Synchrotron-Radiation Model for Meson Production in High-Energy Proton-Proton Collisions*

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A semiclassical model for emission of bosons in high-energy hadron collisions is proposed, based on an analogy with electromagnetic radiation from a radially accelerated moving charge. A simple assumption concerning the shape of the effective classical trajectories leads to absolute predictions with no free parameters for reactions such as $pp \rightarrow pp\pi^0$ and $pp \rightarrow pn\pi^+$. These predictions are in good agreement with available experimental data in the region of validity of the model, which is confined to relatively large momentum transfers.

I. INTRODUCTION

If one adopts a picture of hadrons as extended structures, e.g., a droplet[1] or multiparticle (quark) composite,[2] with no central singularity, then scattering and reaction mechanisms at high energies can be visualized in terms analogous to diffraction or excitation phenomena which take place when a projectile (beam) interacts with a diffuse, semitransparent target.[3] In contrast to older notions involving "core" interactions,[4] it seems that at sufficiently high energies all phenomena are characteristic of such extended, diffuse interactions, with shape parameters (e.g., radius) relatively energy-independent.

This circumstance suggests that a classical description may be appropriate in almost all types of high-energy interactions, since the deBroglie wavelength of a projectile decreases indefinitely (in the laboratory frame) with increasing energy, whereas the effective scattering strength and shape of a target (at rest in the same frame) are relatively independent of the projectile energy. Such an observation has led to many writings on elastic and quasi-elastic two-body reactions wherein the eikonal description, appropriate to small angles and high energy, is applicable.[5] It has also been conjectured that classical ideas are relevant for large-angle elastic scattering at sufficiently high energies.[6]

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[1] N. Byers and C. N. Yang, Phys. Rev. **142**, 976 (1966); N. Byers, Phys. Rev. **156**, 1703 (1967); R. C. Arnold, Phys. Rev. **157**, 1292 (1967).

[2] See, e.g., H. J. Lipkin and F. Scheck, Phys. Rev. Letters **16**, 71 (1966); H. J. Lipkin, invited talk presented at the Second International Conference on High-Energy Physics and Nuclear Structure, Rehovoth, Israel, 1967 (unpublished).

[3] R. P. Feynman, remark in *Proceedings of the Thirteenth Annual International Conference on High-Energy Physics* (University of California Press, Berkeley, 1967).

[4] H. W. Lewis, J. R. Oppenheimer, and S. A. Wouthuysen, Phys. Rev. **73**, 127 (1948).

[5] R. C. Arnold, Phys. Rev. **153**, 1523 (1967), and references therein.

[6] H. A. Kastrup, Phys. Rev. **147**, 1130 (1966).

In the present paper, we propose a new utilization of classical ideas in the high-energy region, to wit, uncorrelated particle production matrix elements using a specific model for the source-current density calculated by integrating along classical orbits followed by the projectile ("inside" the target). The ingredients of the proposed model are developed by analogy to the classical theory of synchrotron radiation, i.e., electromagnetic radiation from a particle uniformly accelerated in a direction perpendicular to its motion.

We compare the predictions of the model to the experimental data of Anderson *et al.*[7] for the reactions $pp \rightarrow ppM^0$, where M^0 is one of the neutral mesons (π^0, η^0, ω^0 in that experiment), and $pp \rightarrow pn\,\pi^+$. The model can be applied to multiple production reactions, but since the computation difficulty is large and there are no satisfactory completely differential cross-section measurements to date we have not done this.

Previous attempts at constructing semiclassical emission theories for particle production have, in the main, been restricted to a "bremsstrahlung approximation"[4,8] which assumes that the wavelength of the emitted meson is large compared to the dimensions of the reaction region. Such theories have not been quantitatively successful, probably because for small meson energies (where such a condition might be met), discrete isobar excitation is an important process which presumably dominates the uncorrelated mechanism described by classical localizable source ideas.

To avoid conflict with low-energy processes (isobar excitation, pole diagrams) we consider the model to be reasonable only when

$$\omega \gg \mu , \tag{1}$$

where ω is the energy of the meson in the frame of the emitting (source) particle, and μ is the meson mass.

To satisfy the requirements that make a classical orbit description possible, we need to have the deBroglie

[7] H. L. Anderson *et al.*, Phys. Rev. Letters **18**, 89 (1967).

