we can propose a reason based on final-state interactions for apparent ρ suppression using semiclassical reasoning in these data. Since the ρ has a short lifetime, and the probability of emission is greatest at smallest M_{23} (small meson energy in the frame of emitting nucleon), there will be a relatively high probability for decay into two pions before escaping very far from the emitting nucleon. The probability for differential rescattering of the final-state pions on the emitter should therefore be important, especially since the Q value in the decay is large, resulting in a wide distribution of decay pion momenta with respect to the emitting nucleon. Such rescattering will cause the given event to appear in a different invariant-mass bin, thus redistributing the ρ events over the spectrum. Such effects are smaller by a large factor for the other mesons, since their lifetimes are much longer.

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Current-Algebra Calculation of Hard-Pion Processes: $A_1 \to \rho+\pi$ and $\rho \to \pi+\pi$†

HOWARD J. SCHNITZER
Brandeis University, Waltham, Massachusetts

AND

STEVEN WEINBERG*
University of California, Berkeley, California

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New techniques are developed for treating the n-point functions of currents, which are interrelated by means of Ward identities obtained from the equal-time current commutation relations. The n-point functions are sorted out so as to define proper vertices which describe the reactions of particles of definite spin. A meson-dominance assumption is made by approximating the proper vertices by simple polynomials in momenta, with the coefficients determined by the Ward identities. The method is discussed in detail for $n=3$ and the currents of chiral $SU(2)\times SU(2)$, and then applied to the decay processes $A_1 \to \rho+\pi$ and $\rho \to \pi+\pi$.

I. INTRODUCTION

MOST of the successful predictions made by current algebra have taken the form of low-energy theorems for soft pions,[1] or equivalent sum rules. However, the scope of current algebra has recently been extended to areas having nothing to do with soft pions, by making use of the additional assumption that the vector and axial-vector currents are dominated by $j=1$ and $j=0$ mesons. In particular, it has been possible to show[2] that $m_{A_1}/m_\rho=\sqrt{2}$, and to derive similar results[3] for the other vector and axial-vector mesons. The idea of meson dominance of the currents has received further support from a successful calculation[4] of the π^+-π^0 mass difference and has led to an estimate[5] of the intermediate boson mass.

With these advances has come a new problem. Several authors[6] have noted that if the chiral $SU(2)\times SU(2)$ currents are saturated by the ρ, A_1, and π mesons, then the A_1-ρ-soft-π vertex is

$$\Gamma_{\nu\lambda}\simeq -2m_\rho^2F_\pi^{-1}g_{\nu\lambda}\,, \tag{1.1}$$

where F_π is the usual pion-decay amplitude and ν and λ are the A_1 and ρ polarization indices. But using (1.1) to calculate the decay rate for $A_1\to\rho+\pi$ would give an A_1 width of about 800 MeV. We are prepared to be tolerant in comparing current-algebra predictions with experiment, but this certainly has to be counted as a

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* Presently Morris Loeb Lecturer at Harvard University. Address during 1967–68: Physics Department, Massachusetts Institute of Technology, Cambridge, Massachusetts.

[1] For a review, see the rapporteur's talk by R. F. Dashen, in *Proceedings of the Eighth International Conference on High-Energy Nuclear Physics* (University of California Press, Berkeley, California, 1967), p. 51.

[2] Steven Weinberg, Phys. Rev. Letters **18**, 507 (1967).

[3] T. Das, V. S. Mathur, and S. Okubo, Phys. Rev. Letters **18**, 761 (1967); H. T. Nieh, *ibid.* **19**, 43 (1967); S. L. Glashow, H. J. Schnitzer, and Steven Weinberg, *ibid.* **19**, 139 (1967).

[4] T. Das, G. S. Guralnik, V. S. Mathur, F. E. Low, and J. E. Young, Phys. Rev. Letters **18**, 759 (1967).

[5] S. L. Glashow, H. J. Schnitzer, and Steven Weinberg, Phys. Rev. Letters **19**, 205 (1967).

[6] D. Geffen, Phys. Rev. Letters **19**, 770 (1967); B. Renner, Phys. Letters **21**, 453 (1966); H. J. Schnitzer (unpublished). See footnote 9 of Ref. 2.

failure. One way out was noted in Ref. 3; the pion in A_1 decay is, in principle, *not* a soft pion, since $|P_\pi \cdot P_\rho| \simeq \frac{1}{2} m_\rho^2$. However, it is one thing to use this as an excuse for our failure to calculate the A_1 width, and another thing to show how to get the right answer. More generally, there are a number of decay processes, such as $\rho \to \pi+\pi$, $K^* \to K+\pi$, $K_A \to K^*+\pi$, etc., which we would like to be able to calculate, but which cannot justifiably be treated by the ordinary methods of current algebra because the pions emitted are not soft.

