

parametrized by Eq. (18), provide a more convenient description than the corresponding eigenphases.

### APPENDIX

In this appendix we derive the form of  $S(E)$  in the vicinity of an isolated resonance. This result has been given previously by Davies and Baranger,<sup>4</sup> who term it the "generalized Breit-Wigner formula."

By only requiring that the form be valid in the vicinity of a resonance, we take its only energy depen-

dence to be in a resonance denominator, so we can write

$$S_{ba}^{\text{BW}}(E) = B_{ba} + \frac{T_{ba}}{E_p - E}, \quad (\text{A1})$$

where  $B_{ba}$  [= "background"],  $T_{ba}$ , and  $E_p = E_0 - i\Gamma/2$  are constants.

The determinant of  $S$  in general has only simple poles; equivalently, the pole occurs in only a single eigenvalue of  $S$ . This requires that  $T_{ba}$  be a dyad; write it  $T_{ba} = t_b t_a$ .

Finally,  $S$  must be unitary,  $S^\dagger S = 1$ , i.e.

$$\sum_b B_{cb}^\dagger B_{ba} + \frac{-i(E - E_0)(B_{cb}^\dagger t_b t_a - t_c^* t_b^* B_{ba}) + [t_c^* t_b^* t_b t_a - \frac{1}{2}\Gamma(B_{cb}^\dagger t_b t_a + t_c^* t_b^* B_{ba})]}{|E_p - E|^2} = \delta_{ca}, \text{ for all } E.$$

This provides the conditions

$$\begin{aligned} B_{cb}^\dagger B_{ba} &= \delta_{ca}, \\ B_{cb}^\dagger t_b &= t_c^*, \\ \sum_b |t_b|^2 &= \Gamma. \end{aligned} \quad (\text{A2}) \quad \begin{aligned} B_{ba} &= e^{2i\beta_a} \delta_{ba} \quad (\beta_a \text{ real}) \text{ and } t_a = e^{i\beta_a} \Gamma_a^{1/2} \\ & \quad (\Gamma_a \text{ real}, \sum \Gamma_a = \Gamma), \text{ so} \end{aligned}$$

It might be noted that

$$\det S^{\text{BW}}(E) = 1 + i\Gamma / (E_p - E) = (E_p^* - E) / (E_p - E). \quad (\text{A3}) \quad S_{ba}^{\text{BW}}(E) = e^{i(\beta_a + \beta_b)} \left[ \delta_{ba} + \frac{i\Gamma_a^{1/2} \Gamma_b^{1/2}}{E_p - E} \right]. \quad (\text{A4})$$

## Superconvergent Dispersion Relations and $I=2$ Electromagnetic Mass Differences of Hadrons

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Explicit calculations have been presented for the  $I=2$  electromagnetic mass difference between particles of a given isomultiplet using the superconvergent-dispersion-relation approach suggested by Harari. We obtain, in particular, the electromagnetic mass differences (i)  $\pi^+ - \pi^0$ , (ii)  $\rho^+ - \rho^0$ , (iii)  $\Sigma^+ + \Sigma^- - 2\Sigma^0$ , (iv)  $N^{*++} + N^{*0} - 2N^{*+}$ , and (v)  $Y_1^{*+} + Y_1^{*0} - 2Y_1^{*0}$ . The agreement of our results with experiments is excellent.

### 1. INTRODUCTION

THE calculation of electromagnetic mass differences between members of various isomultiplets has attracted a lot of attention in recent times. It is well known that attempts to calculate electromagnetic mass differences, taking into account only certain low-lying pole terms in the self-energy diagram, lead to confusing

results. The notorious wrong sign is obtained for the mass differences  $n - p$  and  $K^+ - K^0$ , while the correct sign and magnitude is predicted for the mass difference  $\pi^+ - \pi^0$ . Recently, Harrai<sup>1</sup> put forward a simple criterion based on the use of superconvergent dispersion relations to understand this anomaly. As is well known, in perturbation theory the electromagnetic self-energy of a strongly interacting particle is given by<sup>2</sup>

$$\Delta M = \frac{1}{8\pi^2} \int \frac{T_{\mu\nu}(q^2, \nu)}{q^2 - i\epsilon} g_{\mu\nu} d^4q, \quad (\text{1})$$

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<sup>1</sup> H. Harari, Phys. Rev. Letters **17**, 1303 (1966).

