(\mathbf{r},\mathbf{v},t)$ cannot be linear, the use of linear-response theory as expressed in Eqs. (1)–(5) is correct. The problem is not in the formal use of linear-response theory, but in formulating reasonable approximations for the evolution of a highly localized disturbance. We defer detailed discussion of this point to a later paper, but perhaps Eqs. (1)–(5) are more plausible in the light of the following result.

Consider a fluid in equilibrium with a weak gravitational potential $\Phi(r)$. At $t=0$ this potential is shut off and the initially inhomogeneous density relaxes to a final uniform value n_0. It is easily shown that the evolution of the density is given by

$$n(\mathbf{r},t)-n_0=(n_0/kT)\int d^3r'\Phi(\mathbf{r}')[G(\mathbf{r}-\mathbf{r}',t)-n],$$

where $G(r,t)$ is the double Fourier transform of $S(K,\omega)$ and is the familiar space-time correlation function introduced by Van Hove. This expression is readily checked to be correct at $t=0$ since (Ref. 5, Eq. 5.1)

$$\begin{aligned}\delta n(\mathbf{r})/\delta\Phi(\mathbf{r}')&=(n_0/kT)[\delta(\mathbf{r}-\mathbf{r}')+n_0g(\mathbf{r}-\mathbf{r}')]\\&=(n_0/kT)G(\mathbf{r}-\mathbf{r}',0).\end{aligned}$$

There are no formal limitations in the above result on the degree of localization of $\Phi(\mathbf{r})$.

Finally we consider the relation between the present[6] calculation and the recent work[12,13] on elementary excitations in classical fluids. We have calculated $S(K,\omega)$ directly at a level of approximation corresponding to

[12] Robert Zwanzig, Phys. Rev. **156**, 190 (1967).
[13] Ralph Nossal and Robert Zwanzig, Phys. Rev. **157**, 120 (1967).

the simple eigenfunction considered in Ref. 6. The dispersion relation in this approximation is obtained by examining the zeros of the denominator in Eq. (10). The results of such an examination have been discussed by Zwanzig.[6] Because of the large damping due to thermal motion there is no simple way to relate the dispersion relation to the neutron scattering as given by $S(K,\omega)$. At this level of approximation, however, this is not important since the direct calculation of $S(K,\omega)$ is straightforward.

Recently, Nossal and Zwanzig[12,13] have obtained more realistic variational eigenfunctions and studied the dispersion relations for the elementary excitations. It remains true that dispersion relations and $S(K,\omega)$ are not simply related. In contrast, however, to the earlier results we do not yet have a way to calculate $S(K,\omega)$ at this improved level of approximation. It is suggestive that the quantities appearing in the improved dispersion relations are shear and bulk moduli closely related[14] to the sum rules for

$$\int_{-\infty}^{\infty}\omega^4 S(K,\omega)d\omega$$

which describe short time behavior to order t^4. It is therefore likely that a kinetic description correct to order t^4 will extend the present calculation in a similar way that Nossal and Zwanzig[12,13] have extended Zwanzig's earlier calculation.[6] Work in this direction is now in progress.

ACKNOWLEDGMENTS

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APPENDIX A

We can derive Eq. (9) starting from Eq. (6) by making use of the known short time behavior of

$$N(K,t)=\int_{-\infty}^{\infty}F^{(1)}(K,u,t)du=\int_{-\infty}^{\infty}S(K,\omega)\cos\omega t\,d\omega. \quad \text{(A1)}$$

For a classical fluid it is readily shown that

$$N(K,0)=S(K), \quad \text{(A2)}$$

$$\partial^2N(K,0)/\partial t^2=-\tfrac{1}{2}v_0{}^2K^2. \quad \text{(A3)}$$

We have already built in Eq. (A2) in our initial condition. We can thus use Eq. (A3) to define the effective potential $V(K)$. Differentiating Eq. (6) with respect to time and integrating with respect to u gives

$$\partial^2N(K,t)/\partial t^2=-iK\int_{-\infty}^{\infty}u\frac{\partial F^{(1)}}{\partial t}du. \quad \text{(A4)}$$

Eliminating the time derivative on the right-hand side of Eq. (A4) by using Eq. (6) and evaluating the integral at $t=0$ by using Eq. (8), we obtain

$$\partial^2N(K,0)/\partial t^2=-\tfrac{1}{2}v_0{}^2K^2S(K)[1+2n_0v_0{}^{-2}V(K)]. \quad \text{(A5)}$$

Combining this with Eq. (A3) gives Eq. (9) when we recall that $C(K)$ is defined in terms of $S(K)$ by Eq. (12).