An important clue to the solution of this problem comes to us from a recent work of Schwinger.[7] He studied a phenomenological Lagrangian[8] for ρ, A_1, and π mesons, derived by extending the Yang-Mills theory to chiral $SU(2) \times SU(2)$. The previous soft-pion predictions for[6] $A_1 \to \rho+\pi$ and[9] $\rho \to \pi+\pi$ were recovered off the mass shell, but it was found that on the mass shell, the matrix elements for these processes were reduced, respectively, by factors $\frac{1}{2}$ and $\frac{3}{4}$, the corrections arising of course from the fact that the emitted pions are not soft. This brought the A_1 width into reasonable agreement with experiment, without seriously worsening the situation for $\rho \to \pi+\pi$.

Schwinger's work has led us to reexamine the possible application of current algebra to processes involving hard pions. The problem here is that the usual current-algebra manipulations[10] let us write the matrix elements for pion reactions in terms of calculable equal-time commutator terms, plus unknown "gradient coupling" terms which can only be neglected if the pions are soft. In some cases[11] reasonable models can be used to calculate the gradient-coupling terms, but this possibility is not open to us here. However, although we can not neglect gradient-coupling terms, and have no model with which to calculate them, we will be able to use crossing symmetry to fill this lack. All the decay processes with which we are concerned have matrix elements related to *vacuum* expectation values of time-ordered products of currents, and these are subject to simultaneous Ward identities for each of the spin-1 channels. This provides enough information to do the job.

To be more specific, our method proceeds according to the following steps:

(a) We sort out the contributions of spin-0 and spin-1 poles in the various channels of an n-point function of currents, by defining *proper* vertices which describe reactions among particles of definite spin.

(b) We use the current commutation relation to derive the Ward identities satisfied by the n-point functions.

(c) We rewrite the results of (b) as Ward identities for the proper vertices defined in (a).

(d) We invoke the meson-dominance assumption by approximating the proper vertices as simple polynomials in 4-momenta.

(e) We determine the coefficients in these polynomials by subjecting them to the Ward identities derived in (c).

This method can evidently be applied to any Lie algebra of currents, and to arbitrary n-point functions of these currents. However, in this paper we will content ourselves with a study of the three-point functions

$$\langle T\{A_a{}^\mu, A_b{}^\nu, V_c{}^\lambda\}\rangle_0 \quad \text{and} \quad \langle T\{V_a{}^\mu, V_b{}^\nu, V_c{}^\lambda\}\rangle_0,$$

where $V_a{}^\mu$ and $V_a{}^\mu$ are the currents of chiral $SU(2) \times SU(2)$. Steps (a)–(c), which are essentially exact, will be carried out in Sec. II, and then steps (d) and (e), which rely on the approximation of meson dominance, will be described in Sec. III. The physical information contained in these three-point decay amplitudes include the amplitudes for the decay processes $A_1 \to \rho+\pi$ and $\rho \to \pi+\pi$, as well as the electromagnetic structure of the A_1, π, and ρ. Our predictions are outlined in Sec. IV; they depend upon a single unknown parameter δ; and for $\delta=0$ we find that our decay amplitudes agree precisely with those of Schwinger.[7]

Our study of the vacuum expectation values of time-ordered products of currents has brought us in touch with the purest results of current algebra, results which rely only on the current commutation relations and on the meson-dominance approximation, and which do not depend upon empirical parameters like g_A/g_V. It will be interesting to see whether the extension of this approach to general n-point functions could lead to a satisfactory and self-contained theory of all low-energy reactions among mesons of spin 0 and spin 1.

II. WARD IDENTITIES FOR PROPER VERTICES

We shall apply our method to the three-point function[12] $\langle T\{A_a{}^\mu(x), A_b{}^\nu(y), V_c{}^\lambda(z)\}\rangle_0$. The vector current has a ρ pole, while the axial currents have A_1 and π poles, so that this three-point function describes such different processes as $\rho \to \pi+\pi$, $A_1 \to \rho+\pi$, and $A_1 \to A_1+\rho$. Our first task must be to disentangle their various contributions. We will do this by a sequence of formulas which in turn define the ρ-π-π vertex Γ_λ, the π-A_1-ρ vertex $\Gamma_{\nu\lambda}$, and the A_1-A_1-ρ vertex $\Gamma_{\mu\nu\lambda}$:

$$\int d^4x d^4y\, e^{-iq\cdot x} e^{ip\cdot y} \langle T\{\partial_\mu A_a{}^\mu(x), \partial_\nu A_b{}^\nu(y), V_c{}^\lambda(0)\}\rangle_0 = \frac{i\epsilon_{abc} F_\pi{}^2 m_\pi{}^4 g_\rho{}^{-1}}{(q^2+m_\pi{}^2)(p^2+m_\pi{}^2)} \Delta_\rho{}^{\lambda\eta}(k) \Gamma_\eta(q,p), \quad (2.1)$$

[7] J. Schwinger, Phys. Letters **24B**, 473 (1967).

[8] It was precisely this approach that originally led one of us to the prediction that $m_{A_1}/m_\rho = \sqrt{2}$.