<sup>2</sup> W. N. Cottingham, Ann. Phys. (N. Y.) **25**, 424 (1963).

where  $T_{\mu\nu}(q^2, \nu)\epsilon_\mu\epsilon_\nu$  is the forward (non-spin-flip) Compton scattering amplitude for a virtual photon of mass  $q^2 \neq 0$  and energy  $q_0 = \nu$  with any hadron. In isospace  $\Delta M$  is a sum of two terms  $\Delta M^{(1)}$  and  $\Delta M^{(2)}$ , transforming like a vector and a symmetric traceless tensor of rank two, respectively. One can now decompose  $T_{\mu\nu}(q^2, \nu)$  in terms of invariant amplitudes as

$$T_{\mu\nu}^{(i)}(q^2, \nu) = \sum_{\alpha} K_{\mu\nu}^{\alpha} t_{\alpha}^{(i)}(q^2, \nu), \quad (2)$$

where  $\alpha$  runs over the number of invariant amplitudes which survive in the forward direction and the superscript  $(i)$  indicates the isospin transformation property,  $I=1$  or  $2$  corresponding to  $\Delta M^{(1)}$  and  $\Delta M^{(2)}$ , respectively. The  $K_{\mu\nu}^{\alpha}$ 's denote the kinematical factors associated with the invariant decomposition of the Compton scattering amplitude in the forward direction.

In order to calculate the mass difference  $\Delta M$  from Eq. (1), in practice, one assumes a suitable dispersion relation satisfied by the invariant amplitudes  $t_{\alpha}^{(i)}(q^2, \nu)$  and saturates the dispersion integrals by a few low-lying single-particle states. The question of when the above dispersion relation would need subtractions is of course the standard difficulty in this approach. Harari<sup>1</sup> suggested that the convergence properties of these dispersion integrals can be resolved by looking at the asymptotic behavior of the crossed  $t$ -channel ( $\gamma\gamma \rightarrow H\bar{H}$ ,  $H$  being a hadron) Compton amplitude as predicted by Regge-pole analysis. It is interesting to note that the mass difference  $\Delta M^{(i)}$  is controlled by the Regge-pole exchange in the  $t$ -channel Compton amplitude, having isospin quantum number  $i$  only. Thus, in order to calculate  $\Delta M^{(i)}$ , one needs to know the  $t$ -channel Regge-pole exchange in isospin state  $I=i$ . The resulting asymptotic behavior of the Compton amplitude is  $\nu^{\alpha_i(0)}$ , where  $\alpha_i(0)$  is the  $t=0$  intercept of the Regge trajectory with isospin  $i$ . Since there is no boson trajectory with  $I=2$  known so far, it is reasonable to assume<sup>3</sup> that  $\alpha_{I=2}(0) < 0$ . Thus  $T_{\mu\nu}^{I=2}(q^2, \nu)$  will satisfy an unsubtracted dispersion relation. For the  $I=1$  case, the trajectory of the  $A_2$  meson ( $I=1, C=+1, G=-1$ ) which has  $\alpha_{A_2}(0) \sim 0.4^4$  will govern the behavior of the Compton amplitude. Therefore, one *must* introduce a subtraction in the dispersion relations. In order to avoid calculating this essentially unknown subtraction constant,<sup>5</sup> we confine our attention, in this paper, to  $I=2$  electromagnetic mass differences only.

