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Composite Model of Σ†

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A composite model of Σ as a bound state of $\Lambda\,\pi$ and $N\bar{K}$ is developed in terms of dispersion relations for form factors and an approximate Bethe-Salpeter equation. The effective coupling constants of Σ to its constituents are predicted to be $8.7 \lesssim |g_{\Sigma^0\Lambda\pi^0}| \lesssim 10.7$ and $6.4 \lesssim |g_{\Sigma^0 pK^-}| \lesssim 7.9$ for the range $5.0 \lesssim |g_{\Lambda pK^-}| \lesssim 13.5$. The relative sign of $g_{\Sigma^0\Lambda\pi^0}$ and $g_{\Sigma^0 pK^-}$ is found to be opposite to the relative sign of $g_{pp\pi^0}$ and $g_{\Lambda pK^-}$. The model is further applied to a calculation of the on-shell electromagnetic form factors of the $\Sigma\Lambda$ system. Under the assumption of ρ dominance of the kaon isovector electromagnetic form factor, the $\Sigma\Lambda$ transition moment is determined to be $G_2\,(O) = g_{\Lambda pK^-} g_{\Sigma^0 pK^-} 0.9\times10^{-2}$ nuclear magnetons, whereas the form factor which would vanish in the limit of equal Σ^0 and Λ masses is two orders of magnitude smaller than the transition moment.

1. INTRODUCTION

In this paper a simple composite model of the Σ hyperon is developed. It is based on the conjecture that Σ may be regarded as a bound state of a limited number of other hadrons, specifically of $\Lambda\pi$ and $N\bar{K}$ pairs. In the following paragraphs the basic definitions and assumptions used by the model will be stated, some plausibility arguments presented, and the plan of the calculations outlined.

Various definitions of compositeness have been proposed in a relativistic context. In field theory a particle may be considered composite if its description does not require the field operator of the particle explicitly.[1] A more directly applicable definition of a bound state, provided that it is formed from pairs of other particles, may be stated in terms of the Bethe-Salpeter equation.[2] In field theory this equation is assumed to play, at least in principle, a role analogous to that of the Schrödinger equation in quantum mechanics. In practice, the equation is very involved in the presence of spin, even if a "ladder" approximation is made. Other types of eigenvalue equations characterizing, if not completely describing, composite particles have also been discussed by many authors. In particular, the requirement of vanishing wave-function renormalization constants for composite particles has been investigated in detail in Ref. 3. The requirement of zero vertex function renormalization constants and related conditions, which arise in certain theories, have also been discussed in many works.[4]

In the framework of S-matrix theory, the N/D method[5] has been extensively used for the study of bound states. This method furnishes eigenvalue equations for obtaining the mass and coupling constants of the composite particle by requiring that the T matrix, $T=N/D$, develop a pole due to $\det(D)=0$ in each appropriate channel at the energy and angular momentum corresponding to the mass and spin of the composite particle, and that the residue there be simply related to the square of the effective coupling constant of the particle to the channel in question. Under certain conditions, not fulfilled by Σ,[6] this method leads to a simple relation between the mass of the composite particle and its coupling constant to the lowest-lying channel. Such a relation, which is independent of detailed dynamics, was proposed (for s-wave bound states) by Landau[7] and proved by Nauenberg.[6] The Landau relation is satisfied by the deuteron, so that the validity of the N/D formalism is at least partially demonstrated.

It is possible to obtain Landau's condition in certain cases and for "very small" binding by comparison of the Schrödinger wave function and the dispersion relation satisfied by the electromagnetic form factor of the composite particle with a subtraction allowed.[8] Furthermore, under similar conditions, Blankenbecler and Cook[9] have demonstrated the close connection between the Schrödinger wave function derived from a linear superposition of Yukawa potentials and the corresponding form factor describing the interaction of the composite particle with its constituents and obeying unsubtracted dispersion relations. Their results as well as those of Barton[8] hinge on the dominance of a single

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[1] See, e.g., W. Zimmermann, Nuovo Cimento **10**, 597 (1958); K. Nishijima, Phys. Rev. **133**, B1092 (1964); also Ref. 3.
[2] E. E. Salpeter and H. A. Bethe, Phys. Rev. **84** 1232 (1951).
[3] C. R. Hagen, Ann. Phys. (N. Y.) **31**, 185 (1965); M. M. Broido and J. G. Taylor, Phys. Rev. **147**, 993 (1966), and references therein.
[4] See, e.g., A. Salam, Nuovo Cimento **25**, 224 (1962); R. M. Rockmore, Phys. Rev. **132**, 876 (1963); E. G. P. Rowe, Nuovo Cimento **32**, 1422 (1964); G. Furlan and G. Mahoux, *ibid.* **36**, 215 (1965); W. Güttinger, *ibid.* **36**, 968 (1965); M. Ida, Progr. Theoret. Phys. (Kyoto) **34**, 92 (1965); **34**, 990 (1965); **35**, 692 (1966); J. S. Dowker, Nuovo Cimento **36**, 1050 (1965); T. Yoshimura, *ibid.* **40**, 495 (1965); M. M. Broido and J. G. Taylor, Ref. 3.
[5] G. F. Chew and S. Mandelstam, Phys. Rev. **119**, 467 (1960); J. D. Bjorken, Phys. Rev. Letters **4**, 473 (1960); B. Kayser, Phys. Rev. **138**, B1244 (1965); B. M. Udgaonkar, in *High Energy Physics and Elementary Particles,* (International Atomic Energy Agency, Vienna, 1966), p. 791.
[6] M. Nauenberg, Phys. Rev. **124**, 2011 (1961).
[7] L. D. Landau, Zh. Eksperim. i Teor. Fiz. **39**, 1856 (1960) [English transl.: Soviet Phys.—JETP **12**, 1294 (1961)].
[8] G. Barton, *Introduction to Dispersion Techniques in Field Theory* (W. A. Benjamin, Inc., New York, 1965), p. 193.
[9] R. Blankenbecler and L. F. Cook, Jr., Phys. Rev. **119**, 1745 (1960).

anomalous contribution to the dispersion integral so that, as expected, they are not applicable to Σ. It does not necessarily follow, of course, that dispersion relations for composite form factors can not be used whenever no correspondence to a Schrödinger description can be established.

Nishijima and collaborators[10] have investigated the applicability of unsubtracted dispersion relations to composite form factors by entirely relativistic methods. They have not actually supplied proofs for ordinary dispersion relations satisfied by S-matrix elements. Their framework consists of parametric dispersion relations for Green's functions and their conclusion is, briefly, that such dispersion relations ought to be unsubtracted if they describe composite particles, unless the subtraction constants are calculable or fixed by the universality of an interaction, and subtracted if they pertain to primary interactions of elementary particles. The generalization to ordinary dispersion relations for form factors is not only tempting but also compatible with the notion that the parameters characterizing composite particles ought to be calculable from theory, in contrast to those which describe elementary particles.[10,11]

An investigation of the validity or equivalence of the various approaches to the bound-state problem outlined above is beyond the scope of the present work. Here the following is assumed:

(1) A ladder approximation to the Bethe-Salpeter equation provides a valid description of Σ.

(2) The form factors which specify the interaction of Σ with its constituents obey unsubtracted dispersion relations in which either constituent may be off the mass shell. Further, the electromagnetic form factors of the ΣΛ system obey, with the exception of the Dirac form factor, unsubtracted dispersion relations in which the photon is off-shell, and the same is true of the ΣΣ system. Similar properties may be attributed to any other strong or electromagnetic vertex containing at least one composite particle and an off-shell elementary particle.

(3) Approximately constant behavior can be assigned to certain combinations of form factors involving solely elementary hadrons.

