Octet Dominance in the Current-Current Form of the Weak Interaction*

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It is shown that the f/d ratios of the vector and axial-vector currents play an important role in octet dominance within the current-current theory of weak interactions.

ONE of the most interesting results to emerge from the application of the current algebra and the hypothesis of partially conserved axial-vector current (PCAC) to the current-current theory of weak interactions[1,2] is the intimate relationship[3] between octet dominance and the decay mode $\Sigma^+\rightarrow n\pi^+$. The current-current theory forces the effective interaction to transform as an admixture of octet and 27-plet under $SU(3)$ and the assumptions of the current algebra and PCAC as they have been used by Suzuki and Sugawara imply that the s-wave $\Sigma_+{}^+$ decay is engendered only by the 27-plet. Since this amplitude is known to be small, it follows that the octet part of the Hamiltonian should be much larger than the 27-plet.

One way of testing whether this is indeed the case is to use a method suggested by Sugawara[2] for the evaluation of the amplitudes, $\Sigma_+{}^+$, $\Lambda_-{}^0$, and $\Xi_-{}^-$. These three amplitudes are independent, from the strict symmetry standpoint (under the assumptions of Suzuki and Sugawara), but can be evaluated using experimental form factors if one assumes that the matrix elements appearing in the Suzuki-Sugawara relation can be approximately saturated with a finite number of low-lying intermediate states. If the theory is consistent with experiment, the numbers so obtained should show a suppression of the $\Sigma_+{}^+$ amplitude.

The best candidates for intermediate states are the $J^p=\frac{1}{2}^+$ octet and the $J^p=\frac{3}{2}^+$ decuplet baryons. Chiu and Schechter[4] have shown that the terms arising from the insertion of the decuplet show octet enhancement independently of the form factors. Specifically, assuming only $SU(3)$, they found (for the decuplet intermediate states only)

$$\langle 27\rangle/\langle 8f\rangle=0.14\,,\quad \langle 27\rangle/\langle 8d\rangle=0.17\,.$$

The question arises whether the contribution for the octet intermediate states will also show octet enhancement.

This program of evaluating this contribution by the method suggested by Sugawara has been partially thwarted by the lack of adequate experimental data on the axial-vector form factors. In order to carry out this evaluation at the present time it appears to be necessary to employ some theoretical model for these form factors. On the other hand, the vector form factors can be determined from conserved-vector-current (CVC) theory and the data on the electromagnetic form factors of the nucleons. Thus the matrix elements of the products of vector currents (VV terms) can be evaluated with the insertion of octet intermediate states.

Chiu and Schechter[4,5] used this approach for the VV terms and found that they show some evidence of octet enhancement (i.e., that the $\Sigma_+{}^+$ amplitude is somewhat smaller than the $\Lambda_-{}^0$ or $\Xi_-{}^-$ amplitudes).

In evaluating the products of axial-vector currents they invoked the assumption of approximate invariance under $SU(3)\otimes SU(3)$. The use of this invariance[6–10] has little experimental support and, moreover, there remain some ambiguities with regard to particle assignments in this group structure.

In this work we shall examine the contributions to the nonleptonic amplitudes due to the octet intermediate states in the hope of pointing out the origin of the octet enhancement and the general conditions under which it occurs. We shall then show that these conditions are satisfied in a number of different models for the axial-vector current.

From the Suzuki-Sugawara relation[1,2], for $N\rightarrow N'\pi_b{}^a$,

$$\begin{aligned}A_{nl}=\frac{G\sin\theta\cos\theta}{2G_A}\langle N'(P)|\,&\delta_1{}^a[\{V_2{}^1,V_b{}^3\}+\{A_2{}^1,A_b{}^3\}\\&+(2\leftrightarrow 3)]-\delta_b{}^1[\{V_2{}^a,V_1{}^3\}+\{A_2{}^a,A_1{}^3\}+(2\leftrightarrow 3)]\\&+\delta_2{}^a[\{V_b{}^1,V_1{}^3\}+\{A_b{}^1,A_1{}^3\}]-\delta_b{}^2[\{V_3{}^1,V_1{}^a\}\\&+\{A_1{}^3,A_1{}^a\}]|N(P)\rangle\,,\quad(1)\end{aligned}$$

we obtain, after insertion of octet intermediate states, a sum of products of matrix elements which we refer to as the VV and AA terms.