It can be easily seen that the mass differences (i)  $\pi^+ - \pi^0$ , (ii)  $\rho^+ - \rho^0$ , (iii)  $\Sigma^+ + \Sigma^- - 2\Sigma^0$ , (iv)  $N^{*++} + N^{*-} - 2N^{*+}$ , and (v)  $Y_1^{*++} + Y_1^{*-} - 2Y_1^{*0}$  transform like

$I=2$  in isospace. Definite experimental data exist<sup>6</sup> only for (i), (iii), and (iv);  $\rho^+ - \rho^0$  is highly uncertain,<sup>7</sup> while nothing is known about (v). Our present calculations, based on the superconvergence principle and the dispersion integrals being dominated by a few low-lying states, yield results which are in remarkable agreement with experiments. The numerical values for the mass differences (ii) and (v) could be checked as soon as the experimental data are available. We discuss below the various cases separately.

## 2. CALCULATION OF ELECTROMAGNETIC MASS DIFFERENCES

### A. $\pi^+ - \pi^0$

For the Compton scattering of pions, we express the amplitude  $T_{\mu\nu}^{(i)}$  in the forward direction as

$$T_{\mu\nu}^{(i)}(q^2, \nu) = t_1^{(i)}[q^2 g_{\mu\nu} - q_\mu q_\nu] + t_2^{(i)}[\nu^2 g_{\mu\nu} + (q^2/m_\pi^2) p_\mu p_\nu + (\nu/m_\pi)(p_\mu q_\nu + p_\nu q_\mu)], \quad (3)$$

$p_\mu$  being the 4-momentum of the pion. Using the prescription of Cottingham,<sup>2</sup> we rotate the  $q_0$  integration contour in (1) from the real to the imaginary axis in the complex  $\nu$  plane. We can then write for  $\Delta M$  the form<sup>8</sup>

$$\Delta M^{(i)} = -\frac{1}{4\pi} \int_0^\infty \frac{dq^2}{q^2} \int_{-q}^{+q} d\nu (q^2 - \nu^2)^{1/2} \times [3q^2 t_1^{(i)}(q^2, i\nu) - (q^2 + 2\nu^2) t_2^{(i)}(q^2, i\nu)]. \quad (4)$$

As mentioned before, to calculate the  $I=2$  type of mass differences, we can write unsubtracted dispersion relations for the  $t_{\alpha}$ 's<sup>9</sup>:

$$t_{\alpha}^{I=2}(q^2, \nu) = -\frac{2}{\pi} \int_0^\infty \frac{\text{Im} t_{\alpha}^{I=2}(q^2, \nu')}{\nu'^2 - \nu^2} d\nu'. \quad (5)$$

The low-lying poles in  $\gamma - \pi$  scattering are due to  $\pi$ ,  $\rho$ ,  $\omega$ ,  $\varphi$ ,  $A_1$ , and  $A_2$  contributions. The  $\rho$  meson cannot contribute to the  $\pi^+ - \pi^0$  mass difference because only  $G=-1$  ( $\pi\gamma$ ) states can contribute to the  $I=2$  type mass difference. The coupling constant  $g_{\omega\pi\gamma}$  is known from the electromagnetic decay width  $\Gamma(\omega \rightarrow \pi\gamma) = 1.08$  MeV.<sup>9</sup> Since the decay  $\varphi \rightarrow \pi\gamma$  is not seen, the coupling constant  $g_{\varphi\pi\gamma}$  is probably very small. In fact, from various theoretical models such as the quark model,  $SU(6)$  symmetry, current algebra plus partially conserved axial vector current (PCAC), and vector dominance model, one finds  $g_{\varphi\pi\gamma}$  equal to zero. Therefore we will simply

<sup>6</sup> A. H. Rosenfeld, A. Barbaro-Galtieri, W. J. Podolsky, L. R. Price, Matts Roos, W. J. Willis, and C. G. Wohl, Rev. Mod. Phys. **39**, 1 (1967).