An intuitive argument in favor of the ladder approximation, based on the not too large binding energy of Σ, may be given analogous to that of Ref. 2. Assumptions 2 and 3 are consistent with Ref. 10. The type of dispersion relations valid when a composite particle is

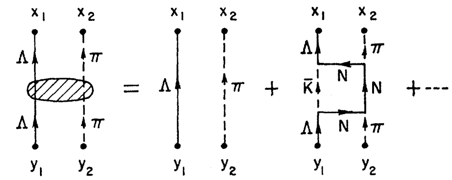

FIG. 1. Bethe-Salpeter ladder.

off-shell is not, however, specified.[12] Further, the last assumption is only a very crude formulation of the behavior of elementary particles, motivated by lack of detailed information. The precise content of this assumption will become evident in the course of the calculations.

Granted the above definition of elementarity and compositeness, the question arises whether any of the known hadrons may be regarded as elementary In the absence of decisive evidence and on account of its simplifying features, the assumption is made in this paper that all hadrons are composite, with the exception of those which are lighter than Σ and stable against strong interactions, namely, the Sakata baryon triplet and the $SU(3)$ pseudoscalar octet.[13] Thus the present model is not in the spirit of "partial bootstrap" models like the one considered by Kayser,[5] nor does it lead to similar predictions after approximations are introduced for calculational purposes.

The distinction between elementary and composite hadrons has important consequences with regard to the forces responsible for the binding of composite hadrons, namely, the dominant contribution is expected to come from the exchanges of elementary rather than of composite hadrons, so that in the particular case of Σ only nucleon exchange need by considered in first approximation (cf. Fig. 1). Qualitatively, this may be seen most easily in the context of dispersion relations for form factors and, by extension, the Bethe-Salpeter equation. The argument runs as follows.

Let the requirement be imposed on the calculation that the graphs used in the dispersion treatment of the $\Sigma\Lambda\pi$ and $\Sigma N\bar{K}$ form factors correspond to the Bethe-Salpeter graphs in a certain approximation. Then the nucleon-exchange ladders of, e.g., Σ^+ lead to the triangular diagrams of Fig. 2 (bosons dispersed) and Fig. 3

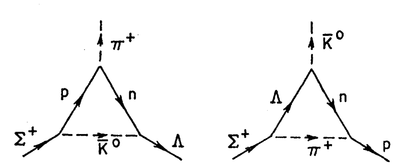

FIG. 2. Graphs for the evaluation of the absorptive parts of the $\Sigma^+\Lambda\pi^+$ and $\Sigma^+ p\bar{K}^0$ form factors with the boson mass dispersed.

[10] K. Nishijima, Phys. Rev. **119**, 485 (1960); **122**, 298 (1961); M. Muraskin and K. Nishijima, *ibid.* **122**, 331 (1961); K. Nishijima, *ibid.* **124**, 255 (1961); E. R. McCliment and K. Nishijima, *ibid.* **128**, 1970 (1962); K. Nishijima, Ref. 1.

[11] L. S. Liu, Phys. Rev. **125**, 761 (1962); B. Barrett and G. Barton, Nuovo Cimento **29**, 703 (1963); G. Barton and B. G. Smith, *ibid.* **36**, 436 (1965); also, Ref. 8., pp. 123 ff.

[12] See the results obtained by M. Ida, Progr. Theoret. Phys. (Kyoto) 34, 92 (1965), concerning the asymptotic behavior of form factors in two-particle unitarity models.

[13] M. Gell-Mann, Phys. Rev. **125**, 1067 (1962).

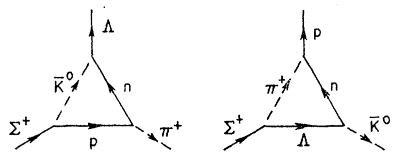

FIG. 3. Graphs for the evaluation of the absorptive parts of the $\Sigma^+\Lambda\pi^+$ and $\Sigma^+p\bar{K}^0$ form factors with the elementary baryon mass dispersed.

(elementary baryons dispersed). The structure of the lower left corner is not important for the comparison of the contributions of different binding agents (N, Σ, K^*, etc.) to the values of the form factors on the mass shell, i.e., to the coupling constants of Σ to its constituents. What is important is that the binding agents, which are the exchanged particles of the ladders, become intermediate, not exchanged particles in the triangular dispersion diagrams, so that in the latter case they are on their mass shell. Now, even if a Born approximation is assumed for the scattering amplitudes occurring in the absorptive parts of the form factors, i.e., even if the two lower corners of the triangular diagrams possess no structure, the upper corner still depends on the dispersion variable. If this corner contains a nondispersed composite particle, it will be damped faster at infinity than one containing only elementary particles, so that the corresponding diagram will contribute less to the dispersion integral. This argument excludes, in particular, Σ exchange in the presence of N exchange. Further, it provides a qualitative explanation for assuming the $N\bar{K}$ system to possess an isospin-1 bound state of spin $\frac{1}{2}^+$, the Σ, but no corresponding isospin-0 state, the Λ, since the latter state cannot be formed via exchange of elementary hadrons.

The above discussion leads directly to the question why the $\Sigma\pi$ channel, which has a low total mass and consists of only two hadrons, both stable against strong interactions, has not been considered by the present model along with $\Lambda\pi$ and $N\bar{K}$. Clearly, if this channel is taken into account, both Σ and Λ can be formed by a $\Sigma\pi$-$N\bar{K}$ nucleon exchange ladder. This already is a difficulty, since Λ is assumed elementary. Further difficulties are associated with the composite nature of one of the constituent particles. Assumption 3, namely, is not applicable to composite form factors, so that it is not easy to introduce a plausible approximation to the Bethe-Salpeter ladders and the corresponding dispersion diagrams if more than one of the vertices in these diagrams is composite. It therefore seems desirable to first try the simplest possibility, i.e., to assume Λ elementary and drop the $\Sigma\pi$ channel.

The essential features of the model have now been described. The plan of the calculations is as follows:

In Sec. 2 unsubtracted dispersion relations for the $\Sigma^+\Lambda\pi^+$ and $\Sigma^+p\bar{K}^0$ form factors are written with the boson masses dispersed and the value of x, the ratio of the coupling constants f_N and f_Λ of Σ^+ to its constituents, is determined. It may be noted that isospin invariance is guaranteed by the input information, so that only Σ^+ need be considered explicitly.

In Sec. 3 the $\Sigma^+\Lambda\pi^+$ and $\Sigma^+p\bar{K}^0$ form factors with the elementary baryon masses dispersed furnish a second value of x. This provides a test for the self-consistency of the calculations.

In Sec. 4 an order-of-magnitude estimate of $|f_\Lambda|$ and $|f_N|$ is attempted with the aid of the normalization condition satisfied by the Bethe-Salpeter wave functions of Σ^+.

Finally, in Sec. 5 the $\Sigma\Lambda\gamma$ vertex is examined and the values of the form factors on the mass shell are determined. The numerical answers are proportional to f_N and may be considered either as predictions of new quantities or, in practice, as an alternative way of fixing the magnitude of the effective coupling constants of Σ to its constituents. The results obtained are discussed in Sec. 6. Before the calculations are presented, some additional remarks on the model will be made.

In evaluating x (Secs. 2 and 3), a Born approximation is assumed for the scattering amplitudes occurring in the absorptive parts of the form factors. Clearly, since this implies that the lower corners of the triangular diagrams in Figs. 2 and 3, and in particular the left corners are structureless, the approximation corresponds to a severe truncation of the Bethe-Salpeter ladders. It is, however, hoped that, even though such a truncation may distort the Bethe-Salpeter wave functions to a degree where they become useless as sources of detailed information on the bound state, it may still lead to acceptable dispersion representations for the form factors describing the interaction of the bound system with its constituents, provided that the truncation can be at least partially offset by some suitable means. The means employed by this paper is the introduction of cutoffs for the dispersion integrals which, as a result of the poorness of the Born approximation, become logarithmically divergent. The presence of cutoffs is a nuisance frequently encountered in actual calculations. Fortunately, in this case the cutoffs can be eliminated in a fairly plausible way (the two-channel nature of the problem being essential for achieving this), so that the final answers do not depend on adjustable parameters.