APPENDIX B

The collision term that we add to the Vlasov equation is the single relaxation-time kinetic model that conserves particle number, momentum, and energy. This was first introduced by Bhatnagar, Gross, and Krook[15] in their work on small amplitude oscillations in gases. With the linearized version of the above model, we have, for the modified Vlasov equation,

$$(i\omega+i\mathbf{K}\cdot\xi v_0)f^{(1)}(\mathbf{K},\xi,i\omega)-i\mathbf{K}\cdot\frac{d\phi_M}{d\mathbf{v}}n_0V(K)$$

$$\times\int f^{(1)}(\mathbf{K},\xi,i\omega)d^3\xi=\alpha[\phi_MZ-f^{(1)}]$$

$$+n_0\phi_M\alpha[2\xi\cdot\mathbf{Q}+\tau(\xi^2-\tfrac{3}{2})]+S(K)\phi_M, \quad \text{(B1)}$$

where

$$\xi=\mathbf{v}/v_0$$

and

$$\begin{pmatrix}Z\\ \mathbf{Q}\\ \tau\end{pmatrix}=\int f^{(1)}(\mathbf{K},\xi,i\omega)d^3\xi\begin{pmatrix}1\\ \xi\\ \frac{2}{3}\xi^2-1\end{pmatrix}. \quad \text{(B2)}$$

The quantity α is a constant and can be identified with a typical collision frequency of the system. Taking ξ_1 to be the component of $\mathbf{K}$ along ξ, we get

$$f^{(1)}(K,\xi,i\omega)=\phi_M[\alpha Z\Lambda+2n_0\alpha Q_i\Lambda\xi_i+n_0\alpha\tau\Lambda(\xi^2-\tfrac{3}{2})$$
$$-n_0C(K)ZiKv_0\xi_1\Lambda+S(K)\Lambda], \quad \text{(B3)}$$

where

$$\Lambda=(\alpha+i\omega+iKv_0\xi_1)^{-1}.$$

By taking the first three velocity moments of (B3), we obtain three coupled equations involving Z, Q_1, and τ. After carrying out the integrations,[16] the final result can be written as

$$\begin{bmatrix}V^R\\ V^I\end{bmatrix}=\begin{bmatrix}C_R & -C_I\\ C_I & C_R\end{bmatrix}\begin{bmatrix}V_0{}^R\\ V_0{}^I\end{bmatrix}, \quad \text{(B4)}$$

[14] P. Schofield, Proc. Phys. Soc. (London) 88, 149 (1966).

[15] P. F. Bhatnagar, E. P. Gross, and M. Krook, Phys. Rev. 94, 511 (1954).

[16] S. Ranganathan, Ph.D. thesis, Cornell University, 1967 (unpublished).

where

$$C_R=\begin{vmatrix} y\gamma_0+C(K)\epsilon_1 & 2y\gamma_1 & y(\gamma_2-\gamma_0/2) \\ y\gamma_1+C(K)\epsilon_2 & 2y\gamma_2 & y(\gamma_3-\gamma_1/2) \\ \frac{1}{3}y(2\gamma_2-\gamma_0)+\frac{1}{3}C(K)(2\epsilon_3-\epsilon_1) & \frac{1}{3}(2y)(2\gamma_3-\gamma_1) & \frac{1}{3}y(2\gamma_4-2\gamma_2+\frac{5}{2}\gamma_0) \end{vmatrix},$$

$$C_I=C_R\begin{bmatrix}\gamma_i\rightarrow\epsilon_i \\ \epsilon_i\rightarrow-\gamma_i\end{bmatrix},$$

$$V^{R,I}=\begin{pmatrix}Z^{R,I}\\ Q_1^{R,I}\\ \tau^{R,I}\end{pmatrix},\quad V_0{}^R=-\frac{S(K)}{Kv_0}\begin{pmatrix}\gamma_0\\ \gamma_1\\ \frac{1}{3}(2\gamma_2-\gamma_0)\end{pmatrix},$$

with $V_0{}^I=V_0{}^R[\gamma_i\rightarrow\epsilon_i]$. Here R and I refer to the real part and the imaginary part, respectively.