[8] A. Bialas and T. Ruijgrok, Nuovo Cimento **39**, 1061 (1965).

wavelength of the projectile (nucleon) much smaller than the rms radius of the target (nucleon) as determined by strong interactions; if we take this from the width $[\sim(2\mu_\pi)^{-1}]$ of the high-energy diffraction cone seen in NN elastic scattering, this means that

$$|\mathbf{p}_1|\gg 2\mu_\pi, \tag{2}$$

where $\mathbf{p}_1$ is the barycentric 3-momentum of the projectile. This is not very restrictive, however, for the following reason.

To make the desired approximation neglecting recoil of the emitting particle as it moves along its orbit during the emission process, we need to have a barycentric momentum much greater than the momentum carried off by the emitted meson. Thus we need

$$|\mathbf{p}_1|\gg\omega. \tag{3}$$

Combined with (1), this results in a restriction always more severe than (2), unless we are actually considering a process of photon emission, where $\mu=0$.

A further restriction on applicability of the model must be kept in mind, although it is difficult to make as precise a statement about it as is given above [(1)–(3)]. Since we will use classical orbits, it is presumably important that the off-shell elastic scattering process which is used be adequately describable as deflection in a real field of force. On the other hand, we know that the near-forward scattering of nucleons at high energies is very conveniently described by absorptive ("shadow") scattering, a diffractive process. This strongly suggests that small momentum transfers should be excluded when applying our model. Since the diffraction pattern seems to show a qualitative change away from the forward (exponential in Δ^2) behavior about $\Delta^2\sim 1.0$ BeV^2, we suggest a restriction to the region of larger momentum transfers:

$$\Delta^2 \gtrsim 1\ \text{BeV}^2. \tag{4}$$

[By Δ^2 we mean the standard 4-momentum transfer variable, $-t=\Delta^2=-(p_1-p_1')^2$, relevant to NN elastic scattering.]

II. REALIZATION OF THE MODEL IN TERMS OF FIELD THEORY: TRANSITION AMPLITUDES AND CROSS SECTIONS

The simplest way of incorporating the ideas of the model into a well-established formalism comes from examining the production amplitude, $A=\langle N(p_1'), N(p_2'); k\lambda, \text{out}\,|\,N(p_1), N(p_2)\text{in}\rangle$, with two nucleons in the initial state, two nucleons and a boson in the final state. The boson momentum is k, its polarization index, if any, is λ. Since we are studying only one-boson production processes, this amplitude will be sufficient for our purposes. Applying the Lehmann-Symanzik-Zimmermann (LSZ) reduction technique to the boson state (taken to be a vector meson for the sake of generality), we find

$$A=i\int d^4x\,\epsilon_\mu(k,\lambda)e^{ik\cdot x}\langle N(p_1')N(p_2'),\ \text{out}\,|\,j^\mu(x)\,|\,N(p_1)N(p_2)\ \text{in}\rangle, \tag{5}$$

where $j^\mu(x)=(\Box+\mu^2)\phi^\mu(x)$ is the source current of the vector-meson field. (We have set $\hbar=c=1$.)

We now make several assumptions about the nature of $j^\mu(x)$ which seem to us to embody the ideas of our model:

(1) The spin behavior of $j^\mu(x)$ is entirely decoupled from that of the initial and final nucleons.

(2) $j^\mu(x)$ can be separated into two parts representing contributions from the two nucleons added coherently with an appropriate phase difference. [Accompanying this is the previously mentioned assumption that the nucleons follow well-defined classical trajectories and scatter only into the forward hemisphere in the center-of-mass (c.m.) system.]

(3) In terms of our model, we may remove $j^\mu(x)$ from the matrix element and express it as a C-number function parametrized with the initial and final nucleon momenta. This leaves us with an expression for the production matrix element which is simply the Fourier transform of $j^\mu(x)$ multiplied by a quasi-elastic (QE) nucleon-nucleon scattering matrix element, to wit,

$$A=i\epsilon_\mu(k,\lambda)\,\tilde{j}^\mu(k)A_{\text{QE}}(p_1'p_2',p_1p_2)\equiv F(k)A_{\text{QE}}. \tag{6}$$

In the case of (pseudo-) scalar fields, the polarization vectors and 4-vector indices are absent.