A second remark concerns the use of the Bethe-Salpeter wave functions in fixing the absolute scale of the effective coupling constants f_Λ and f_N. Each of the two wave functions is proportional to two free-particle propagators (one for each constituent) times a vertex part specifying the interaction of Σ with its constituents. In order to render the normalization integrals tractable, the assumption is made that as far as the vertex part is concerned, the heavy baryon constituent may be treated as real, so that only the dependence on the momentum transferred to the boson need be taken into account. This enables one to express the wave functions in terms of the dispersion representations of Sec. 2 and to carry out integrations in a relatively simple manner. Clearly this, like most of the assumptions made in this paper, hinges on the relatively loose binding of Σ.

2. THE $\Sigma\Lambda\pi$ AND $\Sigma N\bar{K}$ FORM FACTORS WITH THE BOSON MASS DISPERSED

In this section the $\Sigma^+\Lambda\pi^+$ form factor is calculated for low values of the square of the momentum transferred to the pion. The results may be trivially adapted to the $\Sigma^+ p\bar{K}^0$ vertex. Consideration of the dispersion equations on the mass shell yields the ratio of the effective coupling constants of Σ^+ to its constituents.

The $\Sigma\Lambda\pi$ vertex (charge indices are suppressed) may be introduced as follows:

$$\begin{aligned}\Gamma^{rs}(t) &= -(E_\Lambda E_\Sigma/m_\Lambda m_\Sigma)^{1/2}\langle\Lambda^r|\,j_\pi|\Sigma^s\rangle \\ &= F_{\Lambda\pi;\Sigma}(t)\bar{u}^r(p_\Lambda)i\gamma_5 u^s(p_\Sigma)\,,\end{aligned} \tag{2.1}$$

where

$$t=-(p_\Sigma-p_\Lambda)^2. \tag{2.2}$$

Here r and s are spin indices, j_π is the pion current at the origin, and the normalization volume is taken to be unity. $F_{\Lambda\pi;\Sigma}(t)$ is the $\Sigma\Lambda\pi$ form factor whose value on the mass shell defines the $\Sigma^+\Lambda\pi^+$ coupling constant. For convenience the following notation will be used:

$$\begin{aligned}f_\Lambda &= F_{\Lambda\pi;\Sigma}(m_\pi^2)\,,\\ f_N &= F_{p\bar{K};\Sigma}(m_K^2)\,,\\ g_\Lambda &= F_{n\bar{K};\Lambda}(m_K^2)\,,\\ g_N &= F_{p\pi;p}(m_\pi^2)\,.\end{aligned} \tag{2.3}$$

Note that f_N is $\sqrt{2}$ times the commonly used $\Sigma^0 p\bar{K}^+$ coupling constant. g_N is taken to be approximately 13.5. The value of g_Λ is not precisely known and will be treated as a parameter ranging from 5 to 13.5.

According to assumption 2, $F_{\Lambda\pi;\Sigma}(t)$ obeys an unsubtracted dispersion relation. Figure 2, derived from a nucleon exchange Bethe-Salpeter ladder, indicates that only proton-antineutron intermediate states contribute to the absorptive part of F. The coupling of these states to the off-shell pion is described by the pion-nucleon form factor. In principle, the form factor is known, since it belongs to an elementary vertex. However, in practice, this is not the case, so that assumption 3 is here invoked in the form

$$\mathrm{Re}F_{\bar{n}p;\pi}(t)\approx\sqrt{2}g_N\,. \tag{2.4}$$

Further, the single $\bar{K}$ exchange term in the $\Sigma\bar{\Lambda}\to p\bar{n}$ scattering amplitude is taken to be just a Born term. Hence all three vertices of Fig. 2 become structureless. A discussion of this drastic approximation has already been given in Sec. 1.

Standard techniques[14] yield after some calculation the following result for the $\Sigma\Lambda\pi$ form factor:

$$F_{\Lambda\pi;\Sigma}(t_0)=-\frac{1}{\pi}\int_{t_5}^{t_6}dt\frac{\mathrm{Abs}F_{\Lambda\pi;\Sigma}(t)}{t-t_0+i\epsilon}\,, \tag{2.5}$$

$$\mathrm{Abs}F_{\Lambda\pi;\Sigma}(t)=-(16\pi)^{-1}\sqrt{2}g_N g_\Lambda f_N[B(t)+D(t)]\,,$$

where

$$\begin{aligned}B(t)&=[C_1+C_2/t+C_3(1-t_4/t)\Omega/\mathrm{X}]\,H(t)\,,\\ D(t)&=[-\Psi^{1/2}/t+C_3(1-t_4/t)2\Psi^{1/2}/\mathrm{X}]\,\theta(t-t_2)\,,\end{aligned} \tag{2.6}$$

with

$$\begin{aligned}C_1&=\tfrac{1}{2}[-(m_n+m_p)(m_\Sigma+m_\Lambda)+2m_n m_p\\ &\qquad+m_\Sigma^2+m_\Lambda^2-2m_K^2]\,,\\ C_2&=\tfrac{1}{2}(m_n-m_p+m_\Sigma-m_\Lambda)(m_n^2-m_p^2)\\ &\qquad\times(m_\Sigma+m_\Lambda)\,,\\ C_3&=-\tfrac{1}{2}(m_n-m_p+m_\Sigma-m_\Lambda)(m_\Sigma-m_\Lambda)\,,\\ \Psi&=t^2-2(m_p^2+m_n^2)t+(m_n^2-m_p^2)^2\,,\\ \mathrm{X}&=t^2-2(m_\Lambda^2+m_\Sigma^2)t+(m_\Sigma^2-m_\Lambda^2)^2\,,\\ \Omega&=t^2-(m_p^2+m_n^2+m_\Sigma^2+m_\Lambda^2-2m_K^2)t\\ &\qquad-(m_n^2-m_p^2)(m_\Sigma^2-m_\Lambda^2)\,,\\ H(t)&=\mathrm{X}^{-1/2}\ln|[\Omega-(\Psi\mathrm{X})^{1/2}]/[\Omega+(\Psi\mathrm{X})^{1/2}]|\,,\ t_4\leqslant t\\ &=-2(-\mathrm{X})^{-1/2}\tan^{-1}[(-\Psi\mathrm{X})^{1/2}/\Omega]\,,\ t_2\leqslant t\leqslant t_4\\ &=-2\pi(-\mathrm{X})^{-1/2},\quad t_5\leqslant t\leqslant t_2.\end{aligned} \tag{2.7}$$

Here t_4, t_2, and t_5 are the physical, normal, and abnormal thresholds, respectively,

$$\begin{aligned}t_4&=(m_\Lambda+m_\Sigma)^2\,,\\ t_2&=(m_p+m_n)^2\,,\\ t_5&=m_p^2+m_n^2\\ &\qquad+2m_p m_n[-\mu_a\mu_c+(1-\mu_a^2)^{1/2}(1-\mu_c^2)^{1/2}]\,,\end{aligned} \tag{2.8}$$

where

$$\begin{aligned}\mu_a&=(m_K^2+m_p^2-m_\Sigma^2)/(2m_K m_p)\,,\\ \mu_c&=(m_K^2+m_n^2-m_\Lambda^2)/(2m_K m_n)\,.\end{aligned} \tag{2.9}$$

t_6 is a cutoff introduced as a compensation for the logarithmic divergence of the intergal in Eq. (2.5), a result of the poorness of the Born approximation (cf. Sec. 1).