$$\gamma_0=-U, \qquad \epsilon_0=V;$$

$$\gamma_1=-A_1U+B_2V, \qquad \epsilon_1=B_1U+A_1V-1;$$

$$\gamma_2=-A_2U+B_2V-B_1, \qquad \epsilon_2=B_2U+A_2V-A_1;$$

$$\gamma_3=-A_3U+B_3V-B_2, \qquad \epsilon_3=B_3U+A_3V-A_2-\tfrac{1}{2};$$

$$\gamma_4=-A_4U+B_4V-B_3-\tfrac{1}{2}B_1;$$

$$A_1=x, \qquad B_1=y;$$

$$A_2=x^2-y^2, \qquad B_2=2xy;$$

$$A_3=x^3-3xy^2, \qquad B_3=3x^2y-y^3;$$

$$A_4=x^4-6x^2y^2+y^4, \qquad B_4=4x^3y-4xy^3;$$

$$(\sqrt{\pi})\omega(x-iy)=U+iV=(\sqrt{\pi})i/\pi\int_{-\infty}^{\infty}\frac{e^{-t^2}}{x-iy-t}dt,$$

$$x=\omega/Kv_0, \quad y=\alpha/Kv_0.$$

A subroutine to calculate the functions U and V has been constructed. In the earlier treatment, where we did not include any collisions, the value of y was equal to zero and hence we had to approach the imaginary axis in a definite fashion. In the presence of collisions, however, there are no singularities on the imaginary axis.

Since $S(K,\omega)$ is proportional to the real part Z^R of $Z(K,i\omega)$, it can be easily calculated from the matrix equation (B4).

The linearized Euler equations of hydrodynamics for the modified Vlasov equation are given by

$$\partial Z/\partial t+v_0\partial Q_i/\partial x_i=0,$$
$$\partial Q_i/\partial t+v_0(\partial/\partial x_i)[Z+\tau]+(1/v_0)g^{(0)}(x_i,t)=0, \quad \text{(B5)}$$
$$\partial\tau/\partial t+\tfrac{2}{3}v_0(\partial Q_i/\partial x_i)=0,$$

where

$$g^{(0)}(x_i,t)=\frac{1}{m}\int d\mathbf{x}_2\frac{\partial\phi(|\mathbf{x}-\mathbf{x}_2|)}{\partial x_i}Z(\mathbf{x}_2,t),$$

so that $g^{(0)}(K,i\omega)=(1/m)iK\phi(K)Z(K,i\omega)$. The zero-frequency sound speed can be easily calculated and is given by

$$V=(kT/m)^{1/2}[5/3-C(0)]^{1/2}. \quad \text{(B6)}$$

In the case of the Vlasov equation, the "sound" speed was derived to be[6]

$$V_c=(kT/m)^{1/2}[1-C(0)]^{1/2}. \quad \text{(B7)}$$

Comparing the Euler equations (B5) with the Euler equations corresponding to a dilute gas, we note that the self-consistent-field term only contributes to the momentum-balance equation. It does not have any contribution to the heat flow. The hydrostatic pressure and hence the zero-frequency sound speed is thus affected by the self-consistent-field term and the adiabatic sound speed is given by Eq. (B6). Thus, by referring to Eq. (B7) also, it is seen that the sound speed changes from its isothermal value to its adiabatic value on the inclusion of collisions.

As noted before, $S(K,\omega)$ can be computed from the matrix equation (B4), once a proper value for the collision frequency α is assigned. We took $\alpha=2.2\times10^{12}$ sec^{-1} (so that $y=1/K$) and with this value for α, it was seen that collisions do not play an important role in the regime currently explored by neutron-scattering experiments.