[9] K. Kawarabayaski and M. Suzuki, Phys. Rev. Letters **16**, 255 (1966); Riazuddin and Fayyazuddin, Phys. Rev. **147**, 1071 (1966); F. J. Gilman and H. J. Schnitzer, *ibid.* **150**, 3162 (1966); J. J. Sakurai, Phys. Rev. Letters **17**, 552 (1966); M. Ademollo, Nuovo Cimento **46**, 156 (1966).

[10] See, e.g., S. Weinberg, Phys. Rev. Letters **17**, 616 (1966).

[11] In particular, see the discussion of p-wave π-N scattering lengths by H. J. Schnitzer, Phys. Rev. **158**, 1471 (1967).

[12] Here $A_a{}^\mu(x)$ and $V_b{}^\nu(x)$ are axial-vector and vector currents, normalized as in Refs. 2, 5, and 10, with a and b isospin indices running over the values 1, 2, 3, and μ and ν space-time indices running over the values 1, 2, 3, 0. Our metric is $+, +, +, -$.

$$\int d^4x d^4y\, e^{-iq\cdot x} e^{ip\cdot y} \langle T\{\partial_\mu A_a{}^\mu(x), A_b{}^\nu(y), V_c{}^\mu(0)\}\rangle_0$$

$$\equiv \frac{\epsilon_{abc}F_\pi m_\pi{}^2 g_\rho{}^{-1} g_{A_1}{}^{-1}}{(q^2+m_\pi{}^2)} \Delta_{A_1}{}^{\nu\sigma}(p)\Delta_\rho{}^{\lambda\eta}(k)\Gamma_{\sigma\eta}(q,p)$$
$$+\frac{\epsilon_{abc}F_\pi{}^2 m_\pi{}^2 g_\rho{}^{-1}}{(q^2+m_\pi{}^2)(p^2+m_\pi{}^2)} p^\nu \Delta_\rho{}^{\lambda\eta}(k)\Gamma_\eta(q,p)\,, \quad (2.2)$$

$$\int d^4x d^4y\, e^{-iq\cdot x} e^{ip\cdot y} \langle T\{A_a{}^\mu(x), A_b{}^\nu(y), V_c{}^\lambda(0)\}\rangle_0$$

$$\equiv i\epsilon_{abc} g_{A_1}{}^{-2} g_\rho{}^{-1} \Delta_{A_1}{}^{\mu\tau}(q)\Delta_{A_1}{}^{\nu\sigma}(p)\Delta_\rho{}^{\lambda\eta}(k)\Gamma_{\tau\sigma\eta}(q,p)$$
$$+\frac{ig_\rho{}^{-1} g_{A_1}{}^{-1}\epsilon_{abc}F_\pi p^\nu}{(m_\pi{}^2+p^2)} \Delta_{A_1}{}^{\mu\tau}(q)\Delta_\rho{}^{\lambda\eta}(k)\Gamma_{\tau\eta}(p,q)$$
$$+\frac{ig_\rho{}^{-1} g_{A_1}{}^{-1}\epsilon_{abc}F_\pi q^\mu}{(m_\pi{}^2+p^2)} \Delta_{A_1}{}^{\nu\sigma}(p)\Delta_\rho{}^{\lambda\eta}(k)\Gamma_{\sigma\eta}(q,p)$$
$$+\frac{ig_\rho{}^{-1}\epsilon_{abc}F_\pi{}^2 q^\mu p^\nu}{(m_\pi{}^2+q^2)(m_\pi{}^2+p^2)} \Delta_\rho{}^{\lambda\eta}(k)\Gamma_\eta(q,p)\,. \quad (2.3)$$

Here $k\equiv p-q$, and $\Delta_\rho{}^{\mu\nu}(k)$ and $\Delta_{A_1}{}^{\mu\nu}(q)$ are the covariant spin-1 parts of the unrenormalized vector and axial-vector propagators:

$$\Delta_\rho{}^{\mu\nu}(k)\equiv \int d\mu^2 \rho_V(\mu^2)[g^{\mu\nu}+k^\mu k^\nu/\mu^2] \times[\mu^2+k^2]^{-1}, \quad (2.4)$$

$$\langle V_a{}^\mu(x) V_b{}^\nu(0)\rangle_0 = (2\pi)^{-3}\delta_{ab}\int d^4p\, \theta(p^0) e^{ip\cdot x}\rho_V(-p^2) \times[g^{\mu\nu}-p^\mu p^\nu/p^2], \quad (2.5)$$

$$\Delta_{A_1}{}^{\mu\nu}(q)\equiv \int d\mu^2 \rho_A{}^{(1)}(\mu^2)[g^{\mu\nu}+q^\mu q^\nu/\mu^2] \times[\mu^2+q^2]^{-1}, \quad (2.6)$$

$$\langle A_a{}^\mu(x) A_b{}^\nu(0)\rangle_0 \equiv (2\pi)^{-3}\delta_{ab}\int d^4p\, \theta(p^0) \times e^{ip\cdot x}\{\rho_A{}^{(1)}(-p^2)[g^{\mu\nu}-p^\mu p^\nu/p^2] +\rho_A{}^{(0)}(-p^2)p^\mu p^\nu\}\,. \quad (2.7)$$

Furthermore, $g_\rho{}^2$, $g_{A_1}{}^2$, and $F_\pi{}^2$ are defined as the coefficients of $\delta(\mu^2-m_\rho{}^2)$, $\delta(\mu^2-m_{A_1}{}^2)$, and $\delta(\mu^2-m_\pi{}^2)$ in $\rho_V(\mu^2)$, $\rho_A{}^{(1)}(\mu^2)$, and $\rho_A{}^{(0)}(\mu^2)$, respectively. At this point, we have made no approximations.