The integration in Eq. (2.5) was carried out numerically on an IBM 7074 computer for various values of t_6 and t_0 on the mass shell. In view of the similarity between the $\Sigma\Lambda\pi$ and $\Sigma N\bar{K}$ vertices, the cutoff was assumed to be the same for both, when measured in appropriate units. Both absolute (m_p^2) and relative (t_2,t_4) units were considered.

Equation (2.5) and its analog for the $\Sigma N\bar{K}$ vertex

[14] J. D. Jackson, in *Dispersion Relations*, edited by G. R. Screaton (Oliver and Boyd Ltd., Edinburgh, Scotland 1961), p. 1; J. D. Jackson, in *Elementary Particle Physics and Field Theory*, edited by K. W. Ford (W. A. Benjamin, Inc., New York, 1963), p. 263, Appendix D; R. E. Cutkosky, J. Math. Phys. **1**, 429 (1960); R. Blankenbecler and Y. Nambu, Nuovo Cimento **18**, 595 (1961); L. D. Landau, Nucl. Phys. **13**, 181 (1959).

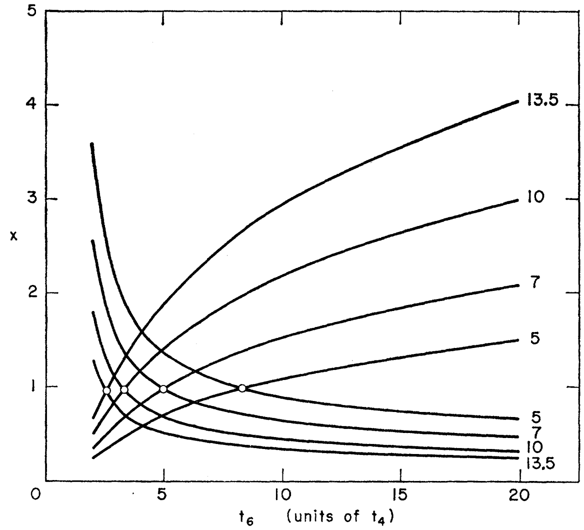

FIG. 4. The ratio of the $\Sigma^+p\bar{K}^0$ to the $\Sigma^+\Lambda\pi^+$ coupling constant x versus the cutoff t_6 of the dispersion integrals corresponding to Fig. 2. t_6 is measured in units of physical threshold. For each value of the magnitude of the $\Lambda p\bar{K}^+$ coupling constant, $|g_\Lambda|$, the intersection of two curves fixes both x and t_6.

yield a set of equations of the form

$$\alpha_1 f_\Lambda + \alpha_2 f_N = 0, \qquad (2.10)$$
$$\alpha_3 f_\Lambda + \alpha_4 f_N = 0,$$

where α_i are cutoff-dependent constants. Each equation yields a certain value for the ratio

$$x = f_N / f_\Lambda. \qquad (2.11)$$

These values are plotted versus t_6 in Fig. 4, with $|g_\Lambda|$ as a parameter. The relative sign of g_Λ and g_N is taken to be negative (x would be negative if the sign of $g_\Lambda g_N$ were reversed). For each value of $|g_\Lambda|$, a self-consistent solution for x is found from the intersection of the two curves, one of which corresponds to the $\Sigma\Lambda\pi$ vertex, and one to the $\Sigma N\bar{K}$ vertex. In this manner the cutoff is eliminated. It is seen that even though the cutoff required to make x self-consistent varies with $|g_\Lambda|$, x is rather insensitive to the simultaneous variation of t_6 and $|g_\Lambda|$.

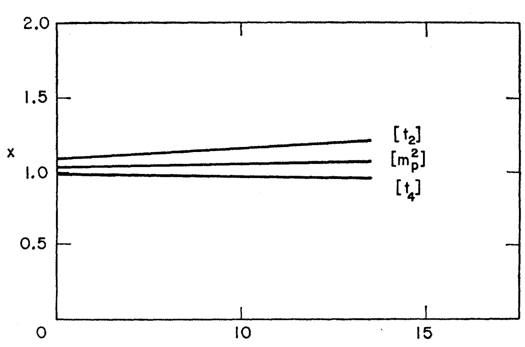

FIG. 5. x as obtained in Sec. 2 versus $|g_\Lambda|$. The three curves correspond to three different choices of cutoff units (normal threshold, m_p^2, physical threshold). The relative sign of g_Λ and the pion-nucleon coupling constant is assumed negative, $g_\Lambda g_N < 0$.

In Fig. 4 the cutoff is measured in units of t_4. Similar plots, not shown here, were also made for units of t_2 and m_p^2. x is shown in Fig. 5 as a function of $|g_\Lambda|$ for these three choices of t_6 units. It is seen that although there is certain degree of arbitrariness in the procedure of the self-consistent elimination of cutoffs, the end result is not very sensitive to it: x lies in the neighborhood of unity for all three choices of units of which t_6 is measured.

3. THE $\Sigma\Lambda\pi$ AND $\Sigma N\bar{K}$ FORM FACTORS WITH THE ELEMENTARY BARYON MASS DISPERSED

In this section the $\Sigma^+p\bar{K}^0$ vertex is considered for low values of the square of the momentum transferred to the proton. The $\Sigma^+\Lambda\pi^+$ vertex is treated analogously. An additional value of x is obtained by methods similar to the ones employed in Sec. 2.

The vertex is introduced as follows:

$$\begin{aligned} G^s(t) &= -(2E_{\bar{K}}E_\Sigma/m_\Sigma)^{1/2}\langle \bar{K}|J_p|\Sigma^s\rangle \\ &= [G_1(t) + i\gamma\cdot(p_\Sigma - p_{\bar{K}})G_2(t)]i\gamma_5 u^s(p_\Sigma), \qquad (3.1) \\ t &= -(p_\Sigma - p_{\bar{K}})^2. \end{aligned}$$

The calculation of the absorptive parts of $G_1(t)$ and $G_2(t)$ proceeds essentially as in Sec. 2. In this case Fig. 3 is used and assumption 3 is employed in the form (cf. Ref. 15)

$$\mathrm{Re}F_1(t) + i\gamma\cdot(p_\Sigma - p_{\bar{K}})\,\mathrm{Re}F_2(t) \approx \sqrt{2}g_N, \qquad (3.2)$$

where F_1 and F_2 are the form factors describing the pion-nucleon vertex with one nucleon off-shell (the analogs of G_1 and G_2). No anomalous threshold appears. The expressions for the absorptive parts of G_1 and G_2 are not written because they are rather lengthy.

It turns out that the dispersion integral for $G_1(t)$ diverges logarithmically. Following the procedure of Sec. 2, an upper limit of integration, t_6, is introduced, the same for $G_1(t)$ and $G_2(t)$, and varied. The ratio $x = f_N/f_\Lambda$ is obtained for various values of g_Λ by considering the dispersion equation on the mass shell. The calculation is repeated for the $\Sigma\Lambda\pi$ vertex. The result is plotted in Fig. 6 versus the common cutoff t_6, measured in units of t_4. The sign of x is again opposite to that of $g_N g_\Lambda$, assumed negative. x is self-consistently determined by the intersection of the $\Sigma\Lambda\pi$ and $\Sigma N\bar{K}$ curves and the cutoff thereby eliminated. x is plotted versus $|g_\Lambda|$ in Fig. 7, for three different choices of t_6 units. Its variation with the choice of units is not negligible, but it tends to compensate for the corresponding variation of x in Sec. 2. The average of the value of x as given in Secs. 2 and 3 is shown in Fig. 8. It may thus be con-

[15] M. Ida, Phys. Rev. **136**, B1769 (1964).

cluded that

$$x \approx \pm 1 \quad \text{for} \quad g_N g_\Lambda \lesssim 0. \tag{3.3}$$

4. THE BETHE-SALPETER WAVE FUNCTIONS

In this section the Bethe-Salpeter wave functions of Σ^+ are introduced. The normalization condition which they are assumed to obey[16] is transformed into an inhomogeneous algebraic equation for the squares of the effective coupling constants f_Λ and f_N. In conjunction with the results of Secs. 2–3 the equation yields the magnitude of the two coupling constants.