<sup>7</sup> See footnote (h) of Ref. 6.

<sup>8</sup> This expression for  $\Delta M$  relates electromagnetic self-energy to the Compton scattering amplitude for spacelike photons, thus enabling one, in principle, to use experimental data on electron scattering directly in Eq. (4).

<sup>9</sup> We have included the below-threshold pole terms also in the dispersion integral in (5), hence the lower limit for  $\nu$  integration is zero in our case.

<sup>3</sup> V. de Alfaro, S. Fubini, G. Furlan, and G. Rossetti, Phys. Letters **21**, 576 (1966).

<sup>4</sup> R. J. N. Phillips and W. Rarita, Phys. Rev. **140**, B200 (1966); V. Barger and M. Olsson, *ibid.* **146**, 1080 (1966).

<sup>5</sup> Recently Cottingham and Gibb [W. N. Cottingham and J. Gibb, Phys. Rev. Letters **18**, 883 (1967)] have shown, using Jost-Lehmann-Dyson representation for  $g_{\mu\nu} T_{\mu\nu}(q^2, \nu)$ , that if the dispersion integral for the Compton amplitude requires a subtraction, the electromagnetic self-energy diverges.

neglect it. Further, since nothing is known about  $A_1 \rightarrow \pi\gamma$  and  $A_2 \rightarrow \pi\gamma$  decay widths, we shall neglect their contributions as well.

The relevant expressions for these pole terms (sum of  $s$  and  $u$  channels) are

$$T_\pi(q^2, \nu) = 2g_{\pi\pi\gamma}^2 [(q^4 + 4m_\pi^2 q^2) + 8m_\pi^2 \nu^2] / (q^4 + 4m_\pi^2 \nu^2), \quad (6)$$

$$T_\omega(q^2, \nu) = \frac{16m_\pi^2 g_{\omega\pi\gamma}^2 (m_\pi^2 + q^2 - m_\omega^2)(q^2 + \nu^2)}{[(m_\pi^2 + q^2 - m_\omega^2)^2 + 4m_\pi^2 \nu^2]}.$$

In Eq. (6), the isospin Clebsch-Gordan coefficients have been left out. For the form factors associated with the coupling constants in (6), we have assumed a vector-meson pole dominance model. Substituting (6) in (4), one can easily evaluate the  $\pi^+ - \pi^0$  mass difference. Our result is  $m_{\pi^+} - m_{\pi^0} = 4.54$  MeV. This value should be compared with the earlier result of Coleman and Schnitzer<sup>10</sup> who get a value of  $\sim 5$  MeV. Recent calculation by Das *et al.*,<sup>11</sup> using the algebra of currents, PCAC, and the superconvergent spectral function sum rules, yields a value  $\sim 4.8$  MeV. The experimental value is  $4.604 \pm 0.014$  MeV.

It should be remarked that in our calculation we needed an infrared cutoff parameter  $\sim (1/400)m_\pi^2$ . Varying this parameter to  $[1/(4 \times 10^6)]m_\pi^2$ , the mass difference alters by about 2.5%. Thus, our calculation is not in any way sensitive to the infrared cutoff procedure even over such a wide range of variation.

### B. $\rho^+ - \rho^0$

The Compton scattering of  $\rho$  mesons can be adequately described in terms of  $\rho$  and  $\pi$  poles. The  $\pi$ -meson pole terms cannot contribute to the  $\rho^+ - \rho^0$  mass difference because only  $G = +1$ ,  $\rho\gamma$  states can give rise to an  $I = 2$  contribution. Since one does not know anything about the higher (electric and magnetic) moments of the  $\rho$  meson, we have restricted ourselves to the simplest  $\rho\rho\gamma$  coupling of the form

$$H_{\rho\rho\gamma} = g_{\rho\rho\gamma} \rho_\mu^i \rho_\nu^j F_{\mu\nu}^k \epsilon^{ijk}, \quad (7)$$