For the (pseudo-) scalar and vector cases which we treat here, we choose the simplest possible expressions for the source current[8,9]:

$$\text{scalar:}\ j(x)=g_S\int d\tau\times\{\delta^{(4)}[x-s_{(1)}(\tau)]+\delta^4[x-s_{(2)}(\tau)]\}, \tag{7}$$

$$\text{vector:}\ j^\mu(x)=g_V\int d\tau\{v_{(1)}{}^\mu(\tau)\delta^{(4)}[x-s_{(1)}(\tau)]+v_{(2)}{}^\mu(\tau)\delta^4[x-s_{(2)}(\tau)]\}.$$

Here $s_{(i)}{}^\mu(\tau)$ is a 4-vector function of proper time which describes the trajectory followed by nucleon (i) during the interaction, and $v^\mu{}_{(i)}(\tau)$ is the 4-vector velocity $ds_{(i)}{}^\mu/d\tau$ of nucleon (i). Note that the expression assumed in the vector case is essentially the static source current for classical electrodynamics, with no anomalous (Pauli) term.

When we Fourier-analyze these currents, we obtain expressions which contain effects due to static fields, i.e., to emission of virtual mesons. In order to isolate the part due to emission of real mesons (the part linear in

[9] Z. Chylinski, Nucl. Phys. 44, 58 (1963).

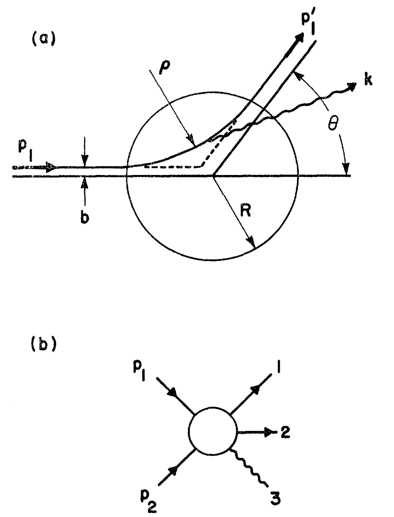

FIG. 1. (a) Geometry for determination of orbits in model. (b) Identification of particles in reactions considered.

the acceleration of the nucleon), we integrate by parts once with respect to time and discard the divergent surface terms, which seems to us to be analogous to self-energy effects in the quantized theory. We finally obtain the expressions (for a single nucleon)

$$\text{scalar: } \tilde{j}(k) = -ig_S \int dt\, e^{i[\omega t - \mathbf{k}\cdot\mathbf{s}(t)]} \frac{d}{dt}\left[\frac{(1-\beta^2)^{1/2}}{\omega - \mathbf{k}\cdot\boldsymbol{\beta}}\right],$$
$$\text{vector: } \tilde{j}^\mu(k) = -ig_V \int dt\, e^{i[\omega t - \mathbf{k}\cdot\mathbf{s}(t)]} \frac{d}{dt}\left[\frac{\beta^\mu}{\omega - \mathbf{k}\cdot\boldsymbol{\beta}}\right], \tag{8}$$

where $\beta^\mu = ds^\mu/dt$, and $\omega = (\mathbf{k}^2+\mu^2)^{1/2}$. This gives us automatically the relation $k_\mu \tilde{j}^\mu(k) = 0$, since $k_\mu \beta^\mu \equiv \omega - \mathbf{k}\cdot\boldsymbol{\beta}$.

We now need an explicit form for the nucleon trajectories, which will give the simplest possible test of the basic idea of the model without necessarily purporting to be a realistic description of the physical event. To avoid introducing new parameters, we assume the following:

(a) The nucleons interact within a region which is spherical with radius R in the center of mass.

(b) In a hypothetical elastic collision, the nucleons enter and leave this region with the same impact parameter b.

(c) Scattering into the backward hemisphere is so small as to be ignorable. (This eliminates problems of a semiclassical interpretation of the exclusion principle, but also limits the applicability of the model to events in which the nucleons emerge in different hemispheres.)

(d) The nucleons travel on circular trajectories within the region of interaction, which are tangent to the asymptotic straight-line paths at boundaries. These conditions are illustrated in Fig. 1(a). The expression which emerges for the radius of curvature of the trajectory ρ is

$$\rho = (R^2 - b^2)^{1/2} \cot\tfrac{1}{2}\theta - b\,, \tag{9}$$

where θ is the c.m. scattering angle. In classical scattering, the impact parameter is related to the differential cross section by

$$2\pi b \frac{db}{d\cos\theta} = \frac{d\sigma^{\text{el}}}{d\cos\theta}. \tag{10}$$

For $b > R$, there is no scattering, so that we can obtain R, under our assumptions, from

$$R^2 = 2\int_0^R b\,db = 2\int_0^1 \frac{d\sigma^{\text{el}}}{d\Omega} d\cos\theta. \tag{11}$$

Hence all the parameters of the model can be obtained from a suitable parametrization of the empirical proton-proton elastic-scattering cross section.