The $\Lambda\pi$ channel is characterized by the superscript 1, the $N\bar{K}$ channel by 2. $\phi^i(x)$, $\psi^i(x)$ denote boson fermion field operators, respectively; $\Delta_F{}^i(x)$, $S_F{}^i{}_{,}(x)$ are free boson, fermion propagators. The Bethe-Salpeter

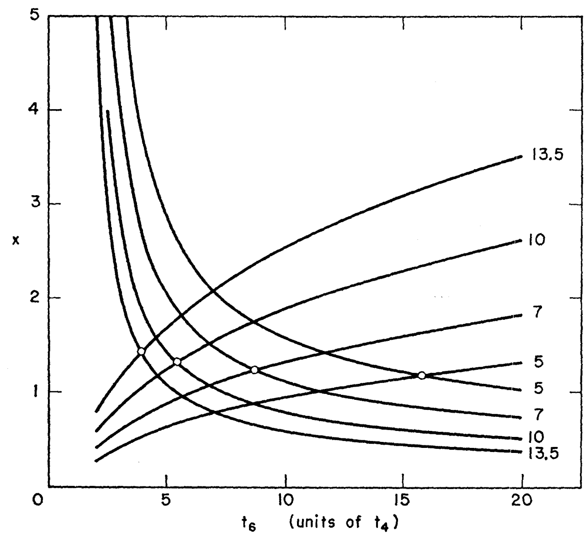

FIG. 6. x versus the cutoff t_6 of the dispersion integrals corresponding to Fig. 3, with $|g_\Lambda|$ as a parameter. t_6 is measured in units of physical threshold.

wave functions of Σ are defined as follows:

$$\chi^i(P,s;x_1,x_2) = \langle 0 | T[\psi^i(x_1)\phi^i(x_2)] | \Sigma^s(P) \rangle. \tag{4.1}$$

The integral equations satisfied by the χ^i may be found by the technique of Ref. 17, starting from nucleon ladder approximations to the two-body propagators $G^{ii}(x_1,x_2;y_1,y_2)$ (cf. Fig. 1 for $i=1$). This approximation (cf. Sec. 1) leads to

$$\chi^i(P,s;x_1,x_2) = \int d^4\xi_1 d^4\xi_2 S_F{}^i(x_1-\xi_1)\Delta_F{}^i(x_2-\xi_2) \times I(\xi_1-\xi_2)\chi^j(P,s;\xi_2,\xi_1), \quad i \neq j, \tag{4.2}$$

[16] R. E. Cutkosky and M. Leon, Phys. Rev. **135**, B1445 (1964).
[17] M. Gell-Mann and F. Low, Phys. Rev. **84**, 350 (1951).

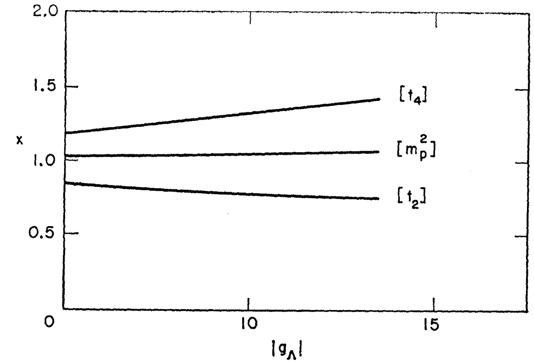

FIG. 7. x as obtained in Sec. 3 versus $|g_\Lambda|$. The three curves correspond to three different choices of t_6 units employed in the elimination of cutoffs. $g_\Lambda g_N$ is assumed negative.

where the interaction is the same for both channels,

$$I(\xi_1-\xi_2) = -g_\Lambda g_N i\gamma_5 S_F{}^N(\xi_1-\xi_2) i\gamma_5. \tag{4.3}$$

The first of a pair of coordinates or momenta in the argument of G^{ii} or χ^i always refers to the relevant fermion, the second to the boson. Let

$$X = \mu_1 x_1 + \mu_2 x_2, \quad x = x_1 - x_2, \quad \mu_1 + \mu_2 = 1. \tag{4.4}$$

Translational invariance implies

$$\chi^i(P,s;x_1,x_2) = \chi^i(P,s;x) \exp iP\cdot X. \tag{4.5}$$

If the Fourier transform of the relative wave function is defined through

$$\chi^i(P,s;x) = (2\pi)^{-4} \int d^4q \quad \chi^i(P,s;q) \exp iq\cdot x, \tag{4.6}$$

then it can be shown[18] that it is related to the off-shell T-matrix element describing the transition of Σ into

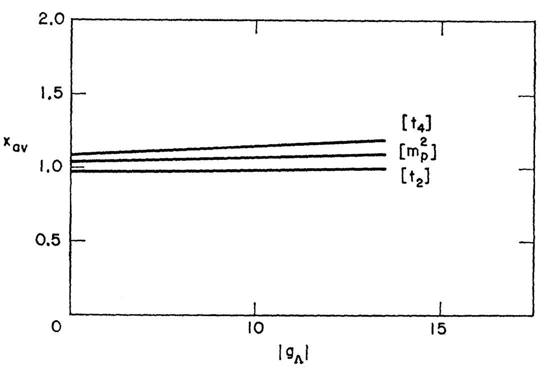

FIG. 8. The average value of x as obtained from Figs. 5 and 7.

[18] R. Blankenbecler, M. L. Goldberger, and F. R. Halpern, Nucl. Phys. **12**, 629 (1959).

the ith channel by

$$\chi^i(P,s;q)=[K^i(P,q)]^{-1}\times[-i(m_\Sigma/E_\Sigma)\Gamma^i(P,q)]u^s(P), \quad (4.7)$$

where

$$T^i(p_1^r,p_2;P^s)=\bar{u}^r(p_1)\Gamma^i(P,q)u^s(P),$$
$$[K^i(P,q)]^{-1}=S_F{}^i(\mu_1P+q)\Delta_F{}^i(\mu_2P-q), \quad (4.8)$$
$$p_1=\mu_1P+q, \quad p_2=\mu_2P-q.$$

The $S_F{}^i$ and $\Delta_F{}^i$ of Eq. (4.8) are Fourier transforms of free propagators. For future use the following quantity is also introduced:

$$I(P,q,k)=-g_\Lambda g_N(2\pi)^{-4}i\gamma_5\times S_F{}^N[(\mu_1-\mu_2)P+q+k]i\gamma_5. \quad (4.9)$$