The next step is to write all the independent Ward identities which follow from the chiral commutation relations[13] and from vector-current conservation. These are

$$(p-q)_\lambda \int d^4x d^4y\, e^{-iq\cdot x} e^{ip\cdot y} \times\langle T\{\partial_\mu A_a{}^\mu(x), \partial_\nu A_b{}^\nu(y), V_c{}^\lambda(0)\}\rangle_0$$
$$=2\epsilon_{adc}\int d^4y\, e^{ip\cdot y}\langle T\{\partial_\mu A_d{}^\mu(0), \partial_\nu A_b{}^\nu(y)\}\rangle_0$$
$$+2\epsilon_{bdc}\int d^4x\, e^{-iq\cdot x}\langle T\{\partial_\mu A_a{}^\mu(x), \partial_\nu A_d{}^\nu(0)\}\rangle_0\,, \quad (2.8)$$

$$(p-q)_\lambda \int d^4x d^4y\, e^{-iq\cdot x} e^{ip\cdot y} \times\langle T\{\partial_\mu A_a{}^\mu(x), A_b{}^\nu(y), V_c{}^\lambda(0)\}\rangle_0$$
$$=2\epsilon_{adc}\int d^4y\, e^{ip\cdot y}\langle T\{\partial_\mu A_d{}^\mu(0), A_b{}^\nu(y)\}\rangle_0$$
$$+2\epsilon_{bdc}\int d^4x\, e^{-iq\cdot x}\langle T\{\partial_\mu A_a{}^\mu(x), A_d{}^\nu(0)\}\rangle_0\,, \quad (2.9)$$

$$(p-q)_\lambda \int d^4x d^4y\, e^{-iq\cdot x} e^{ip\cdot y} \times\langle T\{A_a{}^\mu(x), A_b{}^\nu(y), V_c{}^\lambda(0)\}\rangle_0$$
$$=2\epsilon_{adc}\int d^4y\, e^{ip\cdot y}\langle T\{A_d{}^\mu(0) A_b{}^\nu(y)\}\rangle_0$$
$$+2\epsilon_{bdc}\int d^4x\, e^{-iq\cdot x}\langle T\{A_a{}^\mu(x) A_d{}^\nu(0)\}\rangle_0\,, \quad (2.10)$$

and

$$p_\nu \int d^4x d^4y\, e^{-iq\cdot x} e^{ip\cdot y}\langle T\{\partial_\mu A_a{}^\mu(x), A_b{}^\nu(y), V_c{}^\lambda(0)\}\rangle_0$$
$$=i\int d^4x d^4y\, e^{-iq\cdot x} e^{ip\cdot y} \times\langle T\{\partial_\mu A_a{}^\mu(x), \partial_\nu A_b{}^\nu(y), V_c{}^\lambda(0)\}\rangle_0$$
$$-2\epsilon_{bcd}\int d^4x\, e^{-iq\cdot x}\langle T\{\partial_\mu A_a{}^\mu(x), A_d{}^\lambda(0)\}\rangle_0\,, \quad (2.11)$$

$$q_\mu \int d^4x d^4y\, e^{-iq\cdot x} e^{ip\cdot y}\langle T\{A_a{}^\mu(x), A_b{}^\nu(y), V_c{}^\lambda(0)\}\rangle_0$$
$$=-i\int d^4x d^4y\, e^{-iq\cdot x} e^{ip\cdot y} \times\langle T\{\partial_\mu A_a{}^\mu(x), A_b{}^\nu(y), V_c{}^\lambda(0)\}\rangle_0$$
$$+2\epsilon_{abd}\int d^4y\, e^{ik\cdot y}\langle T\{V_d{}^\nu(y), V_c{}^\lambda(0)\}\rangle_0$$
$$+2\epsilon_{acd}\int d^4y\, e^{ip\cdot y}\langle T\{A_b{}^\nu(y), A_d{}^\lambda(0)\}\rangle_0\,. \quad (2.12)$$

[13] M. Gell-Mann, Physics 1, 63 (1964). The "σ terms" arising from commutators of $A_a{}^0$ with $\partial_\mu A_b{}^\mu$ do not contribute here, because they must carry isospin 0 and/or 2, and $V_c{}^\lambda$ carries isospin 1. Schwinger terms will not contribute here if they are c numbers, as is the case if $V_c{}^\lambda$ and $A_a{}^\mu$ are interpreted as hadron gauge fields; see T. D. Lee, S. Weinberg, and B. Zumino, Phys. Rev. Letters 18, 1029 (1967).