where  $i, j$ , and  $k$  are isotopic indices. The coupling (8) means that we have neglected the isoscalar photon coupling with  $\rho$ . The coupling constant  $g_{\rho\rho\gamma}$  is now known as the charge of the  $\rho$  meson multiplied by a form factor for which we will again use the vector-meson pole dominance model. The  $\rho$ -pole term in  $\gamma\rho$  scattering is given by

$$T_\rho(q^2, \nu) = \frac{2g_{\rho\rho\gamma}^2 (q^2/m_\rho^2) (5m_\rho^2 q^2 - 2q^4 - 10m_\rho^2 \nu^2)}{(k^4 + 4m_\rho^2 \nu^2)}. \quad (8)$$

<sup>10</sup> S. Coleman and H. J. Schnitzer, Phys. Rev. **136**, B223 (1964).

<sup>11</sup> T. Das, G. S. Guralnik, V. S. Mathur, F. E. Low, and J. E. Young, Phys. Rev. Letters **18**, 759 (1967).

Using (8) in (4) as before, we obtain  $m_{\rho^+} - m_{\rho^0} = 3.68$  MeV. Varying the cutoff parameter from  $(1/400)m_\pi^2$  to  $(1/4 \times 10^6)m_\pi^2$  we get  $m_{\rho^+} - m_{\rho^0} = 3.9$  MeV, demonstrating once again the insensitivity of our results to the infrared cutoff. There is no direct experimental evidence for the  $\rho^+ - \rho^0$  mass difference; however, there are two tentative figures for this number,<sup>7</sup>  $m_{\rho^+} - m_{\rho^0} \sim 8$  MeV and  $\sim 3$  MeV. Our result is certainly consistent with these. On the other hand, it is interesting to note that on the basis of  $SU(4)$  symmetry (electromagnetic interaction transforms like a member of the 15-plet), one finds<sup>12</sup>

$$m_{\rho^+} - m_{\rho^0} = m_{\pi^+} - m_{\pi^0}. \quad (9)$$

The present superconvergent dispersion relation calculation is not in gross disagreement with the  $SU(4)$  symmetry result.

### C. $\Sigma^+ + \Sigma^- - 2\Sigma^0$

The nearby poles contributing in  $\gamma\Sigma$  scattering are the  $\Lambda$ ,  $\Sigma$ , and  $Y_1^*$ . For the spin- $\frac{1}{2}$  baryons, we have chosen the charge and magnetic couplings<sup>13</sup>

$$H_{BB\gamma} = g_{BB\gamma} \bar{u}(p) \left[ \gamma_\mu F_1(q^2) + \frac{\sigma_{\mu\nu} q_\nu}{2m_B} F_2(q^2) \right] u(p') \epsilon_\mu(q). \quad (10)$$

In calculating the  $\Sigma$ -pole terms, we have related  $g_{\Sigma\Sigma\gamma}$  and  $g_{NN\gamma}$ , using  $SU(3)$  symmetry,<sup>14</sup> while for the  $\Lambda$ -pole term  $F_1(q^2) = 0$ , and  $F_2(0)$ , which is nothing but  $\mu_{\Sigma^0 \rightarrow \Lambda^0 \gamma}$ , is related to  $\mu_p$ , using  $SU(3)$  symmetry. The  $SU(3)$  symmetry value gives rise to a  $\Gamma(\Sigma^0 \rightarrow \Lambda^0 \gamma)$  which is consistent with the experimental upperbound.