The normalization condition on the Bethe-Salpeter wave functions requires a knowledge of $\chi^i(P,s;q)$ for all values of q. Hence both the boson and fermion constituents must be allowed to be off-shell. However, the approximation which would correspond to the triangular diagrams of the preceding sections, namely,

$$\chi^i(P,s;q)=[K^i(P,q)]^{-1}\int d^4k\times I(P,q,k)\chi^{(1)j}(P,s;k), \quad (4.10)$$

where $\chi^{(1)j}$ is the "first" approximation to χ^j,

$$\chi^{(1)j}(P,s;k)=[K^j(P,k)]^{-1}\times[-i(E_\Sigma/m_\Sigma)^{-1/2}f^ji\gamma_5]u^s(P), \quad (4.11)$$
$$f^j=f_\Lambda \quad \text{for} \quad j=1,$$
$$=f_N \quad \text{for} \quad j=2,$$

leads to integrals which are too involved to be of practical use. For this reason the assumption is made that, as far as the vertex part of the Bethe-Salpeter wave functions is concerned, the heavy baryon constituent may be treated as real. Thus only the lighter boson is allowed to be virtual and $\Gamma^i(P,q)$ in Eq. (4.7) is approximated by

$$\Gamma^i(P,q)=F^i(t)^*i\gamma_5, \quad (4.12)$$
$$t=-(\mu_2P-q)^2.$$

F^i are the $\Sigma\Lambda\pi$ and $\Sigma N\bar{K}$ form factors of Sec. 2. That their complex conjugate is needed may be seen by reducing Eq. (4.10) to (4.7) and (4.12) in the limit of on-shell Λ, N. Since $[K^i]^{-1}$ as well as the adjoint of χ^i have $-i\epsilon$ prescription for going around the poles, the existence of the normalization integrals is guaranteed. The adjoint of χ^i is given by[19]

$$\bar{\chi}^i(P,s;x_1,x_2)=\eta^i[\exp(-iP\cdot X)](2\pi)^{-4}\times\int d^4q \quad \bar{\chi}^i(P,s;q)\exp(-iq\cdot x), \quad (4.13)$$

where η^i is a phase factor and $\bar{\chi}^i(P,s;q)$ is simply the Dirac adjoint of $\chi^i(P,s;q)$ and possesses the same $-i\epsilon$ prescription as the latter.

The derivation of the normalization condition of Ref. 16 may be adapted to the present problem through the introduction of a two-dimensional space labeled by the two channels $\Lambda\pi$ and $N\bar{K}$. Then χ, G, I, and K are understood to be a two-component vector, a 2×2 matrix, a 2×2 matrix with vanishing diagonal elements, and a 2×2 diagonal matrix, respectively. With this notation the arguments proceed as in Ref. 16. There are, however, a few modifications, which take the fermion nature of Σ and the normalization of state vectors into account. With these modifications, the normalization condition reads in momentum space[20]

$$(2\pi)^{-4}\int d^4q\,\bar{\chi}(P,r;q)[\partial_\mu K(P,q)]\chi(P,s;q)$$
$$-(2\pi)^{-4}\int\int d^4qd^4q'\,\bar{\chi}(P,r;q)[\partial_\mu I(P,q,q')]$$
$$\times\chi(P,s;q')=(2m_\Sigma)^{-1}(E_\Sigma/m_\Sigma)^{-1}2iP_\mu\delta_{rs},$$
$$\partial_\mu\equiv\partial/\partial P_\mu. \quad (4.14)$$

The orthogonality of degenerate amplitudes[16] corresponding to different spin orientations, as well as the validity of the $r\neq s$ case in Eq. (4.14) is guaranteed if it is assumed that the $u^s(P)$ of Eq. (4.7) is given formally by the same expressions of spin-up, spin-down spinors as for $P^2=-m_\Sigma{}^2$. With the choice

$$\mu_1=\mu_2=\tfrac{1}{2}, \quad (4.15)$$

the second term in Eq. (4.14) vanishes. Summation over r for $r=s$ leads after some algebra to the following expression:

$$\sum_i\eta^i(2\pi)^{-4}\pi^{-2}\int_{t_5}^{t_6}dt_1\int_{t_5}^{t_6}dt_2\int d^4Q\times N^i(P,Q,t_1,t_2)T^i(P,Q,t_1,t_2)=2iP_\mu, \quad (4.16)$$

where

$$N^1(P,Q,t_1,t_2)=(t_1+Q^2-i\epsilon)^{-1}(t_2+Q^2-i\epsilon)^{-1}\times(Q^2+m_\pi{}^2-i\epsilon)^{-2}[(P-Q)^2+m_\Lambda{}^2-i\epsilon]^{-2},$$
$$T^1(P,Q,t_1,t_2)=F^1(t_1,t_2)[\beta_1P_\mu+\beta_2Q^2P_\mu-Q^4P_\mu+\beta_3Q_\mu+\beta_4P\cdot QQ_\mu+4(P\cdot Q)^2Q_\mu], \quad (4.17)$$

with

$$F^1(t_1,t_2)=\mathrm{Abs}F^1(t_1)\,\mathrm{Abs}F^1(t_2),$$
$$\beta_1=-m_\pi{}^2(m_\Sigma-m_\Lambda)^2,$$
$$\beta_2=-m_\pi{}^2-(m_\Sigma-m_\Lambda)^2, \quad (4.18)$$
$$\beta_3=2m_\Sigma(m_\Sigma-m_\Lambda)(m_\Sigma{}^2+m_\pi{}^2-m_\Lambda{}^2),$$
$$\beta_4=2(m_\Sigma-m_\Lambda)^2-2m_\pi{}^2+4(m_\Sigma{}^2+m_\pi{}^2-m_\Lambda{}^2).$$

[19] K. Nishijima, Progr. Theoret. Phys. (Kyoto) 13, 305 (1955); N. Nakanishi, Phys. Rev. 138, B1182 (1965).

[20] Cf. K. Baumann, P. G. O. Freund, and W. Thirring, Nuovo Cimento 18, 908 (1960); J. Harte, *ibid.* 45, 179 (1966).

The expressions for N^2 and T^2 are obtained from N^1 and T^1 through the replacements $m_\Lambda \to m_N$, $m_\pi \to m_K$, and $F^1(t_1,t_2) \to F^2(t_1,t_2)$.

The cutoff t_6 in Eq. (4.16) is taken from Sec. 2. Thus no arbitrary parameters are introduced. Since the absorptive parts of the form factors F^i are proportional to the coupling constants f^i, Eq. (4.16) reduces, after integration of the left side,[21] to the form

$$\gamma_1 f_\Lambda{}^2 + \gamma_2 f_N{}^2 = 1\,, \tag{4.19}$$

where γ_i depend on t_6 and η^i have been set equal to -1. t_6 is determined by $g_\Lambda \cdot |f_\Lambda|$ and $|f_N|$ are plotted in Figs. 9 and 10, respectively, as functions of $|g_\Lambda|$. For large $|g_\Lambda|$ the choice of cutoff units is not entirely immaterial, but the discrepancies do not significantly damage the crude approximation scheme employed

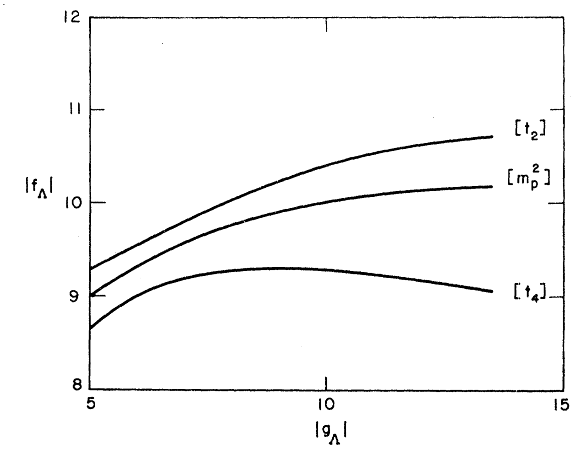

FIG. 9. The magnitude of the $\Sigma^+\Lambda\pi^+$ coupling constant, $|f_\Lambda|$, versus the magnitude of the $\Lambda p\bar{K}^+$ coupling constant, $|g_\Lambda|$. The three curves correspond to those of Fig. 5.