The choice of a suitable expression for the quasi-elastic matrix element is straightforward in the elastic limit, where the momentum carried off by the boson is negligible. However, in our comparison with available data, we are faced with a situation where (3) is not strictly satisfied, i.e., where the momentum transfers of the two nucleons are quite different. In order to circumvent this difficulty, we take

$$A_{\text{QE}} \propto \left(\frac{d\sigma^{\text{el}}}{dt_1}\frac{d\sigma^{\text{el}}}{dt_2}\right)^{1/4}. \tag{12}$$

This amounts to taking the geometric mean of the cross sections for two independent elastic-scattering events. The theoretical grounds for preferring this choice are obscure, but we may say, with benefit of hindsight, that it is a reasonable one. A suitable parametrization of the elastic differential cross section, applicable to the entire kinematic region of interest, is given by Krisch.[10]

Another ambiguity arises in the treatment of pseudoscalar mesons. Although the situation in the classical field-theory case is not clear, it is possible to make an *ad hoc* correction to the production amplitude of the form

$$\bar{u}(p')\gamma_5 u(p) \Big/ \sum_{\text{spins}} \bar{u}(p')u(p)\,,$$

where p and p' are the initial- and final-state momenta, respectively, of the nucleon in question. In the final result, this reduces to a factor of the form

$$(p'\cdot p - M^2)/(p'\cdot p + M^2)\,,$$

before the production cross sections for each nucleon emitting a meson (cross terms drop out in the spin

[10] A. D. Krisch, Phys. Rev. **135**, B1456 (1964). The expression used, consisting of a sum of three exponentials in $\tau = P_\perp^2$ is given in Eq. (5) of this paper.

sums). Such a correction is, however, foreign to the spirit of a semiclassical model, and does not, we have found, improve the fits to experiment. Hence we have omitted this correction in our quantitative comparison with the data.

The cross section that interests us here is completely differential with respect to the final-state variables in the production event, as recorded in the data. Referring to Fig. (1b) for the labeling of particles in the reaction, the cross section we shall calculate is

$$\frac{\partial_\sigma^3}{\partial M_{23}^2\partial t_1\partial\Omega_2^b}=\left(\frac{g^2}{4\pi}\right)\frac{p_2'}{8\pi^2\{\omega+E_2'[1+(p_1'/p_2')\cos\theta_{12}]\}}\times|\tilde{F}(k)|^2\left(\frac{d\sigma^{\mathrm{el}}}{dt_1}\frac{d\sigma^{\mathrm{el}}}{dt_2}\right)^{1/2}, \quad (13)$$

where $M_{23}^2=(k+p_2')^2$, $t_1=(p_1'-p_1)^2$, $t_2=(p_2'-p_2)^2$, Ω_2^b is the solid angle for nucleon (2) in the barycentric system, θ_{12} is the angle between $\mathbf{p}_1'$ and $\mathbf{p}_2'$ in the same system, and $F(k)=g\tilde{F}(k)$. The normalization is a straightforward matter; it is only necessary to construct expressions for the production and elastic cross sections according to a standard prescription,[11] solve for the elastic amplitude, and insert this in the production cross section. The coupling constant g is assumed, by analogy with classical electrodynamics, to be the coefficient which appears before the trilinear coupling term (after renormalization) in a standard Lagrangian theory, i.e., $L_I=g\bar{\psi}\Gamma\psi\phi$, where $\Gamma=1$ or γ_μ.

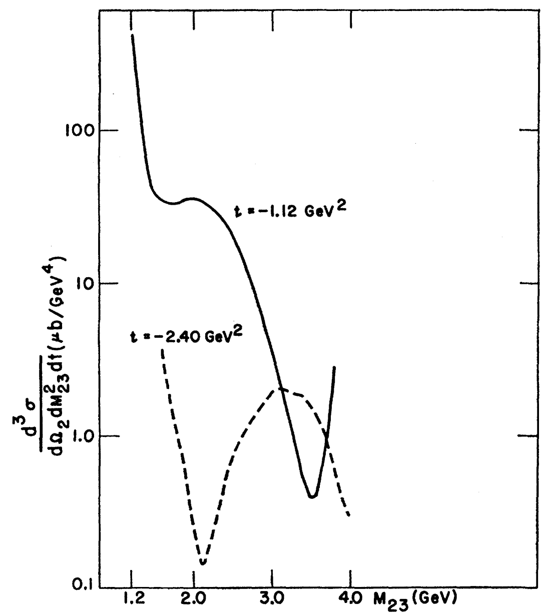

FIG. 3. Predictions of model, for the same kinematics as in Fig. 1, with fixed t values and varying M_{23}.