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Let us begin by reviewing the calculation of the VV terms in the above relation as it was carried out in Ref. 4. These authors wrote the matrix elements of the vector currents,

$$\langle N'(P')|V_j^\mu|N(P)\rangle = -i\frac{M}{(P_0P_0')^{1/2}}\bar{u}(P')\left[\gamma^\mu f_j^V + i\frac{(P+P')^\mu}{2M}g_j^V\right]u(P),$$

where

$$f_j^V = \left(1+\frac{q^2}{4M^2}\right)^{-1} G_E(q^2)\mu[(1-\tfrac{1}{2}\beta)F_j + \tfrac{3}{2}\beta D_j],$$

$$g_j^V = \left(1+\frac{q^2}{4M^2}\right)G_E(q^2)\left\{\left[1+\left(\frac{q^2\mu}{4M^2}\right)(1-\tfrac{1}{2}\beta)\right]F_j + \left[\frac{q^2\mu}{4M^2}(\tfrac{3}{2}\beta)\right]D_j\right\},$$

and where $G_E(q^2)=G_E{}^p(q^2)$ is the proton electric form factor. Also $\mu=\mu_p$, $\mu_n\equiv-\beta\mu_p$, $\beta=\frac{2}{3}$, and F_j and D_j are $SU(3)$ coefficients: $D_1=\frac{1}{2}(D_1{}^2-D_2{}^1)$, etc.

They used an empirical fit for the form factors due to Chan *et al.*,[11] and obtained for the VV terms:

$$(A_{nl})_{VV}=\sum_{ij} A_0[-K_1\{F_i,F_j\} + K_2\{(D_i+\beta F_i),(D_j+\beta F_j)\}], \quad (2)$$

where we use the notations $\{F_i,F_j\}$ and $\{(D_j+\beta F_j),(D_i+\beta F_i)\}$ to stand for symmetrized products of $SU(3)$ coefficient sums over octet intermediate states. The sum on i and j extends over the currents in Eq. (1). The constants K_1 and K_2 are given by[4,5]

$$K_1=\frac{1}{4}\int_1^\infty dx(x^2-1)^{1/2}[G_E(x-1)]^2\simeq(0.0195),$$
$$K_2=\tfrac{1}{4}\mu^2\int_1^\infty dx(x^2-1)^{1/2}(x-1)[G_E(x-1)]^2\simeq(0.076). \quad (3)$$

Making use of the following identities derived by Rosen,[12]

$$\{F_i,F_j\}=(9/5)d_{ijk}D_k-[27]_{ij}+\tfrac{7}{8}\delta_{ij}I,$$
$$\{D_i,D_j\}=-\tfrac{3}{5}d_{ijk}D_k+\tfrac{1}{3}[27]_{ij}+\tfrac{3}{8}\delta_{ij}I,$$
$$\{D_i,F_j\}+\{D_j,F_i\}=2d_{ijk}F_k,$$

we can separate the contributions due to the **8**f, **8**d, and **27** representations in Eq. (2). We find that the first term (proportional to K_1) contains the **8**d and **27** and that its contributions to each of the amplitudes $\Sigma_+{}^+$, $\Lambda_-{}^0$, and $\Xi_-{}^-$ are very nearly equal. The second term (proportional to K_2) contains all three representations but because the **8**f dominates the other two its contribution to the $\Sigma_+{}^+$ amplitude is much smaller than its contributions to $\Lambda_-{}^0$ and $\Xi_-{}^-$. To see more clearly how it comes about that the **27** is smaller than **8**f in the second term we use the identities above to construct the ratio of **27** to **8**f for this term. We find

$$R\equiv\frac{\langle 27\rangle}{\langle 8f\rangle}=(\surd\tfrac{1}{6})\frac{1-3\beta^2}{\beta}.$$

As we have said, $\beta\simeq\frac{2}{3}$ experimentally and hence the ratio is $R=\frac{1}{12}\surd 6$. But since we are interested here in how the dynamics enter the problem, let us consider R as a function of β. Notice that we would have perfect suppression of the **27** ($R=0$) if β were equal to $\surd\frac{1}{3}$, and that we would have no suppression ($R=1$) if β were equal to about 0.30. This seems to suggest that, at least as far as this term is concerned, the **27** contribution is small due to the particular value of the f/d ratio selected by the dynamics.

To pursue this question further we must turn our attention to the more difficult problem of calculating the AA terms. To do this we shall study some models for the axial form factors.

Recent work on the current algebra has led to reproduction of various static $SU(6)$ results; among them the value of $f/d=\frac{2}{3}$ for the axial-vector current. For example, calculations by Oehme[13,14] and also by Wienternitz[15] based upon current algebra lead to the result that $f/d=\frac{2}{3}$ independent of the momentum transfer. Also Pais[16] has shown that if $SU(3)$ invariance holds and if the octet baryons are a part of a "boosted 56" representation of $SU(6)$, then the f/d is $\frac{2}{3}$ independent of q^2 for the pseudoscalar-meson-baryon vertex. If we accept the point of view that the f/d ratio is independent of the momentum transfer then it is tantalizing to adopt the experimental value $f/d=0.58\simeq\surd\frac{1}{3}$ obtained[17] (for zero momentum transfer) from data on semileptonic processes. We also note that if the Nambu form of PCAC[18] is assumed to be valid, then the f/d ratio is the same for both the axial-vector and induced pseudoscalar terms in the matrix elements of the axial-vector current. In light of these results we shall adopt the following assumption for calculating the AA matrix elements:

(A) The f/d ratio of the axial-vector current is independent of the momentum transfer and is the same for the axial-vector and induced pseudoscalar terms.