We will also need formulas for the propagators. For the vector current, (2.4) and (2.5) yield

$$\int d^4y\, e^{ik\cdot y}\langle T\{V_d{}^\nu(y),V_c{}^\lambda(0)\}\rangle_0 = -i\delta_{cd}[\Delta_\rho{}^{\nu\lambda}(k)-C_V\eta^\nu\eta^\lambda], \quad (2.13)$$

where

$$C_V\equiv\int \rho_V(\mu^2)\mu^{-2}d\mu^2, \quad (2.14)$$

$$\eta^\nu\equiv\{0,0,0,1\}. \quad (2.15)$$

For the axial current, we shall use the idea of partial conservation of the axial-vector current (PCAC) to approximate

$$\rho_A{}^{(0)}(\mu^2)\simeq F_\pi{}^2\delta(\mu^2-m_\pi{}^2). \quad (2.16)$$

Together with (2.6) and (2.7), this yields

$$\int d^4x\, e^{-iq\cdot x}\langle T\{A_a{}^\mu(x),A_b{}^\nu(y)\}\rangle_0 = -i\delta_{ab}\left[\Delta_{A_1}{}^{\mu\nu}(q)+\frac{q^\mu q^\nu}{q^2+m_\pi{}^2}F_\pi{}^2 -\eta^\mu\eta^\nu(C_A+F_\pi{}^2)\right], \quad (2.17)$$

$$\int d^4x\, e^{-iq\cdot x}\langle T\{\partial_\mu A_a{}^\mu(x),A_b{}^\nu(0)\}\rangle_0 = -\delta_{ab}F_\pi{}^2m_\pi{}^2q^\nu[q^2+m_\pi{}^2]^{-1}, \quad (2.18)$$

$$\int d^4x\, e^{-iq\cdot x}\langle T\{\partial_\mu A_a{}^\mu(x),\partial_\nu A_b{}^\nu(0)\}\rangle_0 = -i\delta_{ab}F_\pi{}^2m_\pi{}^4[q^2+m_\pi{}^2]^{-1}, \quad (2.19)$$

where

$$C_A=\int \rho_A(\mu^2)\mu^{-2}d\mu^2. \quad (2.20)$$

Next, we evaluate the left sides of the Ward identities (2.8)–(2.12) in terms of the proper Γ vertices defined by (2.1)–(2.3), and evaluate the right sides of these Ward identities by using (2.13) and (2.17)–(2.20). Note that the noncovariant terms on the right sides of (2.10) and (2.12) cancel, in the latter case because of the equality of Schwinger terms[2]:

$$C_V=C_A+F_\pi{}^2. \quad (2.21)$$

With a little effort, we can write the resulting formulas as Ward-like identities for the proper Γ vertices:

$$k^\lambda\Gamma_\lambda(q,p)=2g_\rho C_V{}^{-1}(p^2-q^2), \quad (2.22)$$

$$k^\lambda\Gamma_{\nu\lambda}(q,p)=-2g_\rho g_{A_1}F_\pi C_V{}^{-1}k^\lambda\Delta_{A_1\mu\lambda}{}^{-1}(p), \quad (2.23)$$

$$k^\lambda\Gamma_{\mu\nu\lambda}(q,p)=-2g_\rho g_{A_1}{}^2C_V{}^{-1}\{\Delta_{A_1\mu\nu}{}^{-1}(q) -\Delta_{A_1\mu\nu}{}^{-1}(p)\}, \quad (2.24)$$

$$p^\nu\Gamma_{\nu\lambda}(q,p)=-F_\pi g_{A_1}C_A{}^{-1}\Gamma_\lambda(q,p) +2F_\pi g_\rho g_{A_1}C_A{}^{-1}q^\nu\Delta_{\rho\nu\lambda}{}^{-1}(k), \quad (2.25)$$

$$q^\mu\Gamma_{\mu\nu\lambda}(q,p)=-F_\pi g_{A_1}C_A{}^{-1}\Gamma_{\nu\lambda}(q,p) +2g_\rho g_{A_1}{}^2C_A{}^{-1}\Delta_{\rho\nu\lambda}{}^{-1}(k) -2g_\rho g_{A_1}{}^2C_A{}^{-1}\Delta_{A_1\nu\lambda}{}^{-1}(p). \quad (2.26)$$

III. RESULTS FROM MESON DOMINANCE

Up to this point, we have made no approximations beyond the weak version of PCAC stated in Eq. (2.16). We now, for the first time, invoke meson dominance, which we take to mean here that the momentum-dependence of the three-point function $\langle T\{A_a{}^\mu,A_b{}^\nu,V_c{}^\lambda\}\rangle_0$ arises almost entirely from its ρ, A_1, and π poles. But the definitions (2.1)–(2.3) of the proper Γ vertices display the pole structure of the three-point function explicitly, leaving no poles in Γ. *Hence we may state the assumption of meson dominance concisely as the requirement that the proper vertices be as smooth as possible as functions of 4-momenta*, subject to the requirements of Lorentz invariance, crossing, and the Ward identities.