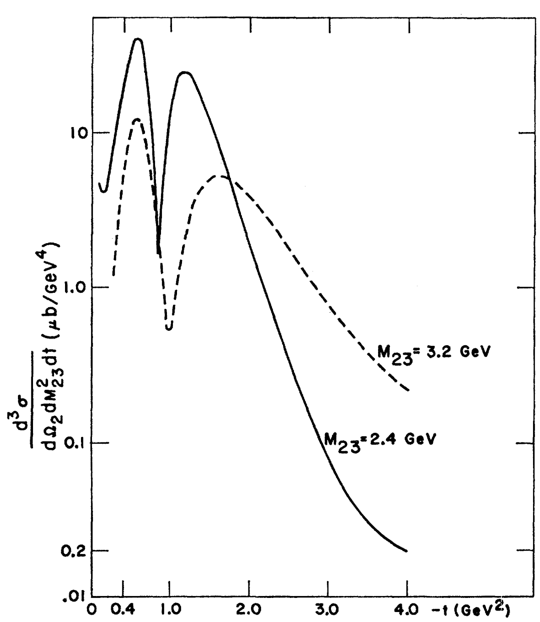

FIG. 2. Predictions of model for the reaction $pp\to pp\pi^0$ at 12.5 GeV/c; completely differential cross section for coplanar configuration, at fixed M_{23} (invariant mass of π^0 and recoil proton) and varying $-t=\Delta^2$ (4-momentum transfer between projectile proton and fast final-state proton); recoil proton taken here at a barycentric angle of $\theta_2=2.75$ rad.

We close this section by remarking how this model differs from previous bremsstrahlung models.[8,9] In these models, a kind of impulse approximation is used, in which the velocity of the nucleon changes suddenly at time t_0, but is constant before and after. This gives the acceleration as proportional to $\delta(t-t_0)$, whence the nonstatic part of $j(x)$ is nonvanishing only at one point in time. Hence the essential difference between these models and our own is that we assume a smooth change in the velocity of the colliding particles, which we deem to give a more realistic description of high-energy collisions. The bremsstrahlung models give a slowly varying $F(k)$, whereas our model yields a rapid decrease at large ω, in agreement with experiment.

We now have all the information needed to construct cross sections for one-meson production in coplanar configurations. The quasi-elastic-scattering matrix element is approximated from the known empirical scattering cross section, and the cross section for the total process deduced by comparing the phase-space factors for the two processes. The detailed comparison with the available data is discussed in Sec. IV.

III. HIGH-ENERGY BEHAVIOR

If one examines the model's predictions for $\omega\gg R^{-1}$, the integrals (8) can be carried out approximately, as in the theory of a charge moving uniformly in a circle, for most meson emission angles. For simplicity, we

[11] See, e.g., J. D. Bjorken and S. D. Drell, *Relativistic Quantum Fields* (McGraw-Hill Book Company, Inc., New York, 1965), Appendix B.

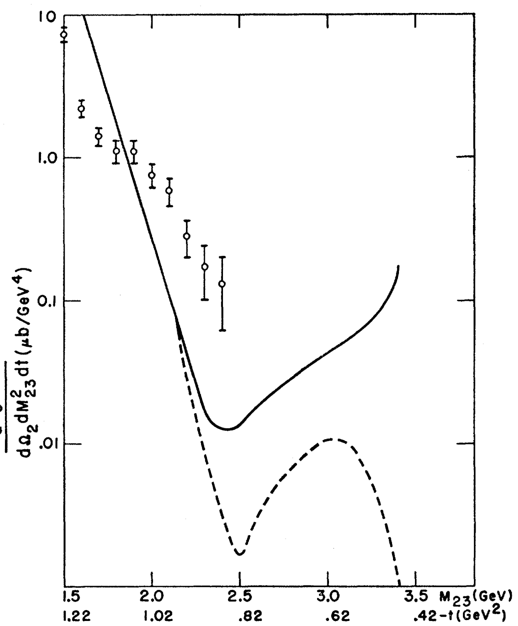

FIG. 4. Comparison of model predictions with data of Ref. 7 for the reaction $pp \to pp\ \pi^0$ (at 12.5 GeV/c) with $\theta_2 = 2.306$ rad; coplanar configuration. Dashed curve shows effect of retaining only the recoil nucleon's contribution.

confine our attention to the coplanar configuration. The most important feature of the integrand, for large meson energies, is the exponential, which is rapidly oscillating along the integration path (the orbit).