For  $Y_1^* \Sigma \gamma$  coupling we have assumed the form

$$H_{B^*B\gamma} = \bar{\psi}_\lambda(p) \left[ \delta_{\lambda\mu} G_1(q^2) + \frac{\gamma_\mu q_\lambda}{m_B} G_2(q^2) \right] \gamma_5 u(p') \epsilon_\mu(q). \quad (11)$$

Here  $G_1 = (1 + m_{B^*}/m_B)G_2$ . For the coupling constants  $G_1, G_2(0)$ , one can either use  $SU(6)$  symmetry and relate them to  $NN\gamma$  couplings, or use the experimental values found by Albright and Liu<sup>15</sup> from an analysis of the high-energy  $N^*$  production process by neutrinos.  $Y_1^* \Sigma \gamma$  couplings can be related to  $N^* N \gamma$  couplings, using  $SU(3)$  symmetry as before. In Eq. (11) we have neglected higher derivative couplings, since they turn out to be rather small in the work of Albright and Liu.<sup>15</sup>

The  $\Sigma, \Lambda$ , and  $Y_1^*$  pole terms in  $\gamma - \Sigma$  scattering can

<sup>12</sup> S. N. Biswas, V. S. Mathur, and R. P. Saxena, University of Delhi (1966) (unpublished).

<sup>13</sup> In what follows, all the form factors will be assumed to have a form  $F(q^2) = F(0)/(q^2 + b)^2$  where  $b \sim 30m_\pi^2$ .

<sup>14</sup> M. Gell-Mann and Y. Ne'eman, *Eightfold Way* (W. A. Benjamin Inc., New York, 1964).

<sup>15</sup> C. H. Albright and L. S. Liu, Phys. Rev. Letters **13**, 673 (1964).

now be written as

$$T_B(q^2, \nu) = \frac{[2q^2(2m_B^2 F_1^2 - q^2 F_2^2) - (8m_B^2 F_1^2 - q^2 F_2^2)\nu^2]}{m_B(k^4 + 4m_B^2 \nu^2)}, \quad (12)$$

$$T_{B^*}(q^2, \nu) = \frac{2(A + B\nu^2 + C\nu^4)}{(X^2 + 4m_B^2 \nu^2)(Y^2 + 4m_B^2 \nu^2)}; \quad (13)$$

where

$$\begin{aligned} X &= m_B^2 + q^2, & Y &= m_B^2 - m_{B^*}^2 + q^2, \\ A &= A_1 X^2 Y + A_2 X Y, & C &= 8m_B^3 B_1 \\ & & & + 4m_B^2 C_2 - 2m_B D_2 (X + Y), \\ B &= C_2 X Y + A_2 4m_B^2 - 2m_B B_2 (X + Y) \\ & & & - 2m_B B_1 X^2 - 4m_B^2 A_1 Y, \end{aligned}$$

$$A_1 = 2(m_B - m_{B^*})G_1^2 - \frac{4}{3} \frac{G_2^2}{m_B^2} (m_B + 2m_{B^*})q^2, \quad (14)$$

$$A_2 = \frac{4}{3} \left( \frac{G_2^2}{m_B^2} (m_B + m_{B^*})q^4 - G_1 G_2 m_B q^2 \right),$$

$$B_1 = 2G_1^2 - \frac{4}{3} \left( \frac{G_2^2 q^2}{m_B^2} - G_1 G_2 \frac{m_{B^*}}{m_B} \right),$$

$$B_2 = -\frac{4}{3} \frac{G_2^2}{m_B^2} [q^2(q^2 + 2m_B(m_B + 2m_{B^*}))] - \frac{4}{3} G_1 G_2 \frac{m_{B^*}}{m_B} (m_B^2 + q^2),$$

$$C_2 = -\frac{4}{3} \frac{G_2^2}{m_B^2} [2m_B q^2 + m_B^2(m_B + 2m_{B^*})] - \frac{4}{3} G_1 G_2 m_{B^*},$$

and

$$D_2 = \frac{4}{3} G_2^2.$$

We now obtain for  $m_{\Sigma^+} + m_{\Sigma^-} - 2m_{\Sigma^0}$  a value of 2.01 MeV. This result should be compared with the experimental value  $1.79 \pm 0.10$  MeV. The Coleman and Schnitzer<sup>10</sup> value for this mass difference is  $\sim 1.5$  MeV.