Using this assumption we write the matrix elements

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of the axial-vector current in the following form:

$$\langle N(P')|A_j^\mu|N(P)\rangle = -iM(P_0P_0')^{-1/2}\bar{u}(P) \times\left\{\gamma^\mu\gamma_5 G_1^A(q^2)+i\frac{q_\mu}{2M}\gamma_5 G_2^A(q^2)\right\}u(P)[D_j+\beta' F_j], \quad (4)$$

where $\beta'=f/d$.

Note that the form of the axial-vector matrix element in Ref. (4) based upon the assumption of approximate $SU(3)\times SU(3)$ invariance is different from Eq. (4).

Under assumption (A) the $SU(3)$ part of the AA terms in the Suzuki-Sugawara relation now take the same form as the terms proportional to K_2 in the VV terms, that is,

$$(A_{nl})_{AA}=A_0K_3\{[D_i+\beta' F_i],[D_j+\beta' F_j]\}.$$

In other words, for the AA terms we have octet enhancement if the f/d ratio is near $\sqrt{\frac{1}{3}}$ irrespective of the nature of the form factors which contribute to the integral K_3.

Combining the VV and AA terms, the total nonleptonic amplitude takes the form (for $\beta=\beta'$)

$$A_{nl}=A_0[-K_1\{F_i,F_j\} +(K_2+K_3)\{(D_i+\beta F_i),(D_j+\beta F_j)\}]. \quad (5)$$

It is clear from this that even though the VV and AA terms separately give rise to octet enhancement, any cancellation between K_2 and K_3 might destroy the enhancement for the total amplitude.

Some indication that this does not occur is given by the Nambu form of PCAC[18] which allows us to write the matrix elements of the axial-vector current in the following form:

$$\langle N(P')|A_j^\mu|N(P)\rangle = -iM(P_0P_0')^{-1/2}\bar{u}(P')\left[\gamma^\mu\gamma_5+i\left(\frac{q_\mu}{2M}\right)\gamma_5\left(\frac{4M^2}{q^2+m_\pi^2}\right)\right] \times G_0^A(q^2)u(P)(D_j+\beta F_j), \quad (6)$$

then the integral K_3 becomes

$$K_3=\int_1^\infty dx(x^2-1)^{1/2} \times\left[(x+2)-\frac{(x-1)[x-1+(m_\pi^2+m_K^2)/2M^2]}{(x-1+m_\pi^2/2M^2)(x-1+m_K^2/2M^2)}\right] \times[G_0^A(x-1)]^2,$$

which is positive since the second factor in the brace is always smaller than the first (assuming the form factor $G_0^A(q^2)$ is real as required by time reversal invariance). Since we have seen that $K_2\cong 4K_1$, it appears likely that the second term in Eq. (5) will be the dominant one regardless of the axial-vector form factors. We suggest that it is the properties of the second term in Eq. (5) which give rise to octet enhancement.

In Eq. (5) we have expressed three nonleptonic decay amplitudes in terms of only two parameters. Therefore, we can write down a sum rule (taking $\beta=\beta'=\frac{2}{3}$);

$$\Lambda_-^0+\Xi_-^-=(\sqrt{\tfrac{2}{3}})\Sigma_+^+,$$

where the experimental values,[19]

$$\Lambda_-^0=1.551\pm0.024,$$
$$\Sigma_+^+=0.008\pm0.034,$$
$$\Xi_-^-=-2.022\pm0.029,$$

give for the ratio

$$|\Lambda_-^0/\Xi_-^-|_{\exp}=0.77\pm0.02$$

to be compared to unity.

The fact that this ratio deviates from unity only by some 20% (and the fact that the decuplet intermediate states would not make the fit worse) seems to suggest that the octet intermediate states form an essential contribution to the saturation of the matrix elements.

We feel that subject to our assumption (A), the following conclusion is in order: Octet enhancement is an inherent part of the current-current form of the weak interaction and it comes about due to dynamical effects which, (i) select an f/d ratio near $\sqrt{\frac{1}{3}}$, and (ii) cause the factor (K_2+K_3) to be at least the same order of magnitude as the factor K_1.

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