here. It may thus be concluded that

$$8.7 < |f_\Lambda| < 10.7\,, \tag{4.20}$$
$$9.0 < |f_N| < 11.2\,,$$

for

$$5.0 < |g_\Lambda| < 13.5\,.$$

5. THE ΣΛγ VERTEX

The assumptions mentioned in Sec. 1 are applied in this section to a dispersion-theoretic determination of the form factors which describe the interaction of a neutral Σ with a Λ and a photon as a function of the squared mass of the latter. Numerical results are obtained for the mass-shell case. The calculation proceeds essentially as in Secs. 2 and 3, except that no divergent integrals appear. The ΣΛγ vertex is in-

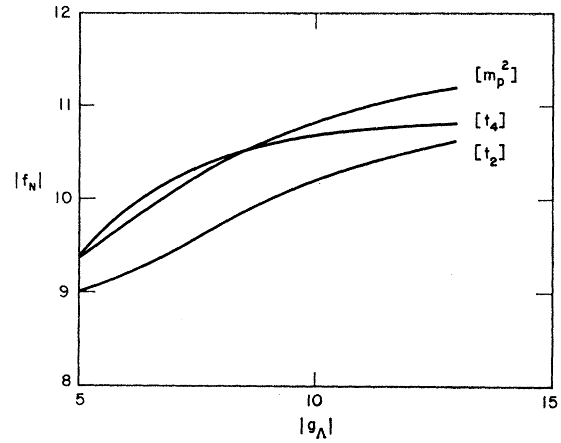

FIG. 10. The magnitude of the $\Sigma^+ p\bar{K}^0$ coupling constant, $|f_N|$, versus $|g_\Lambda|$. The curves correspond to those of Fig. 5.

troduced as

$$\Gamma_\mu{}^{rs}(t) = -(E_\Lambda E_\Sigma / m_\Lambda m_\Sigma)^{1/2} \langle \Lambda^r | j_\mu | \Sigma^s \rangle\,, \tag{5.1}$$
$$t = -(p_\Sigma - p_\Lambda)^2\,.$$

j_μ is the electromagnetic current at the origin. Let

$$K = p_\Sigma - p_\Lambda\,. \tag{5.2}$$

Form factors are introduced as follows:

$$\Gamma_\mu{}^{rs}(t) = -\bar{u}^r(p_\Lambda) \times [G_1(t) i\gamma_\mu + G_2(t) i\sigma_{\mu\nu} K_\nu + G_3(t) K_\mu] u^s(p_\Sigma)\,. \tag{5.3}$$

Only $G_2(t)$ and $G_3(t)$ need be considered since $G_1(t)$ is related to $G_3(t)$ via current conservation. $G_2(0)$ is the ΣΛ transition moment.

The absorptive part of $\Gamma_\mu{}^{rs}(t)$ is evaluated in analogy to the absorptive parts of the $\Sigma\Lambda\pi$ and $\Sigma N\bar{K}$ form factors of Secs. 2 and 3. That is, Σ is assumed to interact with Λ and γ by virtue of the interaction of its constituents. Further, only two-particle intermediate states are kept which, according to Sec. 1, contain only elementary hadrons. C-conjugation invariance, assumed valid, excludes $\pi^0\eta$ intermediate states, and one is left with $\bar{K}^+K^+$, $\bar{K}^0K^0$, $p\bar{p}$, and $n\bar{n}$ pairs. If electromagnetic splittings of strong-interaction parameters are neglected, only the isovector configurations contribute. Thus there are two graphs to be taken into account (Fig. 11). As before, the two lower vertices are assumed structureless (Born approximation) so that their form factors are set equal to their value on the mass shell. The upper vertices are represented by

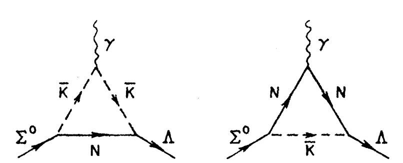

FIG. 11. Graphs for the evaluation of the absorptive parts of the ΣΛ electromagnetic form factors.

[21] A combination of the Feynman parametrization technique [see J. M. Jauch and F. Rohrlich, *The Theory of Photons and Electrons* (Addison-Wesley Publishing Company, Inc., Reading, Massachusetts, 1955), Appendix A5] and numerical integration is employed.

the isovector parts of the kaon and nucleon form factors. These are defined through the equations

$$\langle \bar{K}K \text{ out}|j_\mu|0\rangle = (4E_K E_{\bar{K}})^{-1/2} F_{\bar{K}K;\gamma}(t)(p_{\bar{K}}-p_K)_\mu \quad (5.4)$$

and

$$\langle \bar{N}^\rho N^\sigma \text{ out}|j_\mu|0\rangle = (E_N E_{\bar{N}}/m_N m_{\bar{N}})^{-1/2}\bar{u}^\sigma(p_N) \times[F_1(t)i\gamma_\mu - F_2(t)i\sigma_{\mu\nu}K_\nu]v^\rho(p_{\bar{N}}). \quad (5.5)$$

For the kaon form factor the following *ansatz* is employed:

$$F_{\bar{K}K;\gamma}(t) = -e[1-\alpha_1+\alpha_1 m_1^2/(m_1^2-t)], \quad (5.6)$$

where e is the charge of the proton, m_1 is the effective mass of the ρ meson, taken to be (cf. Ref. 22)

$$m_1 \approx 600 \text{ MeV}, \quad (5.7)$$

and α_1 is a parameter. The nucleon form factors are taken from Ref. 23, where data covering the region -1 to -30 F^{-2} are fitted by the following expressions:

$$\begin{aligned} &F_1(t) = e[\alpha_2+\alpha_3 m_3^2/(m_3^2-t)], \\ &F_2(t) = (e/2m_N)[\alpha_4+\alpha_5 m_5^2/(m_5^2-t)], \\ &m_3 = m_5 \approx 565 \text{ MeV}, \\ &\alpha_2 = 0.19, \quad \alpha_3 = 0.81, \\ &\alpha_4 = -0.48, \quad \alpha_5 = 4.18. \end{aligned} \quad (5.8)$$

The final answer for the on-shell form factors may be written in terms of α_1 as follows:

$$\begin{aligned} G_2(0) &= g_\Lambda(f_N/\sqrt{2})[0.68+0.23\alpha_1]10^{-2}e/2m_N, \\ G_3(0) &= g_\Lambda(f_N/\sqrt{2})[2.65-1.65\alpha_1]10^{-4}e/2m_N. \end{aligned} \quad (5.9)$$

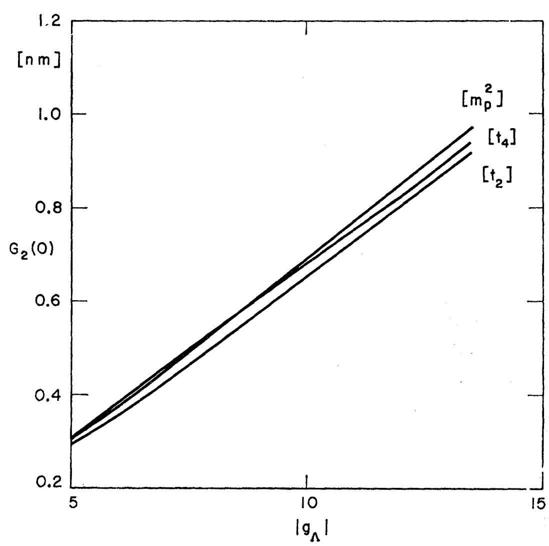

FIG. 12. The $\Sigma\Lambda$ transition moment $G_2(0)$, in nuclear magnetons, versus $|g_\Lambda|$. The three curves correspond to those of Fig. 10.

[22] C. W. Akerlof, W. W. Ash, K. Berkelman, and C. A. Lichtenstein, Phys. Rev. Letters **16**, 147 (1966).
[23] E. B. Hughes, T. A. Griffy, M. R. Yearian, and R. Hofstadter, Phys. Rev. **139**, B458 (1965).