For our purposes it will prove sufficient to apply this meson-dominance assumption only to the proper A_1-A_1-ρ vertex $\Gamma_{\mu\nu\lambda}$ and the proper π-A_1-ρ vertex $\Gamma_{\nu\lambda}$. In this case, our assumption that the proper vertices be "as smooth as possible" evidently means that $\Gamma_{\mu\nu\lambda}$ is approximately *linear* in 4-momenta and that $\Gamma_{\nu\lambda}$ is approximately *quadratic* in 4-momenta, at least for $|p^2|$, $|q^2|$, and $|k^2|$ less than about 1 BeV^2. [Note that (2.23) precludes a $\Gamma_{\nu\lambda}$ simply proportional to $g_{\nu\lambda}$, so that $\Gamma_{\nu\lambda}$ must at least contain quadratic terms.]

The first consequence of this ansatz can be obtained by inspection of Eqs. (2.24) and (2.26); we see immediately that $\Delta_{A_1\nu\lambda}{}^{-1}(p)$ and $\Delta_{\rho\nu\lambda}{}^{-1}(k)$ must be at most quadratic in 4-momenta. We can also use (2.4), (2.6), (2.14), and (2.20) to show that

$$p_\nu\Delta_{A_1}{}^{\nu\lambda}(p)=C_Ap^\lambda,$$
$$k_\nu\Delta_\rho{}^{\nu\lambda}(k)=C_Vk^\lambda.$$

It is a trivial matter to prove that these two facts require the propagators to have the free-field form. [For, if

$$\Delta_{\nu\lambda}{}^{-1}(p)\simeq(\alpha+\beta p^2)g_{\nu\lambda}+\gamma p_\nu p_\lambda,$$

then

$$\Delta^{\nu\lambda}(p)\simeq(\alpha+\beta p^2)^{-1}[g^{\nu\lambda}-\gamma(\alpha+(\beta+\gamma)p^2)^{-1}p^\nu p^\lambda],$$

and

$$p_\nu\Delta^{\nu\lambda}(p)\simeq(\alpha+(\beta+\gamma)p^2)^{-1}p^\lambda.$$

The condition that $p_\nu\Delta^{\nu\lambda}(p)$ be proportional to p^λ requires that $\beta=-\gamma$, yielding a free-particle propagator with mass α/β and coupling constant $1/\beta$.] We, of course, identify the particles described by these propagators as the A_1 and the ρ, so that

$$\Delta_{A_1}{}^{\nu\lambda}(p)\simeq g_{A_1}{}^2[g^{\nu\lambda}+p^\nu p^\lambda/m_{A_1}{}^2][p^2+m_{A_1}{}^2]^{-1}, \quad (3.1)$$

$$\Delta_\rho{}^{\nu\lambda}(k)\simeq g_\rho{}^2[g^{\nu\lambda}+k^\nu k^\lambda/m_\rho{}^2][k^2+m_\rho{}^2]^{-1}, \quad (3.2)$$

with

$$C_A \simeq g_{A_1}{}^2/m_{A_1}{}^2, \quad C_V \simeq g_\rho{}^2/m_\rho{}^2. \tag{3.3}$$

We could have assumed (3.1) and (3.2) as part of our general meson-dominance assumption, but we think it noteworthy that current algebra and the assumption of smoothness of proper vertices force the vector and axial-vector spectral functions to be dominated by *single* one-meson states.

Having deduced (3.1) and (3.2), we will only need our meson-dominance assumption as it applies to the proper A_1-A_1-ρ vertex $\Gamma_{\mu\nu\lambda}$, assumed linear in 4-momenta;

$$\Gamma_{\mu\nu\lambda}(q,p) \simeq \Gamma_1 g_{\mu\nu}(p+q)_\lambda + \Gamma_2(g_{\mu\lambda}k_\nu - g_{\nu\lambda}k_\mu) + \Gamma_3(g_{\mu\lambda}p_\nu + g_{\nu\lambda}q_\mu). \tag{3.4}$$

From (2.24) we then have

$$\Gamma_1 = -\Gamma_3 = +2g_\rho{}^{-1}m_\rho{}^2.$$

It will not be possible to determine the value of Γ_2, so it will be convenient to introduce in its place an unknown dimensionless parameter δ:

$$\Gamma_2 \equiv \Gamma_1(2+\delta).$$

The proper A_1-A_1-ρ vertex is then

$$\Gamma_{\mu\nu\lambda}(q,p) \simeq 2g_\rho{}^{-1}m_\rho{}^2[g_{\mu\nu}(p+q)_\lambda + (g_{\mu\lambda}k_\nu - g_{\nu\lambda}k_\mu) \times (2+\delta) - g_{\mu\lambda}p_\nu - g_{\nu\lambda}q_\mu]. \tag{3.5}$$