We may add in passing that the contribution of other higher resonances like the  $Y_0^*(1405)$ ,  $Y_1^*(1770)$ , etc., for which we can at best give estimates, tends to improve the agreement of our result with experiments.

#### D. ( $N^{*++} + N^{*0} - 2N^{*+}$ ) and ( $Y_1^{*+} + Y_1^{*-} - 2Y_1^{*0}$ )

Here we consider only baryon ( $B$ ) and decuplet baryon resonance ( $B^*$ ) poles. The baryon-pole terms can be directly evaluated by using (12) and the same coupling constants as before to give

$$\begin{aligned} T_B(q^2, \nu) &= \{-2[(m_{B^*}^2 - m_B^2 + q^2) \\ &\times (4G_1^2(m_{B^*} - m_B) + 2(m_{B^*} + 2m_B)G_2^2 q^2 / m_B^2) \\ &- 8m_B \nu^2 (2G_1^2 + G_2^2 q^2 / m_B^2)]\} / \\ &[(m_{B^*}^2 - m_B^2 + q^2)^2 + 4m_B^2 \nu^2]. \quad (15) \end{aligned}$$

For the  $B^*B^*\gamma$  vertex we retain only the charge and magnetic moment couplings (since nothing is known about the higher moments of  $B^*$ )

$$H_{B^*B^*\gamma} = \bar{\psi}_\mu(p) [\gamma_\lambda H_1(q^2) + 2\sigma_{\lambda\nu} q_\nu H_2(q^2)] \psi_\mu(p') \epsilon_\lambda(q). \quad (16)$$

Here  $H_1(0)$  and  $H_2(0)$  may be related to the baryon charge and magnetic moments using  $SU(6)$  symmetry. The relevant pole terms are

$$\begin{aligned} T_{B^*}(q^2, \nu) &= \frac{(32/3)[m_{B^*} q^2 (H_1^2 - 3k^2 H_2^2) - 2m_{B^*} H_1^2 \nu^2]}{(k^4 + 4m_{B^*}^2 \nu^2)}. \quad (17) \end{aligned}$$

Our results for the mass differences are

$$\begin{aligned} N^{*++} + N^{*0} - 2N^{*+} &= 4.79 \text{ MeV}, \\ Y_1^{*+} + Y_1^{*-} - 2Y_1^{*0} &= 5.7 \text{ MeV}. \end{aligned}$$

Experimentally, the former is  $4.81 \pm 5.39$  MeV, while no information is available on the latter.

### 3. REMARKS

To summarize, we give the following table of our results on  $I=2$  type mass differences among hadron isomultiplets based on the use of superconvergent dispersion relations. The fact that all of them show a remarkable agreement with experimental data suggests that the dispersion integrals for  $t_{\alpha}^{I=2}(q^2, \nu)$  do indeed get saturated by the nearby poles. The success of the present calculations indicates that the superconvergent dispersion relations are a powerful tool for investigating hadron dynamics.

TABLE I. Table showing  $E=2$  type mass differences among hadron isomultiplets in MeV with infrared cutoff parameters  $(1/400)m_{\pi^*}^2$  (a) and  $[1/(4 \times 10^6)]m_{\pi^*}^2$  (b).

Mass difference	Present calculation		Experiment
	a	b	
$\pi^+ - \pi^0$	4.54	4.63	$4.604 \pm 0.014$
$\rho^+ - \rho^0$	3.68	3.9	3-8
$\Sigma^+ + \Sigma^- - 2\Sigma^0$	2.01	2.06	$1.79 \pm 0.10$
$N^{*++} + N^{*0} - 2N^{*+}$	4.79	4.83	$4.81 \pm 5.39$
$Y_1^{*+} + Y_1^{*-} - 2Y_1^{*0}$	5.70	5.74	None

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