The curves of Fig. 10 may be used in Eq. (5.9) to yield $G_2(0)$, and $G_3(0)$ for given α_1. For definiteness it is assumed that $g_\Lambda f_N > 0$ and that the ρ dominates the kaon isovector form factor,

$$\alpha_1 \approx 1. \quad (5.10)$$

The results are plotted in Figs. 12 and 13 as functions of $|g_\Lambda|$, where the three curves correspond to the three choices of cutoff units in Secs. 2–4. It is seen that the choice of t_6 units is immaterial. From the curves and the formula which relates $G_2(0)$ to the Σ^0 lifetime,[24] τ_{Σ^0}, it may be concluded that

$$\begin{aligned} &0.29 \lesssim G_2(0) \lesssim 0.98, \\ &0.32\times10^{-2} \lesssim G_3(0) \lesssim 1.07\times10^{-2}, \\ &0.21\times10^{-18} \lesssim \tau_{\Sigma^0} \lesssim 2.36\times10^{-18} \end{aligned} \quad (5.11)$$

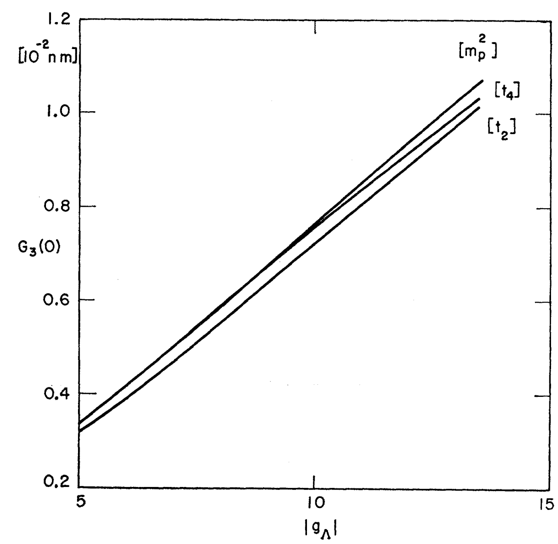

FIG. 13. The form factor $G_3(0)$ of Sec. 5, in 10^{-2} nuclear magnetons, versus $|g_\Lambda|$. The three curves correspond to those of Fig. 10.

for the range

$$5.0 \lesssim |g_\Lambda| \lesssim 13.5.$$

The first two quantities in Eq. (5.11) are measured in nuclear magnetons, the third in seconds.

The formulas of this section could be easily adapted to a calculation of the magnetic moment of Σ^+. Unfortunately, the imput electromagnetic form factors would be very uncertain in this case, so that no numerical calculations have been undertaken.

6. CONCLUSION

The results of the calculations undertaken in this paper under the assumptions discussed in Sec. 1 may be summarized as follows.

[24] J. Dreitlein and H. Primakoff, Phys. Rev. **125**, 1671 (1962).

Let $f_N/\sqrt{2}$, f_Λ, g_N, g_Λ denote $g_{\Sigma^0 pK^-}$, $g_{\Sigma^0\Lambda\pi^0}$, $g_{pp\pi^0}$, $g_{\Lambda pK^-}$, respectively, and $x=f_N/f_\Lambda$. Further let $G_2(t)$ and $G_3(t)$ be the form factors characterizing the $\Sigma^0\Lambda\gamma$ vertex, defined by Eqs. (5.1)–(5.3). Then for the range

$$5.0<|g_\Lambda|<13.5\,, \tag{6.1}$$

the model predicts

$$\begin{aligned} x&\approx\pm1 \quad \text{for} \quad g_Ng_\Lambda\lesssim 0\,,\\ 8.7&<|f_\Lambda|<10.7\,, \\ 9.0&<|f_N|<11.2\,. \end{aligned} \tag{6.2}$$

Further, if ρ dominance of the kaon isovector electromagnetic form factor is assumed, then, in units of $e/2m_N$,

$$\begin{aligned} 0.29&<|G_2(0)|<0.98\,, \\ 0.32\times10^{-2}&<|G_3(0)|<1.07\times10^{-2}\,. \end{aligned} \tag{6.3}$$

TABLE I. Theoretical estimates of the magnitudes of the strong coupling constants g_Λ, f_Λ, and f_N.

Author[a]	$\lvert g_\Lambda\rvert$	$\lvert f_\Lambda\rvert$	$\lvert f_N\rvert$
Aznauryan and Soloviev		12.0	
Cook	8.1		
Cutkosky	10.5–14.6	9.0–13.1	2.0– 9.1
Dalitz		13.5	
Dufour	7.9– 8.7		
deSwart and Iddings		12 –17	
Eberle	7.2– 9.4	0 –13.6	4.2–15.3
Gürsey *et al.*	14.0	9.4	3.8
Kayser		22.4	
Kuo	7.1– 7.5		
Lusignoli *et al.*	7.8		$\lesssim$9.0
Martin and Wali	11.8–14.9	8.6–11.8	1.4– 6.8
B. R. Martin		11.7	
Matsuda and Oneda	10		
Minamikawa			$\lesssim$13
Raman	7.8–14.5	12.5	4.1– 5.7
Rimpault	7.4	3.4	10.5
Umemura and Watanabe	13.5	6.8	
Warnock and Frye	10 –17		
Zovko	9.2		7.3

[a] See Ref. 25.

In order to facilitate the comparison of the above results to those of other authors, Tables I and II have been prepared.[25,26] It is seen that the predictions of the model concerning the magnitudes of f_Λ, f_N, and $G_2(0)$ fall within the range of presently acceptable numbers. The sign of x is the same as that given by $SU(3)$ with an F/D ratio smaller than 1, a currently accepted value.[27] The sign of $G_2(0)$, the $\Sigma\Lambda$ transition moment, is predicted to be positive for $g_\Lambda f_N>0$, in disagreement with $SU(3)$.[28] It should, however, be remarked that this sign is not experimentally measurable.

TABLE II. Theoretical estimates of the $\Sigma\Lambda$ transition moment $G_2(0)$ in nuclear magnetons.

Author[a]	$G_2(0)$ [nm]
Coleman and Glashow[b]	1.65
Dreitlein and Lee	1.3 (for $g_{\Sigma\Sigma\pi}f_\Lambda\approx g_N^2$)
Mathur and Pandit	1.5 –2.0
Pisarenko	1.52–1.82
Primakoff	1.85 (for $g_{\Sigma\Lambda\rho}\approx g_{NN\rho}$)

[a] See Ref. 26.
[b] The sign is taken from Ref. 28.

In conclusion, the predictions of the present model are compatible with existing experimental and much theoretical information. Thus, even though it is not possible in the absence of decisive tests to assert the validity of the model, it is permissible to say that the results obtained allow the possibility that its main features are approximately correct.

Note added in proof. Table I should be supplemented by the following recent estimates of the $\Lambda N\bar{K}$ and $\Sigma N\bar{K}$ coupling constants based on forward dispersion relations for kaon-nucleon scattering: G. H. Davies, N. M. Queen, M. Lusignoli, M. Restignoli, and G. Violini (unpublished): $|g_\Lambda|\approx8.7$, $|f_N|\lesssim11.9$; J. K. Kim [Phys. Rev. Letters **19**, 1079 (1967)]: $|g_\Lambda|\approx14.2$, $|f_N|\approx2.7$; A. D. Martin and F. Poole [Phys. Letters **25**, B343 (1967)]: $|g_\Lambda|\approx7.9$; H. P. C. Rood [Nuovo Cimento **50**, A493 (1967)]: $|g_\Lambda|\approx9.6$. Kim's results indicate that the breaking of $SU(3)$ baryon-pseudoscalar meson couplings is small. If this is indeed the case, the conclusions of the present paper should be modified.

ACKNOWLEDGMENTS

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