Inserting (3.1)–(3.3) and (3.5) in (2.26) gives

$$\begin{aligned}\Gamma_{\nu\lambda}(q,p) \simeq (2F_\pi m_\rho{}^2 m_{A_1}{}^2/g_\rho g_{A_1})[&-g_{\nu\lambda} - m_{A_1}{}^{-2} \\ &\times(g_{\nu\lambda}p^2 - p_\nu p_\lambda) + F_\pi{}^{-2}g_{A_1}{}^2 m_{A_1}{}^{-2}(m_\rho{}^{-2} - m_{A_1}{}^{-2}) \\ &\times(g_{\nu\lambda}k^2 - k_\nu k_\lambda) + (\delta g_{A_1}{}^2/F_\pi{}^2 m_{A_1}{}^4) \\ &\times(g_{\nu\lambda}q\cdot k - q_\lambda k_\nu)],\end{aligned} \tag{3.6}$$

and inserting (3.1)–(3.3) and (3.6) in (2.25) gives

$$\begin{aligned}\Gamma_\lambda(q,p) \simeq 2g_\rho{}^{-1}\{&m_\rho{}^2(p+q)_\lambda \\ &+\tfrac{1}{2}[1 - (g_{A_1}{}^2/F_\pi{}^2 m_{A_1}{}^4)(m_{A_1}{}^2 - m_\rho{}^2)] \\ &\times[k^2(p_\lambda + q_\lambda) - k_\lambda(p^2 - q^2)] \\ &-(\delta g_{A_1}{}^2 m_\rho{}^2/F_\pi{}^2 m_{A_1}{}^4)(p_\lambda q\cdot k - q_\lambda p\cdot k)\}.\end{aligned} \tag{3.7}$$

It is easy to check that (2.22) and (2.23) are then automatically satisfied, provided only that (2.21) is satisfied, i.e.,

$$g_{A_1}{}^2 m_{A_1}{}^{-2} + F_\pi{}^2 = g_\rho{}^2 m_\rho{}^{-2}. \tag{3.8}$$

These formulas become much simpler if we apply the other spectral-function sum rule,[2] which reads here

$$g_\rho \simeq g_{A_1}, \tag{3.9}$$

and use the current-algebra estimate[9] of g_ρ,

$$g_\rho{}^2 \simeq 2F_\pi{}^2 m_\rho{}^2, \tag{3.10}$$

which, with (3.8), yields $m_{A_1}{}^2 = 2m_\rho{}^2$. Applying these approximate formulas to (3.6) and (3.7) gives the proper π-A_1-ρ and π-π-ρ vertices as

$$\begin{aligned}\Gamma_{\nu\lambda}(q,p) \simeq F_\pi{}^{-1}\{&-2m_\rho{}^2 g_{\nu\lambda} - (g_{\nu\lambda}p^2 - p_\nu p_\lambda) \\ &+(g_{\nu\lambda}k^2 - k_\nu k_\lambda) + \delta(g_{\nu\lambda}q\cdot k - q_\lambda k_\nu)\},\end{aligned} \tag{3.11}$$

$$\begin{aligned}\Gamma_\lambda(q,p) \simeq \sqrt{2}F_\pi{}^{-1}m_\rho{}^{-1}\{&m_\rho{}^2(p+q)_\lambda + \tfrac{1}{4}(1+\delta) \\ &\times[k^2(p+q)_\lambda - k_\lambda(p^2 - q^2)]\}.\end{aligned} \tag{3.12}$$

It is also trivial to derive the proper ρ-ρ-ρ vertex by these methods. With the assumption of ρ dominance of the isovector current one obtains

$$\begin{aligned}\Gamma_{\mu\nu\lambda}{}^\rho(q,p) \simeq 2g_\rho{}^{-1}m_\rho{}^2\{&g_{\mu\nu}(p+q)_\lambda + 2(g_{\mu\lambda}k_\nu - g_{\nu\lambda}k_\mu) \\ &-g_{\mu\lambda}p_\nu - g_{\nu\lambda}q_\mu\},\end{aligned} \tag{3.13}$$

which has no free parameters.

IV. DISCUSSION OF RESULTS

It is of particular interest to apply our results to the decays $A_1 \to \rho + \pi$ and $\rho \to \pi + \pi$. From (3.11) one can read off the effective $\pi A_1 \rho$ vertex, which on the particle mass shells is

$$\epsilon_{A_1}{}^\nu(p)\Gamma_{\nu\lambda}(q,p)\epsilon_\rho{}^\lambda(k) \simeq -\epsilon_{A_1}{}^\nu(p)F_\pi{}^{-1} \times\{\tfrac{1}{2}m_\rho{}^2(2+\delta)g_{\nu\lambda} + \delta k_\nu q_\lambda\}\epsilon_\rho{}^\lambda(k), \tag{4.1}$$

where $\epsilon_{A_1}{}^\nu(p)$ and $\epsilon_\rho{}^\lambda(k)$ are the polarization vectors of the A_1 and ρ mesons. Similarly, from (3.12) we obtain the physical $\rho\pi\pi$ vertex,

$$\epsilon_\rho{}^\lambda(k)\Gamma_\lambda(q,p) \simeq \sqrt{2}F_\pi{}^{-1}\tfrac{1}{4}m_\rho(3-\delta)(p+q)_\lambda\epsilon_\rho{}^\lambda(k). \tag{4.2}$$