For conciseness, let ($i\psi$) be the argument of the exponential in (8);

$$\psi \equiv \omega t - \mathbf{k} \cdot \mathbf{s}(t) . \tag{14}$$

Then under the above circumstances one may expand ψ in powers of t and retain only the lowest two terms

$$\psi \cong (\omega/2\gamma^2)(t + t^3\gamma^2/3\rho^2) , \tag{15}$$

where

$$\gamma^{-2} \equiv 1 - \beta^2 .$$

The limits of integration in the integrals (8) can then be extended to infinity, and the resulting functions can be expressed analytically in terms of modified Bessel functions of third-integral orders. For the vector case, retaining only the largest (parallel) polarization term, one obtains with any meson emission angle less than the scattering angle θ

$$F \cong \gamma \xi K_{2/3}(\xi) , \tag{16}$$

where

$$\xi \equiv \omega \rho(\theta)/3\gamma^3 .$$

For large ω, F therefore drops off exponentially in ω. This is a characteristic of the synchrotron-radiation model which is not shared by other production models known to us. It is a consequence of the basic classical-limit ideas discussed in the Introduction, and not of our specific orbit approximations, except for the neglect of recoil. There exist a considerable number of hitherto unanalyzed production spectra which seem to have this characteristic behavior for $\omega \gg \mu$. In fact, this is the feature which has led us to pursue the model with considerable hope of qualitative success, although we are unable at present to deal quantitatively with more complicated reactions.

IV. CALCULATIONS IN THE MODERATE-ENERGY REGION AND COMPARISON WITH AVAILABLE DATA

When numerical integrations of (8) were performed to compare with the data of Anderson *et al.*, it was found that at least in that energy range (12 BeV), in addition to the smooth behavior discussed in Sec. III, the model predicts some drastic oscillations in the production cross sections as a function of invariant momentum transfer and/or invariant meson-nucleon mass (M_{23}). These dramatic fluctuations are illustrated in Figs. 2 and 3; the kinematical configurations (lab angles) are not those found in the experiment of Anderson *et al.*, but were changed to a different value for illustration.

The feature of the model responsible for these oscillations is the assumption of a sharp cutoff boundary for the region of interaction. When the ratio of meson wavelength to radius of curvature is changed, one sees this edge-effect influence through its Fourier transform, (8). We assume, therefore, that in reality—where edges are diffuse—that such dramatic effects will be absent, although there should be some remainder of non-monotonic behavior since one expects some characteristic shape factor of the proton to persist.

It is interesting to note that in the calculations done

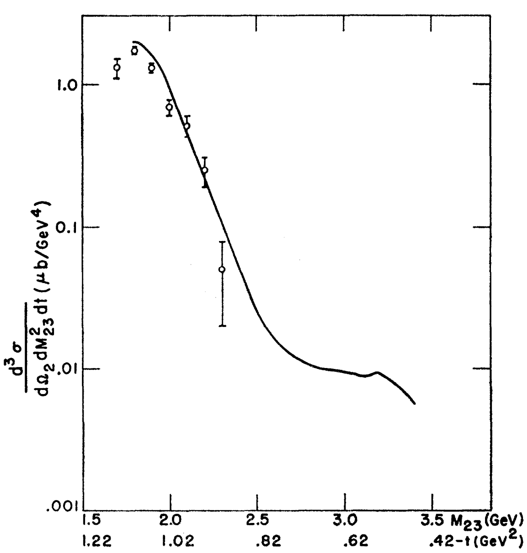

FIG. 5. Comparison of model predictions with data of Ref. 7 for the reaction $pp \to pn\pi^+$; kinematic configuration same as in Fig. 4.

in the region covered by experiment, little trace of such fluctuations is seen even in our crude model; the experimental data seem to show such effects more than our present model, as will be seen shortly.

The problem arises here of how to take into account the relative phase of the two nucleons in the emission process. Unless some selection rule, e.g., charge conservation, can be used, it is impossible to say that one nucleon emits the meson and the other does not. Indeed, Eqs. (7) indicate that both nucleons contribute to each Fourier component of the meson field. The detailed assumptions of the model yield a plausible prescription for treating the relative phase; simply take the formulas seriously and assume that if the nucleons begin to interact at time t_0 the initial phase difference is $\mathbf{k}\cdot[\mathbf{s}_1(t_0)-\mathbf{s}_2(t_0)]$; the phase may be counted naturally throughout the rest of the process. This has been done in the present treatment. Two of the figures show results with both coherent and incoherent sums over the single-nucleon contributions. We find that the present data, because of the particular kinematic arrangement, do not offer a clear choice between these two alternatives.

Turning now to the detailed comparison with experiment, we note first that the coupling constant $g_{\pi NN}$ is known, so that the calculations for $pp\to pp\pi^0$ and $pn\pi^+$ can be done without any free parameters. The model's predictions for these reactions, together with the available data, are shown in Figs. 4 and 5.

In both cases, the correct order of magnitude for the cross sections and the sharp dropoff for large M_{23} are given correctly by the model. The agreement is particularly spectacular for the π^+ reaction. It should be noted that the π^0 data have an indication of a shape effect (as discussed above), whereas the model predicts a monotonic decrease with M_{23} in this kinematic region.

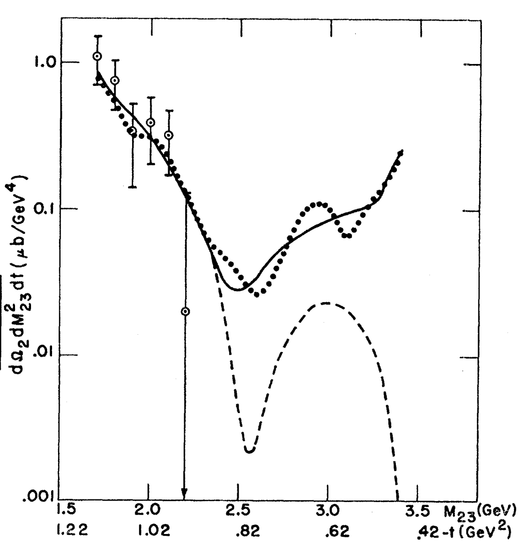

FIG. 6. Comparison of model predictions, using $g_{\eta NN}^2/4\pi=2.7$, with data of Ref. 7 for the reaction $pp\to pp\eta$. Kinematic configuration same as in Fig. 4. Dashed curve shows effect of retaining only the recoil nucleon's contribution. Dotted curve shows effect of including interference term.

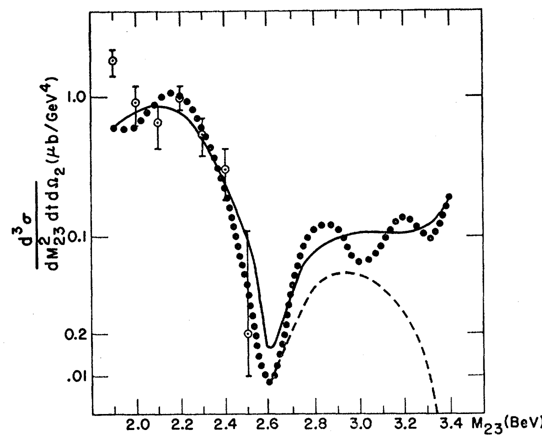

FIG. 7. Comparison of model predictions, using $g_{\omega NN}^2/4\pi=0.5$, with data of Ref. 7 for the reaction $pp\to pp\omega$. Kinematic configuration same as in Fig. 4. Dashed curve shows effect of retaining only the recoil nucleon's contribution. Dotted curve shows effect of including interference term.

In the cases $pp\to pp\eta^0$ and $pp\omega^0$, the relevant coupling constants are not well known. The η^0 data, and the model's prediction normalized to the experiment, are shown in Fig. 6. We find $g_{\eta pp}^2/4\pi=2.7$ is necessary for this fit. This is about as expected on the basis of $SU(3)$ and the usual $\frac{3}{2}$ ratio for D/F in the pseudoscalar-meson–baryon couplings; it is expected that $g_{\eta pp}^2/4\pi \ll g_{\pi NN}^2/4\pi \sim 15$. Before taking our value seriously, however, it should be realized that the background subtraction necessary to yield an absolute cross section here is rather ambiguous; we suggest that a figure 50% larger or smaller might be obtained from the same raw data. This estimate is obtained by looking at Fig. 3(b) of Anderson *et al.*[7] In any case, the spectrum shape seems to agree in Fig. 6.

The comparison of model and experiment for $pp\omega^0$ is shown in Fig. 7. Here again we normalized to the data, resulting in $g_{\omega NN}^2/4\pi=0.5$. This figure is smaller than that determined by Scotti and Wong[12] based on a one-boson exchange plus dispersion-theory fit of low-energy NN scattering, and somewhat lower than estimates based on $SU(6)$ plus universality,[13] or on the fits of Bryan *et al.*[14] based on the nonrelativistic Schrödinger theory of NN scattering. Again, ambiguities in background subtractions may be partially responsible; clearly, the shape of the spectrum fits the model well.