Equations (4.1) and (4.2), together with (2.2) and (2.1), allow us to calculate the A_1 and ρ widths:

$$\Gamma(A_1 \to \rho\pi) = 7.6(25 + 22\delta + 5\delta^2)\ \text{MeV} \tag{4.3}$$

and

$$\Gamma(\rho \to \pi\pi) = 140[(3-\delta)/4]^2\ \text{MeV}, \tag{4.4}$$

in terms of the parameter δ. The presently accepted *experimental* widths are[14]

$$\Gamma(A_1 \to \rho\pi) = 30 \pm 130\ \text{MeV}$$

and

$$\Gamma(\rho \to \pi\pi) = 128 \pm 20\ \text{MeV},$$

which is consistent with a value of $\delta \simeq -\frac{1}{2}$. Because of the uncertainties in the widths and value of F_π,[15] we might reasonably expect δ to lie in a range $-1 \lesssim \delta \lesssim 0$. To compare with other theoretical results, we first note that for $\delta = 0$ our results coincide with those of

[14] See the compilation by A. H. Rosenfeld, N. Barish-Schmidt, A. Barbaro-Galtieri, W. J. Podolsky, L. R. Price, Matts Roos, Paul Soding, W. J. Willis, and C. G. Wohl, University of California Radiation Laboratory Report UCRL-8030 (1967). In particular, footnote (h) to the meson table indicates the uncertainty in the ρ width. Our calculations and those of Refs. 2 and 7 give support to the interpretation of the A_1 as the chiral partner of the ρ, though the experimental situation is far from clear.

[15] There is a question of whether the numerical value of F_π should be taken from the pion-decay rate or the Goldberger-Treiman formula. This leads to a corresponding uncertainty in our predictions. We choose the former, while some of the estimates of Ref. 9 were made with the latter. This causes a corresponding mismatch in numerical values.

Schwinger,[7] which means that the phenomenological Yang-Mills Lagrangian predicts

$$\Gamma(A_1 \to \rho\pi) = 185 \text{ MeV}$$

and

$$\Gamma(\rho \to \pi\pi) = 78 \text{ MeV}.$$

The experimental data do *not* seem to be overwhelmingly in favor of this choice, although this model is not obviously inconsistent with the data. It is also interesting to compare our results with those of Gilman and Harari,[16] who study all the $\pi\rho$ threshold sum rules, and also obtain $m_{A_1}/m_\rho \simeq \sqrt{2}$, but find that the A_1 decay vertex is given entirely by the invariant $k_\nu q_\lambda$. This can be realized in our calculation with the choice $\delta = -2$, which eliminates the invariant $g_{\nu\lambda}$ in (4.1). This also leads to a prediction

$$\Gamma(A_1 \to \rho\pi) = 9 \text{ MeV}$$

and

$$\Gamma(\rho \to \pi\pi) = 215 \text{ MeV},$$

which is quite different from their predicted widths, and in poor agreement with experiment. Thus there does not appear to be an obvious connection between our work and theirs. More precise measurements of the A_1 and ρ widths should help one to select among these models, but perhaps the most useful experiment would be a determination of the ρ spin correlation in A_1 decay since this is directly related to δ.

From (2.1) and (3.12) we can infer the coupling of off-shell ρ mesons to mass-shell pions,

$$G_{\rho\pi\pi}(k^2) = (2m_\rho^2/g_\rho)[1 + \tfrac{1}{4}(1+\delta)k^2/m_\rho^2]. \quad (4.5)$$

From this we find the electromagnetic form factor of the pion to be

$$G_\pi(k^2) = \tfrac{1}{4}(1+\delta) + \tfrac{1}{4}(3-\delta)m_\rho^2/(m_\rho^2+k^2). \quad (4.6)$$

From the naive point of view, one would say that the ρ meson accounted for only $\frac{1}{4}(3-\delta)$ of the total pion charge; however, this is not true, since the result (4.6) just comes from the k^2 dependence of the $\rho\pi\pi$ coupling. We can also write explicit expressions for the electromagnetic properties of the ρ and A_1 mesons, but there is little interest in doing this since they do not seem to be accessible to experimental measurements.

We close with a remark concerning the technical aspects of the calculation. What has evolved here is a systematic way of determining n-point functions of currents, by using the Ward identities obtained from the current commutator relations. In our treatment one must retain the gradient-coupling terms, which generally vanish in calculations with *soft* pions, to obtain predictions which are valid even when the pion is not soft. It is obvious that we do not have to sacrifice energy-momentum conservation, as is the case in some treatments of decay amplitudes using soft-pion techniques.

Note added in proof. We recently learned of the work of J. Wess and Bruno Zumino [Phys. Rev. **163**, 1727 (1967)], which is a study of effective Lagrangians for nonsoft pions sufficiently general to include $\delta \neq 0$. Their results for the three-point functions coincide with our Eqs. (3.5)–(3.7) and (3.13) when we set $g_\rho = g_{A_1}$.

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[16] F. J. Gilman and H. Harari, Phys. Rev. Letters **18**, 1150 (1967).