If $SU(6)$ estimates[13] that $g_{\rho NN}\sim g_{\omega NN}$ are correct, then we have no explanation strictly within our model for the apparent absence of an appreciable ρ peak in the invariant-mass spectrum of Ref. 7; such a peak should appear in our model as a broad hump under the sharp ω peak, with area comparable to the latter. However,

[12] A. Scotti and D. Y. Wong, Phys. Rev. **138**, B145 (1965).
[13] B. Sakita and K. C. Wali, Phys. Rev. Letters **14**, 404 (1965).
[14] R. A. Bryan and R. A. Arndt, Phys. Rev. **150**, 1299 (1966).

we can propose a reason based on final-state interactions for apparent ρ suppression using semiclassical reasoning in these data. Since the ρ has a short lifetime, and the probability of emission is greatest at smallest M_{23} (small meson energy in the frame of emitting nucleon), there will be a relatively high probability for decay into two pions before escaping very far from the emitting nucleon. The probability for differential rescattering of the final-state pions on the emitter should therefore be important, especially since the Q value in the decay is large, resulting in a wide distribution of decay pion momenta with respect to the emitting nucleon. Such rescattering will cause the given event to appear in a different invariant-mass bin, thus redistributing the ρ events over the spectrum. Such effects are smaller by a large factor for the other mesons, since their lifetimes are much longer.

ACKNOWLEDGMENTS

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Current-Algebra Calculation of Hard-Pion Processes: $A_1 \to \rho+\pi$ and $\rho \to \pi+\pi$†

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New techniques are developed for treating the n-point functions of currents, which are interrelated by means of Ward identities obtained from the equal-time current commutation relations. The n-point functions are sorted out so as to define proper vertices which describe the reactions of particles of definite spin. A meson-dominance assumption is made by approximating the proper vertices by simple polynomials in momenta, with the coefficients determined by the Ward identities. The method is discussed in detail for $n=3$ and the currents of chiral $SU(2)\times SU(2)$, and then applied to the decay processes $A_1\to\rho+\pi$ and $\rho\to\pi+\pi$.

I. INTRODUCTION

MOST of the successful predictions made by current algebra have taken the form of low-energy theorems for soft pions,[1] or equivalent sum rules. However, the scope of current algebra has recently been extended to areas having nothing to do with soft pions, by making use of the additional assumption that the vector and axial-vector currents are dominated by $j=1$ and $j=0$ mesons. In particular, it has been possible to show[2] that $m_{A_1}/m_\rho=\sqrt{2}$, and to derive similar results[3] for the other vector and axial-vector mesons. The idea of meson dominance of the currents has received further support from a successful calculation[4] of the π^+-π^0 mass difference and has led to an estimate[5] of the intermediate boson mass.

With these advances has come a new problem. Several authors[6] have noted that if the chiral $SU(2)\times SU(2)$ currents are saturated by the ρ, A_1, and π mesons, then the A_1-ρ-soft-π vertex is

$$\Gamma_{\nu\lambda}\simeq -2m_\rho^2F_\pi^{-1}g_{\nu\lambda}\,, \tag{1.1}$$

where F_π is the usual pion-decay amplitude and ν and λ are the A_1 and ρ polarization indices. But using (1.1) to calculate the decay rate for $A_1\to\rho+\pi$ would give an A_1 width of about 800 MeV. We are prepared to be tolerant in comparing current-algebra predictions with experiment, but this certainly has to be counted as a

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[1] For a review, see the rapporteur's talk by R. F. Dashen, in *Proceedings of the Eighth International Conference on High-Energy Nuclear Physics* (University of California Press, Berkeley, California, 1967), p. 51.

[2] Steven Weinberg, Phys. Rev. Letters **18**, 507 (1967).

[3] T. Das, V. S. Mathur, and S. Okubo, Phys. Rev. Letters **18**, 761 (1967); H. T. Nieh, *ibid.* **19**, 43 (1967); S. L. Glashow, H. J. Schnitzer, and Steven Weinberg, *ibid.* **19**, 139 (1967).

[4] T. Das, G. S. Guralnik, V. S. Mathur, F. E. Low, and J. E. Young, Phys. Rev. Letters **18**, 759 (1967).

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[6] D. Geffen, Phys. Rev. Letters **19**, 770 (1967); B. Renner, Phys. Letters **21**, 453 (1966); H. J. Schnitzer (unpublished). See footnote 9 of Ref. 2.